CHAPTER SIXTEEN

ITRODUCTION TO LINEAR PROGRAMMING

Specific Objectives

At the end of this to the trainee should be able to:
- ➢ Explain the model concepts;
- ➢ Describe the various types of models;
- ➢ State the requirements of LP models;
- ➢ Model decision problems as linear models;
- ➢ Solve LP models.

INTRODUCTION

These are mathematical technique that deals with optimism of linear function variables known as objective functions. It is a method of determine an optimum program of independent activities in view of available resources. Linear programming is a technique of decision making used by managers to allocate limited resources e.g. machinery, raw materials and labor in order to minimize costs or maximize production. Decision variable are the amounts of each product to be made in a given time period. Linear programming assumes that the variable has a linear relationship.

N.B; it is a technique concerned with scarce resources.

Advantages
1. It helps in retaining the optimum use of production factor e.g. efficient use of man power.
2. it improves quality of decision
3. it gives possible practice positively
4. It improves the knowledge and skills of executive.

Limitation
1. it assumes that the activity is infinitely divisible
2. it treats all relationships as linear
3. it takes into account single object only.
4. It can be adopted only under the condition of certainty i.e. when the quantity of resources is known.

Assumption of linear programming
1. linearity
2. Divisibility- assumes that quantities, costs and revenues are infinitely divisible.
3. certainty- the technique assumes that there is certainty in all
4. positive solutions- non negative constraints are introduced to ensure the values are considered
5. Interdependent between demands for products is ignored.

6. Time factor is ignored. It assumes that production is instantaneous.

<u>Application of linear programming</u>
Production department to decide the quantity of pots to be produced subject to limited resources (constraints) e.g. labor, power, machine hours, raw materials etc.
Marketing department: Allocation of salesmen to different sales regions subject to their expected performance.
Human Resource: Scheduling personnel's work hours and job description to either maximize production or minimize cost.

<u>Steps in solving linear programming problems (problem formulation)</u>
1. Identify variables (e.g. product x and product y)
2. Identify the objective (To maximize contribution or to minimize cost), and write down its mathematical presentation in terms of variables.
3. Identify the constraints (i.e. the limited resources shared among the variables), and write down its mathematical representation in terms of variables.
4. Write down the objectives and the constraints in terms of the variables.
These steps apply regardless of the number of the variables.

NOTE: If only two variables are involved, a graphical solution can be used otherwise for multivariable problems, an algebraic method is applied to find the solution.

Example 1:
Long Castling Breweries manufactures two brands of beer, Benko lager and Benoni lager. Benko has a contribution of Sh.4 per unit and Benoni has a contribution of Sh.3 per unit. Benko requires 30 machine minutes and 30 labor minutes to manufacture a unit. Total available machine hours per day are 12hrs whereas total available labor hours per day are 14hrs.

Required:
1. Formulate linear programming model.
2. How much of each brand should Long Castling produce if it wishes to maximize its daily contribution assuming that all the lager produced is sold.

Solution:

1. <u>Formulating a linear programming model</u>

Step 1: <u>Identifying variables:</u>
The variables here are the number of units of Benko and Benoni lager produced by Long castling breweries per day; we can represent them as:

$$X_1 = \text{a unit of Benko lager.}$$
$$X_2 = \text{a unit of Benoni lager.}$$

Step 2: <u>Identify the objective:</u>

Definition: An **objective** is the desired result i.e. optimization of a function dependent on decision variable and subject to some constraints.

The objective of Long Castling Breweries is to maximize daily contribution. Objective function is the formula that will give us the total contribution in a day for both Benko lager and Benoni lager.
The information above can thus be represented in a tabular form as:

(PER DAY)	PRODUCT		Maximum available
	X_1	X_2	hours/day
Machine hours	0.5	0.33	12
Labor hours	0.5	0.5	14
Contribution	4	3	

Objective function = $4X_1 + 3X_2$
The objective is to maximize $4X_1 + 3X_2$

Step 3: <u>Identifying constraints (constraints formulation)</u>

Definition: **Constraints** are circumstances that govern achievement of an objective.
Limitations must be quantified mathematically and they must be linear.

For Long castling breweries we have limited machine hours (12hrs/day), which must be shared among production of Benko and Benoni lagers.
Therefore production must be such that the numbers of machine hours required is less than or equal to 12 hours per day.

$$0.5X_1 + 0.33X_2 \leq 12\text{hrs}$$

Similarly for labor hours we have:

$$0.5X_1 + 0.5X_2 \leq 14\text{hrs}$$

Non-Negativity: It is logical assumption to assume that the company cannot manufacture negative amounts of a product, thus it can only manufacture either zero product or more. Therefore we have: $X_1 \geq 0$
$X_2 \geq 0$ or $X_1, X_2 \geq 0$

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Thus the complete linear programming model is;
  Maximize $4X_1 + 3X_2$

Subject to the constraints;
  $0.5X_1 + 0.33X_2 \leq 12$
  $0.5X_1 + 0.5X_2 \leq 14$
  $X_1 \, X_2 \geq 0$

Solving linear programming problems
The question requires us to optimize (in our case, maximize) the objective (the contribution function), or in simple terms we are required to solve the linear programming model.
Solving linear programming model entails finding the values of variables that satisfy all inequalities simultaneously and optimize the objective.

<u>Graphical solution</u>
This method is used to solve LP models in case where only two variables are involved. For more than two variables (multivariable) then the simplex technique (algebraic method may be used).
Now in solving the problem above we first draw the axis, taking X_1 to be the y axis and X_2 to be the X axis.

Next we plot the scales on each axis to approximate the scales to use them, we consider each constraint equation. We get the value of one of variables putting the other variable to be zero and by substituting the inequality ≤or ≥ with equality sign (=).

For: $0.5X_1 + 0.33X_2 \leq 12$
When $X_1 = 0$
    $0.5(0) + 0.33X_2 = 12$
    $0.33X_2 = 12$
    $X_2 = 12/0.33 = 36$
Therefore point to plot is ~~(36, 0)~~   …   Implying that when $X_2 = 36$ then $X_1 = 0$

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When $X_2 = 0$
$$0.5X_1 + 0.33(0) = 12$$
$$0.5X_1 = 12$$
$$X_1 = 12/0.5 = 24$$
Therefore the point is (0, 24)

For $0.5X_1 + 0.5X_2 \leq 14$
When $X_1 = 0$
$$0.5(0) + 0.5X_2 = 14$$
$$0.5X_2 = 14$$
$$X_2 = 14/0.5 = 28$$
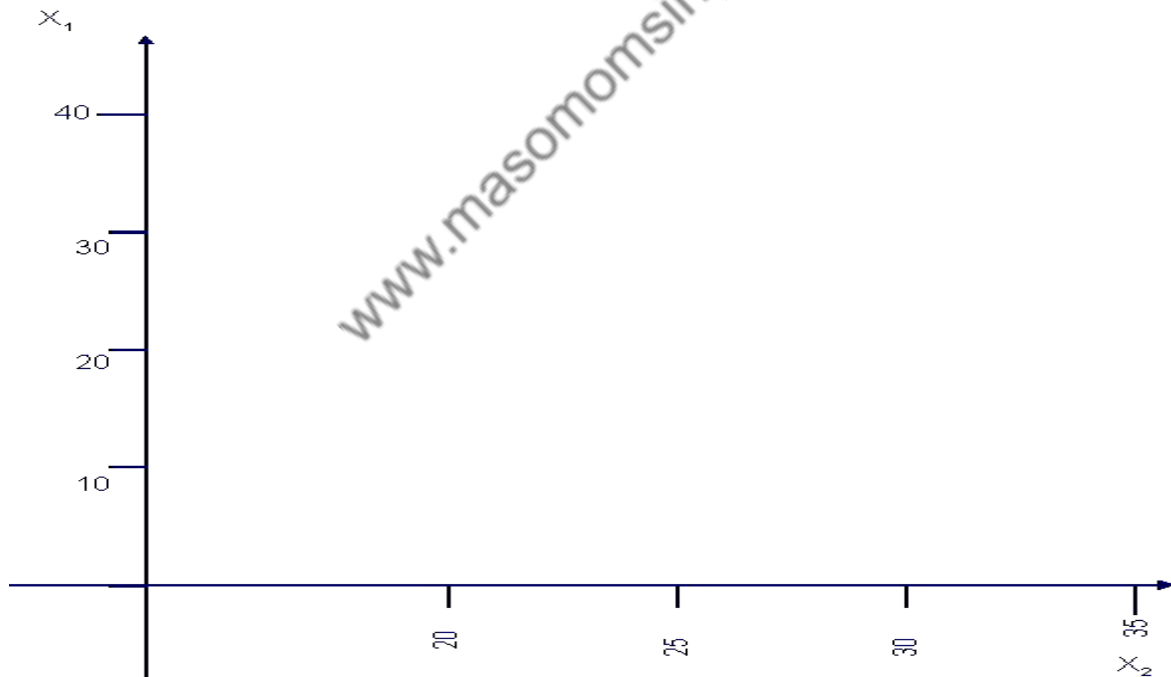Therefore the point is (28, 0)

When $X_2 = 0$
$$0.5X_1 + 0.5(0) = 14$$
$$0.5X_1 = 14$$
$$X_1 = 14/0.5 = 28$$
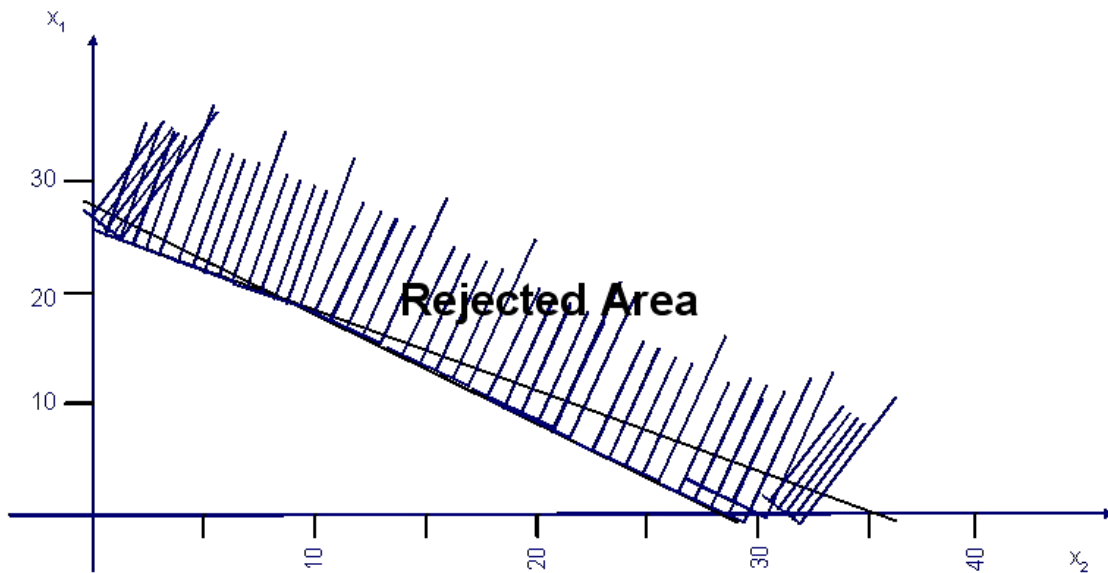Therefore the point is (0, 28)

Comparing these values we see that X_2 ranges between 0 – 28, therefore we can have the graph plotted as:

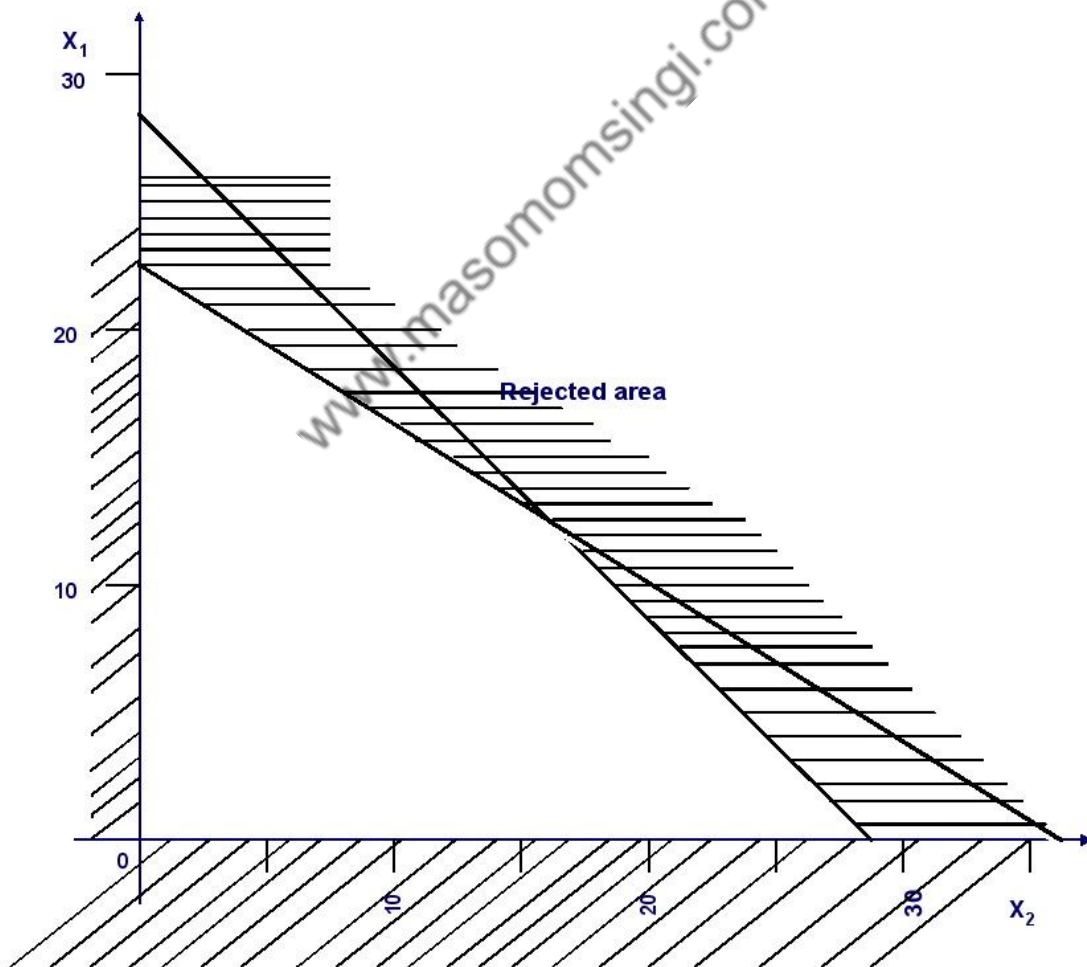Next draw each limitation (constraint) as separate line on the graph.
For $0.5X_1 + 0.33X_2 \leq 12$
The two points that represent this line are (36, 0) and (0, 24). This is plotted as a straight line from 36 on X_2 axis to 24 on X_1 axis.

Now including the Non-Negativity constraints since no negative product can be produced;

$X_1 \geq 0$; $x_2 \geq 0$

We must now consider how to choose the production which will maximize contribution. This we do by plotting a line representing the objective function $(4x_1 + 3x_2)$.

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First choose a convenient point inside the feasible region

$$\text{E.g. } X_2(10) + 3X_1(20) = 40 + 60$$
$$= Sh\ 100$$

All of the other product mixes that give a contribution of Sh.100 lies on the line:

$100 = 4X_1 + 3X_2$ .... (i)
<<This line is called a contribution line>>
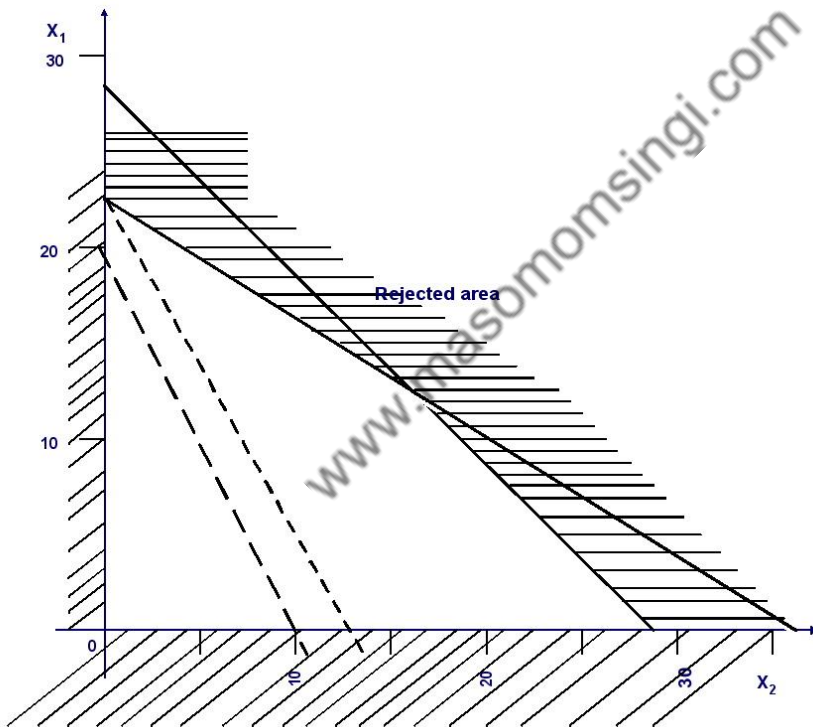
Picking another point, say $X_2 = 10$ ad $X_1 = 20$
Its contribution value is SH 110, thus give a contribution line of

$110 = 4X_1 + 3X_2$ .... (ii)

Plotting these two contribution lines to our graph we get two parallel lines.

Until we reach the last feasible solution(s) before the line moves entirely out of the feasible region.
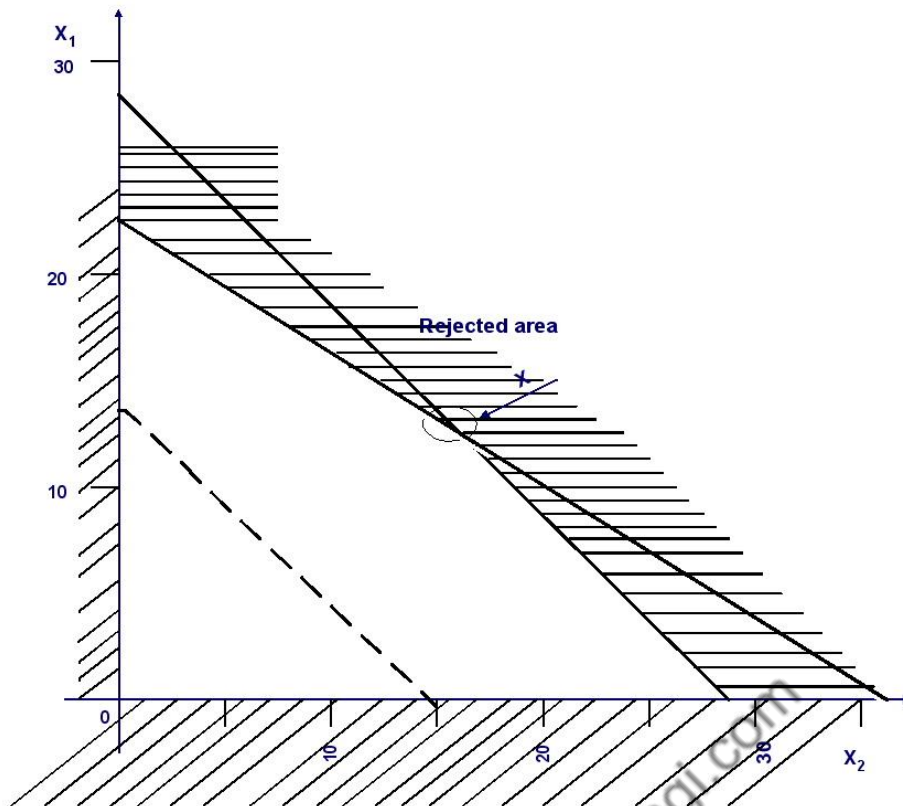
Point X is the last feasible solution. Coordinates of this point give a combination of the two lager's production volumes that fetches the highest contribution.

Coordinates of point X can be read from the graph, but for precision they are calculated by solving simultaneously the equations of the two lines that intersect at point X.

The two constraints are called binding or limiting constraints. They are the resources being fully used thus preventing daily contribution from increasing further.

Therefore to get point x we solve:

$$0.5X_1 + 0.33X_2 = 12.... (i)$$
$$0.5X_1 + 0.5X_2 = 14 .... (ii)$$

Since X is the intersection of these two constraints, solving by deducting (i) from (ii) we get

$$0.17X_2 = 2$$
$$X_2 = 11.76$$

And substituting $X_2 = 11.76$ to equation (i) we get
$$X_1 = 16.24$$

Therefore 11.76 units of Benko lager and 16.24 units of Benoni lager need to be produced for maximum contribution.

$$Contribution = 4(16.24) + 3(11.76) = 100.24$$

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Requirements in linear programming

Assumptions that are made to solve these types of problems are that:.

- Proportionality: all activities in linear programming problems are proportional to the level of decision variables.
- Divisibility: the solution to a linear programming problem does not have to be an integer but for strictly whole number solutions, use integer programming.
- Non-negativity: no decision variable can be negative.
- Additivity: the total of all activities in linear programming problems are assumed to equal to the sum of individual activities.

Special cases in linear programming

- Infeasibility: This is when all constraints don't satisfy a particular point thus there is no feasible solution.
- Redundancy: A constraint is considered redundant if it does not affect the feasible region. This happens in cases of excess resources since it does not limit attainment of the objective.
- Multiple optimal solutions: This occurs when the objective function has the same slope as a binding constraint.

Minimizing problem

Example
A manufacturing company has acquired new machine for producing product P at a rate of 25 units per hour with a 98% rate of efficiency. The company requires producing at least 1800 units of P per day. The 10 old machines that the company has produce 15 units of P with 95% efficiency.

The cost of operating the new machine is Sh. 4 per hour and Sh. 3 per hour for the old ones. The cost incurred due to inefficiency is Sh. 2 per unit; It is government policy that at least 2 of the new machines must be indulged into production.

The company wishes to optimally allocate the machines in order to minimize the total manufacturing cost if the total available hours for production in a day are 8 hours.

Solution
In minimizing problems we use ≥ (greater or equal to) type inequality.

Step 1: Identifying variables.
Since the problem requires us to appropriately allocate the machines in order to minimize the costs thus our variables are the new machines and the old machines, we can let;

X_1 =new machines
X_2=old machines

Step 2: Identify objectives:
The objective is to minimize manufacturing costs.

Total manufacturing cost per machine=Operating cost + (inefficiency rate × number of units × cost of loss)
Therefore cost for new machines
Cost = 4+ (0.02 × 25 × 2) = Sh. 5 per hour
$\quad\quad\quad\quad$ =5 × 8= Sh. 40 per day]
Similarly for old machines
Cost = 3+ (0.05×15×2) = Sh. 4.5 per hour
$\quad\quad\quad\quad$ =4.5×8=Sh. 36 per day
Therefore the objective function is to minimize $40X_1 + 36X_2$
Step 3: Identifying constraint functions.
$\quad\quad\quad\quad X_1 \leq 8$
$\quad\quad\quad\quad X_2 \leq 10$
$(25 \times 8) X_1 + (8 \times 15) X_2 \geq 1800$ (This can be simplified further as)
$200 X_1 + 120 X_2 \geq 1800$ $\quad\quad$ (dividing through by 40)
$5X_1 + 3 X_2 \geq 45$
$X_1 \geq 2$
$X_1, X_2 \geq 0$

Thus the LP model is:
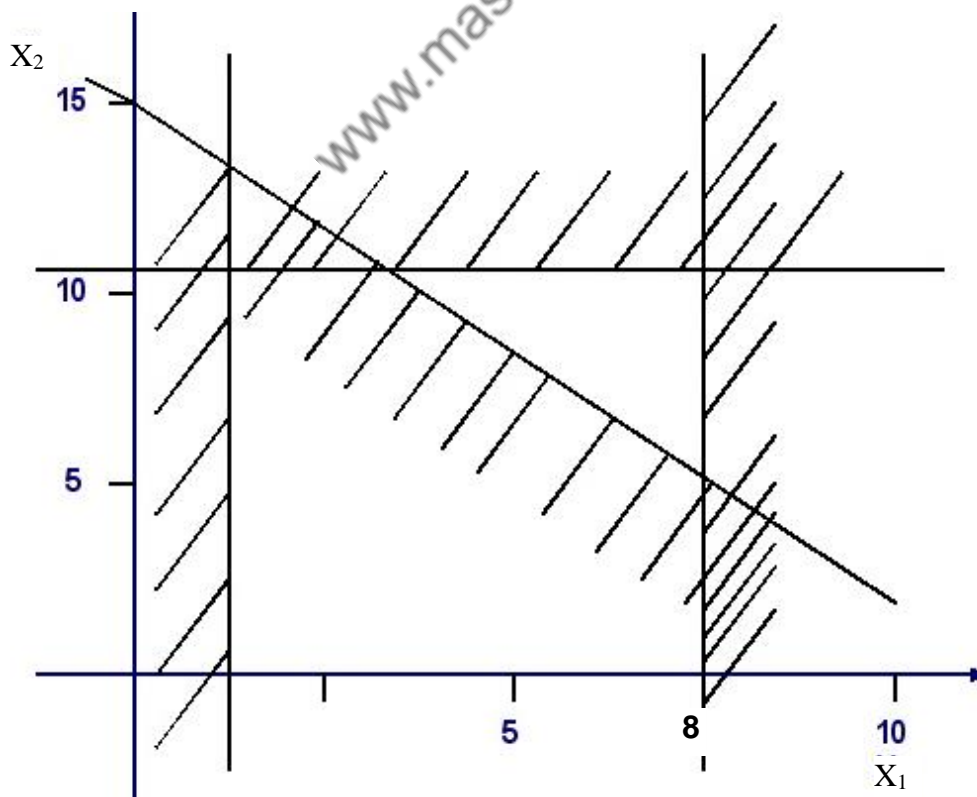Minimize $\quad\quad 40X_1 + 36X_2$
Subject to: $\quad\quad X_1 \leq 8$
$\quad\quad\quad\quad X_2 \leq 10$
$\quad\quad\quad\quad 5X_1 + 3 X_2 \geq 45$
$\quad\quad\quad\quad X_1 \geq 2$
$\quad\quad\quad\quad X_1, X_2 \geq 0$
Plotting this on a graph we get:

The line $X_1 \geq 2$ does not affect the feasible region (doesn't cause reduction of the feasible region), this constraint doesn't limit attainment of the objective, thus its known as a <u>redundant constraint</u>.
Now picking a convenient point inside the feasible region, say (6, 10)

We get a total cost of $600 = (6(40)+10(36))$
Thus the objective function line of $40X_1 + 36X_2 = 600$

Moving this line parallel toward the origin to locate the last apex before the line completely fall off the feasible region, we get:
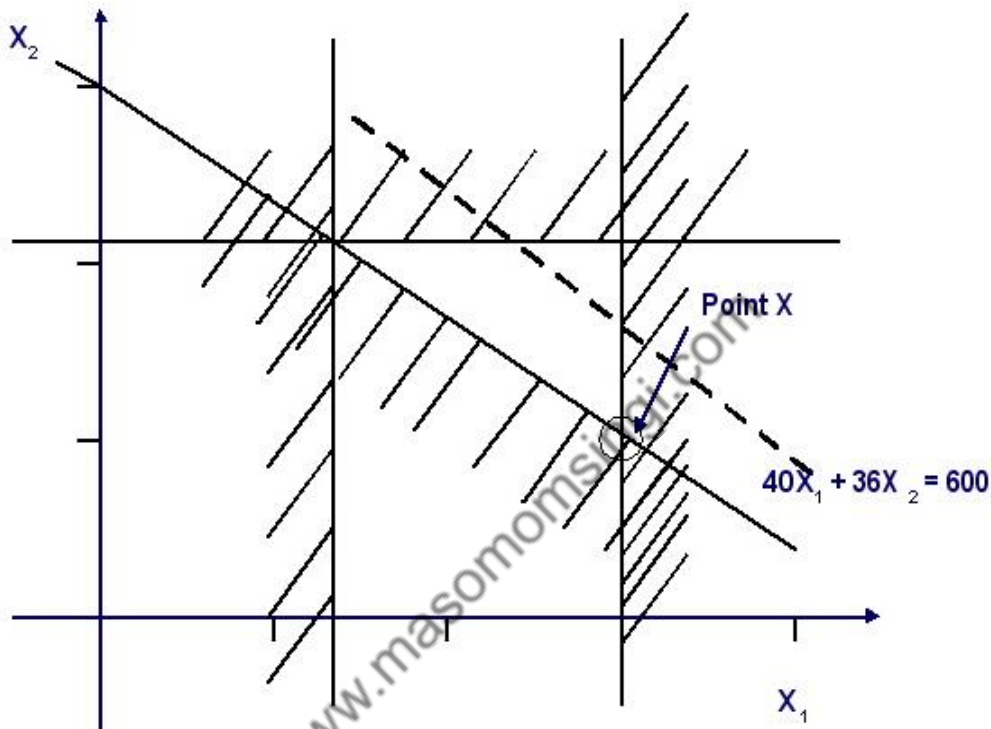
Point X is the point of optimal solution
The binding constraints here are:
$$X_1 \leq 8$$
And $5X_1 + 3X_2 \geq 45$
Solving this to get coordinates of point X
$$5X_1 + 3X_2 = 45 \text{ (when } X_1 = 8)$$
$$40 + 3X_2 = 45$$
$$X_2 = 5/3$$
Thus the solution is $X_1 = 8$
                                $X_2 = 5/3$

<u>Shadow or dual prices</u>
Definition: A shadow price or a dual price is the amount increase (or decrease) of the objective function when one more (or one less) of the binding constraints is made available.
Consider *example 1*.

Maximize $4X_1 + 3X_2$

Subject to: $0.5X_1 + 0.33X_2 \leq 12$ (Machine hours)
                $0.5X_1 + 0.5X_2 \leq 14$ (labor hours)

Starting with machine hours; let's assume that one more machine hour is available (with labor hours remaining constant)

We get:

$$0.5X_1 + 0.33X_2 = 13$$
$$0.5X_1 + 0.5X_2 = 14$$

Solving this simultaneously we get the values of X_1 and X_2 as

$$0.17 X_2 = 1$$
$$X_2 = 5.88$$
$$X_1 = 22.12$$

Thus the contribution is

$$4(22.12) + 3(5.88) = Sh.106.12$$

Comparing this with its original contribution of Sh.100.24 (see example 1) we see increasing machine hours by one unit has increased contribution by Sh.5.88, which is the shadow price per machine hour.

Note: This figure is also arrived at if we assume that machine hours are reduced by 1 unit i.e. 12-1.

Similarly assuming that one more labor hour is made available, and then contribution change is:

$$0.5X_1 + 0.33X_2 = 12$$
$$0.5X_1 + 0.5X_2 = 15$$

Solving this simultaneously gives:

$$0.17 X_2 = 3$$
$$X_2 = 17.65$$
$$X_1 = 12.35$$

Which give a contribution of.

$$4(12.35) + 3(17.65) = Sh.102.35$$

The contribution change is Sh.2.11 which is the shadow price per labor hour.

Note:

The shadow prices apply in so far as the constraint is binding for example if more and more labor hours are available it will reach a point where labor hours are no longer scarce thus labor hours cease to be a binding constraint and its shadow price becomes a zero.(All non-binding constraints have zero shadow price). Logically its senseless to pay more to increase a resource, which is already abundant.

Interpretation of shadow prices

A shadow price of a binding constraint indicates to management how much extra contribution will be gained by increasing a unit of the scarce resource.

In the example above Sh.2.11 is the shadow price for labor hours. This implies that management is ready to pay up to Sh.2.11 extra per hour for the extra hours i.e. say an employee is paid sh.5 per hour and one day he works for two hours extra (overtime), the management is prepared to pay up to sh.7.11 per hour for the two hours overtime worked.

Sensitivity Analysis

Definition: Sensitivity analysis is the test of how certain changes in resources affect the optimal solution.

In sensitivity analysis we consider the effect of additional limiting or non-limiting constraints.
We already know that adding more non-limiting constraints does not change the optimal solution.
We also know that adding more binding constraints affects the objective function.
It is very important for the management to know how much of a limit resource can be made available until it has no effect on the objective function (ie ceases to be a binding resource)

SIMPLEX METHOD

When analyzing linear programming problems with three or more variables the graphical method becomes enadequate, in such cases we employ *simplex method* . Simplex method is an algebraic procedure for solving systems of equations requiring optimization of the objective function..
This method can be applied to any number of variables, the more they are the more complex it becomes to workout a solution on paper. Computer programs e.g. Tora are used to solve the most intricate problems.
The first step in simplex method is conversion of inequalities to linear equations

Example
Consider the linear problem.

$$\text{Maximize} \quad 45x_1 + 80x_2$$
$$\text{Subject to} \quad 5x_1 + 20x_2 \leq 400$$
$$10x_1 + 15x_2 \leq 450$$
$$x_1 \geq 0, x_2 \geq 0$$

Solution.

1. To convert this problem to a system of linear equation, we introduce slack variables to each constraint.

$$Z = 45x_1 + 80x_2$$
$$\text{Subject to} \quad 5x_1 + 20x_2 + x_3 = 400$$
$$10x_1 + 15x_2 + x_4 = 450$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Where the structural variables x_3 and x_4 are slack variables

2. we then place this information in a tabular form known as a **tableau**

Initial tableau

Solution Variabl	Products		Slack Variables		Solution Quantit
	x_1	x_2	x_3	x_4	

e					y
x_3	5	20	1	0	400
x_4	10	15	0	1	450
Z	45	80	0	0	0

The tableau shows that

$x_3 = 400$

$x_4 = 450$

$Z = 0$ (Total contribution)

It also show that unused capacity is at maximum i.e. the value for the slack x_3 and x_4 is 0.

After several operations and when an optimal solution has been attained, these values will change to give an optimal feasible solution.

3. Select the column with the highest value of Z (i.e. 80), then devide the positive numbers in that column (i.e. the x₂ column) into the quantity column.

i.e. $400 \div 20 = 20$

      $450 \div 15 = 30$

Chose the row giving the lowest result (in our case the row with x₃ gives 20) and mark the element falling on the intersection of the selected row and selected column (i.e. 20: selected element)

4. Divide all the elements in the selected row by the value of the selected and change the solution variable to the heading of the identified column (from x₃ to x₂)

Thus the tableau will appear as follows.

Solution Variable	Products		Slack Variables		Solution Quantity
	x_1	x_2	x_3	x_4	
x_2	$\frac{1}{4}$	1	$\frac{1}{20}$	0	20
x_4	10	15	0	1	450
Z	45	80	0	0	0

5. Next we conduct row operations that aim to reduce elements falling in the same column as the previously marked element to zero. These row operations may sometime necessitate multiplying or dividing the selected row with an arbitrary number.

Therefore:

         Row 2 – 15×Row 1

$$\begin{array}{llllll} & x_4 & 10 & 15 & 0 & 1 & =450 \\ 15\times & x_2 & \frac{15}{4} & 15 & \frac{15}{20} & 0 & =300 \\ \hline new & x_4 & 6\frac{1}{4} & 0^* & \frac{3}{4} & 1 & =150 \end{array}$$

Note that the aim was to attain the zero.

$$\begin{array}{llllll} & Z & 45 & 80 & 0 & 0 & = & 0 \\ 80\times & x_2 & 20 & 80 & 4 & 0 & = & 1600 \\ \hline new & Z & 25 & 0^* & -4 & 0 & = & -1600 \end{array}$$

On replacing the new rows to the tableau we get 2nd tableau.

Solution Variable	Products		Slack Variables		Solution Quantity
	x_1	x_2	x_3	x_4	
x_2	$\frac{1}{4}$	1	$\frac{1}{20}$	0	20
x_4	$6\frac{1}{4}$	0	$\frac{3}{4}$	1	150
Z	25	0	-4	0	-1600

Since in the Z row under products column we still have values greater than zero, we conduct another operation.

Taking the column with a Z value of 25, we repeat the process in the same manner.

$$20 \div \frac{1}{4} = 80$$
$$150 \div 6\frac{1}{4} = 24$$

Thus we pick the x_4 row and mark the element $6\frac{1}{4}$, the row solution variable is changed to x_1 and we divide the row by $6\frac{1}{4}$ to convert the marked element to 1.

Therefore; x_1 1   0 $\frac{3}{25}$ $\frac{4}{25}$ = 24

Next we do the row operations

$$\begin{array}{llllll} & x_2 & \frac{1}{4} & 1 & \frac{1}{20} & 0 & =20 \\ (\div 4) & x_1 & \frac{1}{4} & 0 & \frac{3}{100} & \frac{4}{100} & = 6 \\ \hline new & x_2 & 0^* & 1 & \frac{1}{50} & -\frac{1}{25} & = 24 \end{array}$$

$$\begin{array}{llllll} & Z & 25 & 0 & -4 & 0 & =-1600 \\ 25\times & x_1 & 25 & 0 & 3 & 4 & = 600 \\ \hline new & Z & 0^* & 0 & -7 & -4 & =-2200 \end{array}$$

the tableau becomes, 3rd Tableau

Solutio	Products		Slack	Solutio

n Variable	Variables				n Quantity
	x_1	x_2	x_3	x_4	
x_2	0	1	$\frac{1}{50}$	$-\frac{1}{25}$	24
x_1	1	0	$\frac{3}{25}$	$\frac{4}{25}$	24
Z	0	0	-7	-4	-2200

This is the final tableau since the Z row has no values greater than zero thus we have the optimal solution.

Interpretation

- to maximize Z we need to produce 24 units x_2 and 24 units of x_1, we obtain these values from the solutions quantity column

Thus, Z = 24(45) + 24(80) = 3000

- We have zero slack (unused quantities of constraints).

Assume tableau 2 is the final tableau and let the constraint with variable x_3 be labor hours and x_4 are raw materials, the slack would've been interpreted as:
  - 150 units of raw materials were unused
  - To maximize Z we produce 20units of x_2 and none of x_1.
- The value represents in Z row under slack variable column represents **shadow prizes.** Thus the shadow prize for the first constraint with x_3 is 7 and the shadow prize for the second constraint with the variable x_4 is 4.

INTERPRETATION OF COMPUTER GENERATED SOLUTION

Example

Maximize $\quad 25x_1 + 20x_2 + 24x_2$

$where: \ x_1 = Xtragrow, \ x_2 = Youngrow, \ x_3 = Zupergrow$

Subject to $\ 0.3x_1 \qquad + 0.2x_3 \leq 500$

$$0.5x_2 + 0.4x_3 \leq 1000$$

$$0.2x_1 + 0.1x_2 + 0.1x_3 \leq 800$$

$$0.4x_2 + 0.3x_3 \leq 600$$

$$x_1 \qquad\qquad \geq 1500$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

The computer generated solution for this problem is as follows;

Objective value = **71666.7**			
Variable	**Value**	**Obj. Coeff**	**Obj Value Contribution**
X1: Xtragrow	1666.7	25	4166.7
X2: Youngrow	1500	20	30000
X3: Zupergrow	0	24	0

Constraint	RHS	Slack-/Surplus+
1(<)	500	0
2(<)	1000	250-
3(<)	800	316.67-
4(<)	600	0
5(>)	1500	166.7+

Sensitivity Analysis				
Variable	Current obj coeff	Min obj coeff	Max Obj Coeff	Reduced cost
X1: Xtragrow	25	13.50	Infinity	0
X2: Youngrow	20	9.78	Infinity	0
X3: Zupergrow	24	-Infinity	31.67	7.67
Constraint	Current RHS	Min RHS	Max RHS	Dual price
1(<)	500	450	975	83.33
2(<)	1000	750	Infinity	0
3(<)	800	483.3	Infinity	0
4(<)	600	0	800	50
5(>)	1500	-Infinity	1666.7	0

Required
Interpret the data generated by the computer.

Solution.
Table 1:
Objective value, is the solution to objective function (e.g. the solution to this example is 71,436)
The four columns of table 1 are to be interpreted as follows;
- Variable: these are the variables of the model. In our example we have x_1 = Xtragrow, x_2 = Young row and x_3 = Zupergrow
- Value: this is value that the variables assume at optimal solution (to optimize the objective function one needs to produce this amounts of the variables). In our example we are required to produce 1,666.67 of x1 and 1,750 of x2 and none of x3
- Objective coefficient: these are the coefficients of the objective function
- Objective value contribution: this is the value contributed by each variable to the objective function (for x1=25×1,666.67), the total of this is equal to our objective value (i.e. 41,666.67+35,000=76,666.67).

The 3 columns of the second part of table1 can be interpreted as follows;
- Constraints: this is constraints of the model representing the limited resources.
- RHS: the Right hand side value is the limiting value of the constraint. E.g. for the first constraint the maximum amount of material A is 500 tons.

- Slack/surplus: at optimal production not all the materials for some of the constraints will be fully utilized, slack is the amount of material that is left over after production. For constraint 1 and 4 no material remained, this also implies that these are the binding constraints i.e. their adjustment directly affects the objective solution

Sensitivity Analysis
This is the analysis of the effect of adjusting variables or constraint, whether the objective solution will be affected. How much of the objective coefficient (or the maximum available amount of a constraint) can be reduced or increased without affecting the objective solution.
The columns of this table can be interpreted as follows;

Variable: as explained above

Current objective coefficient: this is the value of coefficients of the objective function

Minimum objective coefficient: this is how low the coefficient can be reduced without affecting the optimal basis. The coefficient for x1 can be reduced from 25 to 13.50 (the prize for Xtragrow fall to as low as $13.50 from $25) but the optimal solution will remain the same

Maximum objective coefficient: this is how high the coefficient can be increased without affecting the *optimal basis*

Reduced cost: this is amount by which the coefficient of the variable has to be adjusted with for it to become a basic variable (included to the objective optimal solution). X1 and x2 have 0 reduced costs implying that they already make part of the optimal solution, x3 will require to be increased by 7.67 for it to make part of the basic variable.

The second part of the table is interpreted as follows;
- Constraints: as described above
- Current Right Hand Side: the limiting value of the constraint. E.g. for the first constraint the maximum amount of material A is 500 tons.
- Minimum RHS: the lowest the available amount of the constraint can be reduced without affecting the optimal basis
- Maximum RHS: the highest the available amount of the constraint can be increased without affecting the optimal solution
- Dual price: this is amount increase to the objective contribution due to a unit increase of the available constraint. Since there is only 500 tons of material A, if management decides to increase it by a unit to 501tons then the objective optimal solution will be 76,666.67+83.33.