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TOPIC 2

PORTFOLIO THEORY ANALYSIS

THE MODERN PORTFOLIO THEORY

A portfolio is a combination of assets held by the investor for investment purposes.

Portfolio theory therefore attempts to show an investor how to combine a set of assets to maximise the assets' returns as well as minimise the assets' risk (Risk Diversification).

Diversification is defined as combining assets whose returns are not perfectly positively correlated to reduce the aggregate risk of the total asset holdings (or the portfolio).

A portfolio in financial terms refers to collection of investment opportunities of securities with an aim of minimizing risk.

There are 2 types of risks facing companies

- Financial risk
- Business risk

Financial risks – Are risks associated with borrowing founds i.e using the debt in the capital structure

Business/operating/total risks- Risks associated with declining earnings of the company. They can either be systematic or unsystematic

a) **Systematic risks:** Risks facing all the companies in the industry e.g political risk, inflation etc.
b) **Unsystematic risks:** Risks associated with declining company earnings due to factors affecting specific companies in the industry e.g strikes, changes in mkt price etc.

Assumptions of portfolio Theory

- Investors prefer more wealth
- Investors are risk averse
- Returns on portfolio follow normal distribution
- It assumes only one security exists

- It assumes only one security exists
- It assumes systematic risks
- It assumes there are no economies of scale

THE PORTFOLIO EXPECTED RETURN

If the investor holds only two assets in the portfolio, we can therefore be able to compute the portfolio's expected return (sometimes referred to as the portfolio mean). This will be a weighted average of the expected return of each asset held in isolation, and can be given by the following formula:

$$\bar{R}_P = E(\alpha X_A + ßX_B) \ldots\ldots\ldots (3.a)$$

Where $\bar{R}_P$ is the expected portfolio return

α is the investment in asset A

ß is the investment in asset B

X_A is the expected return of asset A

X_B is the expected return of asset B

Formula 3.a can be simplified as follows:

$$\bar{R}_P = \alpha EX_A + ßEX_B \ldots\ldots\ldots (3.a)$$

Not also that α + ß = 1. This is because all the investor's wealth is invested in either asset A or asset B.

Illustration

Consider two investments, A and B each having the following investment characteristics;

Investment	Expected Return (%)	Proportion
A	10	2/3
B	20	1/3

Required:

Compute the expected return of a portfolio of the two assets.

Using formula (3.b)

Note:

$\alpha = \frac{2}{3}$

$ß = \frac{2}{3}$

$$EX_A = 10$$
$$EX_B = 20$$

$$\bar{R}_P = \alpha EX_A + ßEX_B$$

$$\bar{R}_{A+B} = \frac{2}{3}(10\%) + \frac{1}{3}(20\%)$$

$$= 13.3\%$$

Note that the expected return is a weighted average of the expected return of assets held in isolation.

PORTFOLIO STANDARD DEVIATION

Total risk is the variance (or the standard deviation) of an asset's return

Remember that standard deviation is a measure of risk of the investment. Portfolio standard deviation therefore measures the risk of investing in a combination of assets.

The portfolio standard deviation is not a weighted average of standard deviations of assets held in isolation. This is because of the inter-relatedness of the assets, which reduces the risk when assets are held together. This relationship is measured by the correlation co-efficient (r),

The coefficient of correlation (r) lies between -1 and +1. Therefore $-1 \leq r \leq +1$

If $r_{AB} = +1$, this means that A and B are perfectly positively correlated and therefore the outcomes of A and B move in the same direction at the same time.

If $r_{AB} = -1$, then A and B are perfectly negatively correlated and their results are inversely related. That is they move in opposite directions simultaneously.

If we consider the two asset case, then the portfolio standard deviation (δ_{A+B}) can be given by the following formula.

$$\delta_{A+B} = \sqrt{(\alpha)^2\delta_A^2 + ß^2\delta_B^2 + 2(r_{AB})(\alpha)(ß)(\delta_A)(\delta_B)}$$

Where δ_A is the standard deviation of A

δ_B is the standard deviation of B

r_{AB} is the correlation coefficient of asset A and B

Illustration

Consider two investments A & B each having the following characteristics:

Investment	**Expected Return (%)**	**Proportion**
A	20	$^2/_3$
B	40	$^1/_3$

Required:

Compute the portfolio standard deviation if the correlation coefficient between the assets is

a. 1
b. 0
c. -1

Solution

$$\delta_{A+B} = \sqrt{\left(\frac{2}{3}\right)^2 (20\%)^2 + \left(\frac{1}{3}\right)^2 (40\%)^2 + 2(r_{AB})\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)(20\%)(40\%)}$$

Using formula we can compute the portfolio standard deviation as follows:

$$\sqrt{0.0178 + 0.178 + 0.36(r_{AB})}$$

Note that the standard deviation depends on the correlation coefficient if the proportion of investment is fixed. Therefore if

a. If $r_{AB} = 1$

$$r_{A+B} = \sqrt{(0.036 + 0.036)} = 26.8\%$$

b. If $r_{AB} = 0$ then

$$r_{A+B} = \sqrt{(0.036)} = 18.7\%$$

c. If $r_{AB} = 1$

$$r_{A+B} = \sqrt{(0.036\ +\ 0.036)} = 26.8\%$$

There is no risk at all.

Note: If assets are perfectly negatively correlated then holding them in a portfolio greatly reduces their risk.

COVARIANCE AND THE CORRELATION COEFFICIENT

The relationship between security returns can be measured using the following 2 methods;

- Co-variance
- Co-efficient of correlation

1. Co-variance (Cov)

The covariance between 2 security returns indicates whether there is a positive or negative relationship but it does not give the extend.

Co-variance is computed as follows;

$$COV_{xy} = \sum(R_x - \sum R_x)(R_y - \sum R_y)\ Prob \ldots\ldots in\ the\ presence\ of\ probability$$

$$COV_{xy} = \frac{\sum(R_x - \sum R_x)\left(R_y - \sum R_y\right)}{n-1} \ldots in\ the\ absence\ of\ probability\quad n < 30$$

$$COV_{xy} = \frac{\sum(R_x - \sum R_x)\left(R_y - \sum R_y\right)}{n} \ldots in\ the\ presence\ of\ probability\quad n > 30$$

2. Correlation coefficient (r)

This indicates the extend 1 magnitude of the relationship between the security returns its expressed as follows;

$$r_{xy} = \frac{COV_{xy}}{\delta_x \delta_y} \qquad \delta = \sqrt{\sum(R - \sum R)^2 Prob}$$

THE ACTUAL AND WEIGHTED PORTFOLIO RISK

$$Weighted\ \delta_P = W_x\delta_x + W_y\delta_y$$

$$Expected\ Return\ (ER)\ = \sum(Returns\ \times\ prob)$$

$$Expected\ return\ of\ the\ portfolio\ (ERP) = W_x\sum R_x + W_y\sum R_y$$

$$Actual\ portfolio\ risk\ \ \delta_x = \sqrt{\sum W_x^2\delta_x^2 + W_y^2\delta_x^2 + 2W_xW_yCOV_{xy}}$$

$$\%\ of\ risk\ diversification\ = \frac{Weighted\ \delta_p - Actual\ \delta_p}{weighted\ \delta_p} \times 100\%$$

The covariance is a measure which reflects both the variance of an asset's returns and the tendency of those returns to move up and down at the same time other assets move up or down. The covariance between two asset's return can be given by the following formula:

$$Cov(AB) = \sum_{i=1}^{n}[R_{Ai} - E(R_A)][R_{Bi} - E(R_B)]P_i$$

Where Cov(AB) is the covariance between And B

R_{Ai} is the return on asset A under the ith state.
$E(R_A)$ is the expected return of A
R_{Bi} is the return on asset B under the ith state
$E(R_B)$ is the expected return of B
P_i is the probability of the ith state.

The correlation coefficient can be given by the following formula.

$$r_{AB} = \frac{\text{Cov(AB)}}{\delta_A\delta_B}$$

Illustration:

Four assets have the following distribution of returns.

	Rate of return (%)			
Probability Occurrence	**A**	**B**	**C**	**D**
0.1	10.0%	6.0%	14.0%	2.0%
0.2	10.0	8.0	12.0	6.0
0.4	10.0	10.0	10.0	9.0
0.2	10.0	12.0	8.0	15.0
0.1	10.0	14.0	6.0	20.0

Required:

a). Compute the expected return and standard deviation of each asset.

b). Compute the covariance of asset

i. A and B

ii. B and C

iii. B and D

c). Compute the correlation coefficient of the combination of assets in b above.

Solution

a).

$$E(R_A) = 10(0.1) + 10(0.2) + 10(0.4) + 10(0.2) + 10(0.1) = 10\%$$

$$E(R_B) = 6(0.1) + 8(0.2) + 10(0.4) + 12(0.2) + 14(0.1) = 10\%$$

$$E(R_C) = 14(0.1) + 12(0.2) + 10(0.4) + 8(0.2) + 6(0.1) = 10\%$$

$$E(R_D) = 2(0.1) + 6(0.2) + 9(0.4) + 15(0.2) + 20(0.1) = 10\%$$

$$\delta_A = \sqrt{\sum_{i=1}^{5} [R_{Ai} - E(R_A)]P_i}$$

$$= 0 \; since \; R_{Ai} - E(R_A) = 0\%$$

$$= \sqrt{4.8}$$

$$= 2.2\%$$

$$Likewise \; \delta_C = 2.2\% \; and \; \delta_D = 5.0\%$$

b). i.

$$Cov(AB) = \sum_{i=1}^{5}[R_{Ai} - E(R_A)][R_{Bi} - E(R_B)]P_i$$

Note that since $R_{Ai} - E(R_A) = 0$

$$Cov(AB) = 0$$

ii $Cov(BC)$

$$= (6-10)(14-10)(0.1) + (8-10)(12-10)0.2 + (10-10)(10-10)0.4 + (12-10)(8-10)0.2 + (14-10)(6-10)0.1$$

$$= -4.8$$

iii $Cov(BD)$

$$= (6-10)(2-10)(0.1) + (8-10)(6-10)0.2 + (10-10)(9-10)0.4 + (12-10)(15-10)0.2 + (14-10)(20-10)0.1$$

$$= 10.8$$

Assets B and C tend to move in opposite directions and therefore their covariance is negative while Assets B and D tend to move in the same directions and therefore their covariance is positive.

Returns of A has no correlation with the returns of other assets and therefore the covariance between A and any other asset is zero.

c). Correlation coefficient

i.

$$r_{AB} = \frac{\text{Cov(AB)}}{\delta_A \delta_B} = \frac{0}{(0)(2.2)} = 0$$

ii.

$$r_{AB} = \frac{\text{Cov(BC)}}{\delta_B \delta_C} = \frac{-4.8}{(2.2)(2.2)} = -1.0$$

i.

$$r_{AB} = \frac{\text{Cov(BD)}}{\delta_B \delta_D} = \frac{10.8}{(2.2)(5.0)} = 0.98$$

Therefore Assets B and C are perfectly negatively correlated while B and D have a strong positive correlation.

Illustration 2

June 2013 Question Four A

Betty Muye has inverted 75% of her funds in shares of company x and 25% in shares of company following probability distribution relates to the shares of the two companies

state of company	Probability	Return on company	Return on company
		X shares (%)	Y shares (%)
Broom	0.2	24	5
Steady growth	0.6	12	30
Slump	00.2	0	-5

Required

(i) Expected returns on the shares of companies X and Y. (2 marks)

(ii) Standard deviation of returns on shares of companies X and Y. (2 marks)

(iii) Coefficient of correlation between the returns on shares of companies X and Y. (2 marks)

(iv) Expected portfolio return. (1 mark)

(v) Portfolio risk. (3 marks)

vi) Percentage of risk diversification (3 marks)

Solution

i) Expected returns on the shares of companies X and Y

$$Expected\ return\ (ER)\ = \sum(Returns\ \times\ prob)$$

$ER_x = (24 \times 0.2) + (12 \times 0.6) + (0 \times 0.2) = 12$

$ER_y = (5 \times 0.2) + (30 \times 0.6) + (-5 \times 0.2) = 18$

ii) Standard deviation of returns on shares of companies X and Y

$$\delta = \sqrt{\sum(R - \sum R)^2 Prob}$$

$\delta x = \sqrt{0.2(24 - 12)^2 + 0.6(12 - 12)^2 + 0.2(0 - 12)^2}$

$= \sqrt{57.6}$
$= 7.59$

$\delta y = \sqrt{0.2(5-18)^2 + 0.6(30-18)^2 + 0.2(5-18)^2}$
$= \sqrt{226}$
$= 15.03$

iii) Coefficient of correlation between the returns on shares of company X and Y

$$\mathrm{r} = \frac{COV_{xy}}{\delta_x \delta_y}$$

$COV_{xy} = \sum(R_x - ER_x)(R_y - ER_y)Prob$
$(24-12)(S-18)00.2 = -31.2$
$(12-12)(30-18)0.6 = 0$
$(0-12)(25-18)0.2 = 55.2$
$\sum = -32.2 + 0 + 55.2 = 24$

$Given\ that\ COV_{xy} = \delta_{xy}\ then,$
$r_{xy} = \dfrac{24}{7.59 \times 15.03}$

$r = 0.21$

iv) Expected Portfolio return

$ERP = W_x \sum R_x + W_y \sum R_y$
$(0.75 \times 12) + (0.25 \times 18) = 13.5$

(v) Portfolio risk

$$Actual\ portfolio\ risk\quad \delta_x = \sqrt{\sum W_x^2 \delta_x^2 + W_y^2 \delta_x^2 + 2W_x W_y COV_{xy}}$$

$$= \sqrt{(0.75^2 \times 7.59^2) + (0.25^2 \times 15.03) + (2 \times 0.75 \times 0.25 \times 24)}$$

$$= \sqrt{32.4 + 14.12 + 9}$$

$$= 7.45\%$$

vi) Percentage of risk diversification

$$= \frac{Weighted\ \delta_p - Actual\ \delta_p}{weighted\ \delta_p} \times 100\% = \frac{9.45 - 7.45}{9.45} \times 100\% = 21\%$$

$$Weighted\ \delta_P = W_x\delta_x + W_y\delta_y$$

$$= (0.75 \times 7.59) + (0.25 \times 15)$$
$$= 9.45$$

DETERMINING EFFICIENT SETS AND OPTIMUM PORTFOLIO

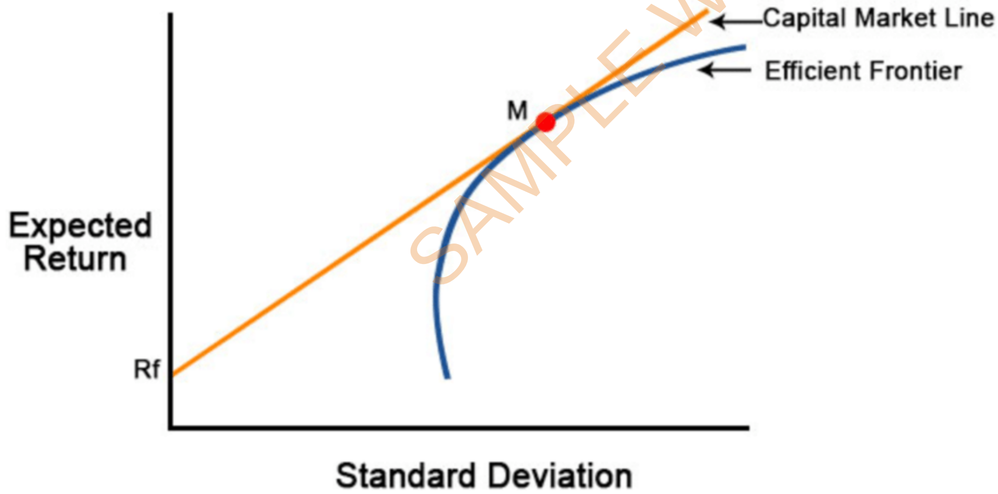
Various investors in the market have different attitudes toward risks; they are classified into 3

i. Risk takes
ii. Risk averse
iii. Risk neutral

The efficient set of portfolios is defined as the collection of portfolios which have the highest attainable expected portfolio rate of return, E[R_P], at each given level of risk or standard deviation, δ_P

Efficient sets portfolio is the combination of portfolio offering the highest return at a given level of risk

When efficient set portfolios are joined together, they form a curve known as efficient frontier curve.

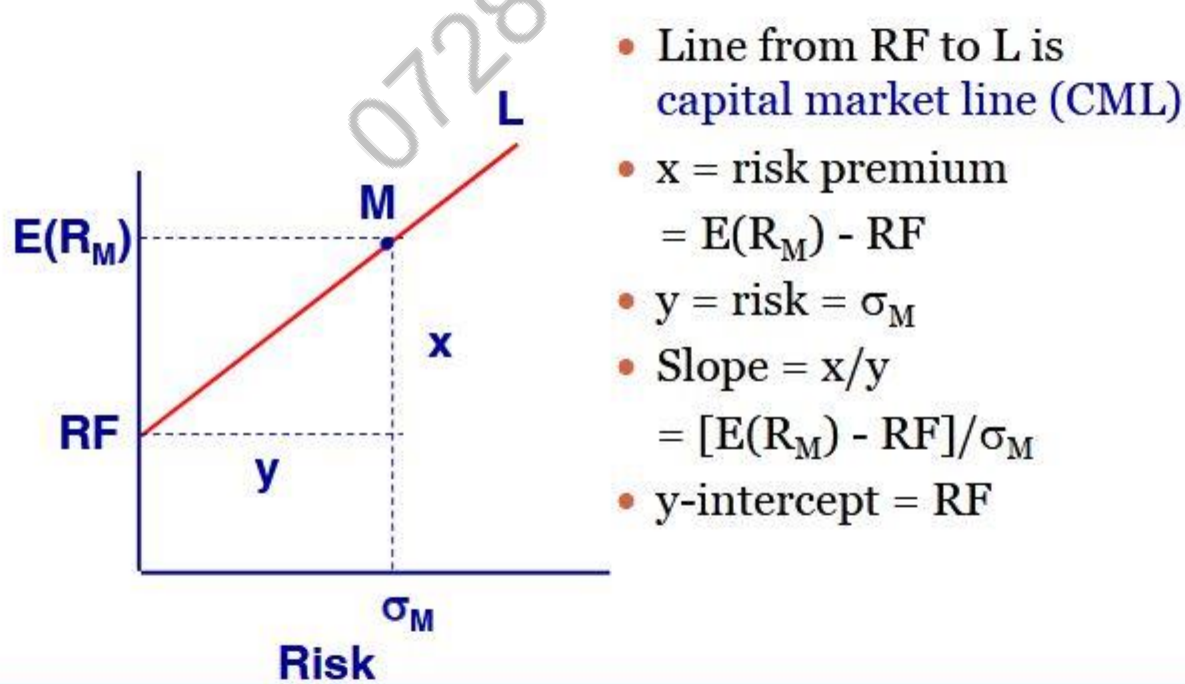


Optimum Portfolio

This is a portfolio located at a point where the efficient frontier curve is target to the highest indifference curve

CAPITAL MARKET LINE (CML)

This is a line originating from Y axis at a risk free rate towards the market portfolion and which is target to the efficient frontier curve



CML Equation

$$R_P = R_F + \left[\frac{ERM - R_F}{\delta_M}\right]\delta_P$$

$$= \frac{ERM - R_F}{\delta_M} \ldots\ldots\ldots\ldots\ldots Risk\ Premium$$

Where

R_F - Risk free rate
ERM - Expected return on the market
δ_M – Standard deviation of the market
δ_P - Standard deviation of the portfolio
R_P - Return on the portfolio

The difference between portfolio return and the expected return of the portfolio forms the basis of determining whether the security has been correctly valued, under valued or oves valued.

The basis of determining whether the security has been correctly valued, under valued or over valued.

Decision criteria

If $ERP > R_P$ (+) ……………………Super efficient or undervalued
If $ERP < R_P$ (-) ……………………Inefficient or overvalued
If $ERP = R_P$……………………….Efficient or correctly valued

Illustration

The following information is provided about securities of the company forming the portfolio

Portfolio	ERP%	Standard deviation%
1	19	8
2	25	12
3	16	6
4	32	16
5	22.5	1.0
6	8	2

The expected return on the market portfolio is 12% and the standard deviation of the market is 4%. the risk is free rate is 5%

Required

a) Using the capital market line advice the investor on which of the above portfolio are efficient, super efficient and inefficient

Portfolio	**ERP**	$R_P = R_F + \left[\frac{ERM - R_F}{\delta_M}\right]\delta_P$	**Comment**
1	19	$5+\left(\frac{12-5}{4}\right)8=19$	efficient
2	25	$5+\left(\frac{12-5}{4}\right)12=26$	inefficient
3	16	$5+\left(\frac{12-5}{4}\right)6=15.5$	super-efficient
4	32	$5+\left(\frac{12-5}{4}\right)16=33$	inefficient
5	22.5	$5+\left(\frac{12-5}{4}\right)100=22.5$	Efficient
6	8	$5+\left(\frac{12-5}{4}\right)2=8.5$	Inefficient

b) Incase of inefficient portfolion in (a) above, determine the standard deviation of the portfolio for the efficiency to be achieved within the expected returns.

Portfolio 2	$25=5+\left(\frac{12-5}{4}\right)\delta_P$	$25=5.1.75\ \delta_P$	$\delta_P= 11.43$
4	$32=5+\left(\frac{12-5}{4}\right)\delta_P$	$32=5.1.75\ \delta_P$	$\delta_P = 15.43$
6	$8=5+\left(\frac{12-5}{4}\right)\delta_P$	$8=5.1.75\ \delta_P$	$\delta_P = 1.71$

MEAN-VARIANCE ANALYSIS

Mean-variance analysis is the process of weighing risk, expressed as variance, against expected return. Investors use mean-variance analysis to make investment decisions. Investors weigh how much risk they are willing to take on in exchange for different levels of reward. Mean-variance analysis allows investors to find the biggest reward at a given level of risk or the least risk at a given level of return.

- Mean-variance analysis is a tool used by investors to weigh investment decisions.

- The analysis helps investors determine the biggest reward at a given level of risk or the least risk at a given level of return.
- The variance shows how spread out the returns of a specific security are on a daily or weekly basis.
- The expected return is a probability expressing the estimated return of the investment in the security.
- If two different securities have the same expected return, but one has lower variance, the one with lower variance is preferred.
- Similarly, if two different securities have approximately the same variance, the one with the higher return is preferred.

$$Return\ per\ unit\ of\ risk\ = \frac{Expected\ portfolio\ return}{Standard\ deviation\ of\ portfolio}$$

Minimum variance portfolio

A minimum variance portfolio indicates a well-diversified portfolio that consists of individually risky assets, which are hedged when traded together, resulting in the lowest possible risk for the rate of expected return.

If we want to find the exact minimum variance portfolio allocation for these two assets, we can use the following equation:

$$W_X = \frac{\delta_y^2 - Cov_{xy}}{\delta_x^2 + \delta_y^2 - 2Cov_{xy}}$$

SHORTCOMINGS/LIMITATIONS OF PORTFOLIO THEORY

- It deals with total risk as measured using standard deviation, however risk can be minimized by holding portfolio
- It uses historical data to analyze portfolio and the probability assigned to each outcome are subjective
- It assumes investors are risk averse, however in practice they are risk takers
- It can only be used to analyze only one security
- It assumes all projects are invisible, however in practice indivisible projects exists