## CHAPTER FOURTEEN

## INVENTORY CONTROL MODELS

## Specific objectives

At the end of this topic the trainee should be able to:

- Define inventory;
- Describe the control systems;
- Calculate the EOQ;
- > Determine safety stock (ss) and re-order level.

## INTRODUCTION

The activities of a business during a financial year combine investment projects in progress with new projects commencing and others terminate within the year. It would appear reasonable to presume, therefore, that business financial reports are presented in the cash-flow mode used to appraise investments, to facilitate comparison of actual with planned cash flows.

Some businesses do make such comparisons as part of their retrospective monitoring of investment decisions, but there is no obligation to do so. Cash-flow accounting, as it is called, has its supporters, but its introduction is frustrated by statutory and non-statutory regulations.

The Companies Act requires limited companies to produce profit and loss accounts and balance sheets in prescribed form. The Inland Revenue assumes that taxable profit has been computed by applying recognized accounting principles. The Accounting Standards Committee recommends the application of standard practices in the measurement of profit and portrayal of a company's financial position in its balance sheet. More compellingly, profit and loss reporting is compatible with the investors' objectives of stable and growing earnings.

Profit is measured conventionally by setting against the sales revenue for a period the costs expired in earning that revenue. That is, sales are *matched* against their relevant costs. Profit is therefore more evenly reported than it would be if all cash receipts and payments, capital and revenue, were fully reflected in the accounts of the period in which they are received and paid.

The management accountant also adopts the *matching principle* when preparing control information in both actual and budgeted form, and also ascertains full product cost as a starting point for setting selling prices.

This outlines the systems and methods used to control the flow of resources through production and service cost centres, for their eventual inclusion in product and period costs.

## MATERIAL CONTROL

It is said that "any fool can sell"—it is buying at the right price that is more critical to the achievement of a satisfactory return on capital employed. Buying price is important of course, but buying the right materials, is equally important if production targets are to be achieved and investment in inventories to be minimized.

## WHAT TO ORDER

This is governed by product specifications, but an efficient buyer will always have his ear to the ground to discover new and substitute materials and components of advantageous quality and price. Other economies can be realized by reducing the variety of materials purchased by *standardization*, e.g. reducing the variety of colors of paint stocked, or by introducing value analysis into the decision process.

## VALUE ANALYSIS

Is a formalized technique involving a rigorous analysis of products at the design stage or at any time during the saleable lives, to determine their value characteristics. These are the attributes that a customer looks for in a product and include its use value (functional qualities), appeal value (color, style etc.) and second-hand value (e.g. trade-in-price). The object of value analysis is to build into the product the optimum of desired value at minimum cost, by introducing the most up-to-date designs, materials and methods of manufacture. No more value need be built into the product than is desired by the customer. For example, moulded plastic bumper bars are now fitted to many cars, because they are cheaper than and equally as functional as chromium-plated steel ones.

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## How much to order

Supposing the estimated annual usage of a component by Harambee Agricultural Machinery Ltd is 20,000 units. Usage is even throughout the year and only one order per annum is placed with the supplier. Because only one delivery is made, average stock will be high, i.e.  $20,000 \div 2 = 10,000$  and consequently stockholding costs will be very high. On the other hand, the costs of ordering will be negligible. If two orders are placed there will be less in stock (i.e. average 5,000), which will reduce holding costs, but ordering costs will increase. Thus, the higher the number of orders placed, the lower are stockholding costs, but the higher are ordering costs.

Stockholding costs include interest on the capital invested in stocks, storage, insurance, rates, security, building maintenance, heating, etc. Ordering costs include buying-department staff costs, receiving and handling.

Assuming that the cost of each Harambee component is £10, that holding cost is 10% of stock value and the cost of placing an order is £1, the total annual cost of stockholding and ordering when different numbers of orders are placed, is as follows:

			62			
Number of orders	4	20	50	100	200	400
Size of order	5,000	1,000	(400	200	100	50
Average Stock (50%	2,500	500	200	100	50	25
order)		S.				
Holding cost	£2,500	£500	£200	£100	£50	£25
Ordering cost (£1 per	£4 📿	£20	£50	£100	£200	£400
order)	25					
Total Annual Cost	£2,504	£520	£250	£200	£250	£425
	2					
5	En la					
1						





Placing 100 orders a year results in the lowest of ordering and holding cost of £200, therefore the economic order quantity is 200 units. The same information is graphed in *Figure 1* above, showing that the *economic* 

order quantity (EOQ) is the point where ordering and holding costs are equal, and total £200.

As costs of ordering and holding stock are equal at the EOQ point, we can build a simple mathematical model to solve the problem, as follows: 20

<u>Q</u> ХН 2	=	A X P Q
Where Q	=	EOQ
H	=	holding cost per unit
Α	=	annual demand
Р	=	cost of placing an order

finally 
$$Q = \sqrt{\frac{2AP}{H}}$$

Using the data in the previous example:

$$EOQ = \sqrt{\frac{2 x 20,000 x I}{I}}$$

$$= \sqrt{40,000}$$
$$= 200$$

Although the model assumes that holding and ordering costs are fixed, this simplification is acceptable given a relatively unchanging level of production activity. In addition, because the total cost curve in the *Figure 1* is relatively flat either side of the EOQ, minor errors and approximation in the variables used in the calculation may not affect the end result significantly.

Practical constraints on the use of the model include restrictions on the available storage space, the availability of quantity discounts (though the model can be modified in this respect), the seasonal nature of supplies, the shelf-life of products and delivery schedules imposed by suppliers.

## WHEN TO ORDER

If deliveries from suppliers normally take two weeks to arrive, then replenishment orders should be placed with them when the level of stocks represents two weeks' supply. For example, if usage is 200 units a week, an order (the EOQ) will be placed when the stock level falls to 400 units. Figure 6(a) illustrates that, with *certain* knowledge of usage and lead time, delivery takes place just as stock is exhausted.



Figure 2(a): Stock levels when usage and replenishment times are known





Lead times and usage may not be stable and provision against running out of stock the becomes necessary (*Figure 2 (b*)). Safety stocks have a cost, however, and this has to be balanced against the cost of running out of stock. `Stock outs' may cause loss of customers and the probability of this happening at various levels of safety stock must be estimated. The point, at which the cost of carrying safety stocks plus the cost of `stock outs' is lowest, indicates the safety stock level. Notice that uncertainty causes the reorder level to be at a higher level to include the required safety stock.

## CONTROLLING MATERIAL FLOW

*Figure 3* outlines the progressive stages in purchasing, issuing and recording materials in a manufacturing concern. An efficient system of documenting and recording is vitally necessary, not only for accounting purposes, but to ensure that the right materials arrive at the right place at the right time.

The *purchase requisition* submitted to the buyer may be triggered automatically if the system is computerised, by a message from the stores ledger that the reorder level has been reached. Other requests to purchase may be raised by the production planning department for new product materials not yet carried in stock, and also by any departmental head for supplies and equipment of any kind.

The buyer, ideally after making *enquiries* of several suppliers, sends a *purchase order*, and eventually the material is received, checked by the good-inwards department as to quality and quantity, and is detailed on a *goods-received note* (GRN). One copy of the GRN goes to the buyer to write off the outstanding order record; one to the accounts department for checking against the order and invoice—the latter authorizing payment to the supplier; and one to the stores department with the materials.



Figure 3: Procedure of material acquisition, recording and issue

A *stores record* is maintained into which the quantity and value of materials received is entered. Issues of materials to production are made by authorized materials requisitions which are also entered into the stores ledger to keep that record up to date continuously, and also into the appropriate job or process const record.

As already indicated, all the above procedures may be integrated into a computerised stock record which can provide information at the press of a button to the storekeeper, buyer, production planner, financial manager or any

other person authorized to key into it. For example, information on slow-moving stock items can be obtained automatically and without delay.

## MATERIAL STORAGE

Sophisticated mathematical models to control economic buying, and systems control the flow of material may all be for nought if the obvious—efficient storekeeping—ignored. Good practice in this respect implies:

- The employment of a well-trained stores staff
- Use of the most efficient equipment-for storage and handling
- Easy access to items—stored in logical order
- Siting of stores convenient to users
- Security against theft and fire
- Protection against deterioration
- A system of continuously checking physical with recorded stocks.

## **INVENTORY PLANNING AND CONTROL**

The major goal of "inventory control" is to discover and maintain the optimum level of investment in all types of inventories, from raw materials and supplies to finished goods that helps to maximize long-run profits.

Two limits must be imposed in controlling inventory levels, because there are two danger points that management usually wants to avoid. They are:

i. That inadequate inventories, disrupts production and may lose sales.ii. That excessive inventories, introduces unnecessary carrying costs and obsolescence risks.

## MOTIVES FOR HOLDING INVENTORIES

If production and delivery of goods were instantaneous, there would be no need for inventories. However in reality, the manufacturing and purchasing processes do not function quickly enough to avoid the need for having inventories. Hence inventories must be maintained so that he or she does not turn to another source of supply. In turn, production operations cannot flow smoothly without having inventories of direct materials, work-in-progress, supplies, etc. In other words inventories must be viewed as cushions:

- i. To absorb planning errors and unforeseen fluctuations in supply and demand. (i.e. precautionary motive).
- ii. To facilitate smooth production and marketing operations.Note: The fundamental questions to which answers are required are:
  - a. How much should the company order (when stocks are re-ordered)?
  - b. When should the company reorder?

How much should the company produce, and when (if internal с. production is involved).

Accordingly regardless of their complexity, all inventory planning models focus on the twin problem of size and timing.

#### 1. CHOOSING ORDER QUANTITY (SIZE—PROBLEM)

The objective of inventory decisions is usually to minimize total inventory costs to the company. Costs are ascribed to all elements which are of interest in reaching its inventory decisions (e.g. purchasing costs, stock out costs etc.), and solutions, are derived based on these costs.

Several inventory planning models exist. These models can be classified into two basic classified into two basic classifications:

- i. Deterministic Models:-whereby all parameters are known with certainty, e.g. lead-time, annual demand, etc.
- ii. Stochastic Models:--in which parameters (particularly demand and lead time) are not known with certainty, but follow known probability distributions (i.e. risks)

### Α. THE DETERMINISTIC MODELS

THE BASIC EOQ MODEL 1.

This is the most simple of all the models discussed. In addition to the general assumptions which relate to all deterministic models (i.e. certainty of all parameters) it is further assumed that:

- Demand is continuous and constant over time. a.
- That suppliers lead time is zero i.e. stocks are delivered b. immediately on the day the order is made.
- That stock-outs are not allowed. c.
- There are *no* bulk quantity discounts. d.

Holding costs per unit, ordering costs per order and costs per e. unit are constant.

Relevant costs of basic EOQ model

The relevant costs that should be considered when determining optimum inventory levels can be classified into two categories:

- Ordering costs. i.
- Holding (Carrying) Costs. ii. TC = Ordering Costs + Holding Costs l.e.
- **Ordering Costs** i.

These are incurred in getting purchased items into the company's inventory or stores, and usually consist of clerical costs of:

- 1. Making the purchase requisition.
- 2. Issuing of a purchase order
- 3. Follow-up action
- 4. Receiving the goods
- 5. Inspection for quality control
- 6. Placing goods in stores
- 7. Paying vendors (Suppliers)

Note:

The basic EOQ model assumes that these costs are fixed constant for each order made.

ii. Carrying costs of inventory

These are costs incurred because the firm has decided to maintain inventories. They usually consist of:

- 1. Stock-out costs
- 2. Insurance costs
- 3. Warehouse and storage costs
- 4. Material handling costs
- 5. Costs of obsolescence

 Total Ordering Cost = Total demand for period x Costs per period Quantity Ordering
 Ordering

 =
 DO Q

 Total Holding Costs =
 Quantity Ordered x Holding Costs per

 2

<u>Q</u>Н 2

unit

Therefore total relevant costs (TC) for any order quantity can be expressed as:

=

 $TC = \frac{DO}{Q} + \frac{Q}{2}H$ 

We can determine a minimum of this total cost function by:

i. Differentiating the above formula with respect to Q and setting the derivative (1st) equal to zero.



$$Q = \sqrt{\frac{2DO}{H}}$$



ii. Equating ordering costs to holding costs.

$$\frac{DO}{Q} = \frac{Q}{2}H$$
$$Q^2 = \frac{2DO}{H}$$

$$Q = \sqrt{\frac{2DO}{H}}$$

## **ILLUSTRATION**

1. Assume a wholesaler has to supply his customer with 40,000 units of a given product every year.

2. Assume that demand is fixed and known

3. Assume the cost of placing each order is Shs 2.00 while the holding cost per unit is Shs 1.00.

Required:

- a. Determine the optimum order size using the basic EOQ model.
- b. Determine total costs incurred at optimum order size.

SOLUTION

a.

$$Q^{2} = \sqrt{\frac{2 \times 40,000 \times 2}{1}}$$

$$Q^{2} = \sqrt{\frac{160,000}{1}} = \sqrt{16 \times 10^{4}}$$

- b. TC =  $\frac{40,000}{400}$  (2) +  $\frac{400}{2}$  (1)
  - = 200 + 200

TC = Shs <u>400</u>

1. EOQ Model with quantity discounts

Circumstances frequently occur where firms are able to obtain quantity discounts for large purchase orders. Buying in bulk has some advantages and disadvantages.

## <u>Advantages</u>

- i. (A saving in purchase rule) Decreases in unit cost, which consists of the total amount of discount for the period.
- ii. A reduction in the total ordering costs because fewer orders are placed to take advantages of the discounts.

## **Disadvantages**

Increased holding cost arising from higher stock levels when large quantities are purchased.

Such as:

- i. Stock out cost
- ii. Insurance.
- iii. Deterioration
- iv. Security etc.

Broadly there are two types of discount structures:

- i. Fixed discount on "all units" when the order placed is for a minimum quantity.
- ii. Variable discounts for given ranges.
- 1. <u>The case of a fixed discount</u>

When evaluating inventory decisions when a fixed discount rate exists, the appropriate procedure is to compare the total costs of the EOQ with the total costs when discounts are taken. The option giving lower costs is then chosen.

## Note:

The Unit (variable) cost (i.e. Purchase Price) behaves in the following manner.

C =	Co Co (1	I - P)	if if	$\begin{array}{l} 0 \leq Q \leq Q_b \\ Q \geq Q_b \end{array}$
Where	P Qb	Co = =	= Disc Brea	basic unit cost without a discount ount rate allowed. ak-point (Quantity)—where discounts

become operational.

In order to determine the optimal ordering quantity, it is necessary to include the costs of the inventory with the carrying ordering costs.

Total costs of Inventory = Total Purchase cost + Total order cost + Total carrying cost

$$\label{eq:transform} \begin{array}{rcl} \mathsf{TC} & = & \mathsf{DC}_o & + \underline{Q^*} \; \mathsf{H} & + & \underline{\mathsf{D}} \; o & & \mbox{If} \; 0 \leq Q \leq Q_b \ (i) \\ & & & & & & & & \\ \end{array}$$

TC = DC. 
$$(1 - P) + QH + DQ$$
 o If  $Q \ge Q_b$  (ii)

Note:

Equation (ii) i.e. with discounts will give a lower TC than equation (i) for the same. The decision whether to go for the discount lies on a trade-off between extra carrying costs vs. a reduction in acquisition costs.

## **ILLUSTRATION**

1. Assume X Ltd purchases a raw material from an outside supplier at cost of Shs 70 per unit.

2. Assume total annual demand for the product is 9,000 units.

3. Assume the holding cost is Shs 40 per unit and the ordering cost is Shs 50 per order.

4. Assume a quantity discount of 3% of the purchase price is available for orders in excess of 1,000 units.

Required:

- a. Calculate the EOQ and the associated costs.
- b. Calculate the total costs if the company purchased in batches of 1,000 units (N.B. It is not wise to buy in batches with more than 1,000 units because of the increase rate in carrying exceed the rate at which ordering costs decline).
- c. Advise the management on the appropriate inventory policy.

## SOLUTION

a. i. 
$$EOQ = \sqrt{\frac{2 x 9,000 x 50}{40}}$$

$$= \sqrt{22,500} = \sqrt{225 \, x \, 10^2}$$

ii. TC = 
$$\frac{9,000}{150}(50) + \frac{150}{2}(40) + 9,000(70)$$

- = 3,000 + 3,000 + 630,000
- = Shs <u>636,000</u>

Q = 1,000 c.

TC = 
$$9,000 (50) + 1,000 (40) + 9,000 (70)(1 - 0.03)$$
  
=  $450 + 20,000 + 611,100$   
= Shs 631,500  
Note: Discount price =  $70 (1 - 0.03) =$  Shs 67.90  
d. Decline in inventory costs =  $631,550 - 636,000$   
= Shs 4,450

# DECISION

The firm saves Shs 4,450 by taking the quantity discount.

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## THE CASE OF VARIABLE QUANTITY DISCOUNTS

In practice, suppliers may offer different discounts for different quantities purchased. For example:

SEGMENT	QUANTITY PURCHASED	UNIT PRICE
1	0 - 500	Shs 100
2	501 — 1,000	Shs 90
3	1001 - 1,500	Shs 80
4	over – 1,500	Shs 70

The best approach to the solution in this case is to apply the price-breaks theorem. This works as follows:

1. For each segment an EOQ is calculated. There are two possible requests:

i. The EOQ is within the quantity segment (i.e. valid) In this case, the EOQ is used as the minimum cost quantity for that

segment.

 The EOQ is outside the quantity segment (i.e. invalid)
 In this case the minimum cost quantity will be the quantity within the segment closest to the EOQ as calculated.

2. Select the quantity that leads to the lowest total inventory costs (i.e. Purchase, Ordering & Carrying).

Illustration:

- 1. Assume a manufacturer uses 3,300 drums of a certain chemical per year.
- 2. Assume delivery costs incurred per order are Shs 40 and inventory carrying costs are estimated to be 30% of stock value.
- 3. Assume the normal cost per drum is Shs 22 but the supplier offers discount of 1.5% on orders for 500 drums or more, and 3% on orders for 1,000 drums or more.

Required:

Determine the order quantity the manufacturer should adopt to minimize total costs.

Solution:

Note:

There are 3 discount levels (0, 1.5% & 3%) and hence 3 segments.

## **STEPS**

1. Calculating the EOQ for each segment.

$$EOQ = \frac{200}{7} drums$$
$$EOQ = \sqrt{\frac{2 x 3,300 x 40}{0.3(22)}}$$

$$= \sqrt{\frac{264,000}{6.6}} = \sqrt{40,000}$$



- For Segment 3 (1,000 or more) c.

$$\begin{array}{rcl} C & = & 22(1 - 0.03) \\ C & = & Shs \, \underline{21.34} \end{array}$$

$$EOQ = \sqrt{\frac{264,000}{0.3(21.34)}}$$

$$=\sqrt{\frac{264,000}{6.40}}=\sqrt{41,237}$$
287

= <u>203</u> drums

	Price	Unit		Valid		
Segment	Break	Quantity	Price		EOQ	Quantity
1	0 - 499	Shs 22		200	200	(Valid)
2	500 - 999	Shs 21.67	202	500	(Invali	id)
3	over 1000	Shs 21.34	203	1,000	(Invali	id)

## 2. <u>Calculate total costs</u>

SEGMENT	UNITS	ORDERED	TOTAL COSTS	
1	200	(3,300 x 22)	+ (200÷2 x 6.6) + (3,300÷200 x 40) =	=
Shs 73.920			- C	
2	500	(3,300 x 21.	67) + (500÷2 x 6.5) + (3,300÷500 x 40)	
= Shs 73.400			à.	
3	1,000	(3,300 x 21.	34) + (1,000÷2 x 6.4) + (3,300÷1000 x ·	40
= Shs 73.754			SI	
			$\sim$	

## **Decision:**

The firm should order 500 drums per order per annum and incur Shs <u>73.400</u> total costs.

## NON-ZERO LEAD TIME (DETERMINING REORDER POINT)

This basic EOQ model assumes that the suppliers lead time is zero (i.e. goods are delivered immediately on the day the order was made). In reality, however, supplies are rarely ordered and received on the same day. Accordingly, orders must be placed some time before stocks reach zero. In world of certainty (when demand is continuous and constant) the reorder point will be the number of days/weeks lead time multiplied by the daily/weekly usage during the period.

i.e. Reorder point = Average daily usage x Lead time in days.



Note:

The reorder point has no cost implications, since it does not affect the EOQ.

## Illustration:

- 1. Assume X Ltd uses 50,000 kg of a raw material annually.
- 2. Assume ordering costs are Shs 160 per order and stock holding costs are Shs 0.25 per kg per annum.
- 3. Assume the purchase price is Shs 20 per kg and no quantity discounts are offered.
- 4. Assume lead time for delivery of orders is 4 weeks.
- 5. Assume working time is 50 weeks a year.

Required:

- a. Calculate the EOQ
- b. Calculate the inventory reorder level.
- c. Calculate the total costs per annum.

**SOLUTION** 

a. 
$$EOQ = \sqrt{\frac{2 \times 50,000 \times 160}{0.25}}$$

$$=\sqrt{\frac{16,000,000}{0.25}}=\sqrt{64,000,000}$$

EOQ = <u>8000</u> kgs

## Make 6.25 orders (50,000÷8,000) per annum.

b. Reorder level = Demand per week x Lead time.

$$=\left(\frac{50,000}{50}\right)x\,4$$

c. Total costs = 
$$(50,000 \times 20) + (8,000 \div 2 \times 0.25) + 50,000 \div 8,000$$
 (160)  
=  $1,000,000 + 1,000 + 1,000$   
= Shs  $1,002,000$ 

## INVENTORY PLANNING & CONTROL UNDER UNCERTAINTY

The basic EOQ model assumes that all the parameters (elements) in the model are certain (i.e. can be predicted accurately in advance). These parameters are:

- i. Demand or usage of stocks
- ii. Lead times.
- iii. Holding costs per unit, ordering costs per order and costs per unit.

In reality however, stock demand, supplies lead times and cost date are not known with certainty. Accordingly to make the models applicable to real situations we must consider uncertainty when planning for inventory levels.

To protect itself from conditions of uncertainty, a firm will maintain a level of safety stocks for raw materials, work-in-progress and finished goods stocks. Thus safety stocks are the amount of stocks that are carried in excess of the expected use during the lead time to provide a cushion against running out of stocks. Thus the reorder point is computed as safety stock plus the average usage during the lead time

i.e. reorder point = Average usage during lead time + safety (buffer) stock.

## DETERMINING THE SAFETY STOCKS LEVEL

1. Uncertainty of demand

Demand is the most troublesome variable to predict accurately. Actually, demand may fluctuate from day to day, from week to week or from

month to month. Thus, the firm takes the risk of running out of stocks if there are sudden increases in demand. Hence safety stock is the extra inventory held as a buffer of protection against the possibility of stock due to higher demand.

However, a larger inventory of safety stock will involve a higher inventory carrying costs, and on the other hand, the higher safety stock will decrease stock-out costs. Therefore one has to make a balance between these two costs in order to find out an optimal safety costs.

Note:

The optimum safety-stock level exists where the costs of carrying an extra unit are exactly counter balanced by the expected stock-out costs. This would be the level that minimizes the annual total stock-out and carrying costs.

Stock-out costs

These are the opportunity costs of running out of stock. They include:

i. The costs of lost customer sales, and therefore lost contribution to fixed costs.

- ii. Potential loss of goodwill with customers whose demand cannot be net.
- iii. Acquiring emergency supplies at higher prices to meet demand.
- iv. Cost production of finished goods, where raw material stock-outs occur.

The computation of safety stocks lingers on demand forecasts. The manager will have some notion (usually based on past experience) of the range of daily demand. That is the probability that exists for usage of various quantities.

Hence total inventory costs will be as follows:

Total inventory costs = Purchase price cost + carrying costs + stock-out cost + order costs.

= Purchase price costs + "normal~ carrying costs (Q÷2 H) + Buffer Stock holding costs (B x H) + Stock-out costs + order costs.

Total inventory costs =  $D \cdot C + Q \div 2 \cdot H + (B \times H) + \text{stock-out costs} + D \div Q \cdot Co$ 

Where:	D	= Total annual demand
Н	=	Holding costs per unit
В	=	Buffer stock
Q	=	EOQ

Note:

1. The normal EOQ formula is used to compute order quantity Q. Hence purchase costs, carrying costs, and ordering costs remain unchanged.

Only buffer stock holding costs and stock-out costs change. Accordingly, the minimization of these two costs will also mean that total/overall costs will be minimized.

2. Stock-out costs = Number of units short x Probability of being short.

= {(Number of units short x Stock-out costs per unit Probability of being short)—Stock-out costs for every usage duration} x Number of orders per year.

3. Buffer Stock-holding costs = Bx H

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## **ILLUSTRATION**

A company has an annual demand for material X of 250 tonnes per annum. Order lead time is 4 days and usage during lead time as shown by past record is

USAGE	PROBABILITY
0	0
1	0.01
2	0.05
3	0.15
4	0.25
5	0.30
6	0.10
7	0.09
8	0.05

The cost per tonne is £20 and stock holding cost is 25% per annum of the stock value. Delivery cost per batch is £4. The costs of stock out also estimated to be £4.

You are required to:

a. Calculate the economic batch quantity and the expected number of orders per annum

b. Ascertain the re-order level taking the information given above into consideration.

Solution:

a. EBQ or EOQ where stock-outs are permitted

$$EBQ \text{ or } EOQ = \sqrt{\frac{2DK}{h}} \sqrt{\frac{h+C_s}{C_s}}$$

Where D is annual demand K is order cost

h is holding cost

and C<sub>s</sub> is stock out cost  $EBQ = \sqrt{\frac{2 \times 250 \times 4}{5}} \sqrt{\frac{5+4}{4}}$ 

$$= \sqrt{400} \sqrt{\frac{9}{4}}$$

= 30 No. of orders =  $\frac{250}{30}$ 

(*Note*: Stock holding cost b = 25% of £20 = £5)

We are required to find out expected demand during lead time.

Usage	Prob	<i>x</i> p( <i>x</i> )
x	p(x)	
0	0	0.00
1	0.01	0.01
2	0.05	0.10
3	0.15	0.45
4	0.25	1.00
5	0.30	1.50
6	0.10	0.60
7	0.09	0.63
8	0.05	0.40
	$\Sigma x p(x)$	4.69
	.0`	

Hence normal usage during lead time is 4.69 tonnes but maximum usage during lead time can be as large as 8 tonnes with a probability of 0.05.

Average usage in four days = 4.69 tonnes and at this level there is no buffer stock. The problem is how much buffer stock we should have, so that the cost of holding stock together with the cost of expected stock-out cost is minimum.

Let	$\mu_1$	=	4.69 tonnes.		
	В	=	Buffer stock		
	S	=	Re-order level (for lead time)	=	μ1 + B
Henc Here	e B B	=	S - μ <sub>1</sub> = S - 4.69		



Hence possible shortages will occur when demand is 5, 6, 7 or 8 units.

Demand	Shortage	Prob p(x)	Expected Value <i>x</i> p( <i>x</i> )
	x		
5	5 - 4.69	0.30	0.31 x 0.30 = 0.93
6	6 - 4.69	0.10	1.31 x 0.10 = 0.131
7	7 - 4.69	0.09	2.31 x 0.09 = 0.2079
8	8 - 4.69	0.05	3.31 x 0.05 = 0.1655
		$\Sigma x p(x)$	0.5974

This is expected stock out per order when demand is 5, 6, 7 or 8

2. Similarly if Re-order level is 5 Shortages will occur if demand is 6, 7, 8

Working in the way as above

Demand	Shortage	Prob p(x)	Expected Stock-
	X		out cost
6	1	0.10	0.10
7	2	0.09	0.18
8	3	0.05	0.15
		$\Sigma x p(x)$	0.43

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Demand	Shortage	Prob p(x)	Expected Stock-
	x		out cost
7	1	0.09	0.09
8	2	0.05	0.10
		$\Sigma x p(x)$	0.19

3. If re-order level is 6, shortage will occur if demand is 7 or 8

If re-order level is 7, shortage if demand is 8

4.

Expected shortage cost =  $(8 - 7) \times 0.05 =$ 0.05 5 If re-order level is 8, there is no probability of shortage a. Expected Annual Shortage = (Expected Stock-out per order) x No. of orders i.e (Expected stock-out per order) x 250 3 3 OF Expected stock-out cost (Expected annual shortage) x b. Shortage cost per item Holding cost Buffer stock x cost of holding per item с. Important Note: If shortage  $C_s$  is not taken into account  $EOQ = \sqrt{\frac{2 x \overline{250 x 4}}{5}}$  $=\sqrt{400} = 20$ and number of orders 250 12.5 = = 20

Using this model, we can still develop a table similar to the table worked out in this problem.

Graph of total cost against re-order level can be drawn using the table. The minimum cost and the re-order level can then read from the graph. Once re-order level is known, safety stock can be worked out. In the next example, the initial EOQ value has been taken using the formula

$$EOQ = \sqrt{\frac{2 D K}{h}}$$

## **ILLUSTRATION**

- 1. Assume a manufacturer has experienced trouble from stock shortages (stock-outs) of raw materials X which is required in a manufacturing process. Usage of X averages 6,000 units per year where a year consists of 50 weeks.
- 2. Assume the costs of ordering each batch of X is Shs 30 and the lead time is 2 weeks (known for certain). The annual holding costs amount to Shs 1 per unit of X held. The cost of a stock-out has been estimated to be Shs 5 per unit short.
- 3. Assume the demand (usage) is unknown. However, the total usage of raw material X over the 2 week lead time is expected to be as follows:

- 63	
Usage in units	Probability
60	0.07
120	0.08
180	0.20
240	0.30
300	0.20
360	0.08
420	0.07

## **Required:**

- a. Calculate the EOQ, and the expected number of orders per annum
- b. Calculate the average usage per 2 weeks (Lead-time)

## <u>SOLUTION</u>

Annual demand	=	6,000 units
Order cost =	Shs 30	)
Holding cost	=	Shs 1
Shortage cost	=	Shs 5 per unit short

$$EOQ = \sqrt{\frac{2 x 6,000 x 30}{1}} = 600 \text{ units}$$

Number orders = 6,000 = 10

Average usage for two weeks = 60 x 0.07 + 120 x 0.08 + 180 x 0.20 + 240 x 0.30 + 300 x 0.20 + 360 x 0.08 + 420 x 0.07

> 240 units =

Re-order Levels	Buffer Stock	Expected Stock- out per order	Expected Annual Shortage	Expected stock-out cost	Holding Cost at Shs 1	Total cost
240	0	34.2	342	1,710	0	1,710
300	60	13.2	132	660	60	720
360	120	4.2	42	210	120	330
420	180	0	0	0	180	180

Expected Stock out per order where re-order level is 240 i.

$$(300 - 240) \times 0.2 + (360 - 240) \times 0.08 + (420 - 240) \times 0.07 = 34.2$$

similarly for re-order level = 
$$300$$
  
(360 -  $300$ ) x 0.08 + (420 -  $300$ ) x 0.07 =  $13.2$ 

and for re-order level = 360 iii.

No shortage when re-order level is 420 iv. Hence total cost is minimum when re-order level is 420 units.

Best policy is to have safety stock of 180 units.

## SENSITIVITY ANALYSIS OF EOQ MODEL

Sensitivity Analysis is concerned with the way in which those results of solutions change in response to change in model parameters.

$$EOQ = \sqrt{\frac{2 D O}{H}}$$

Note:

It is important to appreciate that in formulating our inventory models, we have really been performing a planning exercise. Thus we have made certain assumptions and estimates (e.g. annual demand D, holding costs H, & ordering costs O) and out solutions have obviously been affected by these. When for example calculating a deterministic EOQ value with maximization of total

inventory costs as the objective, expected annual demand (D) is taken into account. If we subsequently find that annual demand has differed from that expected then we will find that the EOQ we selected was not the optimum and, as a result, the total inventory cost was not actually minimum.

## Illustration:

1. Assume X Ltd expected annual demand for 1991 for 62,500 units of raw material X per annum.

2. Assume holding costs are Shs 15 per unit per annum and each order costs Shs 10.

3. Assume that the end of the year actual demand has been found to have been for 90,000 units, not the 62,500 expected.

## Required:

Calculate the additional costs borne by X Ltd through basing the size and frequency of orders on expected figures.

Solution:

a. Calculating EOQ based on expected figures, and the associated inventory costs.

$$EOQ = \sqrt{\frac{2 x 62,500 x 10}{5}}$$
$$EOQ = \sqrt{\frac{1,250,000}{5}} = \sqrt{250,000}$$

= <u>500</u> units

Total inventory costs =  $\frac{62,500}{500} (10) + \frac{500}{2} (5)$ = 1,250 + 1,250 = Shs <u>2,500</u>

b. Calculating the total inventory costs based on actual demand.

$$TC = \frac{90,000}{500} (10) + \frac{500}{2} (5)$$
$$= 1,800 + 1,250$$

#### = Shs <u>3,050</u>

Calculating the total inventory costs had the company forecasted the c. actual demand accurately.

$$EOQ = \sqrt{\frac{2 \times 90,000 \times 10}{5}}$$

$$EOQ = \sqrt{\frac{1,800,000}{5}} = \sqrt{360,000}$$

$$= \frac{600}{5} \text{ units}$$

$$TC = \frac{90,000}{500} (10) + \frac{600}{5} (5)$$

$$= 1,500 + 1,500$$

$$= \text{Shs } 3,000$$

Therefore, X Ltd total inventory costs are actually Shs 50 (3,050 - 3,000) higher than they would have been if the EOQ had been set in accordance with perfect WN.WS information.

Observation:

Hence the change in demand by 44%	<u>(99,000 - 62,500)</u> has result i	in a 20%
<u>(600 - 500)</u>	62,000	500
	02,000	500

Increase in EOQ but only 91.64% (3,050 - 3,000) increase in total costs. 3.000

Hence we can conclude that total cost is relatively insensitive to changes in demand.

## SELECTIVE INVENTORY MANAGEMENT

The inventory of an industrial firm generally comprises thousands of items with diverse prices, usage and lead time, as well as procurement and/or technical problems. It is neither desirable nor possible to exercise the same degree of control over all those items. The organisation should pay more attention and care to those items whose usage value is high and less attention to those whose usage and consumption value is low. The organisation has, therefore, to be selective in its approach to control its investment in various types of stocks and inventories. Such a system is known as `*selective inventory control*' system.

## ABC ANALYSIS (PARETO ANALYSIS)

In ordinary parlance, *ABC analysis* can be best compared with our class society where the population is categorised into Top, Middle and Lower classes. In the case of inventories also, it has been noticed that out of a large number of items (in a million-tonne capacity steel plant there would be usually about 50,000 items of inventory of various types) that are generally held in stock, some of the items are quite significant whereas the others are not that important. Through *ABC* plan which is in fact an analytical approach based on common statistical techniques, the relative importance of the various items is established for the purpose of individual scrutiny and subsequent control. Through this technique `VIP' or the `privileged few' and the `trivial many' are distinguished and treated as such.

ABC analysis contemplates to classify all the inventory items in a number of categories, generally in three categories based on their values. Items of high value but small in number are classified as `A' items which would be under a strict control. `C' items represent relatively small value items and would be under simple control. Items of moderate value and size are classified as `B' items and would attract reasonable attention of the management. Since this plan concentrates attention on the basis of the relative importance of the various items of inventory, it is also known as `control by importance and exception'. As items are classified in order of their relative importance in terms of value, it is also known as the `proportional value analysis'.

It has been found that normal inventory items in most organisations show the following distribution pattern:

- A 5 to 10% of total number of items account for about 70% of the total consumption value. These items may be called "A" items.
- B 10 to 20% account for 20% of total consumption value.
- C the remaining large number of items account for the balance of 15% of the consumption value.

Remark When a detailed scrutiny was conducted in respect of inventories held by the Ford Motor Company, the following results were obtained.

- 1. 9% of the total items (in number) were accounting for 57% of the total value of the inventory. These were classed as `A' items.
- 2. 10% of the items (in number) were found to be accounting for 18% of the total value. These were categorized as `B' items.

3. 81% of the items (in number) were found to be accounting for only 25% of the total value. These were classified as `C' items.

Similarly, when ABC ANALYSIS was done in the case of G.E.C., the results obtained were as under:

`*A' Category*—Items accounting for 8% of the total number but 75% of total value.

*`B' Category*—Items accounting for 25% of the total number and 20% of total value.

*C' Category*—Items accounting for 67% of the total number and only 5% of the total value.

It would thus be observed that substantial and effective controls are made possible if greater attention is focused on `A' category items since these would be covering quite a substantial part of the inventory in terms of shilling value.

Some Remarks:

- 1. There is no hard and fast rule that all the inventory items should be classified only in these three categories. There can be a large number of classifications based on the requirements of the company and the nature of the items. For example `A' items may be further sub-classified as A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, etc. The same principle may be extended to `B' items also or alternatively all the inventory items may be classified into A, B, C, D, E, F, etc.
- 2. All items that the company consumes must be considered together while classifying into ABC classes. Separate classification of raw materials, spares and consumable is not really meaningful.

While classifying as ABC items, what counts is the consumption shillings and not the unit price of an item or its consumptions in terms of units. Thus of the three items given below, the last one is most important since its annual consumption in terms of value is more than the other two.

Item	Price/U	Annual	Annual
	nit	Consumption	Consumption
	Shs	in units	in Shs
Х	20,000	2	40,000
Y	0.02	100,000,000	200,000
Z	1,000	500	500,000

4. Even though, so far we have referred to annual consumption, it is not at all necessary that the consumption figures should be taken only for one year. It can be for 6 months or even 3 months. But the period should be so selected that the consumption figures would be representative.

However, annual figures are far more convenient and are universally followed.

- 5. If a firm follows ABC analysis, it will devote much time and effort on the control of `A' items. For example, extra care will be taken in the determination of minimum, maximum, reorder level, etc. of the `A' items, whereas so much control may not be exercised on `C' items. `A' items may be purchased only once in a year. For `A' items perpetual inventory system may be applied whereas in the case of `C' items, only a bin card may be maintained. In the same way an appropriate accounting method for `B' items may be devised. However, in the classification of items into ABC categories if there are some critical items which are of small value whose non-availability may hamper the production, may in the normal situation, be classified as `C' items but, due to the critical nature of these items extra care may be taken so that these may not go out of stock.
- 6. The objective of classifying inventory items into `A', `B' and `C' categories is to develop policy guidelines for selective control. Such a policy can be designed in a variety of ways. In general `A' items merit a tightly controlled inventory system with periodic attention, and `C' items to subject to loose control with casual attention.

Matur		A it a mar	Ditema	Citome
Nature		Aitems	Bitems	Citems
		(High value)	(Moderate value)	(Low value)
1.	Extent of	Rigid control	Moderate control	Loose control
contr	ol	(close day to day	(Regular review)	(infrequent
		control)		review)
2.	Safety stock	Low safety stocks	Medium safety	Large safety
cover	age		stock	stocks
3.	Frequency of	Frequently	Less frequently	Bulk ordering
order				_
4.	Degree of	Individual posting	Small group	Group postings
postir	ng		postings	
5.	Period of	Every fortnight	Quarterly	Yearly
reviev	N			
6.	Sources of	Good number of	Few reliable	One or two
suppl	ies	sources	sources	sources
7.	Follow up	Vigorous	Periodic	Occasional
8.	Control	Weekly control	Monthly control	Quarterly control
state	ments	statements	reports	reports
9.	Forecasting	Emphasis on	Focus on past	Rough estimate
	-	accurate forecast	trend	-
10.	Level of	Senior	Middle	Stores supervisor

# 7. TABLE SHOWING FEATURES OF ABC ANALYSIS

Nature	A items	B items	C items
	(High value)	(Moderate value)	(Low value)
management	management	management	
11. Lead time	Maximum efforts to reduce lead time	Moderate efforts	Minimum clerical efforts
12. Value % and	80% of the value	15% of the value	5% of the value in 50% of the items
item percentage	in 20% of the	in 30% of the	
(Approximation)	items	items	

## ADVANTAGES OF "ABC ANALYSIS"

The benefits derived from this analysis and its subsequent follow up are summarized below:

Facilities selective control and thereby save valuable time of busy executives.

- Eliminates lot of unnecessary paper work involved in various other control procedures. Tangible savings can be affected in this behalf by following Two-Bin System which is very closely related with this technique.
- Facilitates Inventory Control and control over usage of stores materials which ultimately results in cost control.

LIMITATIONS OF ABC ANALYSIS Although ABC analysis is a fundamental tool for exercising selective control over numerous inventory items, it does not, in its present form, permit precise consideration of all relevant problems of inventory management. For instance, a never-ending problem in inventory management is that of adequately handling thousands of low-value `C' items. Low-value purchases frequently require more items and thereby reduce the time allowance available to purchasing personnel for value analysis, vendor investigation, and other `B' items.

Besides, if ABC analysis is not periodically reviewed and updated, the very approach of control may be defeated. For example, `C' items like diesel oil in a firm, will become most high-value items during power crisis should, therefore, deserve more attention, but this point may be overlooked if classification of items is not reviewed and updated.

The following steps are involved in implementing the ABC analysis:

1. Classify the items of inventories, determining the expected use in units and price per unit for each item.

2. Determine the total value of each item by multiplying the expected units by its unit price.

- Rank the item in accordance with the total value, giving first rank to the 3. item with highest total value and so on.
- 4. Compute the ratios (percentage) of number of units of each item to total units of all items and the ratio of total value of each item total value of all items.

Combine items on the basis of their relative value to form three categories-A, B and C.

The data in the table below illustrates the ABC analysis.

				TABLE	ABC AN	IALYSIS			
Item	Units	% of		Cumulati	Unit	Total	% of		Cumulativ
		total		ve O	price	cost	total		е
1	10,00 0	10		mas	30.4	304,000	38.0 0		
			15	15				7 0	70
2	5,000	5	1		51.20	256,000	32.0 0		
3	16,00 0	16			5.50	88,000	11.0 0		
		J	3 0	45			,	2 0	90
4	14,00 0	14			5.14	72,000	9.00		
5	30,00 0	30			1.70	51,000	6.38		
		J	5 5	100			J	1 0	100
6	15,00 0	15			1.50	22,500	2.81		
7	10,00 0	10			0.65	6,500	0.81		
TOTAL	100,0 00					800,000			

The tabular and graphic representation indicates that "Item A" forms a minimum proportion, 15 per cent of total units of all items, but represents the higher value, 70 per cent. On the other hand, "Item C" represents 55 per cent of the total units and only 10 per cent of the total value. "Item B" occupies the middle place. Items A and B jointly represent 45 per cent of the total units and 90 per cent of the investment. More than half of the total units are item C, representing merely 10 per cent of the investment. Thus, a tighter control should be exercised on "Item A" in order to maximize profitability on its investment. In case of "Item C" simple controls will be sufficient.



# JUST-IN TIME (JIT) INVENTORY MANAGEMENT

JIT is a system whose objective is to produce or to purchase products or components as they are required by customers or for use rather than for stock. A JIT system is a pull system which responds to demand as opposed to a push system in which stocks acts as buyers\* between the different element of the system such as purchasing, production and sales

## JIT PRODUCTION

Is a production system, which is driven by demand for the finished products whereby each component on the production line is produced only when needed for the next stage.

## JIT PURCHASING

\_On the other hand is a purchasing system in which material purchased are contracted so as that the receipt and usage of materials to the maximum extent possible, coincide.

JIT concept can be traced back to the Japanese company whose success in the international market generates interest among many western companies as t60 how this success was achieved.

The implementation of JIT production methods was considered to be pursuit of excellence in all phase of manufacturing systems design and operations. The JIT are to produce the required items at the required quality and in the required quantities, at the precise time that they are required.

JIT seeks to achieve the following goals-

- (1). Elimination of non-value adding activities.
- (2). Zero inventory
- (3). Zero defects
- (4). Batch size of one.
- (5). Zero break-downs
- (6). 100% on time delivery services.

The above goals represent perfection and are most unlikely to be archived in practice. They do however offer targets and create a climate for continuous

(1). Elimination of non-value added activity JIT manufacturing can be described activity to the elimination of waster value to a prediction JIT manufacturing can be described as a philosophy of management, dedicate to the elimination of waste. Waste is defined as anything that does not add

The cycle time involved in manufacturing and selling a product consist of -

- Process time-add values
- Inspection time
- More time
- Queue time
- Storage time

Of these 5 steps only process time actually adds value to the products. All the other activities add cost and No value to the production and therefore are deemed as non-value within the JIT philosophy.

Usually in many companies, process time is less than 10% of total manufacturing, lead and cycle time. Therefore 90% of the manufacturing lead time disassociated with the product, adds cost but no-value to the product by adapting a JIT philosophy and forecasting, on reducing lead time, it is claimed that total cost can be significantly reduced.

The ultimate products with lead-time = processing time, and eliminating all non-value adding activities.

(2). Factory Layout

The first stage of implementing the JIT manufacturing techniques is to rearrange the factory floor away from the batch production functional layout towards a production layout using low lines with a functional plant layout production through a No. of special departments that normally contain a group of similar machines.

Products are processed in stage batches so as to minimize the set times when machine settings are changed between processing batches of different products. Batches move via different and complex routes through the various departments, traveling over much of the factory floor before they are complete.

Each process normally involves a considerable amount of waiting time and which much time is taken transporting items from one process to another.

A further problem is that it is not easy at any point in time to determine what progress has been made on individual batches and therefore detailed cost accumulation records are necessary to track work in progress. This results in long manufacturing cycles and high work in progress levels.

The JIT solution is to reorganize the production process by dividing the many different products that an organization makes into families of similar products or component. All the products is a in a particular group will have similar production requirement and routing. Production is necessary used\* so that each production family is manufactured in a well-defined production cell based on flow line principles. In a production how lines specialist department containing similar markets no longer exist. Instead groups of dissimilar markets are organised into products or component family flow lines that function like an assembly line.

For each production lines the market are placed close together in the order in which they are required by the group of products to be processed. Items in each product family can now move one at time from process to process more easily, thereby reducing wip, and lead time.

The aim is to produce products or component from start to finish without returning to the stock room or stores

(3). <u>Batch size of one</u>.

Set up time is the amount of time required to adjust equipment and to retool for a different product. Long set ups a change over time make the production of batches with a small no. of units uneconomical.

However, the creation of large batches lead to substantial lead time delays and the creation of high inventory levels. The JIT philosophy is to reduce and eventually eliminate set-up times.E.g. by investing in advanced manufacturing technology some machines setting can be adjusted automatically instead of manually.

Alternatively some set up times can be eliminated entirely by reducing products, so that markets do not have to be reset each time a different product has to be made.

If the set up times are approaching zero, then there's no advantage production in batches and therefore the optimal batch size can be one. With a batch size of one, the work can flow smoothly to the next stage without the need for and to schedule the next machine to accept this item.

## JIT PURCHASING ARRANGEMENTS

JIT philosophy also extends to adapting JIT purchasing techniques whereby delivery of material immediately precedes their use. By arranging with suppliers for more frequent deliveries stocks can be out to a **minimum** 

Considerable savings in material handling expenses can be obtained by requiring suppliers to inspect materials before their delivery and guarantee their guality.

This improved service is obtained by giving more business to fewer suppliers and **placing long tern purchasing orders**, therefore the suppliers has an assurance of long term sales and can plan to meet this demand.

Companies that have implemented JIT purchasing techniques have claimed to substantially reduce their investment in raw materials and work in progress stocks.

Other advantages include

- 3. Savings in time from negotiating with fewer suppliers
- Substantial savings in factory space.
   Large quantity discount.
   Savings in time from negotiation
   Reduction in the factory space. 4. Reduction in paper work arising from issuing (long term orders) to a few suppliers rather than individual purchase order to many suppliers.

## JIT and Management Accounting.

Management accountants in many organisations have been criticised because of their failure to change their managing accounting system to reflect the mode from a traditional manufacturing to a JIT manufacturing system. Conventional management accounting systems can encourage behaviour that is inconsistent with JIT phylosopy, management, accounting must support JIT manufacturing by monitoring, identifying, and communicating to decision makers any delays errors and waste in the system.

Modern management accounting systems are now placing greater emphasis on providing information on suppliers relialibility set up times cycle times, percentage of deliveries that are on time and defect rates All these measures are critical in supporting JIT manufacturing philosophy.

## FINANCIAL MANAGER'S ROLE IN INVENTORY MANAGEMENT

The techniques of inventory management, discussed above are very useful in determining the optimum level of inventory and finding answers to the problems of the economic order quantity, the re-order point and the safety stock. The techniques are very essential to economize the use of resources by minimizing

the total inventory cost. Although out treatment of inventory management has been simple, it indicates the broad framework of managing inventories. Many sophisticated techniques have been evolved to handle inventory management problems more efficiently and effectively and the improvements are still continuing. For the majority of the companies, inventory represents a substantial investment. Thus, the goal of the wealth maximization is related to the efficiency with which inventory is managed. Consequently, the financial manager has an important role to play in the management of inventory, although it is not his operating responsibility to control inventory. The financial manager should see that only an optimum amount is invested in inventory. He should be familiar with the inventory control techniques and ensure that inventory is managed well. He should introduce the policies which reduce the lead time, regulate usage and thus, minimize safety stock. The net effect would be to reduce inventory investment and increase the firm's prospects of making more profits.



## **PRACTICE QUESTIONS**

## QUESTION

Boots Ltd. manufactures a range of five similar products, A, B, C, D and E. The table below shows the quantity of each of the required inputs necessary to produce one unit of each product, together with the weekly inputs available and selling prices of each product.

Inputs	А	В	С	D	Е	Weekly inputs available
Raw materials (Kg)	6.0	6.5	6.1	6.1	6.4	35,000 Kgs
Forming (hours)	1.00	0.75	1.25	1.00	1.00	6,000 hours

Firing (hours)	3.00	4.50	6.00	6.00	4.50	30,000 hours
Packing (hours)	0.50	0.50	0.50	0.75	1.00	4,000 hours
Selling price	40	42	44	48	52	
(Sh.)						

The costs of each input are as follows:

Material	Sh.2.10 per Kg
Forming	Sh.3.00 per hour
Firing	Sh.1.30 per hour
Packing	Sh.8.00 per hour

## **Required:**

- a) Formulate this problem as a Linear Programming problem.
   (7 marks)
- b) The problem has been solved using a computer package and the following final tableau of a simplex solution has been produced:

Basis	Α	В	С	D	Е	- A	S	т	U	Value
	1	1.18	1.04	0.46	0 *	0.36	0	0	-2.29	3,357
Α					~Č	S.				
В	0	-0.34	0.23	0.02	c 0 )	-0.18	1	0	0.14	321
Т	0	1.37	2.97	2.28	0	-0.27	0	1	-2.79	9,482
Е	0	-0.09	-0.02	0.52	0	-0.18	0	0	2.14	2,321
Zj	0	1.26	1.06	0.51	0	2.02	0	0		105,791
2			S	5					8.81	
			~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~							

Where A, B, C, D and E are the weekly production levels for the five products; X is the amount of raw material that falls short of the maximum available; S, T an U are the respective number of hours short of maximum weekly input of forming, firing and packing time.

i Use this tableau to find the optimum weekly production plan. (4 marks)

- ii Describe the implications of using this plan in terms of unused resources and overall contribution to profit. (3 marks)
- iii In the context of this problem explain the meaning of "The dual or shadow price of a resource" (3 marks)
- iv There is a proposition that the company manufactures an additional product which would sell at Sh.50 per unit. Each unit will need 6 kg of raw material, one hour of forming time, five hours of firing time and one hour of packing time. Is it s worthwhile proposition?

(3 marks)

(Total: 20 marks)