KASNEB NEW REVISED SYLLABUS

QUANTITATIVE ANALYSIS STUDY NOTES

MASOMO MSINGI

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QUANTITATIVE ANALYSIS

NOTES

FOUNDATIONAL LEVEL

REVISED SYLLABUS

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PAPER NO. 5 QUANTITATIVE ANALYSIS

UNIT DESCRIPTION

This paper is intended to equip the candidate with knowledge, skills and attitudes that will enable the learner to use quantitative analysis tools in business operations and decision making.

LEARNING OUTCOMES

A candidate who passes this paper should be able to:

- Use mathematical techniques to solve business problems.
- Apply set and probability theories in business decision making
- Apply operation research techniques in decision making
- Apply hypothesis testing in analysing business situations
- Apply linear programming to solve practical business problems

CONTENT

1. Mathematical Techniques

1.1 Functions
1.1.1 Definition
1.1.2 Functions, equations, inequalities and graphs; linear, quadratic, cubic, Exponential and logarithmic functions
1.1.3 Application of mathematical functions in solving business problems

1.2 Matrix Algebra

1.2.1 Definition
1.2.2 Types and operations (addition, subtraction, multiplication, transposition and inversion of up to order 3x3)
1.2.3 Application of matrices; statistical modelling, Markov analysis, input-output analysis and general applications

1.3 Calculus

1.4 Differentiation

1.4.1 Definition
1.4.2 Rules of differentiation (general rule, chain, product, quotient)
1.4.3 Differentiation of exponential and logarithmic functions
1.4.4 Turning points (maxima, minima and inflexion)

1.4.5 Application of differentiation to business problems

1.5 Integration

1.5.1 Definition

1.5.2 Rules of integration (general rule)

1.5.3 Integration of exponential and logarithmic functions

1.5.4 Applications of integration to business problems

1.6 Descriptive Statistics

1.6.1 Measures of central tendency: mean: arithmetic mean, weighted arithmetic mean; geometric mean, harmonic mean, median and mode

1.6.2 Measures of dispersion: range, quartile, deciles, percentiles, mean deviation, standard deviation and coefficient of variation

1.6.2.1 Measures of skewness: Pearson's coefficient of skewness, product coefficient of skewness

1.6.2.2 Measures of kurtosis: Pearson's coefficient of kurtosis, product coefficient of kurtosis

2. Probability

2.1 Set Theory

2.2 Definition

2.3 Types of sets

2.4 Set description; enumeration and descriptive properties of sets

2.5 Venn diagrams (order - Venn diagrams precede operation of sets)

2.6 Operations of sets; union, intersection, complement and difference

2.7 Probability Theory and Distribution

2.7.1 Probability Theory

2.7.2 Definitions; event, outcome, experiment, sample space, probability space

2.7.3 Types of events: elementary, compound, dependent, independent, mutually exclusive, exhaustive, mutually inclusive

2.7.4 Laws of probability; additive and multiplicative laws

2.7.5 Conditional probability and probability trees

2.7.6 Expected value, variance, standard deviation and coefficient of variation using frequency and probability

2.7.7 Application of probability and probability distributions to business problems

2.8 Probability Distributions

2.8.1 Discrete and continuous probability distributions Z, F, test statistics (geometric, uniform, normal, t distribution, binomial, Poisson and exponential and chi-square)

2.8.2 Application of probability distributions to business problems

3. Hypothesis Testing and Estimation

3.1 The arithmetic mean and standard deviation

3.2 Hypothesis tests on the mean (when population standard deviation is unknown)

3.3 Hypothesis tests on proportions

3.4 Hypothesis tests on the difference between two proportions using Z and t statistics

3.5 Chi-Square tests of goodness of fit and independence

3.6 Hypothesis testing using R statistical software

4. Correlation and Regression Analysis

4.1 Correlation Analysis

4.1.1 Scatter diagrams

4.1.2 Measures of correlation - product-moment and rank correlation coefficients (Pearson and Spearman) using R software

5. Regression Analysis

5.1.1 Simple and multiple linear regression analysis

5.1.2 Assumptions of linear regression analysis

5.1.3 Coefficient of determination, standard error of the estimate, standard error of the slope, t and F statistics

6. Time series

6.1 Definition of time series

6.2 Components of time series (circular, seasonal, cyclical, irregular/ random, trend)

6.3 Application of time series

6.4 Methods of fitting trend; freehand, semi-averages, moving averages, least-squares methods

6.5 Models - additive and multiplicative models

6.6 Measurement of seasonal variation using additive and multiplicative models

6.7 Forecasting time series value using moving averages, ordinary least squares method and exponential smoothing

7. Linear programming

7.1 Definition of decision variables, objective function and constraints
7.2 Assumptions of linear programming
7.3 Solving linear programming using graphical method
7.4 Solving linear programming using simplex method (basic scenarios)

8. Decision Theory

8.1 Definition
8.2 Decision-making process
8.3 Decision-making environment; deterministic situation (certainty)
8.4 Decision making under risk - expected monetary value, expected opportunity loss, risk using the coefficient of variation, the expected value of perfect information
8.5 Decision trees - sequential decision, the expected value of sample information
8.6 Decision making under uncertainty - maximin, maximax, minimax regret, Hurwicz decision rule, Laplace decision rule.

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TOPIC 1

FUNCTIONS

Functions are often representatives of real phenomena or events. Functions therefore are models. Obtaining a function to act as a model is commonly the key to understanding business in many areas.

Functions may be represented by formulae. There are a number of common ways in which functions are presented and used. We shall consider functions given by formulae, since this provides a natural context for explaining how a function works.

1. Functions of one variable

If you get a job that pays Ksh 200 per hour, the amount of money M that you earn depends on the number of hours (h) that you work, and the relationship is given by a simple function:

$$Money = 200 \times hours\ worked$$

$$M = 200h$$

The formula *M=200h* shows that the money *M* that you earn depends on the number of hours worked. We say that *M* is a function of *h*.

In this context, *h* is a variable whose value we may not know until the end of the week. Once the value of h is known, the formula M=200h can be used to calculate the value of M. To emphasise that M is a function of h, it is common to write M=M(h),so that M(h)=700h.

Example 1

if you work 30 hours, then the function M(30) is the money you would earn. To calculate the amount earned, you need to replace the formula by 30. Thus, M = M(30) = 700×30 = sh. 21,000.

Note:

- It is important to remember that *h* is measured in hours and *M* is measured given is Kenya shillings. The function/formula is not useful unless you state in words the units you are using.

- We would also use different letters or symbols for the variables. Whatever letter/symbol used, it is critical that you explain in word what they mean.

2. Functional notation and substitution

We normally write functions as f(x) and read as "function f of x". We could use other letters such as g, p, H or h and write the function of x as g(x), p(x), H(x) or h(x).

Given f(x) = 3x +5, the value of this function f(x) when x=0 is written as f(0). The values are obtained by substituting.

Example 2

a) Given f(x) = 3x + 5, find i) f(0), ii) f(2) and iii) f(-2).
b) Given that h(x) =2x2 +5x +3, find i) h(0). ii) h(-2) and iii) h(3).
c) Given that G(t) = 2 +3t - 5t2 +t3, find i) g(-1) and g(2).
d) If h(x) = 5 - 2x, find the value of x for which the function is zero.

Solution:

a) i) f(0) =3(0) + 5 = 0 + 5 = 5
ii) f(2) =3(2)+5 = 6 + 5=11
iii) f(-2) = 3(-2) + 5 = -6 +5 = -1

b) i) h(0) = 2(0)2+ 5(0)+3 = 3
ii) h(-2) = 2(-2)2 + 5(-2) +3 =1
iii) h(3) = 2(3)2+ 5(3)+3 = 36

c) i) G(-1) = 2+3(-1)-5(-1)2-2(-1)3 = -4
ii) G(2) = 2+3(2) - 5(2)2 - 2(2)3 = -28

d) h(x) = 5-2x = 0
-2x = - 5 x = 2.5

Example 2. 3

a) If f(x) = 5x -3. find f(x+2).
b) If y = g(x) = 3x - 7. find g(y) in terms of x.

Solution:

a) Replace x by the assigned value (x+2) in the given equation:

$$f(x+2) = 5(x+2) - 3$$

$$= 5x + 10 - 3 = 5x + 7$$

b) Replace x by the assigned value y = 3x - 7 in the equation:

$$g(y) = 3y - 7$$

$$= 3(3x - 7) - 11 = 9x - 21 - 11 = 9x - 32$$

Equations

Equations often express relationships between given quantities (the knowns) and quantities yet to be determined (the unknowns). By convention, unknowns are denoted by letters at the end of the alphabet, x, y, z, w, …, while knowns are denoted by letters at the beginning, a, b, c, d, … . The process of expressing the unknowns in terms of the knowns is called solving the equation. In an equation with a single unknown, a value of that unknown for which the equation is true is called a solution or root of the equation. In a set simultaneous equations, or system of equations, multiple equations are given with multiple unknowns. A solution to the system is an assignment of values to all the unknowns so that all of the equations are true. Two kinds of equations are linear and quadratic.

Inequalities and Graphs

Inequality tells us about the relative size of two values. Mathematics is not always about "equals", sometimes we only know that something is greater or less than. Inequalities show the relationship between two expressions that are not equal to one another. Inequalities are useful when projecting profits and breakeven figures.

Graphs of inequalities

The graph of a function is a set of all points whose co-ordinates (x,y) satisfy the function y =f(x). This means that for each x-value there is a corresponding y-value which is obtained when we substitute into the expression for f(x). So, the graph of a function is a special case of the graph of an equation.

This is a graph of a linear inequality: