## PRACTICE QUESTIONS

## QUESTION ONE

Regal Investments has just received instructions from a client to invest in two shares; one an airline share, the other an insurance share. The total maximum appreciation in share value over the next year is to be maximized subject to the following restrictions:

- the total investment shall not exceed Sh.100,000
- at most Sh. 40,000 is to be invested in the insurance shares
- $\quad$ quarterly dividends must total at least Sh.2,600

The airline share is currently selling for Sh. 40 per share and its quarterly dividend is Sh.1per share. The insurance share is currently selling for Sh. 50 per share and the quarterly dividend is Sh.1.50 per share. Regal's analysts predict that over the next year, the value of the airline share will increase by Sh. 2 per share and the value of the insurance share will increase by Sh. 3 per share. Computer software provided the following part solution output:

Objective Function Value $=5,400$

|  | Variable | Number | Reduced cost |
| :--- | :---: | ---: | ---: |
| Airline shares | 1,500 | 0.000 |  |
| Insurance shares | 800 | 0.000 |  |
|  |  |  | Dual prices |
| Constraint | Slack/Surplus | 0.050 |  |
| Total investment | 0.000 | 0.010 |  |
| Investment in insurance | 0.000 | 0.000 |  |

## Objective Coefficient Ranges

| Variable | Lower limit | Current value | Upper limit |
| :--- | :---: | :---: | :---: |
| Airline share | 2.500 | 3.000 | No upper limit |
| Insurance share | 0.000 | 2.000 | 2.400 |

## Right-hand Side Ranges

| Constraint | Lower limit | Current value | Upper limit |
| :--- | ---: | :---: | ---: |
| Total investment | $96,000.00$ | 100,000 | No upper limit |
| Investment in insurance20,000.00 | 40,000 | $100,000.00$ |  |
| Dividends | No lower limit | 2,600 | $2,700.00$ |

## Required:

e) Formulate the above problem.
f) Explain what reduced cost and dual prices columns above mean.
g) How should the client's money be invested to satisfy the restrictions?
h) Suppose Regal's estimate of the airline shares appreciation is an error, within what limits must the actual appreciation lie for the answer in (c) above to remain optimal?
(Q 6 Dec
2001)

## QUESTION TWO

c) A baker makes two products; large loaves and small round loaves. He can sell up to 280 of the large loaves and up to 400 small round loaves per day. Each large loaf occupies $0.01 \mathrm{~m}^{3}$ of shelf space, each small loaf occupies $0.008 \mathrm{~m}^{3}$ of space, and there is $4 \mathrm{~m}^{3}$ of shelf space available. There are 8 hours available each night for baking, and he can produce large loaves at the rate of 40 per hour, and small loaves at the rate of 80 per hour. The profit on each large loaf is Sh.5.00 and Sh.3.00 profit on the small round loaf.

## Required:

In order to maximize profits, how many large and small round loaves should he produce?
d) Summarize the procedure for solving the kind of quantitative technique you have used to solve part (a) above.
(Q 6 June 2001)

## QUESTION THREE

c) A small company will be introducing a new line of lightweight bicycle frames to be made from special aluminum alloy and steel alloy. The frames will be produced in two models, deluxe and professional. The anticipated unit profits are currently Sh.1, 000 for a deluxe frame and Sh.1,500 for a professional frame. The number of kilograms of each alloy needed per
frame is summarized in the table below. A supplier delivers 100 kilogram's of the aluminum alloy and 80 kilogram's of the steel alloy weekly.

|  | Aluminum alloy | Steel alloy |
| :--- | :---: | :---: |
| Deluxe | 2 | 3 |
| Professional | 4 | 2 |

## Required:

iii) Determine the optimal weekly production schedule.
iv) Within what limits must the unit profits lie for each of the frames for this solution to remain optimal?
d) Explain the limitations of the technique you have used to solve part (a) above.
(Q 6 Dec 2000)

## QUESTION FOUR

a) Define the following terms as used in linear programming:
i) Feasible solution
ii) Transportation problem
iii) Assignment problem
b) The TamuTamu products company ltd is considering an expansion into five new sales districts. The company has been able to hire four new experienced salespersons. Upon analysing the new salesperson's past experience in combination with a personality test which was given to them, the company assigned a rating to each of the salespersons for each of the districts. These ratings are as follows:
c)

| Districts |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 1 | 2 | 3 | 4 | 5 |
| Salespersons | A | 92 | 90 | 94 | 91 | 83 |
|  | B | 84 | 88 | 96 | 82 | 81 |
|  | C | 90 | 90 | 93 | 86 | 93 |
|  | D | 78 | 94 | 89 | 84 | 88 |

The company knows that with four salespersons, only four of the five potential districts can be covered.

## Required:

iii) The four districts that the salespersons should be assigned to in order to maximize the total of the ratings
iv) Maximum total rating.
2002)
d) Explain the value of sensitivity analysis in linear programming problems and show how dual values are useful in identifying the price worth paying to relax constraints.
e) J.A Computers is a small manufacturer of personal computers. It concentrates on production of three models- a Desktop 386, a Desktop 286, and a Laptop 486, each containing one CPU Chip. Due to its limited assembly facilities JA Computers are unable to produce more than 500 desktop models or more than 250 Laptop models per month. It has one hundred and twenty 80386 chips (these are used in Desktop-386) and four hundred 80286 chips (used in desktop 286 and Laptop 486) for the month. The Desktop 386 model requires five hours of production time, the Desktop 286 model requires four hours of production time, and the Laptop 486 requires three hours of production time. J.A Computers have 2000 hours of production time available for the coming month. The company estimates that the profit on Desktop 386 is Sh. 5,000. For a desktop 286 the profit is Sh.3, 400 and Sh.3,000 profit for a laptop 486.

## Required:

Formulate this problem as a profit maximization problem and mention the basic assumptions that are inherent in such models.
f) An extract of the output from a computer package for this problem is given below:

Output solution
$X_{1}=120, X_{2}=200, X_{3}=200$
Dual values Constraints 3150
Constraints 490
Constraints 520
Sensitivity analysis of objective function coefficients:

| Variable | Lower <br> limit | Original <br> value | Upper <br> limit |
| :--- | :--- | :--- | :--- |
| $\mathrm{X}_{1}$ | 100 | 250 | No limit |
| $\mathrm{X}_{2}$ | 150 | 170 | 200 |
| $\mathrm{X}_{3}$ | 127.5 | 150 | 170 |

Sensitivity analysis on R.H.S ranges.

| Constrain <br> ts | Lower <br> limit | Original <br> value | Upper <br> limit |
| :---: | :--- | :--- | :--- |
| 1 | 320 | 500 | No limit |
| 2 | 200 | 250 | No limit |
| 3 | 80 | 120 | 130 |
| 4 | 350 | 400 | 412.5 |
| 5 | 1950 | 2000 | 2180 |

$\mathrm{X}_{1}=$ Monthly production level for Desktop 386.
$X_{2}=$ Monthly production level for Desktop 286.
$X_{3}=$ Monthly production level for Laptop 486.

## Required:

iv) Interpret the output clearly, including optimum product mix, monthly profit, unused resources and dual values
v) Explain the purpose of upper limits and lower limits for the variables $X_{1}, X 2, X 3$ and constraints 1 to 5.
vi) Calculate the increase in profit if the company is able to produce a further 10 CPU 80386 chips.
(Q7 July 2000 Pilot paper)

