PORTFOLIO MANAGEMENT

EQUITIES MANAGEMENT

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Table of contents

* final level

1. Equities Management*

1.1 Principles of equity management*

All asset (and therefore equity) management processes rely on several building blocks which, when combined, form the end product, i.e. the managed portfolio. The three main building blocks relate to:

- Risk control
- Asset return forecast
- Portfolio construction

This section is mainly devoted to the principles underlying the three themes. First, techniques used to assess risk will be described, focussing initially on the definition of risk and then on forecasting it. Second, some general considerations about the predictions of future asset returns will be described. Finally, portfolio construction techniques will be considered describing how they combine the knowledge of the previous two themes to create portfolios.

As this chapter covers both active and passive management for equity portfolios, we want to specify explicitly the self-imposed limits of this discussion:

- We shall focus on a single country framework in this description of equity management, because the problem of allocating equities between different countries is more of a problem of allocating between asset classes than a pure equity management issue. Having discarded the international aspect of active and passive equity management, we will also not address the issue of currency management[1,2].
- Dealing with pure equity management, we shall focus on portfolio management with respect to a benchmark. Within this framework, volatility as a risk measure is not too restrictive. The desired asymmetry in the distribution of absolute returns[3], which is frequently expressed by investors, is addressed more thoroughly through allocation of asset classes with techniques such as portfolio insurance or tactical allocation. Although it is true that investors prefer positive to negative relative returns with respect to a benchmark, it is difficult to find opportunities to realise such preferences. Within this well-defined framework, we will be able to use the mean variance paradigm of modern portfolio theory.

1 Notice that because of all the derivatives and forward contracts on currencies the portfolio's currency management can be largely separated from the management of its underlying assets.

2 International equity management is covered in the chapter on "International investments and value at risk".

3 Or for a positive skewness.

1.1.1 *Risk in operational terms**

In this section we will focus on the risk as perceived by the portfolio manager who has a mandate to manage an equity portfolio with respect to a benchmark. In terms of performance, it is clear that the management has to be assessed based on the difference of return between the managed portfolio and the benchmark, i.e. on what is known as the *active return*[4]. In the mean variance paradigm, the volatility of this active return, measured either in terms of variance or standard deviation, is the relevant concept of risk for the portfolio manager. This risk is called the *active risk*.

The portfolio manager is subject to another risk as he is usually in competition with a peer group of other portfolio managers. To have active returns distinct from the ones of the peer group can be dangerous in case your performance is worse than the peer group average. For the rest of this chapter we will only focus on the active risk described above.

Note that the investor also bears the active risk, i.e. the risk that his portfolio performance may diverge from the performance of the specified benchmark. But in addition unlike a portfolio manager, the investor also bears the risk of the benchmark, i.e. the volatility of the benchmark's return in excess of the risk free rate.

1.1.1.1 *Tracking error in a broad sense**

When thinking about what is relevant to both the investor and the portfolio manager in terms of active return, it should be clear that it is its full probability distribution that should matter. It does really matter to both the investor and the portfolio manager:

- if there is a systematic difference[5] in performance between the portfolio and its benchmark which correspond to the usual average over- or under-performance concept,
- about the volatility[6] of the active return which has been described as the active risk,
- if there is an asymmetry[7] between positive and negative active returns, realising their (not at all surprising) preference for over-performances versus under-performances.

The full probability distribution of the active return is called the *tracking error* in a broad sense. Unfortunately this terminology has been obscured as the term tracking error - is also often used in a more restrictive sense designating only the volatility of the active return assessed in terms of a standard deviation.

1.1.1.2 *Tracking error in a restrictive sense**

The tracking error of a portfolio, in a restrictive sense, is the standard deviation of the difference between the portfolio and the benchmark returns. It is a measure of the volatility of this difference in returns. In other words, it gives an idea how widely or closely the portfolio return fluctuates around the benchmark return.

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4 The terminology of active return is somewhat unfortunate as this difference in return between the managed portfolio and the benchmark is also relevant and therefore used in passive management.

5 This is the first moment of the probability distribution of the active return.

6 This is linked to the second moment of the probability distribution of the active return.

7 This is linked to the third moment of the probability distribution of the active return.

This concept of tracking error can be forward or backward looking:

- The ex-ante or prospective tracking error is forward looking. It is a forecast of the volatility in the return difference between the portfolio and its benchmark for a specified future period of time.
- The ex-post or retrospective tracking error is backward looking. It is a measure of the volatility in the return difference between the portfolio and its benchmark for a specified past period of time.

To say that a portfolio has an ex-ante annual tracking error of 2% means that the standard deviation of the return difference between the portfolio and the benchmark is forecasted to be 2% for the next twelve months. Assuming both normality in the distribution of active return and a zero mean it means that there are:

- 2 chances out of 3 that the difference in return between the portfolio and the benchmark will be between –2% and + 2% over the next twelve months.
- 19 chances out of 20 that the difference in return between the portfolio and the benchmark will be between –4% and +4% over the next twelve months.

Notice that it does not mean this difference in returns is bounded in any way. Indeed the two last statements imply also that there is:

- 1 chance out of 3 that the difference in return between the portfolio and the benchmark will be either below –2% or above + 2% over the next twelve months.
- 1 chance out of 20 that the difference in return between the portfolio and the benchmark will be either below –4% or above +4% over the next twelve months.

Similarly, to say that a portfolio had an ex-post annualised tracking error of 2% over a specified time period means that it has been estimated using the appropriate statistical tools and the data over the specified time period. These statistical tools are briefly described in the following paragraphs.

To illustrate this let us consider that for a given portfolio we have N months of active returns of portfolio P with respect to benchmark B:

$$r_t^{P,B} \quad (t = 1, \ldots, N)$$

We can then estimate the monthly ex-post tracking error of the portfolio with respect to its benchmark, $\sigma_m^{P,B}$, over the N observations using the usual standard deviation estimate formula:

$$\sigma_m^{P,B} = \sqrt{\frac{1}{N-1} \cdot \sum_{t=1}^{N} \left(r_t^{P,B} - \bar{r}^{P,B} \right)^2} \qquad \text{where} \qquad \bar{r}^{P,B} = \frac{1}{N} \cdot \sum_{t=1}^{N} r_t^{P,B}$$

Then we can transform the monthly tracking error into an annualised one, $\sigma_a^{P,B}$, applying the following formula:

$$\sigma_a^{P,B} = \sqrt{12} \cdot \sigma_m^{P,B}$$

This seems to be easy. However one important caveat is necessary. The application of the above statistical formula relies on the assumption that the monthly active returns are independent through time. This is a strong assumption which is often not verified. Examples can easily be found where this assumption is not met. They are characterized by negative (positive) auto-correlations of the active returns which leads, when applying the aforementioned formulas, to a biased estimate of the ex-post tracking error which overstates (understates) it.

*1.1.1.3 The multi-factor model approach**

To make the concept of ex-ante tracking error operational one needs a forecasting mechanism for it. The framework of a multi-factor model (MFM) is especially suited to provide it.

The MFM is based on the assumption that asset returns are determined by their common but distinct sensitivities to the returns of a set of factors[8] and by a return component uniquely specific to each asset.

This can be expressed by the following formula (linear combination) as follows:

$$R_i = \sum_{j=1}^{NF} x_{i,j} \cdot F_j + E_i$$

Where:

- R_i Excess return[9] of an asset i (i = 1, …, N),
- $x_{i,j}$ Exposure (factor-beta respectively factor-loading) of asset i to factor j,
- F_j Excess return of factor j (j = 1, … , NF),
- E_i Specific return of asset i (residual return).

This can be rewritten more concisely using vector notation[10]:

$$R_i = x_i' \cdot F + E_i$$

Where:

$x_i' = \left(x_{i,1}, \dots, x_{i,NF}\right)$ is the 1 x NF vector of asset i exposures,

$F = \begin{bmatrix} F_1 \\ \dots \\ F_{NF} \end{bmatrix} = \left(F_1, \dots, F_{NF}\right)'$ is the NF x 1 vector of factor returns,

8 This response of the asset returns to common factors formalizes the fact, confirmed by evidence, that asset returns tend to move together or, in other words, they are correlated. Indeed beyond their tendency to follow the market, one can easily observe that smaller groups of stocks also tend to move together, but distinctively from the market, such as the stocks of an industry, interest rate sensitive stocks, small cap stocks, etc…

9 The excess return of an asset is its total return minus the risk free rate for the same time period.

10 Notation: x' denotes the vector (matrix) transpose of x.

This can be generalized to a portfolio P that, in this framework, can be expressed as a vector of the weights w_i^P for each asset:

$$w_P^{'} = \left(w_1^P,...,w_N^P\right) \qquad \text{where} \qquad \sum_{i=1}^{N} w_i^P = 1$$

Using basic linear algebra the return of the portfolio can then be expressed as:

$$R_P = x_P^{'} \cdot F + E_P$$

Where:

$$x_{P,j} = \sum_{i=1}^{N} w_i^P \cdot x_{i,j} \quad \left(j = 1,\dots, NF\right) \qquad (j = 1,\dots, NF)$$

$$E_P = \sum_{i=1}^{N} w_i^P \cdot E_i$$

In other words the exposure of the portfolio to factor j is a weighted average of the exposures of the assets in the portfolio. Similarly, the specific return of the portfolio is also a weighted average of the specific returns of its assets.

Given straightforward stochastic assumptions[11] concerning the returns of the factors and the specific return of the asset one can show that the volatility of the portfolio in terms of variance can be expressed as:

$$V\left(R_P\right) = x_P^{'} \cdot W \cdot x_P + s_P^2 \qquad s_P^2 = \sum_{i=1}^{N} (w_i^P)^2 \cdot s_i^2$$

Where:

W is the covariance matrix[12] of vector F, i.e. of the factor returns.

s_i^2 is the variance of asset i specific return.

We will focus in this chapter on active risk, i.e. on the risk of a portfolio relative to its benchmark. Let us assume without loss of generality that the benchmark is also a portfolio B. The active return of the portfolio relative to its benchmark is given by:

$$R_A^{P,B} = R_P - R_B$$

The active risk, defined as the tracking error, is the volatility of the active return expressed in terms of standard deviation:

$$TE^{P,B} = \sqrt{V(R_A^{P,B})}$$

11 The factor returns and the specific returns are assumed to be uncorrelated. In addition, the specific returns of two distinct assets are also assumed to be uncorrelated.

12 Its dimension is NF x NF.

Using the MFM approach to formalise the tracking error, we get:

$$TE^{P,B} = \sqrt{(x'_P - x'_B) \cdot W \cdot (x_P - x_B) + \sum_{i=1}^{N} (w_i^P - w_i^B)^2 \cdot s_i^2}$$

or:

$$TE^{P,B} = \sqrt{\sum_{i=1,j=1}^{NF} (x'_{P,i} - x'_{B,i}) \cdot W_{i,j} \cdot (x_{P,j} - x_{B,j}) + \sum_{i=1}^{N} (w_i^P - w_i^B)^2 \cdot s_i^2}$$

Notice that it is not the purpose in this section to explain how MFM's are determined and estimated. In the next section we will focus on how the MFM approach can be used to forecast the ex-ante tracking error.

*1.1.1.4 Forecasting the tracking error**

In terms of determining the ex-ante tracking error, there are sophisticated statistical techniques to forecast both the covariance matrix of the factor returns and the specific return variance for all stocks. Let us denote respectively these forecasts as $\tilde{W}$ and $\tilde{s}_i^2$ (i=1, …, N). Given these forecasts and knowing the assets weight and the factor exposures of both the portfolio and the benchmark, it is then possible to get a forecast for tracking error or, in other words, to determine the ex-ante tracking error:

$$\tilde{TE}^{P,B} = \sqrt{(x'_P - x'_B) \cdot \tilde{W} \cdot (x_P - x_B) + \sum_{i=1}^{N} (w_i^P - w_i^B)^2 \cdot \tilde{s}_i^2}$$

One final point is worth mentioning. The ex-ante tracking error is a forecast, and as such it may or may not be accurate. To illustrate this, consider a portfolio with a 2% ex-ante tracking error. After a year, the divergence in performance between the portfolio and its benchmark is –4%. Several interpretations of this fact can be forwarded:

- First, the risk forecast was correct and a very improbable (2 standard deviations) event took place.
- Second, the risk forecast was largely under-estimated which makes the –4% under-performance event much less improbable.

Both these scenarios are plausible. However, given the previous facts it is impossible to discriminate between the two. This leads to the observation that it is difficult for a user of a MFM risk model to assess how such a model has performed, i.e. if it is forecasting accurately or if it is under- or over-predicting future levels of risk.

Another benefit of the MFM framework is that it allows a breakdown of both ex-ante and ex-post active risk into components when expressed in terms of variance. Using the previous formula, we have:

$(\tilde{TE}^{P,B})^2 =$

$(x'_{P,1} - x'_{B,1}) \cdot \tilde{W}_{1,1} \cdot (x_{P,1} - x_{B,1})$	direct risk contribution of factor 1
$+ \ldots..$	
$+(x'_{P,NF} - x'_{B,NF}) \cdot \tilde{W}_{NF,NF} \cdot (x_{P,NF} - x_{B,NF})$	direct risk contribution of factor NF
$+ \sum_{i=1, j=1, i \neq j}^{NF} (x'_{P,i} - x'_{B,i}) \cdot \tilde{W}_{i,j} \cdot (x_{P,j} - x_{B,j})$	cross contribution of all factors
$+ \sum_{i=1}^{N} (w_i^P - w_i^B)^2 \cdot \tilde{s}_i^2$	specific contribution

By analysing these different components, one can determine where the active risk comes from, if it is through active exposure to one or several factors or through lack of diversification.

*1.1.1.5 MFM and optimisation**

For most portfolio managers the development of a MFM risk model is not possible because it requires a lot of resources and expertise in order to do so. Fortunately, these risk models are commercially available at a reasonable price. By subscribing, the portfolio manager will periodically[13] receive, for the universe of stocks covered by the risk model, the following data:

- The factor exposure of all stocks to all factors $x_{i,j}$ (i=1, … N; j = 1, …, NF),
- A forecast for the covariance matrix of the factor returns $\tilde{W}$,
- Forecast for the variance of the stocks' specific returns $\tilde{s}_i^2$ (i=1, …, N).

i.e. all the necessary information to be able to forecast a tracking error and its breakdown. Usually the providers of a risk model will also deliver software, which executes all the risk computation, and a tool, called an optimiser, that makes it possible to build portfolios that takes all the information of the related risk model fully into account.

These optimisers solve the following problem[14]:

$$\underset{\{w_i^P : i=1,\ldots,N\}}{\text{MAX}} \ \tilde{R}_A^{P,B} - v \cdot \tilde{V}(R_A^{P,B}) \quad \text{with} \quad \sum_{i=1}^{N} w_i^P = 1$$

Given a forecast of the asset returns, the optimisers identify the set of asset weights that maximise the forecasted return of the resulting portfolio after adjusting for a penalty linked to the forecasted risk (variance) of the same portfolio. The parameter 'v' allows the investor to express how much he values the forecasted risk in terms of returns. By raising it, he will tilt the trade-off between risk and return against taking risk. By doing so, he places a more negative value on risk than before.

13 Usually on a monthly basis.

14 Technically, this maximisation problem is called a quadratic program (as long as the transaction function TC is also quadratic and convex).

We can see here that this approach is very close to the Mean-Variance approach to portfolio selection discussed in the chapter on "Modern portfolio theory I". Indeed the portfolio that maximizes the Sharpe Ratio was explicitely obtained in this chapter as a solution to the same optimization problem. In "Modern portfolio theory I", however, we took the risky assets' expected returns and covariances as given, and focused on the optimal way to diversify the portfolio. In this chapter we take a more applied view by tackling the estimation problem of the returns distribution as well.

*1.1.2 Risk control**

In this section we will consider two major techniques that can be used to control indirectly or directly the active risk of a portfolio. The first one can be easily implemented with a minimum of information while the second requires access to a risk model.

*1.1.2.1 Maximum weight deviation**

This technique consists of controlling the maximum weight deviation, between the managed portfolio and its benchmark, at the single stock level and/or at the level of a group of stocks.

For example, a portfolio manager can decide that no assets in his portfolio can be under- or over-weighted by more than a limit $\underline{w}$. In other words, for all stocks in the portfolio and/or in the benchmark, the following restriction should be respected:

$$\left| w_P^i - w_B^i \right| \leq \underline{w} \quad (i = 1, \ldots, N)$$

For example in laymen terms, the portfolio manager may decide that in absolute terms the active weight of any stocks cannot be over 2.5%.

Similarly, if the universe of assets can be partitioned in NC asset classes C_j (j= 1, …, NC), the portfolio manager can also decide that he will not accept any under- and/or over-weighting above $\underline{c}$:

$$\left| \sum_{i \in C_j} w_P^i - \sum_{i \in C_j} w_B^i \right| \leq \underline{c} \qquad (j = 1, \ldots, NC)$$

These asset classes relate usually to countries, industries, value/growth style, large/middle/small size stocks, i.e. subsets of stocks that are characterized by distinctive return behaviour. By limiting the over- or under-weighting in these asset classes, one limits the volatility of the return difference between a portfolio and its benchmark. As an example, a portfolio manager may not want to be under- and/or over-weighted to a country, to an industry/sector, or to value stocks more than a certain percentage.

Several remarks are necessary:

- This technique is simple because the only information which is needed to implement it are the weights of the stocks both in the portfolio and in the benchmark together with the fact that these stocks do or do not belong to the relevant asset classes.
- The value of constant limits depends on the underlying volatility of the stocks or of the groups of stocks together with the risk aversion of the portfolio manager. For a given risk aversion, the higher the volatility of the asset returns, the lower should be the width of the allowed weights band.

- In terms of risk, the control is only indirect because the volatility of the active return is not targeted per se. What is really controlled through these maximum weight deviations is the sensitivity of the active return to the volatility of the assets. In other words, with unchanged maximum weight deviation, a doubling of the volatility of all assets will result in a doubling of the active return's volatility, i.e. of the active risk. This technique is therefore not foolproof with respect to changes in the volatility of the assets. Nevertheless, with unchanged volatility of assets, smaller maximum weight deviations will result in smaller maximum active risk.
- The imposition of a single maximum active weight deviation for all stocks raises an interesting issue. A single constraint means that one is ready to accept more active risk from a more volatile stock than from a less volatile one. This illustrates again that maximum weight deviation is only a gross proxy for risk.
- The maximum under- and over-weights relative to the benchmark are specified symmetrically. This symmetry is overstated if the portfolio manager cannot be short on any assets. In this case, the maximum under-weights he can achieve on a specific stock is its weight in the benchmark.

*1.1.2.2 Using ex-ante tracking error**

Provided that the portfolio manager has access to a risk model, an easy and efficient way to implement risk control is by setting a maximum limit to the ex-ante tracking error of the managed portfolio:

$$T\tilde{E}^{P,B} \leq \underline{t}$$

Drawing parallel remarks as for the previous technique, one can notice that:

- This technique is simple to implement because the portfolio manager is usually spared the burden of providing a risk model (with all its technicalities and sophistication).
- Again, the value of the maximum limit for the ex-ante tracking error depends on the risk aversion of the portfolio manager.
- Contrary to the previously described technique, the control of the ex-ante tracking error aims directly at the forecasted active volatility of the managed portfolio. Assuming that the forecast is able, to some extent, to track changes in the true volatility, this means that the distance (in terms of weight deviation) of the managed portfolio around the benchmark will shift through time adjusting to these changing volatilities. In times of increasing (decreasing) volatilities, the distance of the managed portfolio will get smaller (larger) with respect to the benchmark.
- Controlling the ex-ante tracking error takes into account (through the risk model) the difference in stocks' volatility and is therefore doing a better job at controlling risk.

Other risk control using a risk model can be considered. Additional constraint on active exposures for one or several factors or on the active specific risk can be added. For example, a portfolio manager may want to constrain the exposure of its portfolio to the i-th factor and impose also a minimum level of active specific diversification:

$$\left| x_{P,i} - x_{B,i} \right| \leq \underline{x} \quad \text{and} \quad \tilde{s}_{P,B} \leq \underline{s}$$

*1.1.2.3 MFM in an equity framework**

Up to now, we have discussed the MFM approach in general terms. In this section, we want to rewrite it in a more suitable form when dealing with equities by being more explicit in terms of the nature of the factors. We will limit the discussion to the case of a single country. Within the framework of a single country equity model, the factors that are usually included are:

- The market,
- Various style factors, like earnings yield, dividend yield, book to price, price momentum, size, earnings revisions, etc.
- And various industry factors

Given that, the MFM can be expressed in vector terms as:

$$R_i = b_i \cdot R_m + y_i' \cdot S + z_i' \cdot I + E_i \qquad i = 1,\ldots, N$$

where:

b_i	is the beta[15] of stock i,
R_m	is the excess return of the market (benchmark)
y_i	is the vector of stock i exposures to the NS style factors,
S	is the vector of the NS style factor returns,
z_i	is the vector of stock i exposures to the NI industry factors,
I	is the vector of the NI industry factors.

As an example we can now present the BARRA Swiss model[16] that was first estimated on a universe of the 250 largest stocks, using data from January 1985 to December 1989. This MFM consists of 8 style factors and 12 industry factors. The following style factors are used to differentiate between stocks:

- Size: large and small,
- Success: recently successful or unsuccessful using price momentum,
- Yield: high or low,
- Volatility: of high or low overall variability,
- Value: cheap or expensive,
- Earnings variability: with low or high earnings or cash-flow fluctuations,
- Growth: with low or high historical growth,
- Financial leverage: with high or low financial leverage.

Each stock belongs to one and only one of the following 12 industries[17]:

- Banking
- Insurance
- Transportation
- Department store

15 i.e. the market factor exposure of stock i.

16 See Beckers S., Cummins P., and Woods C. (1993) for a detailed presentation of the BARRA Swiss model. The above discussion follows their paper quite closely.

17 The industry factor exposures are usually dummy variables (1 or 0) depending on whether the stock belongs to the relevant industry or not. In more sophisticated MFMs (i.e. BARRA USA) a stock may appear in several industries; in this case its industry factor exposures will be ≥ 0, ≤1, and add up to 1. These industry factor exposures are usually not normalised.

- Other Services
- Machinery
- Utilities
- Chemicals, and Pharmaceuticals
- Food and Beverage
- Electronics
- Building and Construction; and
- Miscellaneous

The adjusted R square for all the cross-sectional regressions had a mean of 0.36 over the 5-year estimation time period. All 20 factors were statistically relevant over the same time period. The following tables give some idea of the type of fit, which can be achieved with this methodology.

Factor	Coefficient	T-stat
Size	–0.003	–1.06
Success	0.007	1.51
Yield	0.002	0.47
Volatility	–0.005	–0.94
Earnings Volatility	0.011	1.73
Growth	–0.002	–0.32
Leverage	0.025	4.20
Value	–0.029	–3.93
Banks	–0.042	–4.40
Insurance	–0.035	–5.19
Transport	–0.083	–3.37
Department Stores	–0.043	–2.39
Other Services	–0.049	–4.58
Machinery	–0.055	–3.93
Utilities	–0.111	–5.73
Chemicals	–0.025	–2.76
Food & Beverages	–0.036	–3.29
Electronics	–0.076	–5.45
Building & Construction	–0.055	–3.20
Miscellaneous	–0.040	–3.08

Table 1-1: Swiss BARRA Model, results of August 1992 regression

Factor	F-Stat	% of Time Abs(T-Stat) > 2
Size	7.30	45
Success	4.34	29
Yield	3.79	29
Volatility	3.28	26
Earnings Volatility	2.47	24
Growth	2.31	17
Leverage	2.30	16
Value	2.07	18

Table 1-2: Swiss BARRA Model, significance of factor returns

Note that the industry factors are more significant than the style factors in August 1992. It is not surprising that over time these factors jointly capture the market, which is the strongest source of correlation between stocks. However, there is empirical evidence that these industry factors still have a larger explanatory power than the style factors if the model is estimated with a market factor.

Furthermore, it is usually assumed that the market return is independent of other factor returns, and that the stock specific returns are independent with respect to all factor returns. Given these assumptions, one can show that the variance of the active return can be expressed as:

$$V\left(R_A^{P,B}\right)=\left(b_P-b_B\right)^2\cdot V\left(R_m\right)$$
$$+\left(y_P^{'}\text{-}y_B^{'},z_P^{'}-z_B^{'}\right)\cdot\begin{bmatrix}W_{SS} & W_{SI}\\ W_{IS} & W_{II}\end{bmatrix}\cdot\begin{pmatrix}y_P-y_B\\ z_P-z_B\end{pmatrix}+(s_A^{P,B})^2$$

where

W_{SS}	NS x NS covariance matrix of the style factors,
W_{SI}	NS x NI covariance matrix of the style factors with the industry factors,
W_{IS}	NI x NS covariance matrix of the industry factors with the style factors,
W_{II}	NI x NI covariance matrix of the industry factors.

As an example, we can analyse the tracking error[18] of the SMI with respect to the SPI using this breakdown. The results below are based on a risk analysis using the BARRA Swiss model as of December 29th, 2000. The portfolio (P) analysed is the SMI while the benchmark (B) and the market are the SPI:

- Forecasted beta: 1.05
- Forecasted tracking error: 1.63

The forecasted variance of the active return is broken down as follows:

$$\tilde{V}\left(R_A^{P,B}\right)=2.64$$
$$\left(b_P-b_B\right)^2\cdot\tilde{V}\left(R_m\right)=0.60$$
$$\left(y_P^{'}-y_B^{'}\right)\cdot\tilde{W}_{SS}\cdot\left(y_P-y_B\right)=1.04$$
$$\left(z_P^{'}-z_B^{'}\right)\cdot\tilde{W}_{II}\cdot\left(z_P-z_B\right)=0.90$$
$$2\cdot\left(y_P^{'}\text{-}y_B^{'}\right)\cdot\tilde{W}_{SI}\cdot\left(z_P-z_B\right)=-0.77$$
$$(\tilde{s}_A^{P,B})^2=0.87$$

We can see that there the three larger contributions to the tracking error are, in decreasing order, linked to the active exposure to the style factors, to the industries, and to the lack of diversification[19]. Following are the style factor exposures:

18 More precisely the square of the tracking error.

19 This is not surprising as the SMI has only 29 stocks.

Factor Exposures	SMI	SPI	Difference
Size	0.121	0.015	0.106
Success	-0.039	-0.002	-0.037
Yield	-0.006	0.003	-0.009
Volatility	0.042	0.004	0.038
Earnings Variability	0.047	0.007	0.040
Growth	-0.028	-0.003	-0.025
Leverage	0.034	0.012	0.022
Value	-0.030	-0.006	-0.024

The SMI, as compared to the SPI, is slightly more concentrated on large, recently less successful stocks. It is slightly less exposed than the SPI to stocks with high yield, high value, high growth exposure.

The industry weights are the following:

Industry Weights (%)	SMI	SPI	Difference
Banks	20.30	19.03	1.27
Insurance	15.50	13.49	2.01
Transport	0.31	0.53	-0.22
Department Stores	0.00	0.51	-0.51
Other Services	5.17	6.43	-1.26
Machinery	0.40	1.87	-1.47
Utilities	0.00	0.52	-0.52
Chemical	35.86	33.36	2.50
Food / Beverage	14.10	12.36	1.74
Electronics	6.90	7.55	-0.65
Building / Construction	1.00	1.50	-0.50
Miscellaneous	0.46	2.85	-2.39

We can see that the largest industry weight imbalances are in the chemical, miscellaneous and insurance companies.

*1.1.3 Active and passive management**

In section 1.2, we will consider active and passive equity management separately and study them in detail. Here we will define each briefly.

*1.1.3.1 Definition**

The activity of portfolio management involves transactions in the securities market. Buying and selling of assets and liabilities are inherent to the daily practice of an investment professional. To understand the difference between passive and active management better, it is useful to distinguish between two types of transactions:

- The transactions of the first type are intended to *increase the expected*[20] *return* of the portfolio. Whether the expected improvement in return is stated implicitly or explicitly, quantified or not, realised or not, does not matter in these types of transactions. The important point inherent to these transactions is to forecast improved returns. Improving the expected return of the portfolio through transactions means that the expected returns on the assets bought are higher than those of the assets sold. At least, there should be an expectation or a forecast of a positive return differential between bought and sold assets.
- The transactions of the second type have purposes other than improving the expected return of the portfolio. They may aim at controlling the risk of the portfolio, at raising or investing cash. This does not mean that these types of transactions are not based on forecasts at all. They may be dependent on expectations of risk, volatility etc. but by definition, not on returns.

With the above definitions of transactions, it is now possible to define *active management* as a management method that effectively uses transactions of both types. In other words, it tends to increase the expected return of the managed portfolio either in absolute or relative terms.

In contrast, by definition *passive management* involves only transactions of the second type. It does not need any expectations or forecasts about asset returns.

*1.1.3.2 Active versus passive management**

Although the discussion between the proponents of both passive and active management has been intense and sometimes antagonistic[21], a more realistic position is to recognize that there is room for both in the investment management arena. The following arguments help clarify the situation:

- First, active management is a vital ingredient for the efficiency of the market. It is through all the efforts of all the professionals, involved in active management, looking continuously for potential misevaluation between securities that the market is so (but not perfectly) efficient. A market without active managers would drift like a ship without a rudder in terms of asset relative evaluations. As a matter of fact, such a market would be a paradise for an active manager.
- Second, active managers use costly resources and skills. Generally over the long run, they would not be able to economically sustain their existence if the market was fully efficient. If this were the case, active portfolio managers would not be able to provide positive active returns. In other words, in equilibrium the market has to be optimally but not perfectly efficient in order for the active managers to generate positive active returns and therefore justify their revenues (management fees). These revenues are much needed at least to cover their costs if they intend to stay in business.

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20 And, hopefully, the realised return as well.

21 Most certainly because of underlying commercial reasons.

- Third, we have to remember that in a world composed only of active managers, half of them, in terms of amounts under management, will over-perform and half of them will under-perform. It should not be surprising that there are so many under-performing active managers. Because the market embodies the information's interpretation of all active managers, the reward to any active manager is linked to his relative position and not to his absolute position. In other words, only the best active manager can expect to deliver positive active returns.
- Fourth, there is some empirical evidence of successful active managers who have experienced very long stretches of over-performances.
- Fifth, empirical finance has documented some systematic active strategies which, at least in the past, would have provided positive active returns. These strategies are based on so called anomalies. As it will be described later, it is not clear-cut that these anomalies are inefficiencies.
- Sixth, although active managers are needed to ensure sufficient market efficiency, it does not imply that every manager should be an active one. Sufficient market efficiency can likely be achieved with a substantial fraction of the market being managed passively.

In other words, one has to recognize that active management is necessary for the good working of the market. At the same time there is no evidence that passive management alters this good working as long as it takes an overwhelming fraction of the total market. Given the current situation, there is no sign that such a state has been reached.

The level of average cost relative to the amount managed is lower for passive management than for active management. There are tremendous economies of scale in passive management. This implies substantially lower management fees for passive management. In addition, other studies show that the total cost of transactions is definitely lower for information-free trades linked to passive strategies. Similarly, the choice of an active portfolio manager is certainly more costly and hazardous than the choice of a passive manager. The technologies for passive management are much more easily analysed and linked to the quality of the management, i.e. the size of the ex-post tracking error.

Given these parameters it is perfectly reasonable for an investor, who is sceptical about his own ability to identify a successful active manager, to opt for passive management. At the same time, another investor, who thinks that he is able to identify such an active portfolio manager and that the expected over-performance justifies all the additional costs, is also perfectly rational.

To summarize, the previous arguments should leave the reader with the correct impression that there is room for both types of management and that they are in reality complementary. In sections 1.2.1 and 1.2.2 we will present both active and passive equity management in greater details. However first, we want to show how investors can use combined strategies at the same time involving both passive and active management. In the following paragraphs, we will show how investors (especially large pension funds) have been combining both passive and active management.

*1.1.3.3 Active/passive combinations**

In the past, pension funds of a sufficient size tended to hire many active portfolio managers with similar mandates within an asset class or across asset classes. In many instances the active returns of the different managers would diversify away if they were not sufficiently correlated. The end result was that the aggregate performance was not too far from the benchmark's performance, but with the high cost of active management.

Reacting to this unsatisfactory situation, the funds had the tendency to adopt another investment structure, i.e. the core/satellites approach. With this new structure, they divide their assets into two parts:

- One is composed of a core portfolio that is indexed to the pension fund global strategic benchmark and bears therefore only the cost of passive management.
- The other part is composed of a more parsimonious set of more active and risky portfolios called the satellites. These active portfolios usually have some disjoint subsets of the pension fund global benchmark as reference indices.

With such a structure, the pension fund:

- Lowers the management cost of its assets as it pays passive management fees on a large fraction of them (core indexed portfolio).
- Suffers less from the diversification between its active portfolio managers because each of these managers operate on distinct segment of the fund's global benchmark.

Notice that this structure leaves the pension fund with the important problem of allocating its assets between all its portfolio managers.

1.2 Managing an equity portfolio*

*1.2.1 Active management**

*1.2.1.1 General principles**

Excess return and risk

As already mentioned the purpose of active management is to create portfolios, which are expected to over-perform their respective benchmarks. Note that the expected performance of a portfolio P is a simple weighted average of the assets return:

$$\tilde{R}_P = \sum_{i=1}^{N} w_P^i \cdot \tilde{R}_i$$

One can show that the relative performance of a portfolio with respect to its benchmark can be expressed as:

$$\tilde{R}_A^{P,B} = \tilde{R}_P - \tilde{R}_B = \sum_{i=1}^{N} (w_P^i - w_B^i) \cdot (\tilde{R}_i - \tilde{R}_B)$$

This last formula is interesting as it shows that the only way a portfolio can, on an ex-ante basis, outperform its benchmark is either by being over-weighted in at least one stock which is expected to perform better than the benchmark or by being under-weighted in at least one stock which is expected to perform worse than the benchmark.

Indeed for $\tilde{R}_A^{P,B}$ to be strictly positive, for one asset at least it must be true that:

$$(w_P^i - w_B^i) \cdot (\tilde{R}_i - \tilde{R}_B) > 0$$

which means that either:

$$w_i^P > w_i^B \text{ and } \tilde{R}_i > \tilde{R}_B$$

Or,

$$w_i^P < w_i^B \text{ and } \tilde{R}_i < \tilde{R}_B$$

should be true.

One can also show that the ex-ante tracking error of a portfolio can be formalised as:

$$\tilde{TE}_A^{P,B} = \sqrt{\sum_{i=1;j=1}^{N} (w_i^P - w_i^B) \cdot \tilde{C}_{i,j} \cdot (w_j^P - w_j^B)}$$

where $C_{i,j}$ is the covariance of asset i and j returns.

This last equation shows that a portfolio manager, in order to seek a positive active performance, has to bear some active risk. There is no way to expect to over-perform without creating some distance, in terms of under- or over-weighting some stocks, between the managed portfolio and its benchmark, and therefore inevitably accepting a positive ex-ante tracking error. Note that this risk is taken on all stocks belonging to both the portfolio and the benchmark as long as there is a strict over- or under-weighting. It means that active risk is taken on all stocks that are not in the portfolio but in the benchmark[22]. Finally note that both expected active return and ex-ante tracking error depend on the stock active weight and not on its absolute weight in the portfolio.

Maximizing the information ratio

Active management is based on return forecast for all stocks. Whether these forecasts are implicit or explicit, quantified or not, does not matter as long as the portfolio manager has a clear view of his predictive power and of how much active risk he is taking. Implementing these return forecasts into portfolios raises the question of what principles should be followed.

One simple idea, which comes to mind, is that portfolio construction should be efficient, i.e. that the portfolio manager should build-in as much as possible expected active return per unit of active risk taken. Before adopting this principle, one should make sure that it does not lead the portfolio manager into taking an unwanted level of active risk.

22 As these stocks are all under-weighted.

In the next paragraphs, we will show that the portfolio manager can be efficient and control the level of active risk at the same time, by adopting the underlying principle for active management of maximising the ex-ante information ratio of portfolios. We must now define precisely this new concept. The information ratio of a portfolio P with respect to its benchmark B is defined as the ratio between the portfolio expected active return and its ex-ante tracking error. In other words, we have:

$$I\tilde{R}_A^{P,B} = \frac{\tilde{R}_A^{P,B}}{\tilde{TE}_A^{P,B}}$$

Note that the information ratio is also a forward-looking concept as it is defined as a ratio of two expected values for a specified future time period. This information ratio normalises the expected active return of a portfolio by its level of active risk and can therefore be interpreted as the amount of expected active return per unit of ex-ante active risk[23].

We want to show now, that maximising the ex-ante information ratio is independent of choosing the level of tracking error. We will demonstrate that for any given portfolio we can construct another one with the same information ratio and with any given tracking error.

Let us consider a portfolio P whose ex-ante information ratio and tracking error are respectively IR^* and TE^*. Now, for an arbitrary level of tracking error TE^0, we must show the existence of a portfolio with the same information ratio IR^* but now a level of tracking error equal to TE^0.

Let us consider a portfolio P(n) defined by:

$$P(n) = B + n \cdot (P - B)$$

This new portfolio is just the benchmark plus a multiple n of the active component of portfolio P. One can show that:

$$\tilde{R}_A^{P(n),B} = n \cdot \tilde{R}_A^{P,B}$$

$$\tilde{TE}_A^{P(n),B} = n \cdot \tilde{TE}_A^{P,B}$$

Hence,

$$I\tilde{R}_A^{P(n),B} = I\tilde{R}_A^{P,B} = IR^*$$

From here, if we choose n equal to TE^0 / TE^*, then the portfolio P(n) has the following characteristics:

$$I\tilde{R}_A^{P(n),B} = IR^* \quad \text{and} \quad \tilde{TE}_A^{P(n),B} = n \cdot \tilde{TE}_A^{P,B} = \frac{TE^0}{TE^*} \cdot TE^* = TE^0$$

We have shown that, given any portfolio P, it is possible to construct another portfolio P(n) with the same ex-ante information ratio IR^* and a pre-specified ex-ante tracking error TE^0. This result is important as it gives the portfolio manager the freedom to maximise the ex-ante information ratio and, at the same time, choose the appropriate level of risk he is willing to take.

23 Measured in terms of standard deviation which has the same dimension as the return.

Information ratio maximisation and optimisation

In a previous section, we mentioned that multi-factor risk models are usually delivered with an optimiser, which helps build portfolios by solving the following problem:

$$\underset{\{w_i^P:i=1,\ldots,N\}}{MAX} \tilde{R}_A^{P,B} - v \cdot \tilde{V}(R_A^{P,B}) \quad \text{with} \quad \sum_{i=1}^{N} w_i^P = 1$$

In the following paragraphs, we will show that the use of an optimiser is fully compatible with the principle of maximising the information ratio. The proof to this last statement is simple.

Let us denote the portfolio, which results from the optimisation, by O. By definition of the optimisation, the value for portfolio O of the maximised function, is larger than for any other portfolio P.

$$\tilde{R}_A^{O,B} - v \cdot \tilde{V}(R_A^{O,B}) \geq \tilde{R}_A^{P,B} - v \cdot \tilde{V}(R_A^{P,B}) \qquad \text{for all possible portfolios P}$$

Let us assume that there is a portfolio Q with a strictly higher information ratio than portfolio O. Without lack of generality, we can assume that portfolios Q and O have the same ex-ante tracking error and therefore the same expected variance for its active return:

$$\tilde{IR}_A^{Q,B} > \tilde{IR}_A^{O,B} \quad \text{and} \quad \tilde{TE}_A^{Q,B} = \tilde{TE}_A^{O,B} \quad \text{and} \quad \tilde{V}(R_A^{Q,B}) = \tilde{V}(R_A^{P,B})$$

With identical ex-ante tracking error the only way that portfolio Q can achieve a higher information ratio is by exhibiting a higher expected active return:

$$\tilde{R}_A^{Q,B} > \tilde{R}_A^{O,B}$$

By subtracting $v \cdot \tilde{V}(R_A^{O,B})$ from both side of the previous inequality and recognising that both portfolios Q and O have the same expected variance for their active returns, we get:

$$\tilde{R}_A^{Q,B} - v \cdot \tilde{V}(R_A^{Q,B}) > \tilde{R}_A^{O,B} - v \cdot \tilde{V}(R_A^{O,B})$$

This last inequality contradicts the optimality of portfolio O. Therefore it is not possible to find a portfolio Q with a strictly higher information ratio than the optimal portfolio O. In other words, a portfolio manager can safely use an optimiser and be assured that he is also maximising the information ratio of his portfolios at the same time.

Implementing expected returns into portfolios

As already mentioned the goal for any active manager is to build portfolios that maximise the ex-ante information ratio given their expectation in terms of stocks or groups of stock returns. In the following paragraphs we want to discuss how the portfolio manager's expected returns should be implemented into portfolios in order to create such positive expected active return. We will focus initially on return forecasts at the single stock level.

The key to understanding what a portfolio manager should do is linked to the fact that he can always neutralise the expected active contribution to the performance of a single stock i by making sure that it has the same weight both in the portfolio and in the benchmark ($w_P^i = w_B^i$); if this is the case then the expected active contribution of the stock is indeed equal to zero[24]:

24 Notice that it is truly independent of the expectations of the portfolio manager for the performance of the stock relative to the benchmark.

$$\underbrace{(w_P^i - w_B^i)}_{=0} \cdot (\tilde{R}_i - \tilde{R}_B) = 0$$

In other words, the portfolio manager is in a position where he cannot be obliged to build a stock position with an expected negative active contribution. Within these conditions, his goal is to build portfolios where all stocks have a positive or null expected contribution to the active return:

Given the fact that he can always hold the benchmark and insure a zero expected active return, his goal is basically to build portfolios with a positive expected active return.

$$(w_P^i - w_B^i) \cdot (\tilde{R}_i - \tilde{R}_B) \geq 0 \qquad (i = 1, \ldots, N)$$

The table below summarizes how a portfolio manager should over- or under-weight a stock in a portfolio depending on his expectations about its relative performance:

Relative performance	Over/under-weight	Active contribution
$\tilde{R}_i > \tilde{R}_B$	$w_P^i > w_B^i$	$\underbrace{(w_P^i - w_B^i)}_{>0} \cdot \underbrace{(\tilde{R}_i - \tilde{R}_B)}_{>0} > 0$
$\tilde{R}_i < \tilde{R}_B$	$w_P^i < w_B^i$	$\underbrace{(w_P^i - w_B^i)}_{<0} \cdot \underbrace{(\tilde{R}_i - \tilde{R}_B)}_{<0} > 0$
$\tilde{R}_i = \tilde{R}_B$	$w_P^i = w_B^i$	$\underbrace{(w_P^i - w_B^i)}_{=0} \cdot \underbrace{(\tilde{R}_i - \tilde{R}_B)}_{=0} = 0$
$\tilde{R}_i \,?\, \tilde{R}_B$	$w_P^i = w_B^i$	$\underbrace{(w_P^i - w_B^i)}_{=0} \cdot \underbrace{(\tilde{R}_i - \tilde{R}_B)}_{=?} = 0$

The first two lines of the above table are quite straightforward. In order to create a positive active contribution, a portfolio manager should over-weight a stock, which is expected to over-perform the benchmark. Similarly, he should under-weight a stock, which he expects will under-perform the benchmark.

The third case is less simple. When the portfolio manager expects a stock to perform like the benchmark, the table indicates that he should set its weight at the benchmark's level. This may seem initially surprising as the expected active contribution is equal to zero in all cases whether the stock is over, under or neutrally weighted. So why should this stock be neutrally weighted with respect to the benchmark? The answer can be obtained directly from the principle of the information ratio maximisation. Over- or under-weighting this stock may not affect the expected active return of the portfolio, but it does adversely affect the ex-ante tracking error. In other words, it diminishes the information ratio as it leaves the numerator unchanged while increasing the denominator. This deterioration is in direct violation of its maximisation principle.

The fourth case relates to the fact that the portfolio manager may have no clue whether a stock will out- or under-perform the benchmark. As in the previous case, he should weight this stock neutrally in the portfolio as again he does not want to bear additional risk without being able to expect some additional active return.

Notice that all that has been discussed in this section in terms of implementing expected returns in portfolios at the stock level can be generalized when considering different asset classes. For example, if an asset class[25] is expected to over-perform the benchmark, then it should be over-weighted in the managed portfolio.

Amplitude of the bets

Up to now we have discussed how to implement active bets in terms of whether a stock should be over-, under-, or neutrally weighted. Nothing has been said about the magnitude of these over- or under-weightings. The end response to this question is the maximisation of the information ratio. This principle implies several rules that can help build sound portfolios without the explicit help of quantitative techniques[26]:

- Given two stocks with the same positive (negative) expected relative return to the benchmark and the same confidence in the return forecasts, the less risky should be more over- (under-) weighted.
- Given two stocks with the same positive (negative) expected relative return to the benchmark and presenting the same level of risk, the stock with less confidence in the return forecast should be less over- (under-) weighted.
- Given two stocks presenting the same level of risk and with the same confidence in the return forecast, the stock with the higher positive (negative) expected relative return to the benchmark should be more over- (under-) weighted.

Although these rules give some precision on how to set, in relative terms, the active bets at the stock level, they do not give any indication in terms of their global scaling. This scaling should be done taking into account the risk aversion of the investor so that he feels comfortable given the level of risk taken and the expected active return of the portfolio.

In addition, one has to remember that the implementation of the expected returns into a portfolio has to respect the prevailing risk control rules[27] chosen by or imposed on the portfolio manager.

25 An industry or small cap stocks.

26 For all these rules, the risk is understood in terms of marginal contribution. To simplify, we also assume that there are no transaction costs.

27 See section 1.2.

Pitfalls in portfolio construction

When a portfolio manager chooses to build portfolios without using risk control techniques, he should be aware of some of the pitfalls inherent in such an exercise:

- First, there is a general tendency towards building equally weighted portfolios. It may imply uncompensated risk especially if the benchmark is concentrated in a few stocks. As an example, the tracking error of an equally weighted SMI with respect to the SPI is +5.33%[28]. This is a little bit less than twice the tracking error of the real SMI for which the stock weightings are more in line with the ones of the SPI. Note that the active specific risk comes from the weight difference between the managed portfolio and the benchmark and not from the absolute weights in the managed portfolio. Not holding a stock that counts, for example, for 12% of a benchmark, because it is out of favour, means to say the least a lot of specific risk. The portfolio manager may choose to hold it simply under-weighted in order to avoid excessive risk.
- Sometimes, there is a tendency not to have recommendations for stocks in an undesirable industry. The analyst does not expect that the stocks in this industry will over-perform. Once again, depending on the weight of this industry within the benchmark, the portfolio manager may have to hold some of these undesired stocks just for the sake of diversification and risk avoidance.

*1.2.1.2 Active management using a factor model**

Implementing expected returns

In a framework of MFM, the return forecast part of active management can be formalized in terms of forming expectations for factor returns and for the stock specific returns. Of course, there is no obligation to make all these forecasts. A portfolio manager can, for example, limit himself to forecasting the factor returns and choose not to forecast stock specific returns.

Within the MFM equity framework presented in section 1.2.2 and following the principles described in the previous section, there are four main sources of expected active return that can be built-in into active portfolios:

- *Benchmark timing*[29]: A divergence between the betas of the portfolio and the benchmark is the first source of active return. An active portfolio manager should seek to achieve a portfolio's beta that is larger than the one of the benchmark $(b_P > b_B)$ if he forecasts that the benchmark's return will be positive $(R_m > 0)$. Similarly, he should lower the beta of the portfolio with respect to the benchmark $(b_P < b_B)$ when he expects the benchmark to under-perform the risk free rate $(R_m < 0)$. Finally, if the portfolio manager is not predicting the excess return of the benchmark or he has a zero forecast $(R_m = 0)$, then he should bring the beta of the portfolio to that of the benchmark $(b_P = b_B)$. Otherwise, he would just impose some active risk on the investor without any expectation of a reward for taking it.

28 As of August 30th, 1996.

29 Also known as market timing because most of the time the benchmark is a proxy of the market. Notice that benchmark timing is not a pure equity management problem. It has more to do with asset class management because it is basically an asset class allocation problem between cash and equity.

- *Style factor selection*: Here the source of active return is the difference of both the portfolio's and the benchmark's exposures to the style factors. An active portfolio manager should over-expose the portfolio to the style factor j $\left(y_j^P > y_j^B\right)$ if he expects that this factor is going to exhibit a positive excess return $\left(S_j > 0\right)$. Similarly, he should underexpose it $\left(y_j^P < y_j^B\right)$, if he expects a negative excess return to the style factor $\left(S_j < 0\right)$. If no forecast is made or if it is neutral $\left(S_j = 0\right)$, then both the portfolio and the benchmark should have equal exposure to the factor $\left(y_j^P = y_j^B\right)$, and thus avoiding unrewarded active risk.
- *Industry rotation*: The third source of active return is the industry weight imbalances between the portfolio and the benchmark. An active portfolio manager should overweight the portfolio to the k-th industry factor $\left(z_k^P > z_k^B\right)$ if he predicts that this industry is going to exhibit a positive excess return $\left(I_k > 0\right)$. Similarly, he should underweight it $\left(z_k^P < z_k^B\right)$, if he expects a negative excess return to the industry factor $\left(I_k < 0\right)$. If no forecast is made or if it is neutral $\left(I_k = 0\right)$, then similar weights $\left(z_k^P = z_k^B\right)$ should be used for both the portfolio as in the benchmark.
- *Specific stock selection*: The last source of active return is linked to the specific return of the stocks. An active portfolio manager should overweight a stock i $\left(w_i^P > w_i^B\right)$ if he forecasts his specific reward will be positive $\left(E_i > 0\right)$. If the prediction is negative $\left(E_i < 0\right)$, then he should underweight it $\left(w_i^P < w_i^B\right)$. Once again, no divergence in weight should be applied $\left(w_i^P = w_i^B\right)$ in case of a neutral forecast or of an absence of expectation $\left(E_i = 0\right)$.

The importance of portfolio construction

When a portfolio manager includes a stock in a portfolio with a weight different from the benchmark's weight, he is automatically taking active exposures to:

- the benchmark return,
- the style factor returns[30],
- the industry factor return for the industry to which the stock belongs, and
- the specific return of the stock.

A very important point to note is that the portfolio manager also takes all these active exposures on all the benchmark's stocks that are not included in the portfolio as those are by definition under-weighted. These stocks are as important in terms of describing the active risk taken as the ones that appear in the portfolio.

Notice also that rarely can a portfolio manager find a stock to include in the portfolio that has the correct exposures for the factors whose return he is forecasting and zero exposures for the factors whose return he is not predicting.

For example, he may like a stock because:

30 As long as the stock's exposure to the style factor is different from 0.

- of its specific qualities,
- it belongs to one industry which he expects to over-perform,
- of the quality of the management,

But not like it at the same time because:

- it has a large capitalisation as he expects the small stocks to do better,
- of its small beta (<1) as he expects a bull market.

This example illustrates the dilemma the portfolio manager is facing in terms of positioning this stock in an active portfolio. Indeed this stock provides at the same time both desired and undesired factor exposures. Because of the first set of reasons, the portfolio manager would like to over-weight it, while the second set guides him to under-weight it. Fortunately this dilemma can be solved through a careful construction of the portfolio.

Given the expected returns of the stocks, the work of the portfolio manager is to find a set of stocks, which he expects on an average to over-perform, together with adequate weights, so that the resulting portfolio exhibits as much as feasible:

- positive active exposure to the factors he expects to be positively rewarded,
- neutral active exposure to the factors he does not forecast or he expects to have a zero return,
- negative active exposure to the factors he expects to be negatively rewarded,
- positive or negative stock active weight depending on whether he expects the stock specific return to be positive or negative.
- Adequate specific risk diversification.

Most of the time, it is not possible to achieve all of these. Nevertheless, a careful portfolio construction is important because, without it, a portfolio manager may end up destroying the effects, in terms of returns, of all the good forecasts he made by implicitly taking other negatively rewarded exposures.

One interesting case to mention is when a portfolio manager only forecasts the factor returns and gives up forecasting stock's specific returns. Notice that it is impossible for the portfolio manager to achieve active factor exposure without achieving at the same time active weight on specific stocks. The reason is that there is no existing security that provides pure exposures to factors and has no specific return component. It means that, despite the fact that the portfolio manager is only betting on factors, he has to take not only factor related active risk but also active specific risk. This last source of risk is not expected to be compensated, as the portfolio manager has no view on the stocks' specific returns. When building an active portfolio within this framework, one must be sure that enough specific diversification is ensured so that the noise of the portfolio specific return does not overcome the portfolio factor related return.

<u>Optimisation</u>

Working within a MFM framework usually means that an optimiser is available. As already mentioned, this tool solves the following problem:

$$\underset{\{w_i^P : i=1,\ldots,N\}}{\text{MAX}} \ \tilde{R}_A^{P,B} - v \cdot \tilde{V}(R_A^{P,B}) \qquad \text{with} \qquad \sum_{i=1}^{N} w_i^P = 1$$

The inputs that the portfolio manager should provide to the optimiser are the stocks' expected returns. It can be done usually in two equivalent ways by providing[31]:

- Either directly these expected returns, i.e. $\tilde{R}_i$ (i = 1, …, N)
- Or indirectly the expected factor returns, i.e. $\tilde{S}_j$ (j = 1, …, NS) and $\tilde{I}_k$ (k=1, …, NI), and the expected stock specific returns, i.e. $\tilde{E}_i$ (i = 1, …, N).

Notice that in order to be consistent, i.e. for the optimiser to end-up with the same final optimised portfolios, these two sets of expected returns must satisfy the following conditions:

$$\tilde{R}_i = y_i^{'} \cdot \tilde{S} + z_i^{'} \cdot \tilde{I} + \tilde{E}_i \qquad (i = 1,..., N)$$

One last caveat is necessary. When the quantification of the expected returns does not result from a normalised quantitative process but comes form a more loose assessment of a judgmental process, one has to be careful when using an optimiser. This tool is usually very sensitive to small changes in a single expected return. Due to such a change, one may end up with a surprisingly different portfolio.

*1.2.1.3 Fundamental analysis**

Fundamental analysis is certainly the more prevalent approach toward active investing. Its main goal is to identify over-valued or under-valued securities. We have to be very precise with what we mean by these mispriced securities. For an analyst to declare a stock over- or under-valued, he must have views on that security which must:

- differ from the market view,
- be thought to be closer to the (hard to discover) reality.

Under these conditions, it is likely that the current stock's price reflects the market views and won't embody the analyst views' differences. The hope is that, as future unfolds, the analyst's view will prove to be better than the market views, that the market will realise its mistake and realign the price of the stock with a new stock perception more in line with the analyst's one. This price realignment is equivalent to a temporary abnormal return beyond the market normal expected return of the stock. It is the expectation of this abnormal return, which should correct the analyst's subjective perception of the stock relative misevaluation, which should be implemented into a portfolio through an active weight.

In order for this scenario to unfold favourably, two conditions are to be met:

- First the analyst must be right in his views' regarding differences on the stock,
- Second, new evidence should appear to alert the market on its wrong perception and the market should react correctly to this new information.

This second condition is important as it raises an interesting timing issue, as it is usually difficult to predict when the new evidence will emerge and when the market will adjust to it.

31 We also assumed that the portfolio manager makes no forecast for the market return ($\tilde{R}_m = 0$).

Of course, the analyst can also be wrong. Then, the expected good news will not arrive and the expected abnormal return will not materialize. Having taken an active weight in the stock, it is clear that the portfolio will be exposed to any other news for this security, accepting volatility and therefore risk for no good reason.

It is not the goal of this section to describe the details of fundamental analysis as it is presented in other parts of this program. Nevertheless, one can say that usually the main focus of this discipline is the forecast of earnings and dividends for the current and future (growth) periods. The role of the analyst is to determine if the market is over/under-stating the current earnings and/or their future growth and therefore is over/under evaluating the considered stock. There are of course many ways to get a more precise picture in terms of forecasting the earnings of a specific stock:

- It is clear that a forecast of the macro-economic environment can help to have a better idea of the circumstances under which a firm will operate in the near future. A view on how the real growth of the surrounding economy (and its demand components), the inflation, the interest rates and the currencies will evolve has certainly different implications on different industries and on different firms depending on their activity, their financial leverage, etc.
- Beyond the implications of macro-economic analysis, a view on the structure of an industry can also be very helpful. The fact that an industry is protected or not against the entry of new competitors, the intensity of the existing competition, the position of the firm's product within the industry, the existence of substitute products, the advancement of future products in the development line, are all important factors for the current and future levels of earnings.
- Finally a detailed analysis of the firm is of course unavoidable. A large part of this analysis is linked to a careful study of the financial statements (balance sheet, income statement) of the firm. Although, such a study is based on past data it is nevertheless very useful to better assess the future.

Finally note that all this analysis should also give a better view on the risks that are attached to the studied firms, so that the investor can judge if seeking the expected abnormal return is worthwhile given the risk he must bear.

*1.2.1.4 Top-down / bottom-up**

In this section, we will briefly describe two very general approaches that are used in the practice of fundamental analysis to generate forecasts of stock returns. Finally, we will discuss some of the problems encountered when building portfolios.

Top-down approach

The top-down approach relies first on a broad economic scenario, which describes the evolution of real activity, inflation, the term structure of interest rates and exchange rates for the country considered. From this general scenario provided by the economist, the investment strategist and the analysts derive a financial scenario forecasting the absolute performance of the main asset classes available, i.e.:

- cash
- short-term bonds
- intermediate-term bonds
- long-term bonds
- stocks

From here, they also recommend an asset allocation, i.e. weights for each asset class. Furthermore, the analysts responsible for the specific market, provide:

- an implicit forecast of the relative performance of the different industries or sectors,
- an implicit forecast of the relative attractiveness of the specific stock.

These implicit forecasts are often publicised through industry/sector lists, which have to be under- or over-weighted with respect to the benchmark, and of stocks, which have to be bought or sold. It is at this level that the problem of the portfolio construction starts.

Bottom-up approach

The bottom-up approach relies first, on very detailed and thorough primary analysis of stocks in order to identify those, which should exhibit superior growth in subsequent year(s). As the task of making such analysis could become overwhelming and consume a huge amount of resources if applied indiscriminately to a large universe of stocks, some screening procedure to determine the most likely candidates is generally used first. The aim of the subsequent detailed primary analysis is to detect stocks with solid but under-estimated growth perspective. It will also tend to neglect the short-run noise that surrounds any stock. In that sense this approach has a longer time horizon than the top-down approach. Within that framework the bottom-up analysts tend to concentrate their effort on a segment of the market composed of small capitalisation stocks with a higher growth potential. Because of this concentration on a small segment of the market and a time horizon that usually extends beyond that of the available risk models, the portfolio construction cannot fully benefit from the help of these risk models. Hence the tendency is to create more or less equally weighted portfolios.

*1.2.1.5 Judgmental / quantitative management**

Often in the past, judgmental asset management has been opposed to quantitative management. Nowadays, the distinction between the two is more and more fuzzy as many quantitative techniques (risk controls, screening methods, …) are more frequently used by judgmental asset managers. In the following paragraphs, we will briefly discuss the primary differences between judgmental and quantitative management:

Judgemental management

Judgmental equity management is an area that encompasses very different practices. It is a management style based on both qualitative and quantitative information that usually provides non-quantified forecasts for the stock returns. This lack of quantified prediction means that it is not possible to apply a full optimisation program to build the portfolios. Nevertheless, it is possible to use a risk model to assess their active risks.

Quantitative management

Quantitative equity management is also an area that covers many different techniques. Their common denominator is that they are based solely on quantitative information about the stocks. In their pure form they do not use any qualitative or judgmental information available. In other words, they rely only on a subset of the available data. This fact can be considered as non-optimal and as a handicap. However, the quantitative methods offer several advantages, which hopefully compensate at least for this drawback:

- These methods can be applied systematically on hundreds or even thousands of stocks in a uniform and standardised way[32] both at a point of time and through time using past data.
- They can be applied without emotion through the discipline of not overriding the method.
- They focus more on risk controls. In the case of explicit quantified forecasts, the full use of an optimiser ensures taking into account in a systematic way, the trade-off between expected active return and expected active risk.

*1.2.1.6 Market timing**

Definition

When someone manages a stock portfolio with respect to a pure stock benchmark, he usually has the choice to hold some cash[33]. In other words, he is allowed to invest into two different asset classes even if the benchmark is related to only one. This gives him the possibility to tactically allocate the assets he manages between cash and stocks. This is called market timing.

The principles governing market timing are directly calculated from the general ones of active management, especially those expressed in section 1.2.1.2. Before stating these principles, we must define a few notations. Denoting by w_{CASH} the weight of the portfolio's cash, knowing that the beta of the cash is zero ($\beta_{CASH} = 0$) and assuming that the beta of the stock portfolio is equal to one ($\beta_{STOCK} = 1$), one can show that the beta of the portfolio (β_{PORT}) is equal to:

$$\beta_{PORT} = w_{CASH} \cdot \underbrace{\beta_{CASH}}_{=0} + (1 - w_{CASH}) \cdot \underbrace{\beta_{STOCK}}_{=1} = (1 - w_{CASH})$$

32 The problem of standardising the qualitative output of several analysts (each one with his own bias) is not present as it is the case with judgmental management. It also ensures historical homogeneity as the same method can be applied over time.

33 Up to a maximum pre-specified level most of the time.

Principles

If the portfolio manager believes that he can predict with some success the relative performance of the benchmark with respect to the cash then, following the rule of active management expressed in 1.2.1.2, he should:

- Hold some cash if he forecasts that stocks will under-perform cash. Under these circumstances, we know that the portfolio manager should hold position in the stocks so that the portfolio beta will be smaller than 1. Analysing the previous formula, this is only possible if $w_{CASH} > 0$, i.e. if he holds a positive amount of cash.
- Similarly if he forecasts that stocks will over-perform cash, he must make sure that he will hold a negative amount of cash in order to achieve a portfolio's beta larger than 1. In other words he must borrow money in order to invest more into stocks and get a larger exposure to equities through leverage.
- Finally, if his forecast is that stocks will match cash in terms of performance he must hold no cash so that the beta of the portfolio (=1) will be equal to the benchmark. By doing this, he will have no active risk linked to the benchmark volatility; he must take this position because he cannot expect any reward for taking an active risk. This attitude is the only consistent with the maximisation of the information ratio.

Remarks

Notice that, in most cases, portfolio managers are not allowed to borrow and therefore leverage the managed portfolios. This creates an asymmetrical situation where a portfolio manager can only take active bets when he thinks that stocks will under-perform cash.

One way to break down this asymmetry is for the portfolio manager to hold only stock and to make sure that the beta of the stock portfolio is larger than 1 when he expects the benchmark to outperform cash. By doing so, he will be able to create the desired active exposure to the benchmark.

Notice that forcing the beta of the portfolio to a value larger than 1 may interact, positively or negatively, with his stock selection. For example, he may prefer, for stock selection reasons, stocks with lower betas to stock with higher betas. Forcing the portfolio's beta to a larger value (>1) will push him to diminish the weight of the lower beta stocks and increase the weight of the higher beta stocks. This may well end up with a lower expected active return from stock selection.

When the portfolio manager feel obliged to raise cash because he fears a bad performance of stocks relative to cash, he also has to forgo some expected active return from stock selection because now this stock selection is applied to a smaller fraction of the total portfolio. One way to alleviate this loss is to use a future overlay, if futures linked to the benchmark do exist. If so, the portfolio manager stays fully invested in stocks (and does not give up any expected return from stock selection) and sells futures to position the beta of the portfolio correctly with respect to his forecast.

*1.2.1.7 Industry allocation**

As, within a country, the most powerful factors explaining risk are the industry/sector related ones, it is not surprising that the subject of industry allocation is usually[34] treated separately from stock selection.

Notice that that performances can vary a great deal between industries. For example, there was a 100% difference in absolute performance between the best performing sector in the US in 2000, i.e. utilities (+59.5%), and the worst performing, i.e. technology (-40.0%). In terms of relative price behaviour, industries exhibit on average both price momentum on a one month basis and price reversal on a half year basis. This suggest that rotation between industries should occur much more frequently than for style factors.

The sensitivity of industries to the business cycle is also unequal. Some industries, linked to consumer discretionary spending (like the automobile industry), experience very quick and sharp decrease in sales in case of an economic downturn. The consumer can easily postpone the purchase of a new car if he feels threatened by it or if he suffers the prevailing economic conditions. For other more direct consumer goods, any postponement immediately affects the consumer well-being and is usually handled much less abruptly. In other words, different industries have different sales sensitivities to the vagaries of the business cycle. This is not the only channel through which the business cycle affects an industry's earnings. Their earnings are affected differently depending on the ratio of their fixed costs to their variable costs and on their financial leverage. A similar decrease in sales could cause different decreases in earnings depending on these ratios.

The different industry responses to the business cycle are indirectly exemplified by their different return sensitivities to the market (beta), as limited list shows below:

- Securities & Asset Management: 1.38
- Semiconductors: 1.23
- Electronic Equipment: 1.08
- ….
- Grocery Stores: 0.76
- Tobacco: 0.73
- Electric Utility: 0.62

*1.2.1.8 Stock selection**

Judgmental management

In terms of predicting returns, most judgmental processes stop short of providing explicit quantified return forecasts. They usually provide either an ordinal ranking of the stock universe or more explicitly a list of stock recommendations. Using this information the portfolio manager then constructs portfolios following the principles described earlier.

In this section we will not go into further details on how such rankings and lists are established because judgmental stock analysis techniques are presented throughout in different chapters of this program[35].

34 Especially within a top-down approach.

35 See for example sections 1.2.1.3 and 1.2.1.10

Quantitative management

In this section we will present several quantitative stock selection techniques. We will first consider screening methods based on a single criterion. Then, we will generalise the method to take into account several criteria by showing how these criteria can be aggregated into a composite criterion in practice.

Single criterion screening selection

The single criterion screening management proceeds along the following lines:

- Given a financial characteristic of a stock (i.e. some kind of a factor exposure) at time t, the stocks in the investment universe are ranked in declining order,
- These stocks are then separated into deciles in declining rank order,
- The average performance[36] of each decile is finally computed for the period t to t+1.
- This exercise is then repeated for all time periods t to t+1 (t=1, …, T) given the availability of data.

At the end of this procedure, one can test if there are statistically significant differences in the average returns of the different deciles. If, for example, the difference in average return between the upper and the lower decile(s) is significantly strictly positive, this is an indication that the stocks of the upper deciles have a tendency to outperform the stocks in the lower deciles.

The art is to identify a stock characteristic for which the performance of the upper decile(s) is consistently better than the performance of the lower decile(s)[37]. The forecast part of this method is based on the assumption that the past positive relative performance of the upper decile(s) over the lower decile(s) will still hold in the future. It is a naive forecast in the sense that past behaviour is simply expected to extend to the near future. The main risk is that this may not hold true.

The idea then is to build a portfolio of stocks from the upper decile(s), and to discard stocks from the lower decile(s) in an existing portfolio. From here, we can consider different ways of constructing the portfolio depending on whether the manager tries to forecast explicitly or quantify or not the active return of each stock.

A first approach is to establish a set of desirable stocks belonging to the upper decile(s) and a list of the undesirable stocks from the lower decile(s). The list of undesirable stocks is used to discard stocks already in the portfolio. The portfolio can then be built along these lines using either heuristic rules or an optimiser in order to control risk.

A second approach is to make an explicit forecast for the stock of each decile. For example, the expectation, however naive, that the average past performance of each decile is going to prevail in the near future can be considered. The manager will give to each stock, a forecast equal to the average past performance of its decile. Then he can use an optimiser to its full extent to build the portfolio and balance the risk taken with respect to the forecasted active return. One of the dangers of this second approach is the rather crude way of forecasting returns and its very slow response to any quick change in trend of the decile returns.

36 Usually on an equally weighted basis.

37 In a way, it is not too far from identifying a single factor with a positive trend in cumulative returns and taking its asset exposures as the criteria.

Composite screening selection

Up till now we have considered screening management based on a single criterion. In real life there are many criteria, which exhibit exploitable return patterns. This raises the question of how to aggregate these different criteria into a single composite criterion. Unfortunately, there is no really good answer to this question. This is because the discriminative power of each criterion is asserted individually without taking into account the other criteria. It may indeed happen that the discriminating powers of the different criteria are correlated. If the manager is not cautious he may believe that these criteria explain more than they really do.

A common way of getting a composite score is to add for each stock the decile to which it belongs for each criterion. The following table is an example of this procedure:

Stock	Criterion 1 (decile)	Criterion 2 (decile)	Composite Score
A	1	2	3
B	3	6	9
C	2	3	5
D	6	9	15
...	...	...	...

The composite score can then be used to build a portfolio following the first approach for single criterion screening. As already mentioned this aggregation procedure is not risk free. The following examples illustrate this:

- If two criteria are highly correlated, it means that together they do not bring significantly more discriminative power than each single one. However, the aggregating scheme will give a double weight to what is really a single criterion. The result will be overexposure to this underlying criterion.
- If two uncorrelated criteria have very distinct discriminative power[38], the composite score gives identical weight to the two criteria, which is at odds with the fact that one is more powerful than the other.

Another way to aggregate criteria is to add the explicit forecast determined for each single criterion following the second approach described in the previous section. This method is also subject to the same criticism if the individual criteria are correlated. However, for uncorrelated criteria the addition of the explicit forecast takes into account in a reasonable way the difference in discriminating power between single criteria.

38 For example, the difference in average past performance between the first and last decile for the first criterion is +5% while the same figure for the second criterion is +2%.

*1.2.1.9 Anomalies**

Definition

If the stock market is efficient, stocks should not exhibit any systematic differences in returns over time other than those called by the asset pricing (risk) model. In other words, if, for example, the CAPM is the relevant risk model, these differences should disappear once the stock betas are taken into account in order to explain them.

Over the last twenty years, many empirical studies have shown that many systematic differences in stock returns, which are not explained by the CAPM, were statistically significant over time. These findings raise questions about market efficiency. Unfortunately, it is not possible to answer these questions unambiguously. As already suggested, any significant systematic differences in stock returns over time can be explained:

- either by differences in the stocks risk characteristics inherent to the asset pricing model,
- and/or by market inefficiencies.

As it is not possible to ascertain which is the true asset pricing model, all tests designed to detect these systematic differences in returns are conditional on the chosen risk model. In other words, when such a systematic difference is detected, it could be due to:

- a market inefficiency,
- and/or an asset pricing model misspecification.

Within these conditions, it is not possible to ascertain that it is indeed a market inefficiency. Hence, these disturbing empirical evidences are called “anomalies” and not inefficiencies.

Typology

In this section, we will briefly present a non exhaustive list of the anomalies that have been brought to the attention of the financial community. We will focus on the case of the US capital market as it is the best documented. For most of these anomalies, similar patterns in stock returns have been detected in other major stock markets. Here is a summary of the major anomalies for the US market that were identified in the past, and which have or have not disappeared since their discovery. These anomalies are linked to:

Firm Size:
At the beginning of the eighties, it was shown that the market value of a firm was a criterion that could help discriminate between stock returns or, more precisely, that smaller firms exhibited substantial over-performances with respect to large firms. Since that time, the evidence is more mixed as there have been extended periods of time where large firms over-performed small firms. It does not contradict the fact that firm size is a discriminating criterion in terms of stock return differences even if the direction of those is now less clear. Further studies have also shown that the size anomaly happens usually in the first half of January and may be linked to the tax-loss sales that occur in December in the US.

Seasonality:
Several seasonality anomalies have been detected through empirical studies. They show that stock returns are on average not constant over time or, in other words, that they exhibit seasonal patterns. The time scope for these seasonal effects goes from the year, the week, even to the day:

- Within a year, stocks tend to perform better in January than during the other months. This January effect mentioned for the previous anomaly is mostly concentrated on small firms' stocks.
- Within a week stocks on average show negative returns on Monday[39] and positive returns the other days of the week.
- Intra-day stock prices exhibit interesting patterns depending on the day of the week. A large fraction of the difference in returns between the days of the week occurs in the first hour of trading. In addition, the last hour of trading is on average characterized by positive returns.

Evaluation criteria:
Financial evaluation criteria have shown in the past discriminating power with respect to future stock returns. Stocks with higher earnings yield (lower P/E) or book-to-price ratio have shown higher returns than stocks with lower earnings yield (higher P/E) or book to price ratio in the past[40].

Relative past performances:
Stock relative past performances can also help predict future relative performance. These forecasts depend crucially on the time periods which are used to assess the relative past performances:

- When the relative past performance is evaluated over a time period shorter than a month, the over(under)-performing stocks tend in average to under(over)-perform over the subsequent same period of time.
- When the relative past performance is evaluated over a time period longer than a month but shorter than a year, the over(under)-performing stocks tend on average to still over(under)-perform over the subsequent same period of time.
- When the relative past performance is evaluated over a time period longer than three years, the over(under)-performing stocks tend on average to under(over)-perform over the subsequent same period of time.

In other words, in the first and third cases stock prices seems to be subject to price reversal. Adopting a "contrarian" strategy by selling winning stocks and buying losing ones one should be able to improve performance. In the second case stock prices seem to exhibit price momentum. Here, the successful strategy consists of buying the winners and selling the losers.

39 Notice that the return on Monday is computed based on the closing prices between Friday and Monday. In other words, the so-called Monday return includes the week-end.

40 During the time period beginning at the end of 1997 and ending in March 2000, a very strong opposite phenomena took place, characterized by the over-performance of the lower earnings yield (or book-to-price ratio) stocks. Again, these evaluation criteria were still very powerful in terms of discriminating stock returns, even if many market participants were surprised by the reversal in relative performance.

Earnings surprise:
When a firm announces its results and its earnings, there is scope for a surprise in the sense that the published figures may differ (positively or negatively) from what the market[41] was expecting. In the past, the stocks with positive (negative) surprises, i.e. announced results better (worse)than expected, showed on average positive (negative) relative performance over quite some time[42] after the publication of the results. Although a large fraction of these relative price movements occurred right after the time of the announcement, it was surprising to see that these relative performances would go on for quite some time. Nowadays, this anomaly has almost completely vanished. The price adjustments following the publication of the results occur almost immediately. After this initial adjustment, there is on average no difference in relative performance between the stocks that have experienced a positive or a negative surprise.

Change in expected earnings:
Over the past, positive (negative) changes in the market expectations of earnings[43] have been followed by subsequent patterns of positive (negative) relative performance over the next few months.

All these difference in return patterns, which have been detected in the past, were not of course systematic through time. These results focus on the statistically significant averages over the estimating period. It does not mean that month after month these differences in returns were observed. Since the time of their publications, some of these anomalies vanished[44], some persisted, and some reversed[45].

All the above anomalies have been described by the difference in subsequent performance between two groups of stocks distinguished by a (financial) characteristic. This suggests some simple active strategies[46] in order to create positive active performance. If an anomaly persists in the future, it would be enough to over-weight the stock expected to over-perform and under-weight the stocks expected to under-perform. In other words, it would suffice to tilt the portfolio toward the desired (financial) characteristic. The risk for such a strategy is that the anomaly could disappear or reverse, creating in this last case active under-performance. Notice that the single criterion screening[47] described earlier is one way to exploit these anomalies.

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41 Institutions like I/BE/S or First Call collect analyst's earnings estimates and provide summary statistics of those estimates. The average estimate is a prime candidate for a proxy of the market view on the expected earning.

42 Several months.

43 As measured for example by the I/B/E/S or First Call consensus average.

44 Maybe the inefficiency disappeared as expected when systematically exploited.

45 For example, the earning yield anomaly in the US did reverse strongly from the end of December 1997 to March 2000. During this time period high earning yield stocks severely under-performed low earning yield stocks.

46 Which have been exploited with some success by some portfolio managers.

47 See section 1.2.1.8.

*1.2.1.10 Technical analysis**

Definition

The technical approach to investment relies on the idea that prices change following trends and patterns that reflect changes in investors' attitudes toward economic, financial, and psychological forces. There is also the premise that these changes in investor's attitude are sufficiently stable through time so that these trends and pattern in prices repeat. Technical analysis, in its simplest form[48], studies past stock prices to identify these recurring trend changes and price patterns at an early stage in order to predict future price movements. It allows the investor to maintain an active investment position until there is enough evidence to identify a tend reversal.

Efficiency and fundamental analysis

Notice that the idea that there are recurring patterns in price movements can help the investor to beat the market is at odd with the concept of market efficiency. From the efficiency point of view, such patterns would soon be exploited and consequently the arbitrage opportunity would quickly vanish. Recently, technical analysis has received some moderate support from behavioural finance that outline that investors do not always behave rationally and persist in their irrationality. The empirical evidence from some anomalies (price momentum, price reversal) is also giving some support to the notion of price predictability based on past prices. Underlying technical analysis is also the notion that fundamental analysis is not necessary for successful investment strategies. Technical analysis does not focus on the underlying economic forces at work. It limits itself to the study of the market assessment of those as revealed by price changes.

Techniques used

As already mentioned, technical analysis aims at identifying changes in trends as soon as possible in order for the investor to still be able to bet on them. As prices never progress without ups and downs, some distinctions in trends are useful. Typically, three different trends are considered:

- Primary trends that usually extend over a period of time between 1 and 2 years. They reflect the investor's view toward the underlying forces unfolding in the economic cycle.
- Intermediate trends, known to last from 3 weeks to half a year, are temporary interruptions of the primary movements. They are important to determine if a primary trend may be reversing.
- Short-term trends, which last from 1 to 3 weeks, are of a more random nature and therefore more difficult to identify.

48 More sophisticated technical analysis also uses indicators other than past prices, such as sentiment indicators (from insiders, advisory services), flow-of –funds indicators (cash positions in mutual funds, pension funds, new equity offering, margin debt, …), and market structure indicators (volume, market breadth).

Describing all methods used in technical analysis to determine trend changes is certainly beyond the scope of this section. We will briefly present a broad typology of those:

- Peak-and-Trough Progression: this method is linked to the idea that primary upward (downward) trends are characterized by a series of rising (falling) peaks and troughs for the intermediate trends. Any interruption in such a series is then interpreted as a signal of a change in the primary trend.
- Price patterns: here, the underlying idea is that change in trends usually occurs through transition periods that are signalled by very specific price patterns. These very specific price formations, once completed, are thought to be signals of either a reversal in trend or a continuation of it. There are numerous such price patterns. Let us just mention a few of them like the Head and Shoulder, Double Tops and Bottoms, Broadening Formations, Triangles, Flags, Pennants, Gaps, …
- Moving averages: moving averages are smoothing statistics that make trends more readable when applied to price time series. Change in direction in a medium term moving average (200 days) or a crossover of the price are thought to be indicators of a change in trend. More sophisticated techniques using exponential moving averages and bands[49] around these averages fall into this category.
- Momentum: in order to get more timely signals of price trend changes, market technicians have come up with indicators of price dynamics in terms of acceleration or deceleration. The idea behind these indicators is that usually an upward (downward) trend experiences some loss of upward (downward) momentum before going through a reversal. A typical momentum indicator is the Rate of Change (ROC) statistic that measures the percentage price change over a predefined period of time. A rising (declining) ROC is usually interpreted as a bullish (bearish) signal. More sophisticated indicators such as the Relative Strength Indicator (RSI), the Moving-Average Convergence Divergence (MACD) oscillator and the Stochastic indicators are other momentum instruments for the market technicians.
- Point and Figures Charting: instead of representing graphically the time series of prices using the x-axis for the time and the y-axis for the price, some technicians use another way to represent such an evolution, called a point and figure chart. These charts differ from the standard one in two ways. First only price movements of a minimum amplitude are recorded on the graph. Second, time advances of one "unit" when there is a price reversal. These charts are interpreted in similar ways as the others.

The list of methods given above is by no mean exhaustive. These methods can also be used in combination and are basically timing techniques. They are used at the single stock level, at the indices level, and for interest and exchange rates.[50]

Market technicians also like to use data on volumes because they are thought to be an indicator of the intensity of the changes in the attitudes of the investors. Usually volume increase on rallies and decreases on declines. So, for example, lower volume on price advances can be interpreted as an early warning for a trend reversal. However, this signal has to be confirmed by similar indications received directly from the prices before acting upon it.

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49 Such as the Bollinger bands.

50 In a recent article, Scaillet and Bajgrowicz (2010) found that accounting for transaction costs, it was virtually impossible to construct strategies based on a vast combination of technical trading rule that were able to generate significant risk adjusted returns. SCAILLET O. and BAJGROWICZ P., 2008, "Technical trading revisited: persistence tests, transaction costs, and false discoveries" Swiss Finance Institute DP 2008.5

*1.2.1.11 Specialized management**

Some active portfolio managers specialize in specific segments of the universe of stocks, such as value stocks, growth stocks, or small cap stocks.

In essence, active management on these segments proceeds along the same general principles. It does not mean that discriminating criteria or factors (in terms of subsequent returns) between stocks may not have exactly the same power[51] for each segment. Here are some examples of anomalies that work differently depending on which stock universe subset is used:

- Earning yield works better for value stock than for growth stocks.
- Earnings revision works better for growth[52] stock than for value stocks.
- Price momentum seems to work best with small (growth) cap stocks.
- Short-term price reversal is more effective for small growth stocks.

The investment approach may also matter. For example, a bottom-up analysis for small cap stocks seems more appropriate.

*1.2.1.12 Enhanced indexing**

Return enhanced indexing covers several active investment methods wherein the portfolio manager tries to improve the performance of an indexed fund by taking very limited and controlled active risks. No explicit or quantified forecasts for returns are used. The portfolio construction is then used only to control risk. This type of active management is usually less aggressive in terms of achieving an active position.

Arbitrage between the equity and futures markets

This method enhances returns and relies on the potential mispricing between the equity spot market and the futures market. It is a pure arbitrage technique, which implies that the risk taken is nearly non-existent.

Depending on the relative valuation of the futures market with respect to the equity spot market, the portfolio manager will switch the indexed portfolio to the cheapest alternative, provided that the divergence in valuation is large enough to compensate for the transaction costs.

For example, let us assume that a futures contract exhibits a sufficient[53] discount with respect to its fair value. The portfolio manager will sell the equity index portfolio and replace it with the inexpensive futures contract. By doing so, he secures a positive excess return equal to the discount of the futures contract as he will be able to realise the future's contract at fair value when it expires.

51 Hence the specialization.

52 Especially small cap stocks.

53 The discount is assumed to be large enough to cover the cost of selling and buying back the equity indexed portfolio.

But this is only the minimum excess return he is guaranteed to get over the life of the futures contract. Later it may happen that the relative valuation of the futures contract may exhibit a sufficient premium, prior to its expiration. In this case the portfolio manager will reverse his position by closing the futures contract and buying back the equity portfolio. The excess return achieved is therefore equal to the discount at the purchase of the futures contract, plus the premium at the sale, minus the transaction costs. Note that this was effectively obtained before the expiration of the futures contract.

In order to implement this arbitrage, several conditions must be met:

- The futures contract should be on the index used. If this is not the case, the tracking error of the futures contract with respect to the index used[54] has to be taken into account when assessing whether the futures contract is sufficiently mispriced. Most of the time this tracking error is too large to exploit the actual futures discount or premium.
- The actual equity portfolio should be very tightly indexed. Otherwise, the tracking error widens the futures mispricing needed for the arbitrage to occur without taking too much risk. Practically, a full replication method should be used.
- The liquidity of the stocks and the trade organisation at the stock exchange should be compatible with the execution of the arbitrage. This means that the trade on all stocks in the index can be executed with little market impact and with sufficient synchronisation with the futures trades.

This return enhancing method was quite positively rewarding in the US during the eighties. Later, the pricing of the futures became more efficient, thereby limiting the excess returns provided by this technique.

Finally, note that this method can to some extent also be applied when using synthetic replication. The portfolio manager can then play on the different mispricings of the futures of different maturities and rollover to the cheapest one, if transaction costs are covered by the spread between the futures prices. Nevertheless, if all futures are at a premium at one time, the portfolio manager can only minimise a negative active return.

Use of an evaluation model

Another method to improve the performance of an index portfolio relies on using an equity evaluation model at the same time. Based on this model, the portfolio manager makes a list of (very) undesirable stocks that must be excluded. Then, while constructing the indexed portfolio, using a stratified or an optimisation sampling technique, he uses only the stocks not excluded. For example the optimisation program becomes:

$$\underset{\{w_i^P \geq 0: i=1,\ldots,N\}}{\text{MIN}} \tilde{V}(R_A^{P,B})^{T,T+1} \quad \text{with} \sum_{i=1}^{N} w_i^P = 1$$

and $w_i^P = 0$ for all stock i in the list of excluded stocks.

With this method, there is an implicit forecast based on the evaluation model that the stocks on the list will have a negative return. As these forecasts are not quantified, they are implemented through zero weight constraints when constructing a portfolio. The resulting tracking error increase gives a measure of the active risk taken.

54 And not the underlying index of the future.

The zero weight constraint on an undesired stock can be too abrupt if the weight of the stock in the index is large. In this case, the risk implication can be very detrimental especially at the specific level. The solution for the indexer is to simply underweight the stock to a lesser extent:

$$0 < w_j^P = w_j^B - e \qquad \text{for a large undesired stock j (e>0).}$$

Finally, notice that the role of the valuation model is simply to set a list of (very) undesirable stocks. This imposes no constraint of a minimal output on the valuation model. Even pure qualitative judgements can be implemented.

Tilted indexing

Tilted indexing is based on the evidence that some factors show a long trend of positive returns. For example, in the US, there were extended periods when stocks with high dividend yields, or stocks with high earnings yield were outperforming the market much of the time.

Let us assume that the risk factor 1 has exhibited this trend pattern. The tilting method is simply based on the idea that the positive trend will continue in the future. Then by forcing a positive active exposure of the indexed portfolio towards this factor, the manager hopes to improve the performance of the indexed portfolio. The optimisation technique can be summarised as:

$$\underset{\{w_i^P \geq 0 : i = 1, \ldots, N\}}{\text{MIN}} \quad \tilde{V}(R_A^{P,B})^{T,T+1} \qquad \text{with } \sum_{i=1}^{N} w_i^P = 1$$

$$\text{and } x_{P,1} = x_{B,1} + e \qquad \text{where } e > 0.$$

Let us assume that the first risk factor exhibited an average annualised positive return of 100 basis points in the past. If the indexer actively exposes the indexed portfolio with respect to the first factor by 0.5, he can then hope for an excess return on his portfolio of 50 basis points a year, provided that the first factor behaves in the future as it did, on average, in the past.

The forecast part of this method is the assumption that the past positive trend will continue in the future. The active risk is the increase in the forecasted tracking error due to the additional constraint imposed on the optimisation. The potential problem with this method is the very crude forecast for the return of the relevant factor. The investor is at the mercy of a trend reversal.

Options overwriting

The last method of enhancing return of an indexed portfolio that we will take up is the overwriting of out-of-the-money call options. With this method, the indexer manages the indexed portfolio as usual. But in addition, he writes out-of-the-money call options on the index, or on sub-indices, or on stocks he holds.

Let us consider the writing of call options on the index. By doing so, the portfolio manager bets on the following scenario:

- First, he gets the time value of the calls he sells,
- Second, he hopes that the index will be lower than the strike price of the calls at their expiration.
- Two outcomes to this scenario should be considered:
- First, the index is lower than the strike price of the expiration of the call. In this case, the option is worthless and is not exercised at expiration. The performance of the indexed portfolio is improved through the proceeds of the sale of the call.
- Or, the index is higher than the strike price of the calls at expiration. In this case, the call will be exercised. The indexed portfolio performance will be penalised by the difference in value between the level of the index at expiration and the strike price of the calls[55].

One can easily see that the essence of this return enhancing method is timing forecast. The portfolio manager forecasts that the index will not move above the strike price of the sold calls. The risk he takes can be represented as an asymmetrical beta of the indexed portfolio. Considered at the time of expiration and below the strike price, the beta of the portfolio is unaltered by writing[56] the calls and is equal to the index beta. Above the strike price, it is negatively penalised by the positive beta of the calls and therefore strictly below the beta of the index. In other words, a benchmark's risk arises when it rises above the strike price of the calls.

Within this framework, the access to a benchmark's timing model can help. But, this is an asset class allocation problem beyond the scope of the discussion of this section.

There can be another motivation to sell calls on the index. The calls can be expensive because the market attributes a high implicit volatility to the calls. The manager of the indexed portfolio can bet that the implicit volatility will decline before their expiration and that he will be able to buy them back at a lower price. Nevertheless, he takes the benchmark's asymmetrical timing risk as long as he is short on the calls.

The manager can eliminate the drawback of being a (market) directional method through the sale of volatility swaps depending on his expectations of market volatility changes. As these swaps have a zero delta, the success of this method does not depend on the direction of the market. The portfolio manager does not need to have any market timing abilities. However, he must be able to forecast the change in market volatility with some success.

*1.2.2 Passive management**

*1.2.2.1 Stock index portfolios**

The purpose of indexed portfolios is to *replicate the performance of a predetermined benchmark*, usually an index, as closely as possible by means of a managed portfolio. In other words, their formal goal is to have both an expected active return and an ex-ante tracking error as close as possible to zero:

55 Multiplied by the number of calls.

56 The delta and therefore the beta of the calls are zero.

$$\tilde{R}_A^{P,B} \approx 0 \qquad \text{and} \qquad T\tilde{E}_A^{P,B} \approx 0$$

Most of the time, the targeted benchmark (or index) is also a portfolio[57] managed by a committee. Given the adopted methodology and rules, the committee changes the composition of the index through time by:

- introducing or deleting a stock,
- reflecting a stock capital operation,
- introducing a new country,
- and so on...

All these changes are without cost in the idealised virtual world of a benchmark, which is implicitly characterised by zero transaction costs. They nevertheless imply costly turnover if one tries to fully replicate these changes in actual portfolios. Because of these changes in composition of the benchmark, there is a tendency for a *negative drag*[58] *on the performance of an indexed portfolio* with respect to its benchmark ($\tilde{R}_A^{P,B} < 0$). The importance of this drag depends on the internal turnover of the benchmark and the transaction costs with which the investor is confronted.

The most important decisions that the investor and the portfolio manager have to take when indexing are:

- the choice of the indexing technology,
- the choice of the benchmark, and
- the degree of closeness of the performance replication.

These three questions will be discussed in the following sections.

*1.2.2.2 Specialized index funds**

Besides plain vanilla indexing, an investor, depending on his investment goals, may require his indexer to practise his art on more specialized benchmarks. The following paragraphs describe some of the specialization asked by investors in terms of indexation.

<u>Customized benchmarks</u>

Some investors want a customized benchmark, usually derived from an official index, which is then modified through a set of very specific rules. The following list enumerates in a generic way some of these rules:

57 The most noticeable exceptions are the equally weighted benchmarks. These benchmarks that cannot be considered as portfolios should be avoided, as they imply continuous or periodical restructuring of the indexed portfolio, which imply some drag on the performance with respect to the benchmark because of transactions costs.

58 This drag can be alleviated if there are cash in- or outflows to the indexed portfolio especially if synchronised with the important changes in the benchmark's composition.

- Stock exclusion
- Stock over- or under-weighting scheme
- Industry/sector exclusion
- Industry/sector over- or under-weighting scheme
- Country/region exclusion
- Country/region over- or under-weighting scheme

In other instances, some investors want GDP weighted international indices instead of the usual market capitalization-weighting scheme.

Some customisation is also linked to existing regulations. For example, some funds[59] are subjected to the following restrictions:

- No single stock position can exceed 10%.
- The weight of all stock positions that are above 5% cannot exceed 40%.

As there are numerous indices that do not satisfy these restrictions, it is not possible to index properly if these indices are chosen as benchmarks. One solution is to customize these indices through the application of the restrictions.

In other cases, customized indices are derived from existing indices through the exclusion of stocks that are thought to be unethical or incompatible with the Sharia by the investor.

Once the benchmark is determined, there is no special problem managing the related indexed portfolios in most of these cases.

<u>Completeness portfolio</u>

When a large pension fund has mandated the fraction of its assets (corresponding to its satellite active portfolios) to several active portfolio managers indicating to each of them a benchmark, it is possible to determine the composition of the aggregate benchmark for all the active managers. It may be that this aggregate active benchmark[60] diverges from the pension fund's global strategic benchmark. In this case, the pension fund is taking unwanted long-term risk because no reward is warranted for this divergence. One way to solve this problem is to set a specific benchmark for the indexed core portfolio so that the aggregate benchmark for all active and passive portfolio managers coincides now with the overall strategic benchmark. In other words, the benchmark for the passive core holding is tilted from the strategic benchmark so that it corrects the divergence of the aggregate active benchmark. This special benchmark and the core holding portfolio, which is indexed to it, are called the completeness benchmark and portfolio.

59 Like the Luxemburg SICAV part 1.

60 And not the aggregate portfolio.

In terms of indexing a completeness portfolio, two remarks should be made:

- In essence the completeness benchmark is dynamic. Its composition depends on the overall strategic benchmark, the relative sizes and benchmarks of the active portfolios. Notice that the relative sizes of the active portfolios change through time due to market forces and also because of asset allocation changes between the active managers. Within these conditions the indexer has to simultaneously determine the completeness benchmark and index the portfolio to it.
- Sometimes, because of the overlapping of the active managers' benchmarks, some stocks of the overall strategic benchmark may be already over-weighted in the aggregate active benchmark. In this case the completeness benchmark and portfolio must be short in these stocks.

Style portfolios

Portfolios can also be indexed to more specialized sub-indices representing a style like small cap, value or growth. Again there is not much to say in terms of indexing, except perhaps in the case of small caps. Most of the small cap indices or sub-indices include a large number of stocks that are usually not very liquid. This fact makes the indexing more difficult and, therefore, one must expect larger ex-ante and ex-post tracking errors.

Socially Responsible Investment (SRI)

Some investors have always been particularly concerned with the social impact of their financial investments. As early as 1971 some mutual funds have started to propose investment strategies with exclusion criteria offering a guaranty that some sectors (such as military, tabacco, etc…) would not be included in the portfolio. While initially confined to a limited number of investors (religious groups, universities), SRI represents now approximately 11% of total assets under management in the United States.[61]

SRI funds are available under many forms such as Exchange Traded Funds, mutual funds or structured products. Some funds apply standard methods (discussed above) in addition to exclusion criteria while others are more specifically focused on the interaction between ethical and social performance.

*1.2.2.3 Indexing technology**

There are four basic technologies available for indexing. Their use depends on how closely the investor wants to track the benchmark, the size of the indexed portfolio, and the number of stocks and their weight distribution in the benchmark. Note that in all cases no forecast of absolute or relative returns, at the benchmark or the stock level, will be used; therefore indexation qualifies as passive management.

Exhaustive sampling[62]

Exhaustive sampling consists of fully replicating the benchmark as a portfolio. The indexed portfolio is constructed following these rules:

61 Social Investment Forum, 2007 Report on Socially Responsible Trends in the United Sates.

62 Also known as full replication.

- it has the same stocks as the benchmark,
- the weight of each stock is identical to that in the benchmark.

In other words, we have:

$$w_P^i = w_B^i \qquad (i = 1, \ldots, N)$$

Given these conditions one can show that:

$$\tilde{R}_A^{P,B} = 0 \qquad \text{and} \qquad T\tilde{E}_A^{P,B} = 0$$

With this method, every time the composition of the benchmark changes or there is some in- or out-flow of money from the indexed portfolio, theoretically the indexer should trade in all the stocks in the benchmark. In practise this cannot always be fully implemented, especially when the index encompasses a large number of stocks[63]. Thus, we can see several drawbacks of exhaustive sampling:

- The indexer can be required to trade on a potentially very illiquid segment of the stock market. This can significantly increase the cost of establishing or rebalancing an indexed portfolio.
- If the amount of managed money is small with respect to the number of stocks in the benchmark, it can imply very small trades for specific stocks when establishing or rebalancing the indexed portfolio. These trades can be very costly to implement because of their small size or the round lot constraints.
- Depending on the amount managed and if the number of stocks is large, the administrative costs associated with the transactions and with the monitoring of the index may be significant.
- Nevertheless, the exhaustive sampling technology offers several advantages:
- Little information is required, because the portfolio manager only needs to know the list of the stocks and their weights in the benchmark.
- The tracking error is minimal as both the indexed portfolio and the benchmark coincide. The only divergence in performance is mainly related to the drag linked to the changes in the benchmark and the associated costly turnover.

This method is often used when either the amount of managed money is large with respect to the index, or when the index is composed of a small number of stocks (i.e. SMI, CAC-40, DAX-30 etc.). In the second case, not using an exhaustive sampling may imply too much specific risk, related to the stocks that have been discarded from the indexed portfolio. This may result in a large tracking error.

63 Large single country indices include several hundred stocks, while the number of stocks in large international indices can reach several thousand.

Stratified sampling[64]

The stratified sampling technology tries to overcome the drawbacks of the exhaustive method by limiting the number of stocks in the indexed portfolio. It cleverly allows the indexer to avoid the illiquid segment of the benchmark and offers better control on the minimum size of positions and trades in the managed portfolio. On the other hand, there will be a significant difference between the compositions of the indexed portfolio and the benchmark, and this contributes to a tracking error.

Constructing an indexed portfolio requires the following steps:

- First the universe of the benchmark's stocks is partitioned into cells based on the stock's characteristics[65]. The weight of each cell is then computed, given the weights of the stocks in the benchmark.
- Next each stock whose weight is above a threshold weight is included in the indexed portfolio with its own weight in the benchmark. At this point, most of the cells of the indexed portfolio will exhibit a deficit in weight because of the stocks[66] that have been excluded.
- Finally, the deficit in weight of each cell is filled by choosing heuristically[67] among the remaining stocks available for these cells and using the threshold weight for each of these stocks.

The rationing in the number of stocks comes in this third phase where the included stocks are over-weighted with respect to the benchmark and therefore take the place of other stocks. The higher the threshold weight, the smaller is the number of stocks in the indexed portfolio.

Referring to section 1.2.1 one can interpret the stratified sampling technology as a simple weight deviation risk control technique because the active weight for each cell of the partition is limited by the threshold weight.

Some of the advantages of the stratified sampling method are:

- It alleviates the drawbacks of the exhaustive sampling approach by avoiding the illiquid segment of the benchmark, by keeping trades above a reasonable size[68] and by limiting administrative costs.
- The information needed is still limited: in addition to what was required for the exhaustive sampling, the relevant characteristics necessary for the cell attribution should now be available and maintained.

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64 The stratified sampling is historically the first method used to create indexed portfolios without owning all the stocks composing the benchmark. It was an answer to the problems associated with exhaustive sampling when the amount of managed money is not sufficient to reasonably accommodate the number of stocks in the benchmark.

65 Typically in a single country indexed portfolio, the characteristics used most frequently are the industry/sector and the market capitalisation. In an international framework, the country should also be included.

66 Whose benchmark weight is below the threshold weight.

67 Liquidity of the stock is of prime concern here, hence a tendency to choose among the largest market capitalisation available.

68 As no trade is smaller than the threshold weight.

Some of its major drawbacks are:

- There is a tracking error inherent to the method, but no ex-ante measure of it is provided.
- It is practically impossible to implement all the characteristics, suggested for example by a multi-factor model, to partition the universe of the benchmark's stocks into cells. In other words, the risk control provided by this method is limited to a very small number of factors, usually the country allocation, industry/sector allocation and, to a certain extent, the size factor.
- While rebalancing an indexed portfolio, there is no way to know if the improvement induced by the trades required by stratified sampling is large enough to justify the transaction costs.

Historically, this technology has been and is still being used to manage indexed portfolios based on broad or partially illiquid benchmarks, when the size of the actual portfolio is not large enough to overcome the drawbacks of the exhaustive sampling.

Optimised sampling

The availability of risk models has provided a solution to the drawbacks of the stratified sampling approach.

First, by applying this technology to the indexed portfolio with respect to the benchmark, it is now possible to have a forecast of the volatility of the active risk, i.e. an ex-ante measure of the tracking error of the indexed portfolio.

Second, it is also possible to use the optimiser to control the ex-ante tracking error. As indexing does not imply any forecast of returns ($\tilde{R}_A^{P,B} = 0$), the optimiser solves the following problem:

$$\underset{\{w_i^P \geq 0: i=1,\ldots,N\}}{\text{MIN}} \tilde{V}(R_A^{P,B})^{T,T+1} \qquad \text{with} \sum_{i=1}^{N} w_i^P = 1$$

Furthermore a constraint must be added by the indexer, which limits the number of stocks, whose weight is strictly positive, to NP[69]. Notice that this last constraint is necessary. Otherwise, the optimiser will simply replicate the benchmark as the optimal indexed portfolio. By modulating the parameter NP, the indexer is then able to choose the ex-ante tracking error. The larger the parameter NP is, the smaller will be the active volatility.

Another possibility for the indexer is to constrain the optimisation on a subset of the benchmark's stocks that excludes the illiquid ones. By doing so, the indexer will bear some tracking error as all excluded stocks will be under-weighted. At the same time he will be able to avoid a benchmark segment that is very illiquid.

Optimised sampling is clearly an improvement with respect to stratified sampling. It allows the indexer to explicitly take into account all the factors relevant to the MFM. It gives him an overview of the trade-off in terms of risk between the different factors. Finally, the indexer can explicitly take into account transaction costs while rebalancing an indexed portfolio.

69 With this constraint, the minimisation problem is no longer a pure quadratic program. It is usually solved through heuristic algorithms.

The information needed for the development of a MFM implies a much larger cost than the one inherent in the previous methods. However, this cost has been reduced to a large extent because of the availability of commercial risk models.

Synthetic replication

With synthetic replication, we leave the world of physical stocks, for one of stock index futures and cash instruments. The method consists simply of taking the right exposure to the market, through a combined position of futures and cash[70]. Its advantages are:

- The implementation of the indexed portfolio and the cash in- or out-flows to or from it, is very quick, as the indexer trades with a few contracts and not with several hundred stocks depending on the index.
- The implementation is cheaper because of lower transaction costs, lower custody charges, lower transactions taxes and no withholding taxes.

Nevertheless, the synthetic replication method is not without drawbacks. Some of the drawbacks are:

- A futures contract may not exist for the index selected by the investor. He may have to use a futures contract on another closely related index. Nevertheless, the correlation between the two indices may be less than perfect, which implies a tracking error.
- Periodically, the futures contract used for indexing should be rolled forward, incurring a recurring cost. Because futures may trade at different prices other than their fair value, there is an additional source of tracking error. Furthermore, to estimate the fair value of a stock index futures contract one needs to forecast accurately the dividends of the stocks belonging to the index. This is another source of uncertainty and of tracking error.
- In order to track the index closely, the cash involved in the management of futures should yield the same interest rate as the one used for the fair value computation. This can be an additional source of tracking error.

This method is exposed to several sources of tracking error, which are not really quantifiable. However, its ease of implementation makes it useful in the management of a conventional large indexed fund, especially when dealing with the short-term fluctuations in cash.

*1.2.2.4 Systematic biases in indexing[71]**

One concern expressed in the previous paragraphs is that when building indexed portfolios one should avoid the illiquid segments, if any, of the reference indices. Also, some sampling techniques can lead to discard a large number of very small stocks from the indexed portfolio.

This means that occasionally the indexed portfolio is systematically biased with respect to its benchmark in the sense that there is a subset of stocks, usually very small and illiquid, which will never or barely be represented in the managed portfolio. At this point, one must note that, in terms of risk controls, the ex-ante tracking error is not the only concern that an investor

70 Notice that by doing so, you take the credit risk of future exchange and of cash borrowers.

71 Notice that the issue described in this section is also valid for active management.

must have. If, for example, the discarded stocks represent some 5% of the reference index and assuming that on average they are going to over-perform the other stocks over several years systematically at a yearly rate of 5%, one must expect a systematic cumulative tracking error of some –25 basis points on an annual basis. In addition these yearly ex-post tracking error figures may be large with respect to the ex-ante annual tracking error. This can lead one to think erroneously that either the risk model is under-predicting risk or that the indexed portfolio has been struck by a long streak of very negative events. In reality, when assessing the ex-post active performance of a portfolio, it is important to note that the average difference in returns has to be taken into account as it is not always equal to zero.

*1.2.2.5 Rebalancing when the index changes**

To make the point clearer in this section, we will consider a case where the indexing is implemented through exhaustive sampling. When the stock composition of the reference index changes, the indexer has no choice but to mechanically replicate these changes in the portfolio. Otherwise a strictly positive tracking error would be created. Let us assume that the index change consists of the addition of a new stock which will represent 5% of the index at the time of its introduction.

If the indexer wants to act by the rules and create no divergence in performance between the indexed portfolio and its benchmark, he has to sell 5% of all existing positions in the portfolio and buy for 5% of the new stock at the time the reference index changes, usually at the closing of the previous working day. In other words, he must execute all these trades at the closing prices of the day before the change. By doing so, the indexed portfolio will perfectly track[72] the reference index.

Notice that this behaviour is expected from every indexer who operates by the rules. Given the amounts usually under indexed management, it is easy to realise that the trading conditions for these stocks will be beyond hectic as the sizes of the intended trades will be far beyond the usual liquidity. In addition, it is likely that the bought stocks will temporarily shoot up and that the sold stock will temporarily decline in price. If this is the case, the willingness to perfectly index, while not creating any ex-post tracking error, nevertheless puts the investor into a position to buy stock at temporarily higher prices and sell stocks at temporarily lower prices.

The temptation is then to abandon momentarily a pure indexing strategy and to implement the trades around the time of the closing but not at the closing, with the hope that the trades can be executed at better prices. Notice that this is an active strategy betting on the realisation of the extreme price movements described before. Temporarily the composition of the indexed portfolio will diverge from its benchmark creating an ex-ante tracking error. If the strategy is successful, the indexed portfolio will over-perform temporarily.

This potential source of active return has not escaped notice. Many market participants[73] are playing this strategy by buying, for example, the added stocks in advance and reselling them, expectedly at a profit, to the indexers following a pure strategy and therefore providing them with the needed liquidity.

72 Except for transaction costs.

73 Such as active managers, arbitrageurs, hedge funds, brokers, …

Not surprisingly, these strategies are becoming more and more widely used. In some instances, the closing sessions have been characterized by reverse price movement because of excessive liquidity. This illustrates that there is always a risk in active strategies.

*1.2.2.6 Benchmark choice**

In this section, we want to discuss some of the criteria to be used when choosing a benchmark for indexing. One key question is linked to the relative importance the investor puts on two, sometimes conflicting, investment goals:

- The first is the extent of the stock exposure that he wants. Is an exposure to only large caps sufficient? Or is an exposure to mid and small caps needed?
- The second issue is linked to the desired liquidity of the indexed portfolio. Does the investor want to be able to expand and/or reduce its amount of money indexed quickly or not?

These two questions are interrelated, because usually including smaller and smaller stocks in a portfolio is synonymous to moving toward a less liquid segment of the market.

For an investor whose priority is the liquidity of his indexed portfolio, a preferred benchmark is one that is concentrated on large liquid stocks. The existence of a future's contract, with that benchmark as the underlying asset, would of course be a plus, as futures allows quick, cheap and efficient ways to get in and out of the underlying assets. For such investors, they would choose benchmarks like the ones suggested in the following table depending on which stock market they seek exposure.

Stock Market	Benchmark
Switzerland	SMI
Germany	DAX 30
France	CAC 40
USA	S&P 500
Japan	NIKKEI 225
Europe	STOXX 50
World	FTSE Multinationals

One drawback to this solution is that the investor is neglecting a large segment of the market composed of perfectly valid stocks as investment vehicles. For investors who are sensitive to this last issue, the choice of the benchmark is leaning toward more broadly defined indices, such as the ones mentioned in the table below:

Stock Market	Benchmark
Switzerland	SPI
Germany	FAZ
France	SBF
USA	RUSSELL 1000, 2000, 3000 WILSHIRE 5000
Japan	TOPIX, TSE 2nd section
Europe	MSCI Europe, FTSE Europe, STOXX Europe
World	MSCI World, FTSE World, STOXX World

A second important element of discussion when choosing the benchmark for an indexed portfolio is the amount of money that will be invested in it. When the amount of money to be invested in the indexed portfolio is relatively small and the index includes a large number of stocks, it is not clear at all that the indexed portfolio will be able to deliver the performance of the smaller caps segment of the index[74]. Large indices sometimes have a large tail of small stocks. For example, the MSCI World Index[75], which encompasses more than 1600 stocks, has nearly 75 percent of the names with a benchmark weight less than 5 basis points. All these small stocks represent altogether some 22.5% of the index. For a 50 million USD portfolio, which is already a non-negligible size, it is unrealistic to think that the indexer would be able to build a portfolio that will adequately represent all these small stocks. The indexed portfolio will not be able to represent all the specific returns of these stocks and therefore show some ex-ante tracking error. If, for any reason, these small stocks behave differently in a systematic fashion from the other ones for some time, the indexed portfolio may experience a systematic divergence in performance from its benchmark ($R_A^{P,B} \neq 0$) which may surprise the investor given the ex-ante tracking error. In other words, by choosing a broad based index, the investor may fool himself by believing that he will get the performance of this benchmark. In reality he may get only the performance of the subset of the index, i.e. the index less the tail of the smaller stocks. A more realistic approach for this investor would be to choose a less broadly defined index that will be better tracked by the indexer.

Another good example for this can be found in the Swiss stock market. The broader index, the SPI, which encompasses some 230 stocks as of December 31st, 2004, is characterized by a segment of some 150 illiquid stocks. Indexing with respect to the SPI and avoiding these illiquid stocks creates an ex-ante tracking error of around 20 basis points. If the investor now chooses to index with respect to the SPI Large and Mid (88 stocks), he will own an indexed portfolio very similar to the other one, but with a much smaller tracking error. In this case, choosing different benchmarks implies more different tracking errors than different managed portfolios.

The choice of a benchmark has clear implications in terms of the choice of the indexing technology. With a broad based index, the exhaustive sampling technique and the synthetic replication method are less likely to be adopted. Contrarily, with a narrowly defined index, the only real technological choice is to go for full replication.

For example, if an investor on the Swiss equity market chooses the SPI as the benchmark, he basically discards the use of both the exhaustive sampling technique because of the existence of a very illiquid segment of the index and the synthetic replication methods because the SMI future has a tracking error which is very large with respect to the SPI.

*1.2.2.7 Choice of the tracking error**

As already suggested in the previous section, the choice of a benchmark has implications for the applied indexing technology and therefore for the tracking error. Choosing narrowly defined benchmarks usually implies very small tracking errors because of the choice of a fully replicating technique. We also showed that choosing between two indices, one being more broadly defined than the other, may imply not very different indexed portfolios but significantly different tracking errors.

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74 This discussion is valid under the assumption that the investor wants a segregated account. If the investor would go for an indexed fund, the issue is not relevant.

75 As of December 31st, 2000.

Often, a new indexed portfolio is built from an existing, usually active, portfolio. It means that this existing portfolio has to be restructured at a cost as transactions are not free. For the investor, there is always the temptation to consider the trade-off that exists between tightening the ex-ante tracking error and saving on transaction costs. Notice that the two parts of this trade-off do not have the same time horizon because the risk linked to the tracking error is not limited in time and the transaction cost is a one shot event. So, there is a false sense of comparability.

If the investor cedes to the temptation of saving transaction costs and therefore accepting a higher ex-ante tracking error, he must be aware that the indexed portfolio is likely to partially inherit some of the active bets of the previous portfolio. And this may be an undesirable feature.