CHAPTER 11: INTRODUCTION TO MODELING

Introduction to Mathematical modeling

A **mathematical model** is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed **mathematical modeling**. Mathematical models are used in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (such as computer science, artificial intelligence), as well as in the social sciences (such as economics, psychology, sociology, political science).

A model may help to explain a system and to study the effects of different components, and to make predictions about behavior.

Hypothesis, Model, Theory & Law

In common usage, the words hypothesis, model, theory, and law have different interpretations and are at times used without precision, but in science they have very exact meanings.

Hypothesis

It is a **limited statement** regarding the cause and effect in a specific situation, which can be tested by experimentation and observation or by statistical analysis of the probabilities from the data obtained. The outcome of the test hypothesis should be currently unknown, so that the results can provide useful data regarding the validity of the hypothesis.

Sometimes a hypothesis is developed that must wait for new knowledge or technology to be testable. The concept of atoms was proposed by the ancient Greeks, who had no means of testing it. Centuries later, when more knowledge became available, the hypothesis gained support and was eventually accepted by the scientific community, though it has had to be amended many times over the year. Atoms are not indivisible, as the Greeks supposed.

Model

A *model* is used for situations when it is known that the hypothesis has a limitation on its validity.

Several classes of models

Three classes appear :

- Models which come from laws of physics: this is the case for gravitation laws, Maxwell equations (waves), Navier-Stokes equations (fluid dynamics), and so on;
- Model which come from empirical laws, such as air resistance for a movement: this laws are of empirical nature;
- Models which use statistical laws, for instance that fit a line between several points and assume the response to be linear.

Theory & Law

A *scientific theory* or *law* represents a hypothesis (or group of related hypotheses) which has been confirmed through repeated testing, almost always conducted over a span of many years.

Generally, a theory is an explanation for a set of related phenomena, like the theory of evolution or the big bang theory.

The word "law" is often invoked in reference to a specific mathematical equation that relates the different elements within a theory. Pascal's Law refers an equation that describes differences in pressure based on height. In the overall theory of universal gravitation developed by Sir Isaac Newton, the key equation that describes the gravitational attraction between two objects is called the law of gravity.

These days, physicists rarely apply the word "law" to their ideas. In part, this is because so many of the previous "laws of nature" were found to be not so much laws as guidelines, that work well within certain parameters but not within others.

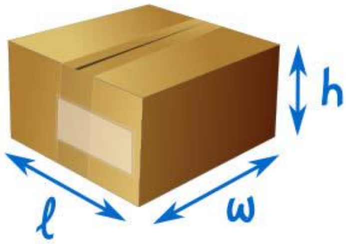
General rules of mathematical modeling

- Look at how others model similar situations; adapt their models to the present situation.
- Collect/ask for background information needed to understand the problem.
- Start with simple models; add details as they become known and useful or necessary.
- Find all relevant quantities and make them precise.
- Find all relevant relationships between quantities ([differential] equations, inequalities, case distinctions).
- Locate/collect/select the data needed to specify these relationships.
- Find all restrictions that the quantities must obey (sign, limits, forbidden overlaps, etc.). Which restrictions are hard, which soft? How soft?
- Try to incorporate qualitative constraints that rule out otherwise feasible results (usually from inadequate previous versions).
- Find all goals (including conflicting ones)
- Play the devil's advocate to find out and formulate the weak spots of your model.
- Sort available information by the degree of impact expected/hoped for.
- Create a hierarchy of models: from coarse, highly simplifying models to models with all known details. Are there useful toy models with simpler data? Are there limiting cases where the model simplifies? Are there interesting extreme cases that help discover difficulties?
- First solve the coarser models (cheap but inaccurate) to get good starting points for the finer models (expensive to solve but realistic)
- Try to have a simple working model (with report) after 1/3 of the total time planned for the task. Use the remaining time for improving or expanding the model based on your experience, for making the programs more versatile and speeding them up, for polishing documentation, etc.
- Good communication is essential for good applied work.
- The responsibility for understanding, for asking the questions that lead to it, for recognizing misunderstanding (mismatch between answers expected and answers received), and for overcoming them lies with the mathematician. You cannot usually assume your customer to understand your scientific jargon.
- Be not discouraged. Failures inform you about important missing details in your understanding of the problem (or the customer/boss) - utilize this information!
- There are rarely perfect solutions. Modeling is the art of finding a satisfying compromise. Start with the highest standards, and lower them as the deadline approaches. If you have results early, raise your standards again.
- Finish your work in time.

Construct generalized models

Mathematics can be used to "model", or represent, how the real world works.

Example: how much space is inside this cardboard box?



We know three measurements:

- **l** (length),
- **w** (width), and
- **h** (height),

and the formula for the volume of a cuboid is:

Volume = l × w × h

So we have a (very simple) mathematical model of the space in that box.

Accurate?

The model is not the same as the real thing.

In our example we did not think about the thickness of the cardboard, or many other "real world" things.

But hopefully it is **good enough to be useful**.

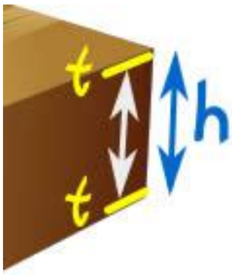
If we are charged by the volume of the box we send, we can take a few measurements and know how much to pay.

It can also be useful when deciding which box to buy when we need to pack things.

So the model is useful!

But maybe we need more accuracy, we might need to send hundreds of boxes every day, and the thickness of the cardboard matters. So let's see if we can **improve the model**:

The cardboard is "t" thick, and all measurements are outside the box ... how much space is inside?



The inside measurements need to be reduced by the thickness of each side:

- The inside length is **l-2t**
- The inside width is **w-2t**,
- The inside height is **h-2t**

and now the formula is:

Inside Volume = (l-2t) × (w-2t) × (h-2t)

Now we have a **better** model. Still not perfect (did we consider wasted space because we could not pack things neatly, etc ...), but better.

So a model is not reality, but should be good enough to be useful.

Playing With The Model

Now we have a model, we can use it in different ways:

Example: Your company uses 200x300x400 mm size boxes, and the cardboard is 5mm thick.

Someone suggests using 4mm cardboard ... how much better is that?

Let us compare the two volumes:

- Current Volume = (200-2×5) × (300-2×5) × (400-2×5) = 21,489,000 mm^3
- New Volume = (200-2×4) × (300-2×4) × (400-2×4) = 21,977,088 mm^3

That is a change of:

(21,977,088-21,489,000)/21,489,000 ≈ 2% more volume

So the model is **useful**. It lets us know we will get 2% more space inside the box (for the same outside measurements).

But there are still "real world" things to think about, such as "will it be strong enough?"

Thinking Clearly

To set up a mathematical model we also need to think clearly about the facts!
Example: on our street there are twice as many dogs as cats. How do we write this as an equation?

- Let D = number of dogs
- Let C = number of cats

  Now ... is that: 2D = C   or   should it be: D = 2C

Think carefully now!

  The correct answer is D = 2C

(**2D = C** is a common mistake, as the question is written "twice ... dogs ... cats")

Here is another one:

Example: You are the supervisor of 8-hour shift workers. They recently had their break times reduced by 10 minutes but total production did not improve.

At first glance there is nothing to model, because there was no change in production.

But wait a minute ... they are working 10 minutes more, but producing the same amount, so **production per hour** must have dropped!

Let us assume they used to work 7 hours (420 minutes):

  Change in production per hour = 410/420 = **0.976...**

  Which is a **reduction of more than 2%**

But even worse: the first few hours of the shift are not be affected by the shorter break time, so it could be a 4 or 5% reduction later in the shift.

You could recommend:

- looking at production rates for every hour of the shift
- trying different break times to see how they affect production

Steps into Model Building
A Bigger Example: Most Economical Size, OK, let us have a go at building and using a mathematical model to solve a real world question.

Your company is going to make its own boxes!

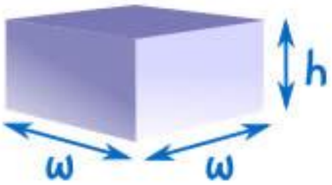
It has been decided the box should hold **0.02m^3** (0.02 cubic meters which is equal to 20 liters) of nuts and bolts.

The box should have a square base, and double thickness top and bottom.

Cardboard costs **$0.30** per square meter.

It is up to you to decide the most economical size.

Step One: Draw a sketch!
It helps to sketch out what we are trying to solve!



The base is square, so we will just use "w" for both lengths

The box has 4 sides, and double tops and bottoms.

The box shape could be cut out like this (but is probably more complicated):

Step Two: Make Formulas.
Ignoring thickness for this model:

$$\text{Volume} = w \times w \times h = w^2h$$

And we are told that the volume should be $0.02m^3$:

$$w^2h = 0.02$$

Areas:

$$\text{Area of the 4 Sides} = 4 \times w \times h = 4wh$$

$$\text{Area of Double Tops and Bases} = 4 \times w \times w = 4w^2$$

Total cardboard needed:

$$\text{Area of Cardboard} = 4wh + 4w^2$$

Step Three: Make a Single Formula For Cost
We want a single formula for cost:

$$\text{Cost} = \$0.30 \times \text{Area of Cardboard}$$

$$= \$0.30 \times (4wh + 4w^2)$$

And that is the cost when we know width **and** height.

That could be hard to work with ... a function with two variables.

But we can make it simpler! Because width and height are already related by the volume:

$$\text{Volume} = w^2h = 0.02$$

... which can be rearranged to ...

$$h = 0.02/w^2$$

... and that can be put into the cost formula ...

$$\text{Cost} = \$0.30 \times (4w \times \mathbf{0.02/w^2} + 4w^2)$$

And now the cost is related directly to **width only**.

With a little simplification we get:

$$\text{Cost} = \$0.30 \times (0.08/w + 4w^2)$$

Step Four: Plot it and find minimum cost

What to plot? Well, the formula only makes sense for widths greater than zero, and I also found that for widths above 0.5 the cost just gets bigger and bigger.

So here is a plot of that cost formula for **widths between 0.0 m and 0.55 m**:

Plot of $y = 0.3(0.08/x+4x^2)$
x is width, and **y** is cost

Just by eye, I see the cost reaches a minimum at about **(0.22, 0.17)**. In other words:

- when the width is about **0.22 m** (x-value),
- the minimum cost is about **$0.17** per box (y-value).

In fact, looking at the graph, the width could be anywhere between 0.20 and 0.24 without affecting the minimum cost very much.

Step Five: Recommendations

Using this mathematical model you can now recommend:

- Width = 0.22 m
- Height = $0.02/w^2 = 0.02/0.22^2 = 0.413$ m
- Cost = $\$0.30 \times (0.08/w + 4w^2) = \$0.30 \times (0.08/0.22 + 4 \times 0.22^2) = \$0.167$

Or about 16.7 cents per box

But any width between 0.20 m and 0.24 m is fine.

You might also like to suggest improvements to this model:

- Include cost of glue/staples and assembly
- Include wastage when cutting box shape from cardboard.
- Is this box a good shape for packing, handling and storing?
- Any other ideas you may have!

Predicting the Future

Mathematical models can also be used to forecast future behavior.

Example: An ice cream company keeps track of how many ice creams get sold on different days.

By comparing this to the weather on each day they can make a mathematical model of **sales versus weather**.

They can then predict future sales based on the weather forecast, and decide how many ice creams they need to make ... ahead of time!

Computer Modeling

Mathematical models can get very complex, and so the mathematical rules are often written into computer programs, to make a computer model.

Examples include:

- Weather prediction
- Economic Models (predicting interest rates, unemployment, etc)
- Models of how large structures behave under stress (bridges, skyscrapers, etc)
- Many more ...

If you become an expert in any of those you will have a job for life!

Meaning of mathematical Model pseudo-code

Pseudo-code is an informal way of programming description that does not require any strict programming language syntax or underlying technology considerations. It is used for creating an outline or a rough draft of a program. Pseudo-code summarizes a program's flow, but excludes underlying details. System designers write pseudo-code to ensure that programmers understand a software project's requirements and align code accordingly. **Pseudo-code** is an informal high-level description of the operating principle of a computer program or other **algorithm**.

An algorithm is a procedure for solving a problem in terms of the steps to be executed and the order in which those steps are to be executed. An algorithm is merely the sequence of steps taken to solve a mathematical problem and may be Regarded as a pseudo-code

Logical models, Statistical models and Other Models

Logic model

A **logic model** (also known as a logical framework, theory of change, or program matrix) is a tool used by funders, managers, and evaluators of programs to evaluate the effectiveness of a program. They can also be used during planning and implementation. Logic models are usually a graphical depiction of the logical relationships between the resources, activities, outputs and outcomes of a program. While there are many ways in which logic models can be presented, the underlying purpose of constructing a logic model is to assess the "if-then" (causal) relationships between the elements of the program.

In its simplest form, a logic model has four components:

Inputs	Activities	Outputs	Outcomes/impacts
what resources go into a program	*what activities the program undertakes*	*what is produced through those activities*	*the changes or benefits that result from the program*
e.g. money, staff, equipment	e.g. development of materials, training programs	e.g. number of booklets produced, workshops held, people trained	e.g. increased skills/ knowledge/ confidence, leading in longer-term to promotion, new job, etc.

Following the early development of the logic model in the 1970s by Carol Weiss, Joseph Wholey and others, many refinements and variations have been added to the basic concept. Many versions of logic models set out a series of outcomes/impacts, explaining in more detail the logic of how an intervention contributes to intended or observed results. This will often include distinguishing between short-term, medium-term and long-term results, and between direct and indirect results.

Some logic models also include assumptions, which are beliefs the prospective grantees have about the program, the people involved, and the context and the way the prospective grantees think the program will work, and external factors, consisting of the environment in which the program exists, including a variety of external factors that interact with and influence the program action.

University Cooperative Extension Programs have developed a more elaborate logic model, called the Program Action Logic Model, which includes six steps:

- **Inputs** (what we invest)
- **Outputs:**
  - **Activities** (the actual tasks we do)
  - **Participation** (who we serve; customers & stakeholders)
  - **Engagement** (how those we serve engage with the activities)
- **Outcomes/Impacts:**
  - **Short Term** (learning: awareness, knowledge, skills, motivations)
  - **Medium Term** (action: behavior, practice, decisions, policies)
  - **Long Term** (consequences: social, economic, environmental etc.)

Advantages

By describing work in this way, managers have an easier way to define the work and measure it. Performance measures can be drawn from any of the steps. One of the key insights of the logic model is the importance of measuring final outcomes or results, because it is quite possible to waste time and money (inputs), "spin the wheels" on work activities, or produce outputs without achieving desired outcomes. It is these outcomes (impacts, long-term results) that are the only justification for doing the work in the first place. For commercial organizations, outcomes relate to profit. For not-for-profit or governmental organizations, outcomes relate to successful achievement of mission or program goals.

Uses of the logic model

Program planning - helps managers to 'plan with the end in', rather than just consider inputs (e.g. budgets, employees) or just the tasks that must be done.

Performance evaluation - used in government or not-for-profit organizations, where the mission and vision are not aimed at achieving a financial benefit. In such situations, where profit is not the intended result, it may be difficult to monitor progress toward outcomes. A program logic model provides such indicators, in terms of output and outcome measures of performance.

The logic model and other management frameworks

There are numerous other popular management frameworks that have been developed in recent decades. This often causes confusion, because the various frameworks have different functions. It is important to select the *right tool for the job*. The following list of popular management tools is suggested to indicate where they are most appropriate (this list is by no means complete).

Organizational assessment tools: Fact-gathering tools for a comprehensive view of the as-is situation in an organization, but without prescribing how to change it:

Strategic planning tools: For identifying and prioritizing major long-term desired results in an organization, and strategies to achieve those results:

Program planning and evaluation tools: For developing details of individual programs (what to do and what to measure) once overall strategies have been defined:

Performance measurement tools: For measuring, monitoring and reporting the quality, efficiency, speed, cost and other aspects of projects, programs and/or processes:

Process improvement tools: For monitoring and improving the quality or efficiency of work processes:

Process standardization tools: For maintaining and documenting processes or resources to keep them repeatable and stable:

Statistical models

A **statistical model** is a class of mathematical model, which embodies a set of assumptions concerning the generation of some sample data, and similar data from a larger population. A statistical model represents, often in considerably idealized form, the data-generating process.

The assumptions embodied by a statistical model describe a set of probability distributions, some of which are assumed to adequately approximate the distribution from which a particular data set is sampled. The probability distributions inherent in statistical models are what distinguishes statistical models from other, non-statistical, mathematical models.

A statistical model is usually specified by mathematical equations that relate one or more random variables and possibly other non-random variables. As such, "a model is a formal representation of a theory" (Herman Adèr quoting Kenneth Bollen).

All statistical hypothesis tests and all statistical estimators are derived from statistical models. More generally, statistical models are part of the foundation of statistical inference.

Formal definition

In mathematical terms, a statistical model is usually thought of as a pair ($\mathcal{S},\mathcal{P}$), where $\mathcal{S}$ is the set of possible observations, i.e. the sample space, and $\mathcal{P}$ is a set of probability distributions on $\mathcal{S}$.

The intuition behind this definition is as follows. It is assumed that there is a "true" probability

distribution induced by the process that generates the observed data. We choose to represent a set (of distributions) which contains a distribution that adequately approximates the true

distribution. Note that we do not require that contains the true distribution, and in practice that is rarely the case. Indeed, as Burnham & Anderson state, "A model is a simplification or approximation of reality and hence will not reflect all of reality"—whence the saying "all models are wrong".

The set $\mathcal{P}$ is almost always parameterized: $\mathcal{P} = \{\mathcal{P}_\Theta : \Theta \in \Theta\}$. The set Θ defines the parameters of the model. A parameterization is generally required to have distinct parameter values give rise to distinct distributions, i.e. $\mathcal{P}_{\Theta 1} = \mathcal{P}_{\Theta 2} \Rightarrow \Theta_1 = \Theta_2$. Must hold (in other words, it must be injective). A parameterization that meets the condition is said to be *identifiable*.

An example

Height and age are each probabilistically distributed over humans. They are stochastically related: when we know that a person is of age 10, this influences the chance of the person being 5 feet tall. We could formalize that relationship in a linear regression model with the following form: $\text{height}_i = b_0 + b_1\text{age}_i + \varepsilon_i$, where b_0 is the intercept, b_1 is a parameter that age is multiplied by to get a prediction of height, ε is the error term, and i identifies the person. This implies that height is predicted by age, with some error.

An admissible model must be consistent with all the data points. Thus, the straight line ($\text{height}_i = b_0 + b_1\text{age}_i$) is *not* a model of the data. The line cannot be a model, unless it exactly fits all the data points—i.e. all the data points lie perfectly on a straight line. The error term, ε_i, must be included in the model, so that the model is consistent with all the data points.

To do statistical inference, we would first need to assume some probability distributions for the ε_i. For instance, we might assume that the ε_i distributions are i.i.d. Gaussian, with zero mean. In this instance, the model would have 3 parameters: b_0, b_1, and the variance of the Gaussian distribution.

We can formally specify the model in the form ($\mathcal{S}, \mathcal{P}$) as follows. The sample space, $\mathcal{S}$, of our model comprises the set of all possible pairs (age, height). Each possible value of $\Theta = (b_0, b_1, \sigma^2)$ determines a distribution on $\mathcal{S}$; denote that distribution by $\mathcal{P}_\Theta$. If Θ is the set of all possible values of Θ, then $\mathcal{P} = \{\mathcal{P}_\Theta : \Theta \in \Theta\}$.. (The parameterization is identifiable, and this is easy to check.)

In this example, the model is determined by (1) specifying $\mathcal{S}$ and (2) making some assumptions relevant to $\mathcal{P}$. There are two assumptions: that height can be approximated by a linear function of age; that errors in the approximation are distributed as i.i.d. Gaussian. The assumptions are sufficient to specify $\mathcal{P}$—as they are required to do.

General remarks

A statistical model is a special class of mathematical model. What distinguishes a statistical model from other mathematical models is that a statistical model is non-deterministic. Thus, in a statistical model specified via mathematical equations, some of the variables do not have specific values, but instead have probability distributions; i.e. some of the variables are stochastic. In the example above, ε is a stochastic variable; without that variable, the model would be deterministic.

Statistical models are often used even when the physical process being modeled is deterministic. For instance, coin tossing is, in principle, a deterministic process; yet it is commonly modeled as stochastic (via a Bernoulli process).

There are three purposes for a statistical model, according to Konishi & Kitagawa.

- Predictions
- Extraction of information
- Description of stochastic structures

Graphical Representation of statistical Data

Statistics is a special subject that deals with large (usually) numerical data. The statistical data can be represented graphically. In fact, the graphical representation of statistical data is an essential step during statistical analysis.
Statistical surveys and experiments provides valuable information about numerical scores. For better understanding and making conclusions and interpretations, the data should be managed and organized in a systematic form. A graph is the representation of data by using graphical symbols such as lines, bars, pie slices, dots etc. A graph does represent a numerical data in the form of a qualitative structure and provides important information.

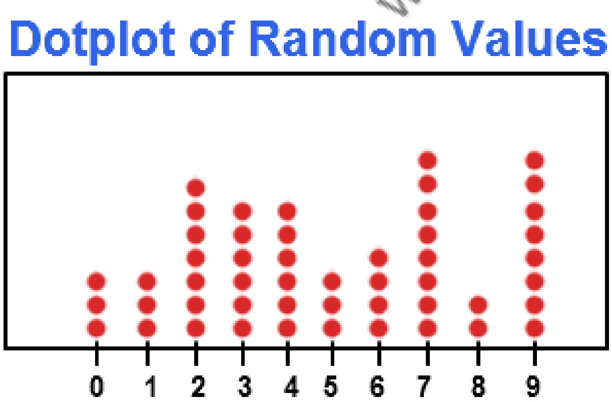
Let us go ahead and study about various types of graphical representations of the data.

<u>Dot Plots</u>

The dot plot is one of the most simplest ways of graphical representation of the statistical data. As the name itself suggests, a dot plot uses the dots. It is a graphic display which usually compares frequency within different categories.

The dot plot is composed of dots that are to be plotted on a graph paper.

A dot plot may look like:



In the dot plot, every dot denotes a specific number of observations belonging to a data set. One dot usually represents one observation.

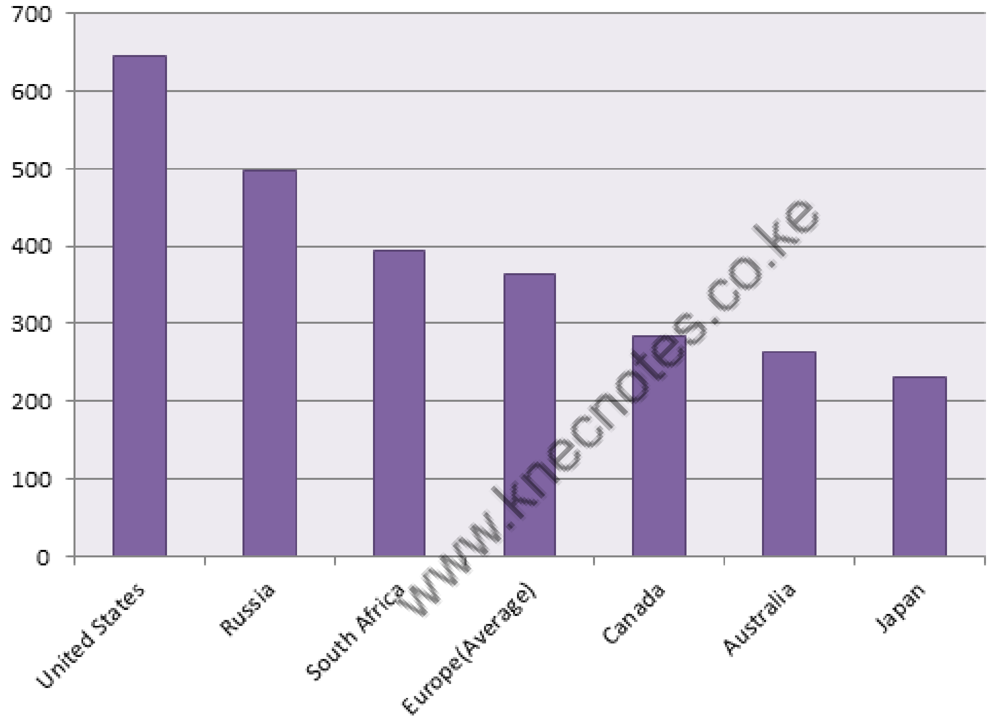
These dots are to be marked in the form of a column for each category. In this way, the height of each column shows the corresponding frequency of some category.
The dot plots are quite useful when there are small amount of data is given within the small number of categories.

Bar Graph

A bar graph is a very frequently used graph in statistics as well as in media. A bar graph is a type of graph which contains rectangles or rectangular bars. The lengths of these bars should be proportional to the numerical values represented by them. In bar graph, the bars may be plotted either horizontally or vertically. But a vertical bar graph (also known as column bar graph) is used more than a horizontal one.

A vertical bar graph is shown below:

Number of students went to different states for study:



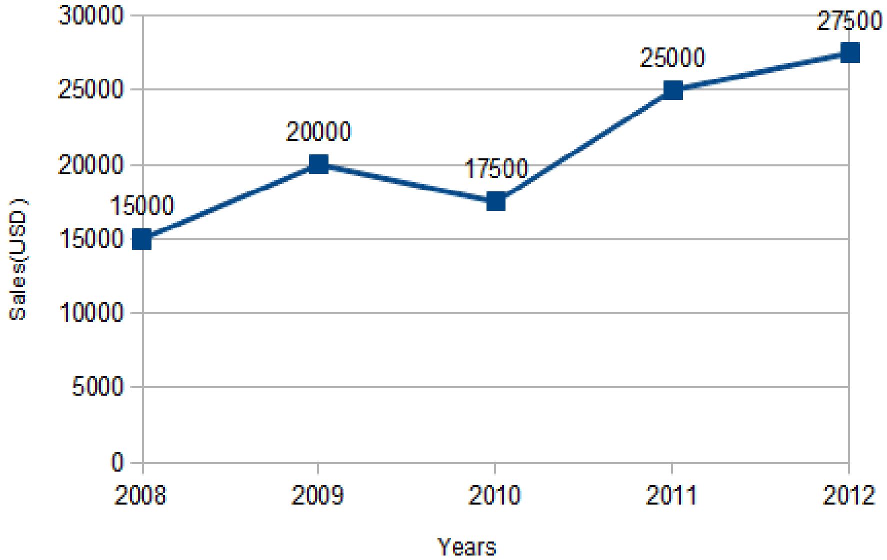
The rectangular bars are separated by some distance in order to distinguish them from one another. The bar graph shows comparison among the given categories.

Mostly, horizontal axis of the graph represents specific categories and vertical axis shows the discrete numerical values.

Line Graph

A line graph is a kind of graph which represents data in a way that a series of points are to be connected by segments of straight lines. In a line graph, the data points are plotted on a graph and they are joined together with straight line.

A sample line graph is illustrated in the following diagram:

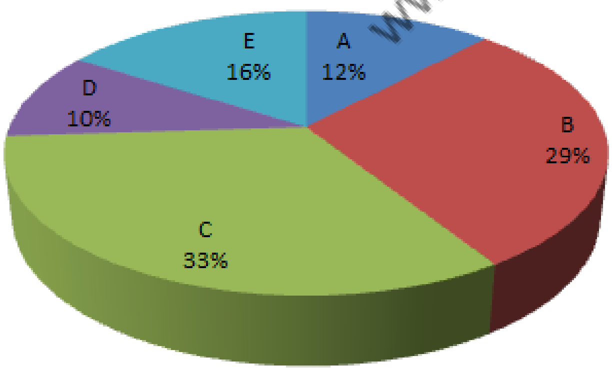


The line graphs are used in the science, statistics and media. Line graphs are very easy to create. These are quite popular in comparison with other graphs since they visualize characteristics revealing data trends very clearly. A line graph gives a clear visual comparison between two variables which are represented on X-axis and Y-axis.

Circle Graph

A circle graph is also known as a pie graph or pie chart. It is called so since it is similar to slice of a "pie". A pie graph is defined as a graph which contains a circle which is divided into sectors. These sectors illustrate the numerical proportion of the data.

A pie chart are shown in the following diagram:

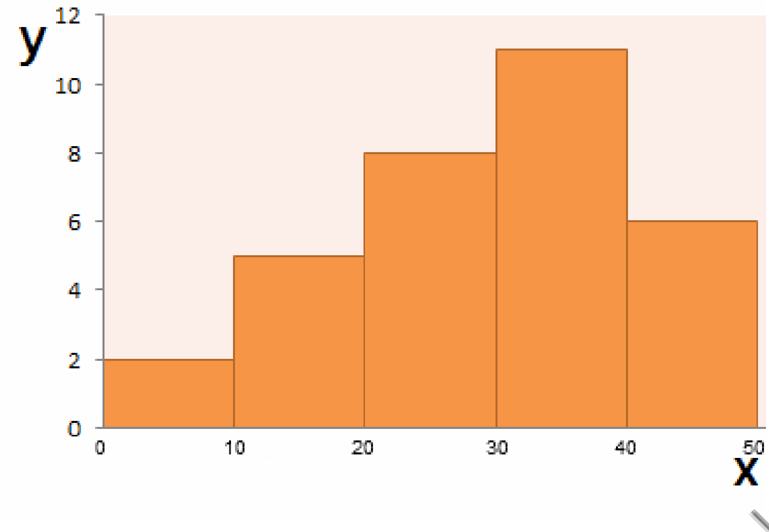


The arc lengths of the sectors, in pie chart, are proportional to the numerical value they represent. Circle graphs are quite commonly seen in mass media as well as in business world.

Histogram and Frequency Polygon

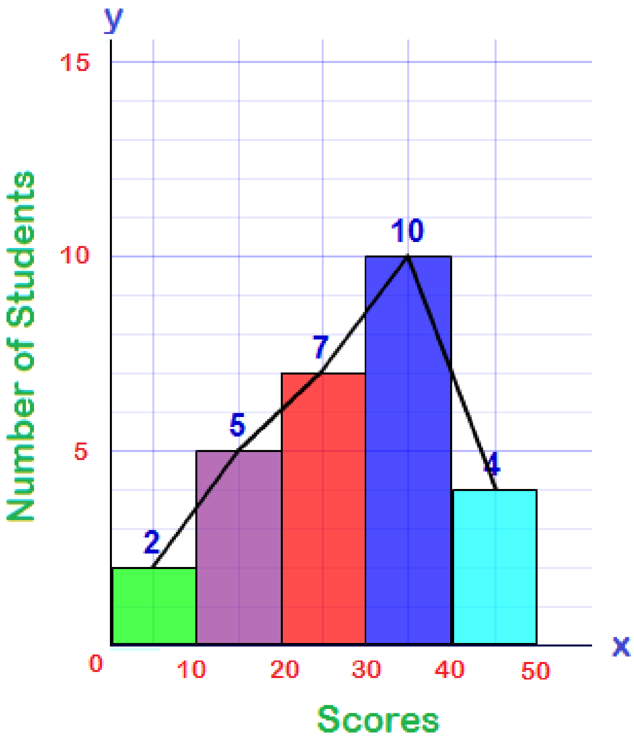
The histograms and frequency polygons are very common graphs in statistics. A histogram is defined as a graphical representation of the mutually exclusive events. A histogram is quite similar to the bar graph. Both are made up of rectangular bars. The difference is that there is no gap between any two bars in the histogram. The histogram is used to represent the continuous data.

A histogram may look like the following graph:



The frequency polygon is a type of graphical representation which gives us better understanding of the shape of given distribution. Frequency polygons serve almost the similar purpose as histograms do. But the frequency polygon is quite helpful for the purpose of comparing two or more sets of data. The frequency polygons are said to be the extension of the histogram. When the midpoints of tops of the rectangular bars are joined together, the frequency polygon is made. **Let us have a look at a sample of**

frequency polygon:



<u>Examples</u>

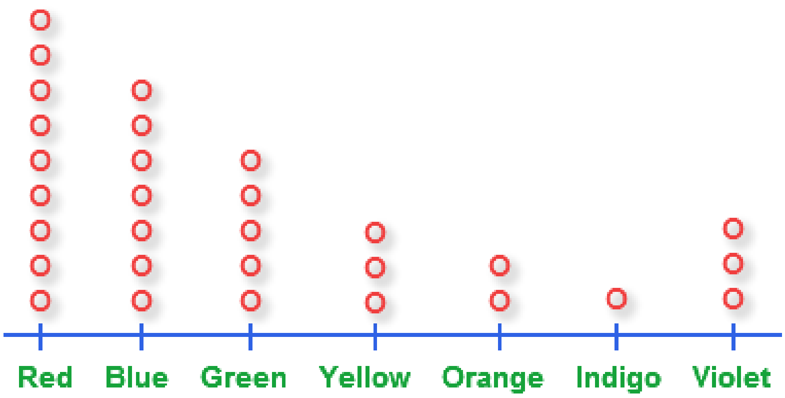
Few examples of graphical representation of statistical data are given below:

Example 1: Draw a dot plot for the following data.

Favorite Colors	**Red**	**Blue**	**Green**	**Yellow**	**Orange**	**Indigo**	**Violet**
Number of Students	9	7	5	3	2	1	3

Solution:

The line graph for the following data is given below:

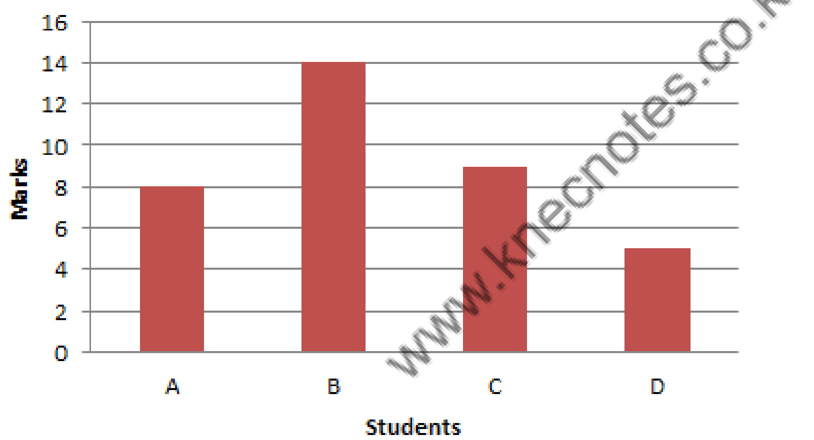


Example 2: Plot a bar graph from the data given below.

Students	A	B	C	D
Marks	8	14	9	5

Solution:

The following bar graph is obtained:

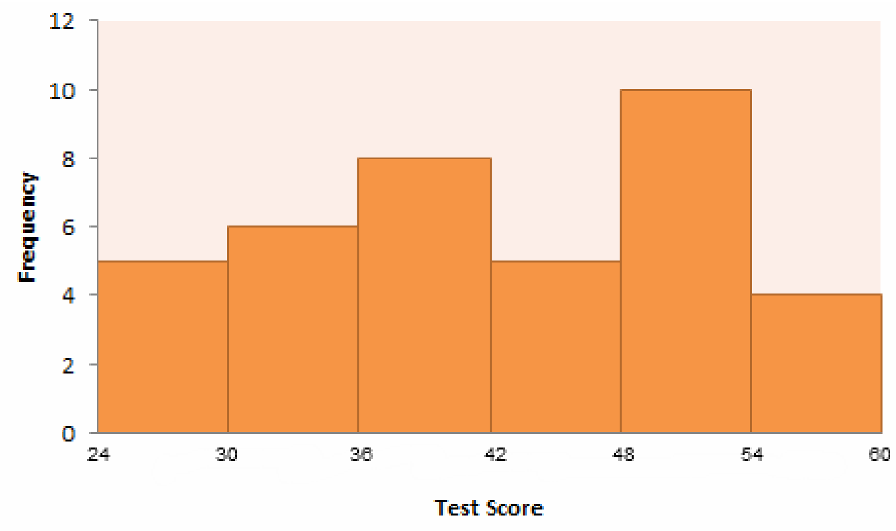


Example 3: Draw a histogram from the given data.

Test Score	24-30	30-36	36-42	42-48	48-54	54-60
Frequency	5	6	8	5	10	4

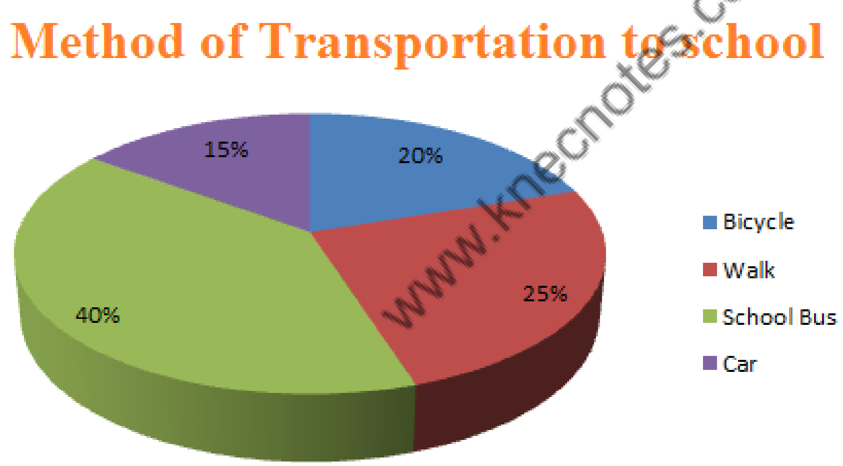
Solution:

We drew the following histogram:



Example 4: The percentages of students who use the different methods of transportation are as follows:
40% go by school bus
25% go by walk
20% go by bicycle
and rest 15% go by car. Draw a pie chart.

Solution: The pie graph of the above data is:



Spatial Model

Spatial modeling is an analytical process conducted in conjunction with a geographical information system (GIS) in order to describe basic processes and properties for a given set of spatial features.

The objective of spatial modeling is to be able to study and simulate spatial objects or phenomena that occur in the real world and facilitate problem solving and planning.

Spatial modeling is an essential process of spatial analysis. With the use of models or special rules and procedures for analyzing spatial data, it is used in conjunction with a GIS to properly analyze and visually lay out data for better understanding by human readers. Its visual nature helps researchers more quickly understand the data and reach conclusions that are difficult to formulate with simple numerical and textual data.

Spatial analysis or **spatial statistics** includes any of the formal techniques which study entities using their topological, geometric, or geographic properties. Spatial analysis includes a variety of techniques, many still in their early development, using different analytic approaches and applied in fields as diverse as astronomy, with its studies of the placement of galaxies in the cosmos, to chip fabrication engineering, with its use of "place and route" algorithms to build complex wiring structures. In a more restricted sense, spatial analysis is the technique applied to structures at the human scale, most notably in the analysis of geographic data.

Manipulation of information occurs in multiple steps, each representing a stage in a complex analysis procedure. Spatial modeling is object-oriented with coverage and concerned with how the physical world works or looks. The resulting model represents either a set of objects or real-world process.

For example, spatial modeling can be used to analyze the projected path of tornadoes by layering a map with different spatial data, like roads, houses, the path of the tornado and even its intensity at different points. This allows researchers to determine a tornado's real path of destruction. When juxtaposed with other models from tornadoes that have affected the area, this model can be used to show path correlations and geographical factors.

Symbolic Modelling

"We define Symbolic Modelling as a process, which uses Clean Language to facilitate people's discovery of how their metaphors express their way of being in the world."

Features of a Symbolic Model

- It contains a set of representations (or symbols) of something.
- It processes and manipulates those representations based on a set of rules programmed into the model.
- The rules operate on the representations according to their 'shape' or syntax, not according to what they represent (their semantics).

Let us take an example. The symbol '1' is character. In normal text, it represents the number one. When you read this text, you probably read it as a one. But this is a matter of interpretation, not a property of the symbol itself. For example, we could use the same symbol to represent the state of being 'on'. In fact, we do use it in this way on certain appliances — switches often have '0' and '1' marked on them to represent that the appliance is 'off' and 'on' respectively. The interpretation of the symbol (its semantics) is independent of the shape of the symbol (its syntax).