Moment coefficient of Kurtosis $\frac{M4}{S4}$

Example

Find the moment coefficient of the following distribution

X	f
12	1
14	4
16	6
18	10
20	7
22	2

X	F	xf	(x-m)	$(x-m)^2$	$(x-m)^2f$	$(x-m)^4f$
12	1	12	-5.6	31.36	31.36	983.45
14	4	56	-3.6	12.96	51.84	671.85
16	6	96	-1.6	2.56	15.36	39.32
18	10	180	.4	0.16	1.60	0.256
20	7	140	2.4	5.76	40.32	232.24
22	2	44	4.4	19.36	38.72	749.62
	30	528			179.20	2,676.74

$$M = \frac{528}{30} = 17.6$$

$$\sigma^2 = \frac{179.20}{30} = 5.973$$

$$\sigma^4 = 35.677$$

$$M4 = \frac{\sum(x-m)^4 f}{\sum f} = \frac{2,676.74}{30} = 89.22$$

Moment coefficient of Kurtosis = $\frac{89.22}{35.677}$ = 2.5

Note Coefficient of kurtosis can also be found using the method of assumed mean.

CHAPTER FIVE

CORRELATION AND REGRESSION

Specific Objectives

At the end of the topic the trainee should be able to:

- Draw the scatter diagram;
- Differentiate between the various forms of correlation;
- Determine the correlation coefficient and interpret;
- Determine the coefficient of determination and interpret;
- Apply the linear regression models.

Introduction

When the relationship is of a quantitative nature, the appropriate statistical tool for discovering and measuring the relationship and expressing it in a brief formula is known as a correlation. This is an important statistical concept which refers to interrelationship or association between variables. The purpose of studying correlation is for one to be able to establish a relationship, plan and control the inputs (independent variables) and the output (dependent variables).

In business one may be interested to establish whether there exists a relationship between the

i. Amount of fertilizer applied on a given farm and the resulting harvest

ii. Amount of experience one has and the corresponding performance

iii. Amount of money spent on advertisement and the expected incomes after sale of the goods/service

There are two methods that measure the degree of correlation between two variables these are denoted by **R** and **r**.

(a) **Coefficient of correlation** denoted by r, this provides a measure of the strength of association between two variables one the dependent variable the other the independent variable r can range between +1 and – 1 for perfect positive correlation and perfect negative correlation respectively with zero indicating no relation i.e. for perfect positive correlation y increase linearly with x increment.

(b) Rank correlation coefficient denoted by R is used to measure association between two sets of ranked or ordered data. R can also vary from +1, perfect positive rank correlation and -1 perfect negative rank correlation where O or any number near zero representing no correlation.

Significance of the study of correlation

- ➢ Most of the variation shows some kind of relationship between price and supply income and expenditure. With the help of correlation analysis the degree of relationship existing between the variable can be measured.
- ➢ The value of one variable can be estimated once it has been known that they closely related. It can be done with the help of regression analysis.
- ➢ It contributes to the economic behavior, aids in locating the critically important variable on which other depend.
- ➢ Nature has been found to be multiplying of interrelated force.
- ➢ It helps in determining the degree of relationship between two or more variable.

Types of correlation

- Positive and negative correlation
- Simple, partial and multiple
- Linear and non linear.

Positive and negative correlation

If both the variable is varying in the same direction i.e. if one variable is increasing the other on an average is also increasing or if one variable is decreasing the other on an average is also decreasing. Correlation is said to be positive. On the other hand if they are varying in the opposite directions i.e. as one variable is increasing the other is decreasing or vice versa, correlation is said to be negative.

Positive correlation

X	T
10	15
12	20
15	22
18	25
20	37

negative correlation

X	T
20	40
30	30
40	22
60	15
80	16

Simple partial and multiple correlations

The distribution between simple, partial and multiple correlations are based upon the number of variable studied.

When only two variables are studied it is a problem of simple correlation. When three or more variables are studied it is a problem of either

Multiple or partial correlation.

In multiple correlations three or more variables are studied simultaneously.

Linear and non linear correlation

The distinction between linear and non linear correlation is based upon the constancy of the ratio of change between the variables. If the amount of change in one variable tends to bear constant ratio to the amount of change in the other variable then the correlation is said to be linear. If such variable are plotted on a graph paper, all the plotted points would fall on a straight line. Correlation would be non linear or curvilinear, If the amount of change in one variable does not bear constant ratio to the amount of change in the other variable.

Scatter diagram

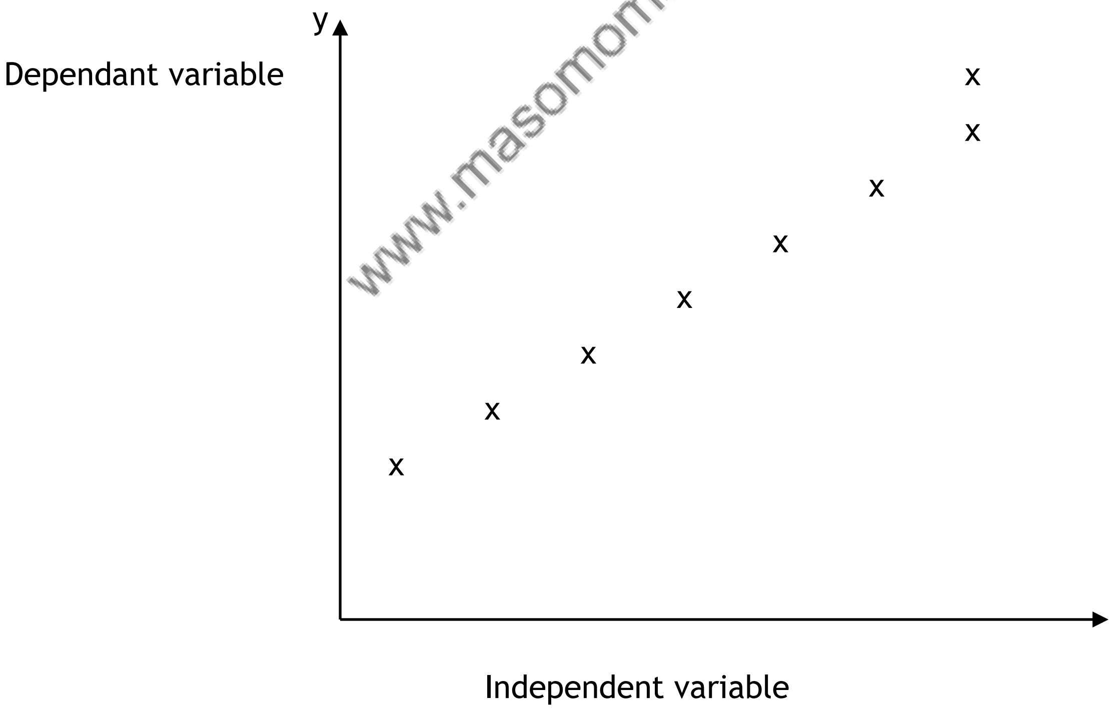
The simplest device for studying correlation in two variables is a special type of dot chart called dotogram or scatter diagram.

A scatter graph is a graph which comprises of points which have been plotted but are not joined by line segments

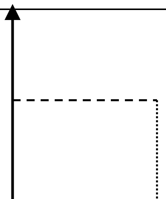
The pattern of the points will definitely reveal the types of relationship existing between variables

The following sketch graphs will greatly assist in the interpretation of scatter graphs.

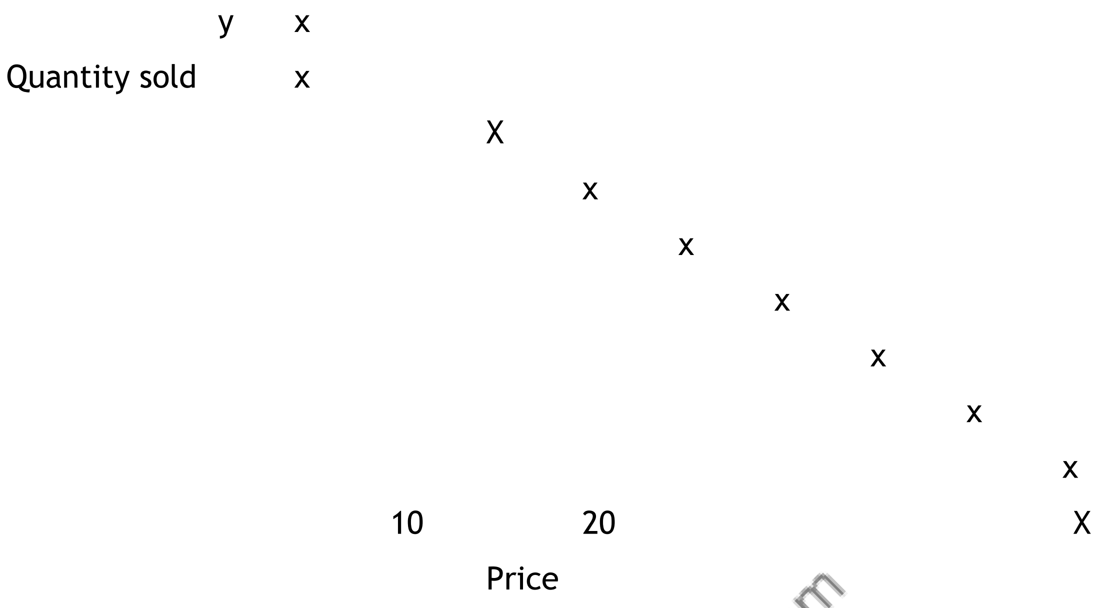
Perfect positive correlation



NB: For the above pattern, it is referred to as perfect because the points may easily be represented by a single line graph e.g. when measuring relationship between volumes of sales and profits in a company, the more the company sales the higher the profits.

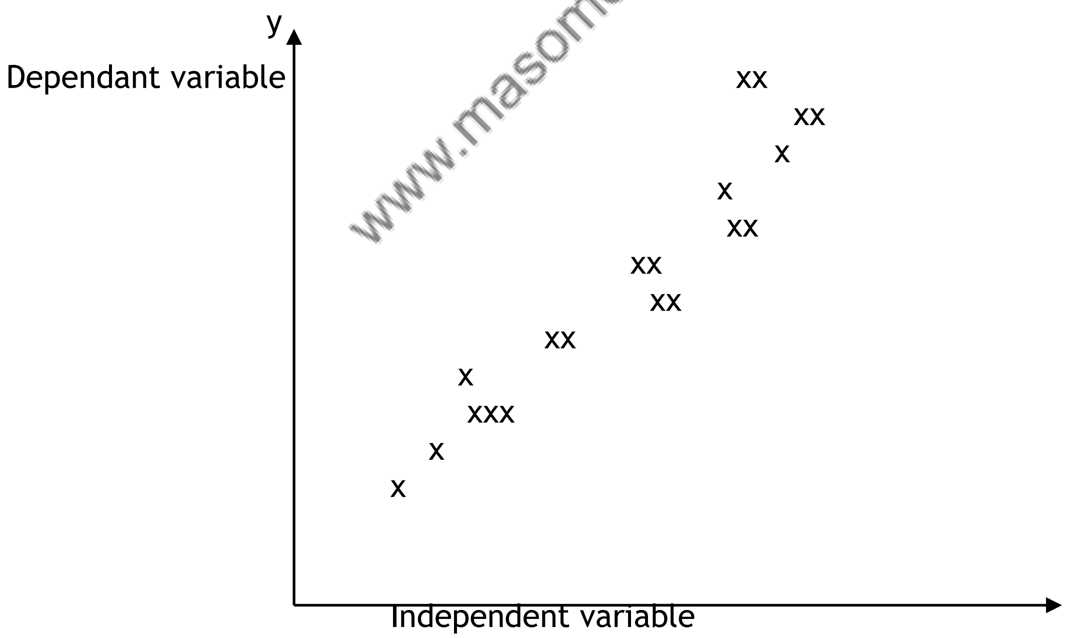


Perfect negative correlation

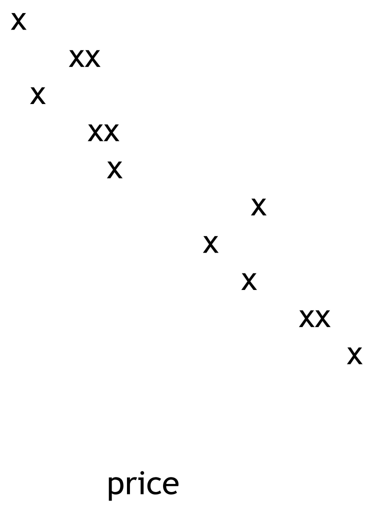


This example considers volume of sale in relation to the price, the cheaper the goods the bigger the sale.

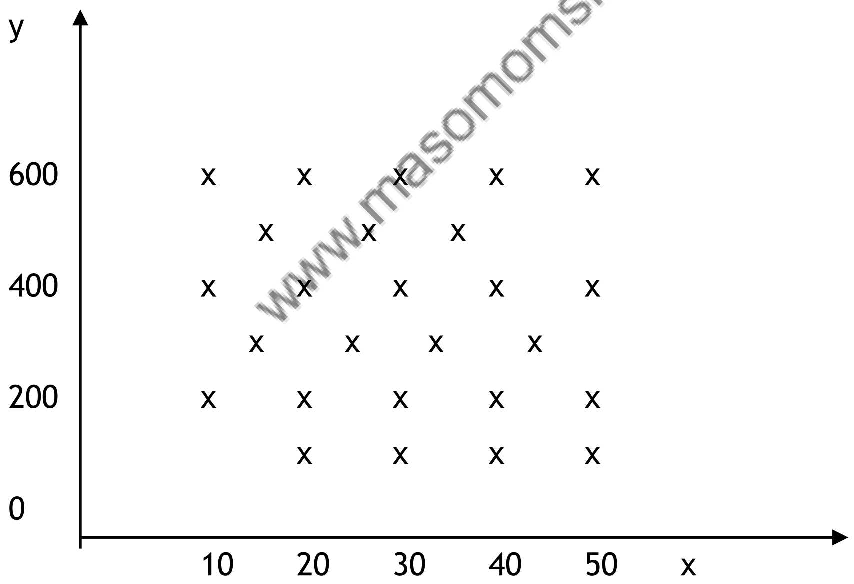
High positive correlation



High negative correlation



No correlation



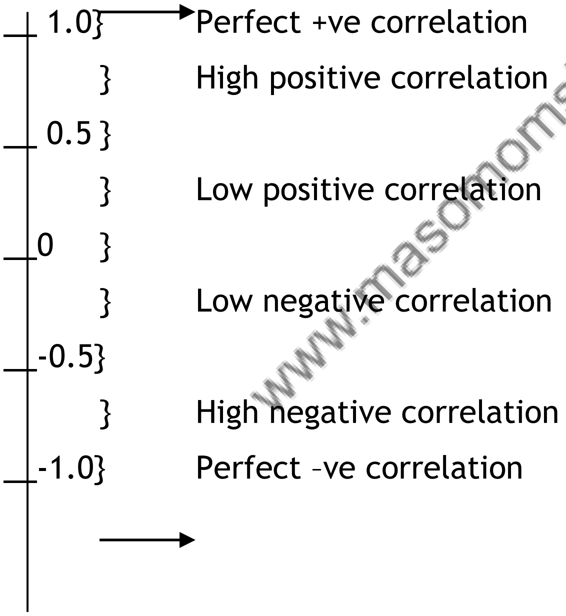
h) Spurious Correlations

in some rare situations when plotting the data for x and y we may have a group showing either positive correlation or –ve correlation but when you analyze the data for x and y in normal life there may be no convincing evidence that there is such a relationship. This implies therefore that the relationship only exists in theory and hence it is referred to as spurious or non sense e.g. when high pass rates of student show high relation with increased accidents.

Correlation coefficient

These are numerical measures of the correlations existing between the dependent and the independent variables. These are better measures of correlation than scatter graphs (diagrams).The range for correlation coefficients lies between +ve 1 and –ve 1. A correlation coefficient of +1 implies that there is perfect positive correlation. A value of –ve shows that there is perfect negative correlation. A value of 0 implies no correlation at all.

The following chart will be found useful in interpreting correlation coefficients.



There are usually two types of correlation coefficients normally used namely;-

Merits of scatter diagram

- It is simple and non mathematical method of studying correlation between the variables.
- It is not influenced by the size of extreme values whereas most of the mathematical methods of findings correlation are influenced by extreme values.

Limitation

- Exact degree of correlation can not be established between the variable

Product Moment Coefficient (r)

It gives an indication of the strength of the linear relationship between two variables.

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2} \times \sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

Note that this formula can be rearranged to have different outlooks but the resultant is always the same.

Example

The following data was observed and it is required to establish if there exists a relationship between the two.

X	15	24	25	30	35	40	45	65	70	75
Y	60	45	50	35	42	46	28	20	22	15

Solution

Compute the product moment coefficient of correlation (r)

X	Y	X^2	Y^2	XY
15	60	225	3,600	900
24	45	576	2,025	1,080
25	50	625	2,500	1,250
30	35	900	1,225	1,050
35	42	1,225	1,764	1,470
40	46	1,600	2,116	1,840
45	28	2,025	784	1,260
65	20	4,225	400	1,300
70	22	4,900	484	1,540
75	15	5,625	225	1,125
$\sum X = 424$	$\sum Y = 363$	$\sum X^2 = 21{,}926$	$\sum Y^2 = 15{,}123$	$\sum XY = 12{,}815$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2} \times \sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{10 \times 12{,}815 - 424 \times 363}{\sqrt{\left(10 \times 21{,}926 - 424^2\right)} \times \sqrt{\left(10 \times 15{,}123 - 363^2\right)}}$$

$$= \frac{-25,762}{\sqrt{(39,484)} \times \sqrt{(19,461)}} = -0.93$$

The correlation coefficient thus indicates a strong negative linear association between the two variables.

Interpretation of r – Problems in interpreting r values

NOTE:

- A high value of r (+0.9 or – 0.9) only shows a strong association between the two variables but doesn't imply that there is a causal relationship i.e. change in one variable causes change in the other it is possible to find two variables which produce a high calculated r yet they don't have a causal relationship. This is known as spurious or nonsense correlation e.g. high pass rates in QT in Kenya and increased inflation in Asian countries.
- Also note that a low correlation coefficient doesn't imply lack of relation between variables but lack of linear relationship between the variables i.e. there could exist a curvilinear relation.
- A further problem in interpretation arises from the fact that the r value here measures the relationship between a single independent variable and dependent variable, where as a particular variable may be dependent on several independent variables (e.g. crop yield may be dependent on fertilizer used, soil exhaustion, soil acidity level, season of the year, type of seed etc.) in which case multiple correlation should be used instead.

The Rank Correlation Coefficient (R)

Also known as the spearman rank correlation coefficient, its purpose is to establish whether there is any form of association between two variables where the variables arranged in a ranked form.

$$R = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

Where d = difference between the pairs of ranked values.

n = numbers of pairs of rankings

Example

A group of 8 accountancy students are tested in Quantitative Techniques and Law II. Their rankings in the two tests were.

Student	Q. T. ranking	Law II ranking	d	d^2
A	2	3	-1	1

B	7	6	1	1
C	6	4	2	4
D	1	2	-1	1
E	4	5	-1	1
F	3	1	2	4
G	5	8	-3	9
H	8	7	1	1
				$\sum d^2 = 22$

d = Q. T. ranking – Law II ranking

R = 1 - $\frac{6\sum d^2}{n\left(n^2-1\right)} = 1 - \frac{6\times 22}{8\left(8^2-1\right)}$

= 0.74

Thus we conclude that there is a reasonable agreement between student's performances in the two types of tests.

NOTE: in this example, if we are given the actual marks then we find r. R varies between +1 and -1.

Tied Rankings

A slight adjustment to the formula is made if some students tie and have the same ranking the adjustment is

$\frac{t^3-t}{12}$ where t = number of tied rankings the adjusted formula

becomes

R = 1 - $\frac{6\left(\sum d^2 + \frac{t^3-t}{12}\right)}{n\left(n^2-1\right)}$

Example

Assume that in our previous example student E & F achieved equal marks in Q. T. and were given joint 3rd place.

Solution

Student	Q. T. ranking	Law II ranking	d	d^2
A	2	3	-1	1
B	7	6	1	1
C	6	4	2	4

D	1	2	-1	1
E	3 ½	5	-1 ½	2 ¼
F	3 ½	1	2 ½	6 ¼
G	5	8	- 3	9
H	8	7	1	1
				$\sum d^2 = 26\frac{1}{2}$

$$R = 1- \frac{6\left(\sum d^2 + \frac{t^3-t}{12}\right)}{n\left(n^2-1\right)} \quad = \quad 1 - \frac{6\left(26\frac{1}{2} + \frac{2^3-2}{12}\right)}{8\left(8^2-1\right)} \qquad \text{since } t = 2$$

$= 0.68$

NOTE: It is conventional to show the shared rankings as above, i.e. E, & F take up the 3rd and 4th rank which are shared between the two as 3½ each.

Coefficient of Determination

This refers to the ratio of the explained variation to the total variation and is used to measure the strength of the linear relationship. The stronger the linear relationship the closer the ratio will be to one.

$$\text{Coefficient determination} = \frac{\text{Explained variation}}{\text{Total variation}}$$

Example (Rank Correlation Coefficient)

In a beauty competition 2 assessors were asked to rank the 10 contestants using the professional assessment skills. The results obtained were given as shown in the table below

Contestants	1st assessor	2nd assessor
A	6	5
B	1	3
C	3	4
D	7	6
E	8	7
F	2	1
G	4	8
H	5	2
J	10	9
K	9	10

REQUIRED

Calculate the rank correlation coefficient and hence comment briefly on the value obtained

			d	d2
A	6	5	1	1
B	1	3	-2	4
C	3	4	-1	1
D	7	6	1	1
E	8	7	1	1
F	2	1	1	1
G	4	8	-4	16
H	5	2	3	9
J	10	9	+1	1
K	9	10	-1	1

$\Sigma d^2 = 36$

∴ The rank correlation coefficient R

$$R = 1 - \frac{6\sum d^2}{n\left(n^2-1\right)}$$

$$= 1 - \frac{6\times 36}{10\left(10^2-1\right)}$$

$$= 1 - \frac{216}{990}$$

$$= 1 - 0$$

Comment: since the correlation is 0.78 it implies that there is high positive correlation between the ranks awarded to the contestants. 0.78 > 0 and 0.78 > 0.5

Example

Contestant	1st assessor	2nd assessor	d	d^2
A	1	2	-1	1
B	5 (5.5)	3	2.5	6.25
C	3	4	-1	1
D	2	1	1	1
E	4	5	-1	1
F	5 (5.5)	6.5	-1	1

G	7	6.5	-0.5	0.25
H	8	8	0	0

Σd^2 = 11.25

Required: Complete the rank correlation coefficient

$$\therefore R = 1 - \frac{6\sum d^2}{n\left(n^2-1\right)} = 1 - \frac{6\times 11.25}{8\left(63\right)}$$

$$= 1 - \frac{67.5}{504}$$

= 1 – 0.13

= 0.87

This implies high positive correlation

Example (Rank Correlation Coefficient)

Sometimes numerical data which refers to the quantifiable variables may be given after which a rank correlation coefficient may be worked out.

Is such a situation, the rank correlation coefficient will be determined after the given variables have been converted into ranks. See the following example;

Candidates	Math	r	Accounts	r	d	d2
P	92	1	67	5	-4	16
Q	82	3	88	1	2	4
R	60	5(5.5)	58	7(7.5)	-2	4
S	87	2	80	2	0	0
T	72	4	69	4	0	0
U	60	5(5.5)	77	3	-2.50	6.25
V	52	8	58	7(7.5)	0.5	0.25
W	50	9	60	6	3	9
X	47	10	32	10	0	0
Y	59	7	54	9	-2	4

Σd^2 = 43.5

$$\therefore \text{Rank correlation } r = 1 - \frac{6\sum d^2}{n\left(n^2-1\right)}$$

$$= 1 - \frac{6\times 43.5}{10\left(10^2-1\right)} = 1 - \frac{261}{990}$$

= 0.74 (High positive correlation between mathematics marks and accounts)

Example

(Product moment correlation)

The following data was obtained during a social survey conducted in a given urban area regarding the annual income of given families and the corresponding expenditures.

Family	(x)Annual income £ 000	(y)Annual expenditure £ 000	xy	x^2	Y^2
A	420	360	151200	176400	129600
B	380	390	148200	144400	152100
C	520	510	265200	270400	260100
D	610	500	305000	372100	250000
E	400	360	144000	160000	129600
F	320	290	92800	102400	84100
G	280	250	70000	78400	62500
H	410	380	155800	168100	144400
J	380	240	91200	144400	57600
K	300	270	81000	90000	72900
Total	4020	3550	1504400	1706600	1342900

Required

Calculate the product moment correlation coefficient briefly comment on the value obtained

The produce moment correlation

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\times\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

Workings:

$\overline{X} = \frac{4020}{10} = 402$ $\qquad$ $\overline{Y} = \frac{3550}{10} = 355$

$$r = \frac{10(1,504,400) - (4020)(3550)}{\sqrt{10(1,706,600) - 4020^2}\times\sqrt{10(1,342,900) - (3550)^2}}$$

= 0.89

Comment: The value obtained 0.89 suggests that the correlation between annual income and annual expenditure is high and positive. This implies that the more one earns the more one spends.

REGRESSION

BASIC CONCEPTS

This is a concept, which refers to the changes which occur in the dependent variable as a result of changes occurring on the independent variable. Knowledge of regression is particularly very useful in business statistics where it is necessary to consider the corresponding changes on dependant variables whenever independent variables change. It should be noted that most business activities involve a dependent variable and either one or more independent variable. Therefore knowledge of regression will enable a business statistician to predict or estimate the expenditure value of a dependant variable when given an independent variable e.g. consider the above example for annual incomes and annual expenditures. Using the regression techniques one can be able to determine the estimated expenditure of a given family if the annual income is known and vice versa.

Difference between correlation and regression analysis

There are two important points of difference between correlation and regression analysis

1. Correlation coefficient as a measure of degree of relationship between x and y. while regression analysis is to study the nature of relationship between the variables.
2. The cause and effect relation is clearly indicated through regression analysis while correlation is merely a tool of ascertaining the degree of relationship between two variables.

The general equation used in simple regression analysis is as follows

$$y = a + bx$$

Where y = Dependant variable

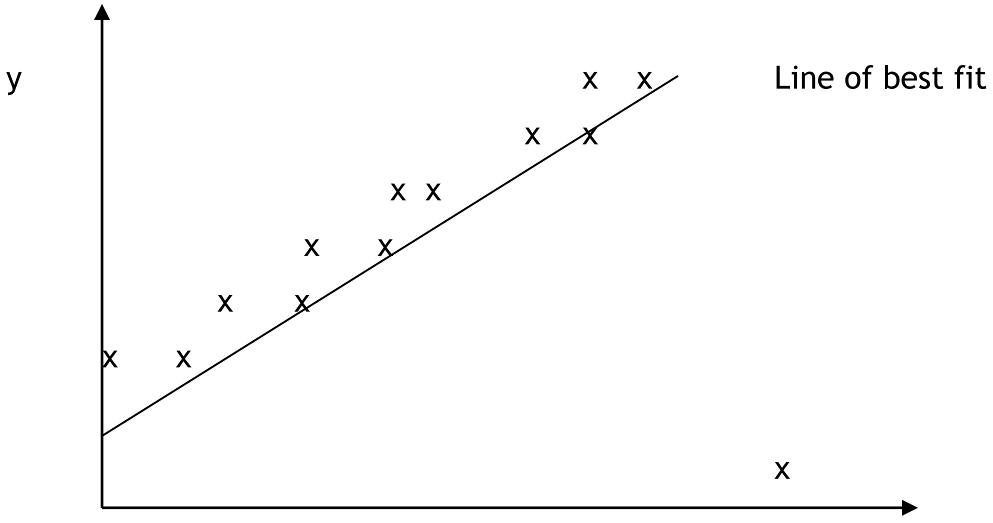
a= Interception y axis (constant)

b = Slope on the y axis

x = Independent variable

i. The determination of the regression equation such as given above is normally done by using a technique known as "the method of least squares'.

Regression equation of y on x i.e. y = a + bx



The following sets of equations normally known as normal equation are used to determine the equation of the above regression line when given a set of data.

$\Sigma y = an + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

Where Σy = Sum of y values

Σxy = sum of the product of x and y

Σx = sum of x values

Σx^2= sum of the squares of the x values

a = The intercept on the y axis

b = Slope gradient line of y on x

NB: The above regression line is normally used in one way only i.e. it is used to estimate the y values when the x values are given.

Regression line of x on y i.e. x = a + by

- The fact that regression lines can only be used in one way leads to what is known as a regression paradox
- This means that the regression lines are not ordinary mathematical line graphs which may be used to estimate the x and y simultaneously
- Therefore one has to be careful when using regression lines as it becomes necessary to develop an equation for x and y before doing the estimation.

The following example will illustrate how regression lines are used

Example

An investment company advertised the sale of pieces of land at different prices. The following table shows the pieces of land their acreage and costs

Piece of land	(x)Acreage Hectares	(y) Cost £ 000	xy	x^2
A	2.3	230	529	5.29
B	1.7	150	255	2.89
C	4.2	450	1890	17.64
D	3.3	310	1023	10.89
E	5.2	550	2860	27.04
F	6.0	590	3540	36
G	7.3	740	5402	53.29
H	8.4	850	7140	70.56
J	5.6	530	2969	31.36
	Σx =44.0	Σy = 4400	Σxy= 25607	Σx^2 = 254.96

Required

Determine the regression equations of

i. y on x and hence estimate the cost of a piece of land with 4.5 hectares

ii. Estimate the expected average if the piece of land costs £ 900,000

$\Sigma y = an + b\Sigma xy$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

By substituting of the appropriate values in the above equations we have

$4400 = 9a + 44b$ …….. (i)

$25607 = 44a + 254.96b$ …….. (ii)

By multiplying equation …. (i) by 44 and equation …… (ii) by 9 we have

$193600 = 396a + 1936b$ …….. (iii)

$230463 = 396a + 2294.64b$ …….. (iv)

By subtraction of equation …. (iii) from equation …… (iv) we have

$36863 = 358.64b$

$102.78 = b$

by substituting for b in …….. (i)

$4400 = 9a + 44(102.78)$

$4400 – 4522.32 = 9a$

$-122.32 = 9a$

$-13.59 = a$

Therefore the equation of the regression line of y on x is

$Y = 13.59 + 102.78x$

When the acreage (hectares) is 4.5 then the cost

$(y) = -13.59 + (102.78 \text{ x } 4.5)$

$= 448.92$

= £ 448, 920

Note that

Where the regression equation is given by

$y= a + bx$

Where a is the intercept on the y axis and

b is the slope of the line or regression coefficient
n is the sample size

then,

intercept a = $\frac{\sum y - b\sum x}{n}$

Slope b = $\frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$

Example

The calculations for our sample size n = 10 are given below. The linear regression model is

y = a + bx

Table

Distance x miles	Time y mins	xy	x^2	y^2
3.5	16	56.0	12.25	256
2.4	13	31.0	5.76	169
4.9	19	93.1	24.01	361
4.2	18	75.6	17.64	324
3.0	12	36.0	9.0	144
1.3	11	14.3	1.69	121
1.0	8	8.0	1.0	64
3.0	14	42.0	9.0	196
1.5	9	13.5	2.25	81
4.1	16	65.6	16.81	256
Σx = 28.9	**Σy = 136**	**Σxy = 435.3**	**$\Sigma x^2 =$ 99.41**	**$\Sigma y^2 =$ 1972**

The Slope b = $\frac{10\times 435.3 - 28.9\times 136}{10\times 99.41 - 28.9^2} = \frac{422.6}{158.9}$

= 2.66

and the intercept a = $\frac{136 - (2.66\times 28.9)}{10}$

= 5.91

We now insert these values in the linear model giving

y = 5.91 + 2.66x

or

Delivery time (mins) = 5.91 + 2.66 (delivery distance in miles)

The slope of the regression line is the estimated number of minutes per mile needed for a delivery. The intercept is the estimated time to prepare for the journey and to deliver the goods which is the time needed for each journey other than the actual traveling time.

PREDICTION WITHIN THE RANGE OF SAMPLE DATA

We can use the linear regression model to predict the mean of dependant variable for any given value of independent variable

For example if the sample model is given by

Time (min) = 5.91 + 2.66 (distance in miles)

Then if the distance is 4.0 miles then our estimated mean time is

$\acute{Y} = 5.91 + 2.66 \times 4.0 = 16.6$ minutes

Multiple Linear Regression Models

There are situations in which there is more than one factor which influence the dependent variable

Example

Cost of production per week in a large department depends on several factors;

i. Total numbers of hours worked
ii. Raw material used during the week
iii. Total number of items produced during the week
iv. Number of hours spent on repair and maintenance

It is sensible to use all the identified factors to predict department costs

Scatter diagram will not give the relationship between the various factors and total costs

The linear model for multiple linear regression if of the type; (which is the line of best fit).

$y = \alpha + b_1x_1 + b_2x_2 + \ldots\ldots\ldots + b_nx_n$

We assume that errors or residuals are negligible.

In order to choose between the models we examine the values of the multiple correlation coefficient r and the standard deviation of the residuals α.

A model which describes well the relationship between y and x's has multiple correlation coefficient r close to ±1 and the value of α which is small.

Example

Odino chemicals limited are aware that its power costs are semi variable cost and over the last six months these costs have shown the following relationship with a standard measure of output.

Month	Output (standard units)	Total power costs £ 000
1	12	6.2
2	18	8.0
3	19	8.6
4	20	10.4
5	24	10.2
6	30	12.4

Required

i. Using the method of least squares, determine an appropriate linear relationship between total power costs and output

ii. If total power costs are related to both output and time (as measured by the number of the month) the following least squares regression equation is obtained

Power costs = 4.42 + (0.82) output + (0.10) month

Where the regression coefficients (i.e. 0.82 and 0.10) have t values 2.64 and 0.60 respectively and coefficient of multiple correlation amounts to 0.976

Compare the relative merits of this fitted relationship with one you determine in (a). Explain (without doing any further analysis) how you might use the data to forecast total power costs in seven months.

Solution

a)

Output (x)	**Power costs (y)**	**x^2**	**y^2**	**xy**
12	6.2	144	38.44	74.40
18	8.0	324	64.00	144.00
19	8.6	361	73.96	163.40
20	10.4	400	108.16	208.00
24	10.2	576	104.04	244.80
30	12.4	900	153.76	372.00
Σx = 123	**Σy = 55.8**	**Σx^2 = 2705**	**Σy^2 = 542.36**	**Σxy= 1,206.60**

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$= \frac{6\times1206.6 - 123\times55.8}{6\times2705 - (123)^2}$$

$$= \frac{376.2}{1101} = 0.342$$

a $= \frac{1}{n}$ (Σy - bΣx)

$= \frac{1}{6} \times (55.8 - 0.342) \times 123$

= 2.29

∴ (Power costs) = 2.29 + 0.342 (output)

b. For linear regression calculated above, the coefficient of correlation r is

$$r = \frac{(6\times1206.6)-(123\times55.8)}{\sqrt{6\times2705-123\times123}\sqrt{6\times542.36-55.8\times55.8}}$$

$$= \frac{376.2}{\sqrt{1101\times140.52}}$$

= 0.96

This show a strong correlation between power cost and output. The multiple correlations when both output and time are considered at the same time are 0.976.

We observe that there has been very little increase in r which means that inclusion of time variable does not improve the correlation significantly

The value for time variable is only 0.60 which is insignificant as compared with a t value of 2.64 for the output variable

In fact, if we work out correlation between output and time, there will be a high correlation. Hence there is no necessity of taking both the variables. Inclusion of time does improve the correlation coefficient but by a very small amount.

If we use the linear regression analysis and attempt to find the linear relationship between output and time i.e.

Month	**Output**
1	12
2	18
3	19
4	20
5	24
6	30

The value of b and a will turn out to be 3.11 and 9.6 i.e. relationship will be of the form

Output = 9.6 + 3.11 × month

For this equation forecast for 7th month will be

Output = 9.6 + 3.11 × 7
= 9.6 + 21.77
= 31.37 units

Using the equation, Power costs = 2.29 + 0.34 × output
= 2.29 + 0.34 × 31.37
= 2.29 + 10.67
= 12.96 i.e. £ 12,960

Non Linear Relationships

If the scatter diagram and the correlation coefficient do not indicate linear relationship, then the relationship may be non – linear

Two such relationships are of peculiar interest

$y = ab^x$ *and* $y = ax^b$

Both of these can be reduced to linear model. Simple or multiple linear regression methods are then used to determine the values of the coefficients

i. **Exponential model**

$y = ab^x$

Take log of both sides

log y = log a + log b^x

log y = log a + xlog b

Let log y = Y and log a = A and log b = B

Thus we get Y = A + Bx. This is a linear regression model

ii. **Geometric model**

$y = ax^b$

Using the same technique as above

Log y = log a + blog x

Y = A + bX

Where Y = log y
A = log a
X = log x

Using linear regression technique (the method of least squares), it is possible to calculate the value of a and b