STANDARD LECTURE NOTES FOR STATISTICS FOR DIPLOMA IN

SOCIAL WORK &COMMUNITY

DEVELOPMENTMODULE II

QUANTITATIVE TECHNIQUES

Course Outline

1. Introduction to Quantitative Techniques
2. Fundamentals of Mathematics and Statistics.
3. Data Collection
4. Measures Of Central Tendency
5. Measures Of Variation /Depression
6. Correlation Analysis
7. Regression Analysis
8. Index Number
9. Time Series Analysis
10. Probability
11. Sampling
12. Test of Hypothesis
13. Linear Programming

GENERAL OBJECTIVES

At the end of this course unit, the trainee should be able to;-

- broaden his/her knowledge in mathematical application;
- understand and appreciate the role of quantitative methods in decision making;
- collect and organize statistical data for management;
- analyze quantitative data for management decision making;
- Apply quantitative methods in solving business problems.

Introduction

Def; quantitative techniques are those techniques which provides the decision maker with a systematic and powerful means of analysis and help, based on quantitative data in exploring policies for achieving pre-determined goals Involves the use of numbers, symbols and other mathematical expressions.
They are essentially helpful in supplementing to judgment and intuition. These techniques evaluate planning factors of alternatives as and when they arise rather than prescribe courses of action. They are particularly relevant to problems of complex business enterprises.
Classification of Q.T

a) Statistical Techniques

Are those techniques which are used in conducting the statistical inquiry concerning a certain phenomenon? They include statistical methods beginning from the collection of data till the task of interpretation of the data collected.

b) Programming Techniques

Are the model building techniques used by decision maker?

Quantitative techniques

Statistical techniques | programming techniques

Statistical techniques	Programming techniques
Methods of collecting data	Linear programming
Classification and tabulation of collected data	Decision theory
	Theory of games
Probability theory and sampling	Simulation
Correlation and regression analysis	a) monte carlo technique
Index numbers	b) system simulation
Time series analysis	queuing (waiting line) theory

Interpretation and extrapolation
Survey techniques and methodology
Ratio analysis
Statistical quality control
Analysis of variance
Statistical inference and interpretation
Theory of attributes.

inventory planning
network analysis/ PERT
integrated production model
others;
non-linear programming
the theory of replacement
quadratic programming
Parametric programming etc.

QT and Business management

Production Management

- Selecting building site fro a plant, scheduling and controlling its development and designing its layout
- Locating within the plant and controlling movement required materials and finished goods inventories.
- Scheduling and sequencing production by adequate preventive maintenance with optimum number of operatives by proper allocation of machines
- Calculating optimum product mix.

Personnel Management

- Optimum manpower planning
- No of persons to be maintained on permanent or full time roll
- The no. of persons to be kept in work pool intended fro meeting the absenteeism.
- Optimum manner of sequencing and routing of personnel to a variety of jobs
- Studying personnel recruiting procedures, accidents rates and labor turnover

Market Management

- Where distribution and warehousing should be located the size , quantity to be stocked and choice of customers.
- Optimum allocation of sales budget to direct selling of promotional expenses.
- Choice of different media of advertising and bidding strategies.
- Financial management
- Finding long range capital requirement as well as how to generate theses requirements
- Determining optimum replacement policies
- Working out a profit plan for the firm
- Developing capital investment plans
- Estimating credit and investment risks.

Limitation of Q.T s

1. the inherent limitation concerning mathematical expressions

2. high costs involved in the use of QTs
3. They do not take into consideration the intangible factors i.e. non-measurable human factors.
4. Quantitative techniques are just the tools of analysis and not the complete decision making process.

Role of QT in business and industry

1. they provide a tool for scientific analysis

these techniques provides executives with a more precise description of the cause and effect relationship and risks underlying the business operations in measurable terms and this eliminates the conventional
intuitive and subjective basis on which management used to formulate their decisions.

2. they provide solution for various business problems

Are used in the field of production, procurement, marketing, finance and other allied fields. Problems like how best can managers and executive4s allocate available resources to various products so that in a given time the profits are maximized or costs are minimized. Is it possible for an enterprise to arrange the time and quantity of orders of its stocks such that the overall profit with given resources is maximized?

3. they enable proper development of resources

E.g. programmed evaluation and review techniques (PERT) enables us to determine earliest and the latest time fro each of he events and activities and thereby helps in the identification of the critical path
All these helps in deployment of resources from one activity to another to enable the project completion on time.

4. They help in minimizing waiting and servicing time.

The queuing theory helps management in minimizing the total waiting of servicing costs. It also analyses the feasibility of adding facilities and thereby helping to take correct and profitable decision.

5. They enable management to decide when to buy and how much to buy.

The main objective of inventory planning is to achieve balance between the costs of holding stock and benefits of holding stock. Helps in determining when to buy and how much to buy.

6. They assist in choosing an optimum strategy.

In a competitive situation game theory helps to determine optimum strategy which maximizes profits or minimizes loses but adopting optimum strategy.

7. they render great help in optimum resource allocation
8. they facilitate the process of decision making
9. Through various QTs management can know the reactions of the integrated business systems.

CHAPTER ONE

FUNDAMENTALS OF MATHEMATICS AND STATISTICS

Specific Objectives

At the end of this topic, the trainee should be able to:

- ❖ Form and solve algebraic equations.
- ❖ Apply the various techniques of counting to solving management decision problems;
- ❖ Applying set theory to business decision problems;
- ❖ Derive and apply the binomial theorem to business problems;
- ❖ Evaluate mathematical series.

ALGEBRAIC EQUATIONS

Algebra

Algebra is a branch of mathematics in which, instead of using numbers, we use letters to represent numbers.

We all know that 2+3=5.

Suppose, though, that we substitute letters for the first two numbers, so that:

$2 = a$

$3 = b$

We can then write:

$a + b = 5$

All that has happened is that we have replaced the numbers with letters. However, a number is a specific quantity – e.g., 5 is more than 4, but less than 6 – whereas a letter can be used to represent any number. Thus in the above expression, 'a' could be 4 and 'b' could be 1. We only know that they are 2 and 3 respectively because we defined them as such before.

The main consequence of this is that algebra uses general expression and gives general results, whereas arithmetic (using numbers) uses definite numbers and gives definite results. Arithmetic is specific whereas algebra is general.

Equations

An equation is an expression with an equal sign (=)
Equations are classified into two main groups' linear equations and non linear equations. Examples of linear equations are

$x + 13 = 15$
$7x + 6 = 0$

Non linear equations in the variable x are equations in which x appears in the second or higher degrees. They include quadratic and cubic equations amongst others. For example

$5x^2 + 3x + 7 = 0$ (quadratic equation)

$2x^3 + 4x^2 + 3x + 8 = 0$ (cubic equation)

The solution of equations or the values of the variables for which the equations hold is called the roots of the equation or the solution set.

Solution of linear equations.

Supposing M, N, and P are expressions that may or may not involve variables, then the following constitute some rules which will be useful in the solution of linear equations

Rule 1: Additional rule

If $M = N$ then $M + P = N + P$

Rule 2: Subtraction rule

If $M = N$, Then $M - P = N - P$

Rule 3: multiplication rule

If $M = N$ and $P \neq O$ then $M \times P = N \times P$

Rule 4: Division rule

If $P \times M = N$ and $P \neq O$
And $N/P = Q$ Q being a raterial number then
$M = N/P$

Example

i. Solve $3x + 4 = - 8$

ii. Solve $\frac{y}{3} = - 4$

Solutions

i. $3x + 4 = -8$

$3x + 4 - 4 = - 8 - 4$ (by subtraction rule)

$$3x = -12 \quad \text{(simplifying)}$$

$$\frac{3x}{3} = -\frac{12}{3} \quad \text{(by division rule)}$$

$$x = -4 \quad \text{(simplifying)}$$

ii. $3 \times \frac{y}{3} = -4 \times 3$

$$y = -12 \quad \text{(simplifying)}$$

<u>Solutions of inequalities</u>

The solutions sets of inequalities frequently contain many elements. In a number of cases they contain infinite elements.

<u>Example</u>

Solve and graph the following inequalities

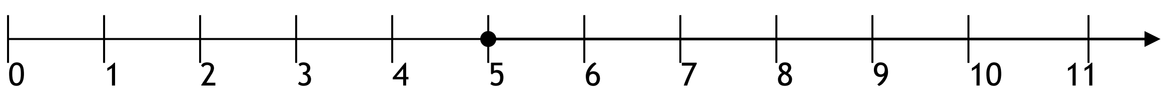
$x - 2 > 2$; $x \subset w$ (where x is a subset of w)

<u>Solution</u>

$x - 2 > 2$ so $x - 2 + 2 > 2 + 2$

Thus, $x>4$

The solution set is infinite, being all the elements in w greater than 4



<u>Example</u>

Solve and graph

$3x - 7 < -13$;

<u>Solution</u>

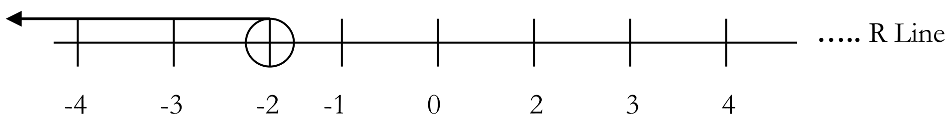
$3x - 7 < -13$

$\Rightarrow 3x - 7 + 7 < -13 + 7$

$\Rightarrow 3x < -6$

$\frac{3x}{3} < \frac{-6}{3}$

$x < -2$



Rules for solving linear inequalities

Suppose M, M_1, N, N_1 and P are expressions that may or may not involve variables, then the corresponding rules for solving inequalities will be:

Rule 1: Addition rule

If $M > N$ and $M_1 > N_1$

Then $M + P > N + P$ and

$M_1 + P > N_1 + P$

Rule 2: Subtraction Rule

If $M < N$ and $M_1 \geq N_1$

Then $M - P < N - P$ and

$M_1 - P \geq N_1 - P$

Rule 3: Multiplication rule

If $M \geq N$ and $M_1 > N_1$ and $P \neq 0$

Then $MP \geq NP$; $M_1P > N_1P$

$M(-P) \leq N(-P)$ and $M_1(-P) < N_1(-P)$

Rule 4: Division

If $M > N$ and $M_1 < N_1$ and $P \neq 0$

Then $M/P > N/P$: $M_1/P < N_1/P$

$M/(-P) < N/(-P)$: and $M_1/(-P) > N_1/(-P)$

Rule 5: Inversion Rule

If $M/P \leq N/Q$ where $P, Q \neq 0$

$M_1/P > N_1/Q$

Then $P/M \geq Q/N$ and $P/M_1 < Q/N_1$

Note: The rules for solving equations are the same as those for solving equations with one exception; when both sides of an equation is multiplied or divided by a negative number, the inequality symbol must be reversed (see rule 3 & Rule 4 above).

<u>Example</u>

Solve and graph the following:

i. $7 - 2x > -11$;

ii. $-5x + 4 \leq 2x - 10$;

iii. $-3 \leq 2x + 1 < 7$;

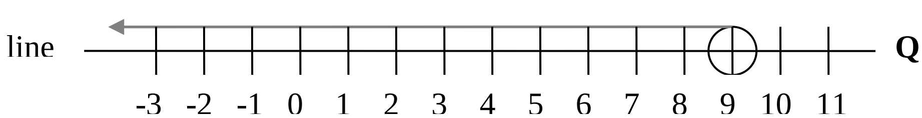
<u>Solutions</u>

i. $7 - 2x > -11$

$-2x > -18$ (subtraction rule)

$\frac{-2x}{-2} < \frac{-18}{-2}$ (bydivision rule)

$x < 9$

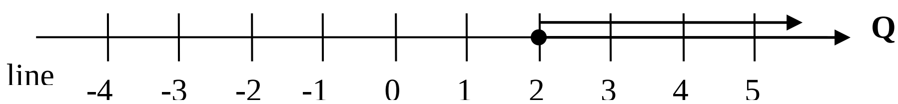


ii. $-5x + 4 \leq 2x - 10$

$-7x + 4 \leq -10$ (by subtraction rule)

$-7x \leq -14$ (by subtraction rule)

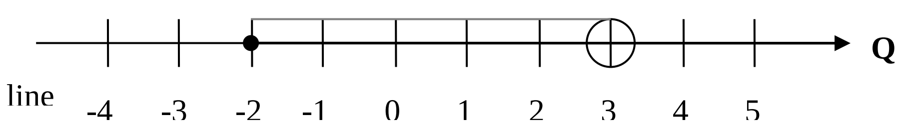
$x \geq 2$ (by division rule)



iii. $-3 \leq 2x + 1 < 7$

$-4 \leq 2x < 6$ (by substraction rule)

$-2 \leq x < 3$ (by division rule)



Linear inequalities in two variables: relations

An expression of the form

$Y \geq 2x - 1$

Is technically called a relation. It corresponds to a function, but different from it in that, corresponding to each value of the independent variable x, there is more than one value of the dependent variable y

Relations can be successfully presented graphically and are of major importance in linear programming.

Solutions of linear simultaneous equations.

Two or more equations will form a system of linear simultaneous equations if such equations be linear in the same two or more variables.

For instance, the following systems of the two equations is simultaneous in the two variables x and y.

$$2x + 6y = 23$$

$$4x + 7y = 10$$

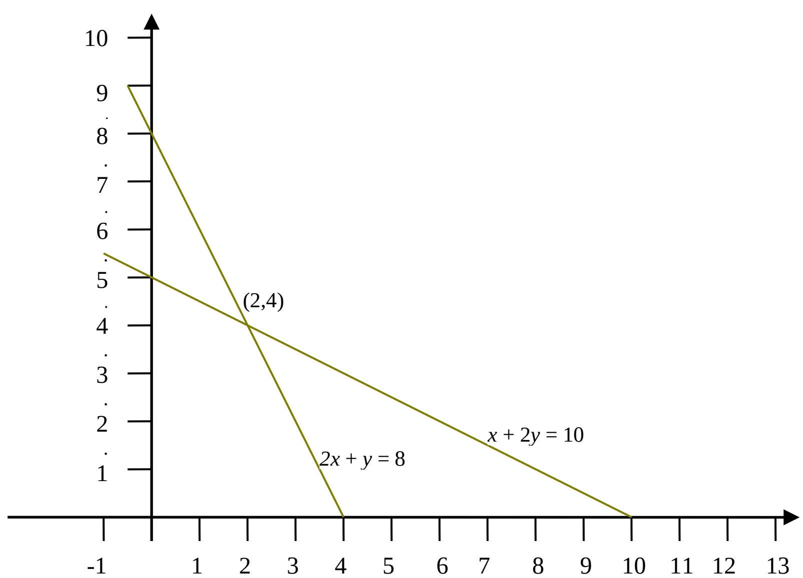
The solution of a system of linear simultaneous equations is a set of values of the variables which simultaneously satisfy all the equations of the system.

Solution techniques

a) The graphical technique

The graphical technique of solving a system of linear equations consists of drawing the graphs of the equations of the system on the same rectangular coordinate system. The coordinates of the point of intersection of the equations of the system would then be the solution.

Example



The above figure illustrates:

Solution by graphical method of two equations

$$2x + y = 8$$

$$x + 2y = 10$$

The system has a unique solution (2, 4) represented by the point of intersection of the two equations.

b) The elimination technique

This method requires that each variable be eliminated in turn by making the absolute value of its coefficients equal in the equations of the system and then adding or subtracting the equations. Making the absolute values of the coefficients equal necessitates the multiplication of each equation by an appropriate numerical factor.
Consider the system of two equations (i) and (ii) below

$2x - 3y = 8$ …….. .... (i).

$3x + 4y = -5$ …….. .... (ii).

Step 1

Multiply (i) by 3

$6x - 9y = 24$ …… .... (iii).

Multiply (ii) By 2

$6x + 8y = -10$ …… .... (iv).

Subtract (iii) from (iv).

$17y = -34$ …….. (v).

$\therefore \quad y = -2$

Step 2

Multiply (i) by 4

$8x - 12y = 32$ ……. .... (vi)

Multiply (ii) by 3

$9x + 12y = -15$ ….. .... (vii)

Add (vi) to (vii)

$17x = 17$ …….. (viii)

$\therefore \quad x = 1$

Thus $x = 1$, $y = -2$ i.e. {1,-2}

c) <u>The substitution technique</u>

To illustrate this technique, consider the system of two equations (i) and (ii) reproduced below

...$2x - 3y = 8$ …….. (i).

...$3x + 4y = -5$ …… (ii).

The solution of this system can be obtained by

a) Solving one of the equations for one variable in terms of the other variable;
b) Substituting this value into the other equation(s) thereby obtaining an equation with one unknown only
c) Solving this equation for its single variable finally
d) Substituting this value into any one of the two original equations so as to obtain the value of the second variable

Step 1

Solve equation (i) for variable x in terms of y

$2x – 3y = 8$

$x= 4 + 3/2\ y$ (iii)

Step 2

Substitute this value of x into equation (ii). And obtain an equation in y only

$3x + 4y = -5$

$3\ (4 + 3/2\ y) + 4y = -5$

$8\ ½\ y = -\ 17$ ……. (iv)

Step 3

Solve the equation (iv). For y

$8½y = -17$

$y = -2$

Step 4

Substitute this value of y into equation (i) or (iii) and obtain the value of x

$2x – 3y = 8$

$2x – 3(-2) = 8$

$x = 1$

<u>Example</u>

Solve the following by substitution method

$2x + y = 8$

$3x – 2y = -2$

<u>Solution</u>

Solve the first equation for y

$y = 8 - 2x$

Substitute this value of y into the second equation and solve for x

$3x - 2y = -2$

$3x - 2(8-2x) = -2$

$x = 2$

Substitute this value of x into either the first or the second original equation and solve for y

$2x + y = 8$

$(2)(2) + y = 8$

$y = 4$

TECHNIQUES OF COUNTING

Permutations

This is an order arrangement of items in which the order must be strictly observed

Example

Let x, y and z be any three items. Arrange these in all possible permutations

1^{st}	2^{nd}	3^{rd}	
X	Y	Z	
X	Z	Y	
Y	X	Z	Six different permutations
Y	Z	X	
Z	Y	X	
Z	X	Y	

NB: The above 6 permutations are the maximum one can ever obtain in a situation where there are only 3 items but if the number of items exceeds 3 then determining the no. of permutations by outlining as done above may be cumbersome. Therefore we use a special formula to determine such permutations. The formula is given below

The number of permutations of ‘r’ items taken from a sample of ‘n’ items may be provided as ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ *where;*! = factorial

e.g.

i. $^{3}P_{3} = \frac{3!}{(3-3)!}$

$= \frac{3\times 2\times 1}{0!}$ **note**; 0! = 1

$= \frac{6}{1} = 6$

ii. $^{5}P_{3} = \frac{5!}{(5-3)!}$

$= \frac{5\times 4\times 3\times 2\times 1}{1\times 2}$

$= 60$

iii. $^{7}P_{5} = \frac{7!}{(7-5)!}$

$= \frac{7\times 6\times 5\times 4\times 3\times 2\times 1}{2\times 1}$

$= \frac{5040}{2}$

$= 2520$

Example

There are 6 contestants for the post of chairman secretary and treasurer. These positions can be filled by any of the 6. Find the possible no. of ways in which the 3 positions may be filled.

Solution

Chairman	Secretary	Treasurer
6	5	4

Therefore the no of ways of filing the three positions is 6 x 5 x 4 = 120

$^{6}P_{3} = \frac{6!}{(6-3)!}$

$= \frac{6\times 5\times 4\times 3\times 2\times 1}{3\times 2\times 1}$

$= \frac{720}{6}$

$= 120$

Combinations

A combination is a group of times in which order is not important.
For a combination to hold at any given time it must comprise of the same items but if a new item is added to the group or removed from the group then we have a new combination

Example

3 items x, y and z will have 6 different permutations but only one combination.
The following formula is usually used to determine the no. of combinations in a given situation.

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Example

i. $$^{8}C_{7} = \frac{8!}{7!(8-7)!}$$

$$= \frac{8!}{7!1!} = \frac{8\times 7!}{1\times 7!}$$

= 8

ii. $$^{6}C_{4} = \frac{6!}{4!(6-4)!}$$

$$= \frac{6!}{4!2!} = \frac{6\times 5\times 4!}{4!\times 2\times 1}$$

= 15

iii. $$^{8}C_{3} = \frac{8!}{3!5!}$$

$$= \frac{8\times 7\times 6\times 5!}{3\times 2\times 1\times 5!}$$

= 56

<u>Application</u>

Example
There is a committee to be selected comprising of 5 people from a group of 5 men and 6 women. If the selection is randomly done. Find the possibility of having the following possibilities (combinations)

i. Three men and two women
ii. At least one man and at least one woman must be in the committee
iii. One particular man and one particular woman must not be in the committee (one man four women)

Solution

i. The committee size = 5 people
   The group size = 5m + 6w

∴ assuming no restrictions the committee can be selected in ${}_{11}C_5$

The committee has to consist of 3m & 2w

∴ these may be selected as follows.

${}^5C_3 \times {}^6C_2$

P(*committee 3m and 2w)*

$= \frac{{}^5C_3 \times {}^6C_2}{{}^{11}C_5}$ *note that this formula can be fed directly to your scientific calculator and attain a solution.*

$$= \frac{\frac{5!}{3!2!} \times \frac{6!}{4!2!}}{\frac{11!}{5!6!}}$$

$$= \frac{5\times4\times3\times2\times1}{3\times2\times1\times2\times1} \times \frac{6\times5\times4!}{2\times1\times4!} \times \frac{5\times4\times3\times2\times1\times6!}{11\times10\times9\times8\times7\times6!}$$

$$= \frac{27}{77}$$

ii. P(*at least one man and at least one woman must be in the committee*)

The no. of possible combinations of selecting the committee without any woman = ${}^5C_5$

The probability of having a committee of five men only

$$= \frac{{}^5C_5}{{}^{11}C_5} = \frac{1}{462}$$

The probability of having a committee of five women only

$$= \frac{{}^6C_5}{{}^{11}C_5} = \frac{\frac{6!}{5!1!}}{\frac{11!}{5!6!}}$$

$$= \frac{6\times5!}{5!1!} \times \frac{5!6!}{11\times10\times9\times8\times7\times6!}$$

$$= \frac{1}{77}$$

∴ P (*at least one man and at least one woman*)

= 1 - {P (*no man*) + P (*no woman*)}

$= 1 - \left\{\frac{1}{77} + \frac{1}{462}\right\}$

$= 1 - \frac{(6+1)}{462}$

$= 1 - \frac{7}{462}$

$= \frac{455}{465}$

iii. P(one particular man and one particular woman must not be in the committee would be determined as follows

The group size = 5m + 6w
Committee size = 5 people

Actual groups size from which to
Select the committee = 4m + 5w
Committee = 1m + 4w

The committee may be selected in $^9C_5$
The one man may be selected in $^4C_1$ ways
The four women may be selected in $^5C_4$ ways

$\therefore$ P (*committee of 4w1man*).

$$= \frac{^5C_4 \times {^4C_1}}{^9C_5}$$

$$= \frac{\frac{5!}{4!1!} \times \frac{4!}{1!3!}}{\frac{9!}{4!5!}}$$

$$= \frac{5\times4!}{1!4!} \times \frac{4\times3!}{1\times3!} \times \frac{4!5!}{9\times8\times7\times6\times5!}$$

$= \frac{10}{63}$

<u>SETS THEORY</u>

Introduction

<u>Sets and set theory</u>

A set is a collection of distinct objects. We may consider all the ocean in the world to be a set with the objects being whales, sea plants, sharks, octopus etc, similarly all the fresh water lakes in Africa can form a set. Supposing A to be a set

A = {4, 6, 8, 13}

The objects in the set, that is, the integers 4, 6, 8 and 13 are referred to as the members or elements of the set. The elements of a set can be listed in any order. For example,

$$A = \{4, 6, 8, 13\} = \{8, 4, 13, 6\}$$

Sets are always precisely defined. Each element occurs once and only once in a set.

The notion $\in$ is used to indicate membership of a set. $\notin$ represents non membership. However, in order to represent the fact that one set is a subject of another set, we use the notion $\subset$. A set "S" is a subject of another set "T" if every element in "S" is a member of "T"

Example

If $A = \{4, 6, 8, 13\}$ then

i) $4 \in \{4, 6, 8, 13\}$ or $4 \in A$; $16 \notin A$

ii) $\{4, 8\} \subset A$; $\{5, 7\} \not\subset A$; $A \subset A$

<u>Methods of set representation</u>

Capital letters are normally used to represent sets. However, there are two different methods for representing members of a set:

i. The descriptive method and
ii. The enumerative method

The descriptive method involves the description of members of the set in such a way that one can determine the elements of the set without difficulty.

The enumerative method requires that one writes out all the members of the set within the curly brackets.

For example, the set of numbers 0, 1, 2, 3, 4, 5, 6 and 7 can be represented ass follows

$P = \{0, 1, 2, 3, 4, 5, 6, 7\}$, enumerative method

$P = \{X/x = 0, 1, 2\ldots7\}$ descriptive method

Or

$P = \{x/0 \le x \le 7\}$ *where x is an integer.*

<u>Types of sets</u>

a) Finite and infinite sets

A set can be classified as a finite or infinite set, depending upon the number of elements it has. A finite set has a finite number of elements whereas an infinite set has an infinite number of elements.

For example, set P below has ten elements and is therefore a finite set. Set S, on the other hand, is an infinite set since it has an infinite number of elements.

P = {2, 4, 6…20}

S = {1, 3, 5…}

b) Universal set

The term refers to the set that contains all the elements that an analyst wishes to study.
The notation U or ξ is generally used to denote universal sets

c) The null set or empty set

This is a set which contains no elements. It is normally designated by a Greek letter Ø, or { }.
The sets Ø and {Ø} are not the same thing since the former has no elements in it, while the later has one element in it, namely zero

d) Equal or equivalent sets

Two sets C and D are said to be equal if every member of set C belongs to D and every member of set D also belongs to C

e) Complement of a set

The complement of set A is written as A′. This set contains all those elements of universal set which are not in A

f) Intersection and union

$B \cap C$ Denotes the intersection of B and C. it is the set containing all those elements, which belong to both B and C

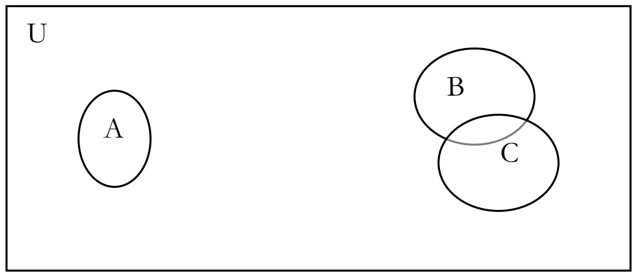
If B = {5, 8, 11, 20, 25} and C = {1, 3, 5, 7, 9, 11, 13}

Then $B \cap C$ = {5, 11}

$B \cup C$ = {1, 3, 5, 7, 8, 9, 11, 13, 20 25}

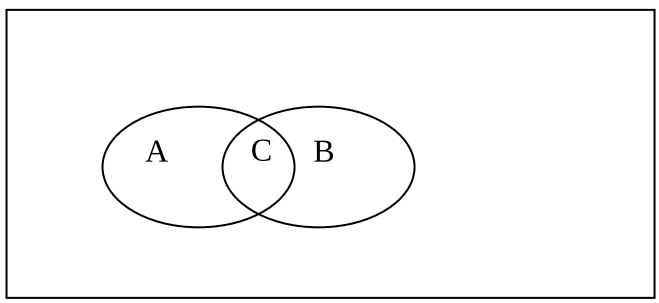
Set Operations and Laws

A simple way of representing sets and relations between sets is by means of the Venn diagram. Venn diagram consists of a rectangle that represents the universal set. Subjects of the universal set are represented by circles drawn within the rectangle, or the universe.
Suppose that the universal set is designated by U and the sets A, B and C are subject of U.The Venn diagram below can be used to illustrate the sets as follows



Venn diagram below representing the intersection of set A and B or $A \cap B$ = C is illustrated as follows

<u>Intersection of sets</u>



U

<u>Example:</u>
You are given the universal set

T = {1, 2, 3, 4, 5, 6, 7, 8}

And the following subjects of the universal set:

A = {3, 4, 5, 6,}

B = {1, 3, 4, 7, 8}

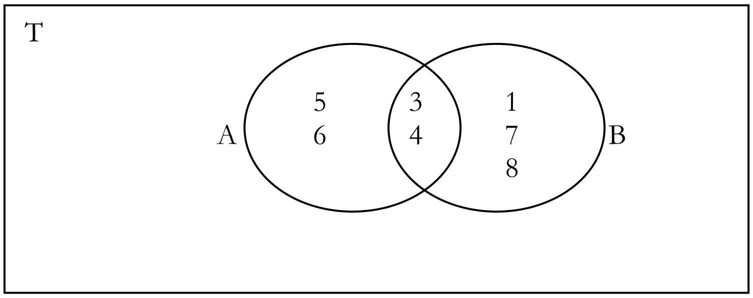
Determine the intersection of A and B

<u>Solution</u>
The intersection of A and B is the subject of T, containing elements that belong to both A and B

$A \cap B$ = {3, 4, 5, 6,} $\cap$ {1, 3, 4, 7, and 8}

= {3, 4}

Or



U

<u>Example</u>

Consider the following universal set T and its subjects C, D and E

T = {0, 2, 4, 6, 8, 10, 12}

C = {4, 8,}

D = {10, 2, 0}

E = {0}

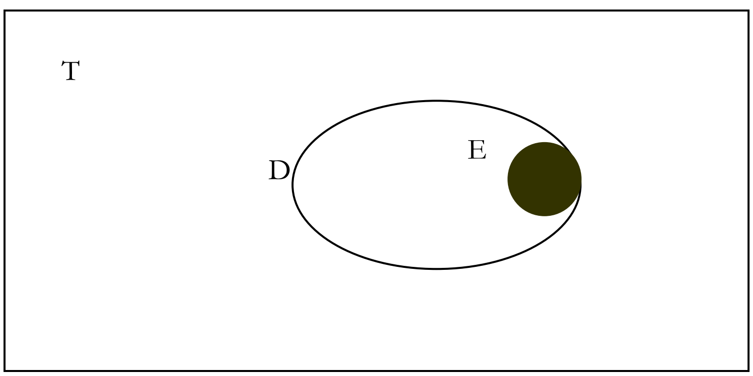
Find

i) $D \cap E$

ii) $C \cap D \cap E$

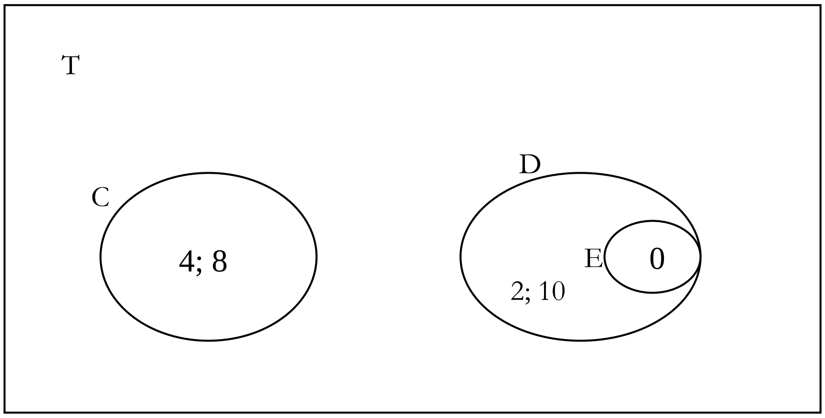
<u>Solution</u>

<u>ii)</u> $D \cap E$ = {10, 2, 0} ∩ {0} = {0}



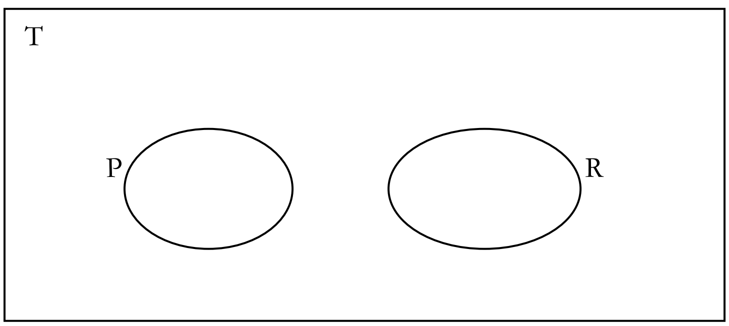
$D \cap E$ = Shaded area

ii) $C \cap D \cap E$ = {4, 8} ∩ {10, 2, 0} ∩ {0} = { } = Ø



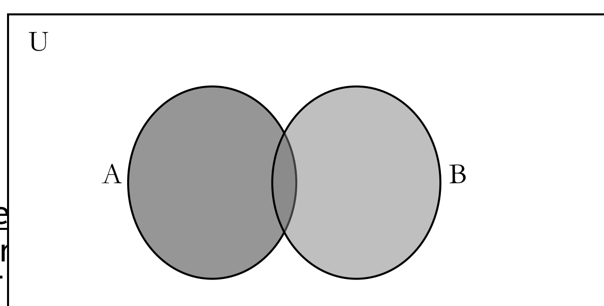
Mutually exclusive or disjointed sets

Two sets are said to be disjointed or mutually exclusive if they have no elements in common. Sets P and R below are disjointed



Disjointed sets are represented by a null set in this case
$P \cap R = Ø$

The union of sets
Venn diagram representing the union of sets A and B or $A \cup B$ = Shaded area is illustrated below;-



Example
Consider ... nd C below:
T
A = {a, d}
B = {b, c, f}
C = {a, c, e, f}

Find

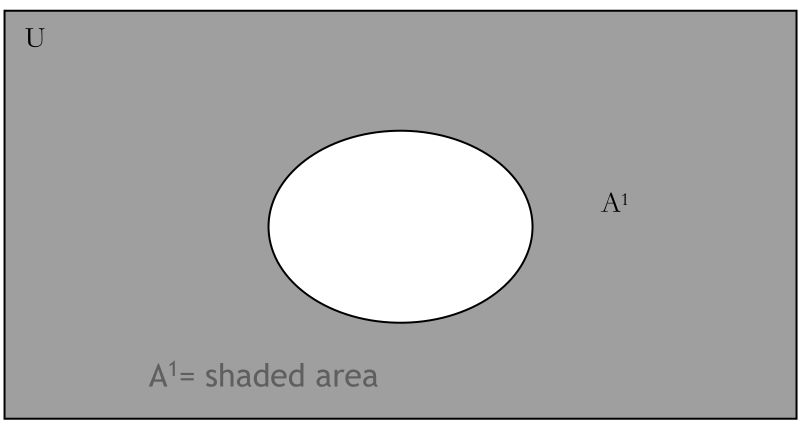
i) $A \cup B$
ii) $A \cup C$
iii) $B \cup C$
iv) $A \cup B \cup C$

Solution

<u>i)</u> $A \cup B$ = {a, d} ∪ {b, c, f} = {a, b, c, d, f}
<u>ii)</u> $A \cup C$ = {a, d} ∪ {a, c, e, f} = {a, c, d, e, f}
<u>iii)</u> $B \cup C$ = {b, c, f} ∪ {a, c, e, f} = {a, b, c, e, f}
<u>iv)</u> $A \cup B \cup C$ = {a, d} ∪ {b, c, f } ∪ {a, c, e, f} = {a, b, c, d, e, f} = T

<u>Complement of a set</u>

Venn diagram representing the complement of a set say A represented by A^1 is illustrated below.



<u>Example</u>

For the universal set T = {1, 2, 3, 4, 5} and its subset A ={2, 3} and B ={5, }

Find

ii) A^1
iii) $(A^1)^1$
iv) $(B^1)^1$

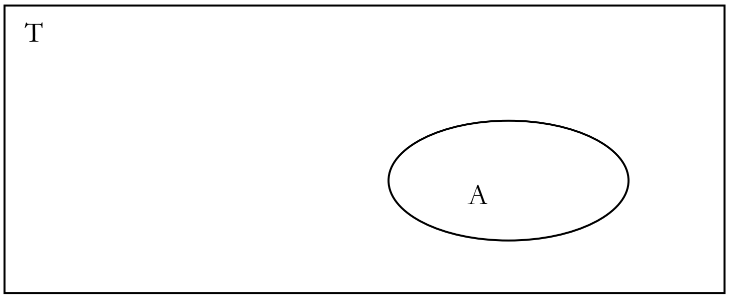
<u>Solution</u>

i) $A^1 = \{2, 3\}^1 = \{1, 4, 5\}$
ii) $(A^1)^1 = (\{2, 3\}^1)^1 = \{1, 4, 5\}^1 = \{2, 3\} = A$

iii) $(B^1)^1 = (\{5\}^1)^1 = \{1, 2, 3, 4\}^1 = \{5\} = B$

Laws of Set Algebra

From the following Venn diagram where T is the universal set and A its subset, we can deduce a number of laws.



i) $A \cup \emptyset = A$
ii) $A \cup T = T$
iii) $A \cup A = A$
iv) $A \cap A = A$
v) $A \cap T = A$
vi) $A \cup A^1 = T$
vii) $A \cap A^1 = \emptyset$
viii) $(A^1)^1 = A$

Applications

Example 1

Of the 20 girls in a form, 16 play hockey 12 play tennis and 4 play basketball. Every girl plays at least one game and two plays all the three. How many play two and only two games.

Solution

N (ξ) = 20