PORTFOLIO MANAGEMENT

PERFORMANCE MEASUREMENT AND EVALUATION

PORTFOLIO MANAGEMENT

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* final level

1. Performance measurement and evaluation

1.1 Introduction to performance measurement and evaluation

This chapter provides an overview of performance measurement and evaluation and is organized as follows. **Chapter 1.1** explains how performance evaluation fits into the portfolio management process. **Chapters 1.2 to 1.5** describe in more detail its major activities: performance measurement, performance attribution, performance presentation and performance appraisal. **Chapter 1.2** thus explains the measurement of returns earned by investment portfolios and benchmarks, describes the measurement of risks taken to make these returns and discusses measures for risk and return efficiency to identify investment skills.[1] **Chapter 1.3** covers the measurement of the contributions to return and risk due to specific investments and the attribution of the excess return or value added and of the active risk to portfolio management decisions. **Chapter 1.4** describes different aspects to consider when presenting return and risk figures. Finally, **chapter 1.5** illustrates performance evaluation as part of the overall investment controlling process and explains the process of analyzing and interpreting investment performance to produce valuable feedback into the portfolio management process.

Performance is one of the words whose definition is very flexible since everyone uses the concept while letting the context take care of the definition. Nevertheless, in general terms, performance is the result of activities (for example of an organization) over a given period. Based on this definition, one can say that in general terms, performance evaluation is the process of quantifying and qualifying the efficiency and effectiveness of past and future action. More concretely, performance evaluation is the process of measuring how well organizations are managed against their targets and the value they generate for their stakeholders.

In portfolio management, performance evaluation is an integral part of the portfolio management process. It covers all recurring or periodic monitoring and controlling activities with respect to measuring, analyzing, reporting, supervising or reviewing the results of the sum of all portfolio management decisions – the investment performance. Performance evaluation provides information about the return and risk of investment portfolios over a specified investment period and gives feedback to various stakeholders about the effectiveness of the portfolio management process in meeting investment targets.

1 In **this chapter**, the term "investment portfolio" is used in a broad way and might cover a single portfolio, multiple portfolios managed according to a specific investment strategy, or all portfolios managed by a certain portfolio manager. In addition, the term “portfolio manager” is used in a comprehensive way to refer not only to a single portfolio manager but also to all kinds of investment management organizations.

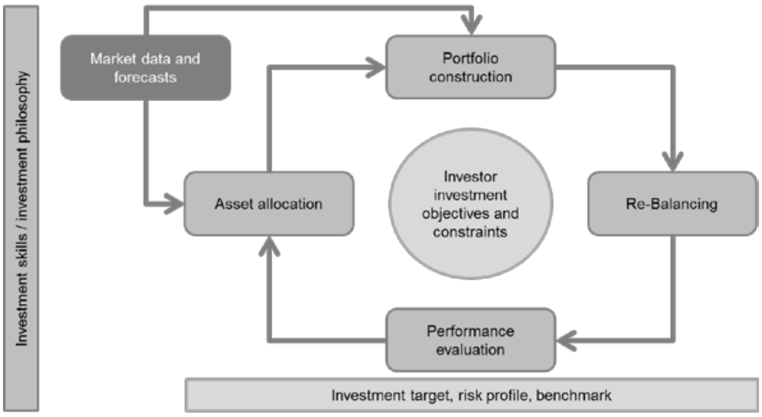

Figure 1-1: Performance evaluation as part of the portfolio management process

The portfolio management process illustrated in Figure 1-1 may in reality be quite complex because of the large number of possible financial instruments, decision makers involved (such as the research team, asset allocation team and specialists in the various investment categories) as well as not always transparent levels of the decision making. This complexity often means that the results of the portfolio management process and its determining factors are not always apparent. Performance evaluation generates transparency with this respect and identifies the contributions to return and risk of the individual investment decisions and of the responsible decision makers. The information gained and the conclusions thereof are important feedback into the portfolio management process notably to enhance future investment performance.

The major elements of performance evaluation are shown in Figure 1-2. They concern investment performance and as such focus not only on the return but also on the risk as well as its relation to the return, i.e. the risk-adjusted return. In doing so, performance evaluation considers not only past action but also future action. One can therefore use it to analyze the past (ex post) but also the expected (ex ante) performance. Furthermore, one can do the different analyses on an absolute basis, i.e. analyzing the performance of an investment portfolio or process in isolation, or on a relative basis, i.e. analyzing the performance of an investment portfolio or process in comparison to a benchmark or an investment target.

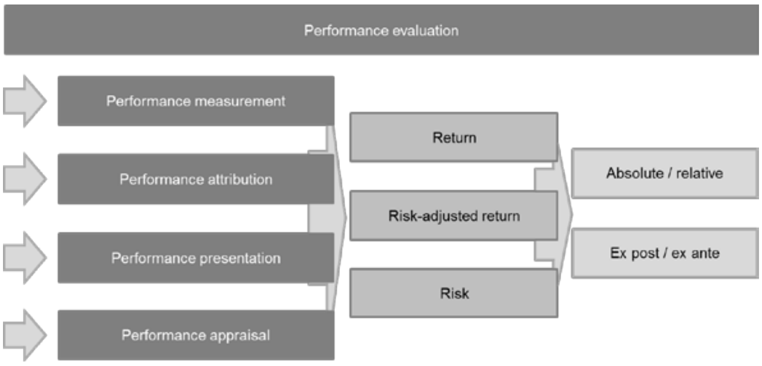

Figure 1-2: Major activities and focus areas of performance evaluation

The major activities of performance evaluation provide answers to various questions in relation to investment performance, with each activity addressing different aspects or questions:

- **Performance measurement** is the process of measuring the historical as well as the expected return and risk of investment portfolios. It answers questions such as, "What was the past investment performance and what may be expected in the future?"
- **Performance attribution** is the process of identifying and measuring the historical as well as expected return and risk contributions of the individual steps of the portfolio management process as well as of the financial instruments that have been used. It answers questions such as, "How did the investment portfolio produce its past performance and what are the sources of expected future performance?"
- **Performance presentation** is the process of illustrating and providing information on the performance of investment portfolios. It focuses on the presentation of the returns achieved and the risks taken within an investment portfolio over some specified measurement period. It answers questions such as, "What investment performance information should be presented and in which way?"
- **Performance appraisal** uses all information produced by the preceding performance evaluation activities, and is the process of analyzing and interpreting the performance of investment portfolios. It focuses on activities which identify and quantify investment skills. It answers questions such as, "Was the observed investment portfolio's performance the result of investment skill or luck?"

1.2 Performance measurement

1.2.1 Return measurement

1.2.1.1 Introduction to return measurement

1.2.1.1.1 Definition of investment return

A rate of return is the benefit one receives from an investment portfolio over a specific period expressed as a percentage, where the benefit covers income as well as capital gains or losses. In general, the return for a single period t is the ratio of gain and loss to the (average) invested capital:

$$R_{p,t} = \frac{\text{Gain and loss}}{\text{Invested capital}}.$$

Where: $R_{p,t}$ = Simple return of a portfolio for a single period t.

Gain and loss as well as the invested capital are determined by the valuation of the investment portfolio and by the value of external cash flows to and from the investment portfolio. For measurement periods without any external cash flow the simplified equation for the return of a single period t is:

$$R_{p,t} = \frac{MV_{end,t} - MV_{begin,t}}{MV_{begin,t}}.$$

Where: $MV_{end,t}$ = Market value at the end of period t,
$MV_{begin,t}$ = Market value at the beginning of period t.

1.2.1.1.2 External versus internal cash flows

Return measurement distinguishes between two types of cash flows:

- **External cash flows** are contributions to and withdrawals from an investment portfolio that have no impact on the gain and loss but change the relevant invested capital, e.g. contributions and withdrawals of cash or deliveries of securities. Often, the portfolio manager has no influence on the amount and the timing of these external cash flows, which tend to be determined by the investor. We need to bear this in mind when selecting the adequate return measurement.
- **Internal cash flows** are cash flows within an investment portfolio, which have an impact on the gain and loss but do not change the relevant invested capital, e.g. buy and sell transactions or corporate actions like dividend or coupon payments. Internal cash flows are typically the result of the portfolio manager’s decisions.

As illustrated in Figure 1-3, depending on the analysis in mind, the definition of the investment portfolio universe can change and be for example a single portfolio, a sub-portfolio, a sector, an asset class or even a single investment. A change in the investment portfolio universe may therefore also result in a change of the classification of the relevant cash flows.

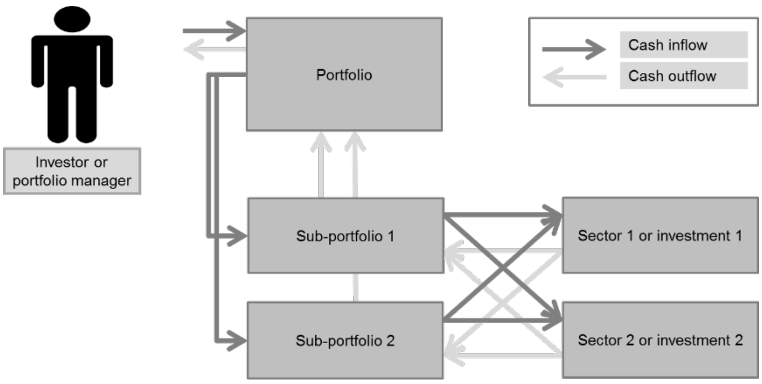

Figure 1-3: Cash flows within the return calculation

For example, if one considers a multi-asset class portfolio, a cash transfer from the equity sub-portfolio to the fixed income sub-portfolio is an external cash flow for both sub-portfolios but an internal cash flow for the overall multi-asset class portfolio. Additionally, the change in classification of a cash flow is a technique used for grossing up returns (e.g. for fees or taxes).

1.2.1.1.3 Gain and loss measurement

Gain and loss are defined by the relevant measurement period as well as by the beginning market value, the ending market value and the relevant external cash flows of an investment portfolio.

To calculate the gain and loss of an investment portfolio, we need the following inputs:

- **Beginning and ending market values of the investment portfolio**, which are equal to the respective market (or fair values) of the investment portfolio at the relevant valuation date.
- **Number and market (or fair value) of all external cash flows**. These will depend on the actual investment portfolio as well as the intended analysis.

The gain and loss is dependent on the intended analysis but independent from the return calculation methodology used. We can calculate it for a single period t as follows:

$$\text{Gain and loss} = MV_{end,t} - MV_{begin,t} - NCF_t = MV_{end,t} - MV_{begin,t} - \Sigma\tilde{C}_d + \Sigma\widetilde{W}_d\,.$$

Where: NCF_t = Net external cash flow for period t,
$\tilde{C}_d$ = External cash inflow at date d,
$\widetilde{W}_d$ = External cash outflow at date d.

Exhibit 1-1: Gain and loss measurement

31st of March: $MV_{begin,t}$ = 100.00 EUR,

30th of April: $MV_{end,t}$ = 160.00 EUR, and

10th of April: $\tilde{C}_d$ = 50.00 EUR.

$$=> \text{Gain and loss} = 160.00 - 100.00 - 50.00 = +10.00\,.$$

1.2.1.1.4 Fees and taxes within return measurement

A **net return** is the return after deduction or net of relevant fees and taxes. Due to the wide range of different fees and taxes, there are different kinds of net returns used in the industry. On the other hand, a **gross return** is a return before deduction or gross of relevant fees and taxes. Similar to the net return, different kinds of gross returns are used in the industry. It is industry best practice to present returns that are at least net of transaction costs.

Exhibit 1-2: Gain and loss net and gross of management fee and non-reclaimable withholding taxes

Same situation as in Exhibit 1-1 but in addition, we have the following information:

20th of April: Dividend payment received = 6.50 EUR (after deduction of 3.50 EUR non-reclaimable withholding taxes), and

30th of April: Management fee paid = 5.00 EUR.

$\Rightarrow \text{Gain and loss (net)} = 160.00 - 100.00 - 50.00 = +10.00$, and

$\text{Gain and loss (gross)} = 160.00 - 100.00 - 50.00 + 3.50 + 5.00 = +18.50.$

In Exhibit 1-2, in order to calculate the gross return instead of the return net of management fees and non-reclaimable withholding taxes, we have reclassified the management fee and the non-reclaimable withholding taxes from internal cash flows to external flows.

1.2.1.1.5 Average invested capital

Invested capital is the basis for the return calculation and is the average of the money invested over the relevant measurement period. The average invested capital depends on the external cash flows (amount and timing) transferred to or withdrawn from the investment portfolio. External cash inflows increase the average invested capital, while external cash outflows decrease it.

In general, the average invested capital is defined by the sum of the beginning market value of the relevant measurement period and the time-weighted external cash flows. The average invested capital for a single period t is calculated as follows:

$$AIC_t = MV_{begin,t} + wNCF_t = MV_{begin,t} + \Sigma w\tilde{C}_d - \Sigma w\tilde{W}_d \,.$$

Where:
- AIC_t = Average invested capital for period t,
- $wNCF_t$ = Weighted net external cash flow for period t,
- $w\tilde{C}_d$ = Weighted external cash inflow at date d,
- $w\tilde{W}_d$ = Weighted external cash outflow at date d.

In practice, there are various methodologies for calculating the time-weight of the external cash flows. It should be noted that the different time-weights but also the different (implicit) assumptions for reinvestment or financing the external cash flows and for compounding can lead to different returns even though the observed gains and losses are equal. For example, the average invested capital and therefore the return may be substantially different if one neglects or 0-weights the external cash flows instead of day weighting them.

Exhibit 1-3: Average invested capital net of management fee and non-reclaimable withholding taxes using day weighting of external cash flows

Same data as in Exhibit 1-2 but day weighting of the external cash flows:

$$=> \mathrm{AIC_t} = 100.00 + 50.00 * \frac{30 - 10}{30} = 133.33\,.$$

The day-weight for the external cash flow depends on the period of time the money was available for investment during the relevant measurement period. In Exhibit 1-3, the external cash inflow was available for investment for 20 out of 30 days within April or for 2/3rds of the time.

1.2.1.1.6 Simple versus continuously compounded returns

Taking into consideration the above, the general formula for calculating the return of an investment portfolio for a single period t is:

$$\mathrm{R_{p,t}} = \frac{\mathrm{MV_{end,t}} - \mathrm{MV_{begin,t}} - \Sigma\tilde{\mathrm{C}}_\mathrm{d} + \Sigma\widetilde{\mathrm{W}}_\mathrm{d}}{\mathrm{MV_{begin,t}} + \Sigma \mathrm{w}\tilde{\mathrm{C}}_\mathrm{d} - \Sigma \mathrm{w}\widetilde{\mathrm{W}}_\mathrm{d}}\,.$$

$\mathrm{R_{p,t}}$ is called a **simple return** as it implicitly assumes a linear interest calculation or compounding, meaning that interest is paid at the end of the measurement period and only on the initial amount, i.e. no interest on interest. Simple returns are adequate when calculating returns over a single period. As will be discussed in **chapter 1.2.1.1.8**, when calculating returns over multiple periods, just adding up simple returns can bring about misleading results, since they are not additive.

By contrast, a **continuously compounded return** (CCR) is a return that assumes continuous compounding, meaning that interest is paid for very short sub-periods (momentarily) and that the interest is again reinvested at the same return. CCRs also have interesting statistical properties: they are additive and symmetric, have no basis effect, and therefore are often used when calculating risk figures. For example, a simple return of –50% is offset by a return of +100% whereas the respective CCR of –69.31% is offset by a return of +69.31%, i.e. the returns are symmetrical.

One can easily transform a simple return into a continuously compounded return and vice versa:

$$\mathrm{r_{p,t}} = \ln\left(1 + \mathrm{R_{p,t}}\right) \quad \text{and} \quad \mathrm{R_{p,t}} = \exp\left(\mathrm{r_{p,t}}\right) - 1\,.$$

Where: $\mathrm{r_{p,t}}$ = Continuously compounded return of a portfolio for a single period t.

Exhibit 1-4: Simple and continuously compounded return measurement

Same situation as in Exhibit 1-3:

$$=> \mathrm{R_{p,t}} = \frac{160.00 - 100.00 - 50.00}{100.00 + 50.00 * \frac{30 - 10}{30}} = \frac{10.00}{133.33} = +7.50\%\,,$$

$$\mathrm{r_{p,t}} = \ln(1 + 7.50\%) = +7.23\%\,, \text{and}$$

$$\mathrm{R_{p,t}} = \exp(7.23\%) - 1 = +7.50\%\,.$$

1.2.1.1.7 Base versus local currency return measurement

Base currency describes the currency the presented performance information is based on. Normally, this is the currency the investor is thinking in (the "home" currency) or, more generally, the currency in which the performance of the investments is measured. By contrast, **local currency** describes the currency of denomination of an international investment. To value an investment portfolio or to determine its gain and loss, one should convert all investments and external cash flows to the base currency of the investment portfolio using appropriate exchange rates. After having done this conversion, one can then apply all return measurement methodologies.

A return in local currency is the return of an investment ignoring the impact of fluctuations in the currency exchange rates. For instance, in the case of the stock ABC, which is denominated in USD, the local currency return would be the return expressed in USD, i.e. without converting into the base currency. The return in base currency also includes the currency return for the period. The return in base currency for a single period t is calculated as follows:

$$R_{p,t}^{BC} = \left(1 + R_{p,t}^{LC}\right) \times \left(1 + C_t^{BC/LC}\right) - 1 \,.$$

Where: $R_{p,t}^{BC}$ = Return of a portfolio for period t in base currency,
$R_{p,t}^{LC}$ = Return of a portfolio for period t in local currency,
$C_t^{BC/LC}$ = Currency return for period t.

Exhibit 1-5: Base versus local currency returns

31st of March: $MV_{begin,t}$ = 100.00 USD (75.00 EUR) with EUR/USD = 0.750 , and

30th of April: $MV_{end,t}$ = 110.00 USD (88.00 EUR) with EUR/USD = 0.800.

$$=> R_{p,t}^{LC} = \frac{110.00 - 100.00}{100.00} = +10.00\% \,,$$

$$C_t^{BC/LC} = \frac{0.800 - 0.750}{0.750} = +6.66\% \,, \text{and}$$

$$R_{p,t}^{BC} = (1 + 10.00\%) \times (1 + 6.66\%) - 1 = +17.33\% \,.$$

1.2.1.1.8 Multi-period return measurement

The returns discussed so far were single period returns. Usually returns are calculated for longer periods than a day or a month by compounding the single period returns into multi-period or cumulative returns.[2] A multi-period return for a specific measurement period is derived from the returns of the individual sub-measurement periods (not necessarily of the same length) taking compounding into consideration. Depending on the use of simple or continuously compounded returns, the multi-period return is calculated on a geometric or arithmetic basis:

$$R_{p,tot} = \prod_{\text{for all sub-periods } t} \left(1 + R_{p,t}\right) - 1 \,.$$

Where: $R_{p,tot}$ = Multi-period or cumulative simple return of a portfolio.

2 The calculation of multi-period returns is only relevant for time-weighted rate of returns, which are discussed in **chapter 1.2.1.3**. Money-weighted rate of returns, which are discussed in **chapter 1.2.1.2**, are always calculated just for a single period like a day, month, year or since inception.

$$r_{p,tot} = \sum_{\text{for all sub-periods } t} r_{p,t} \,.$$

Where: $r_{p,tot}$ = Multi-period or cumulative continuously compounded return of a portfolio.

In addition, multi-period returns often are presented as average returns for specific standard length periods like a month or a year. For example, the annualized return is the geometric mean (using simple returns) or arithmetic mean (using continuously compounded returns) of a multi-period return for a 1-year period and is calculated as follows:

$$\text{Simple return p.a.} = \left(1 + R_{p,tot}\right)^{\left(\frac{1}{\text{Number of years}}\right)} - 1 \text{, and}$$

$$\text{Continuously compounded return p.a.} = r_{p,tot} \times \frac{1}{\text{Number of years}} \,.$$

Exhibit 1-6: Multi-period and annualized return measurement

Simple returns: 1st quarter: +5.00%, 2nd quarter: +10.00%, 3rd quarter: –5.00%, 4th quarter: –8.00%, and 5th quarter: +10.00%.

$$\begin{aligned} => R_{p,tot} &= (1+5\%) \times (1+10\%) \times (1-5\%) \times (1-8\%) \times (1+10\%) - 1 \\ &= +11.04\% \,, \end{aligned}$$

$$\text{Simple return p.a.} = (1+11.04\%)^{\left(\frac{1}{5/4}\right)} - 1 = (1+11.04\%)^{\left(\frac{1}{1.25}\right)} - 1 = +8.74\% \text{, and}$$

$$\text{Continuously compounded return p.a.} = \frac{\ln(1+11.04\%)}{5/4} = \frac{10.47\%}{1.25} = +8.38\% \,.$$

1.2.1.2 Money-weighted rate of return measurement

1.2.1.2.1 Impact of external cash flow timing on the return

The time value of money and especially the movements of the financial markets can cause the timing of external cash flows to have a significant impact on the gain and loss and on the return of an investment portfolio.

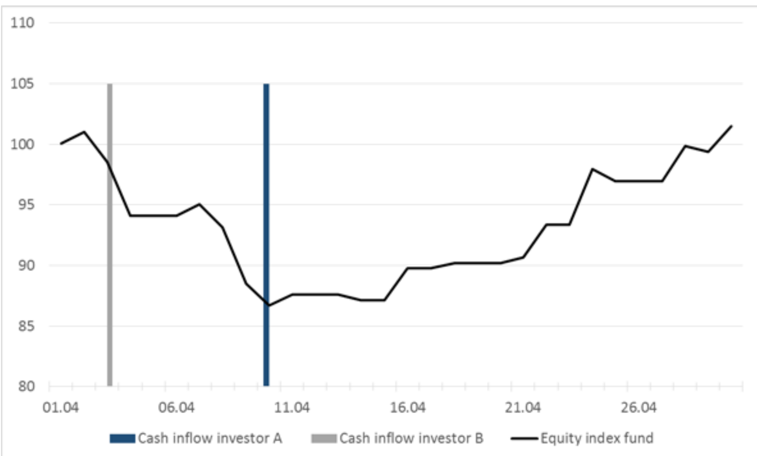

Figure 1-4: Impact of external cash flow timing

For example, let us consider two investors, A and B, who invested each 100 EUR into an equity index fund on March 31. After a decline in the market, both investors invested an additional 50 EUR into the equity index fund: investor A on April 10 and investor B already on the 3rd of April. Figure 1-4 shows the development of the equity index fund throughout April and highlights the dates when the two investors made the additional investments. The graph shows that investor A had better (market) timing than investor B as he or she made the additional investment at a lower index value than investor B. Neglecting the time value of money, it becomes obvious that because of good timing, investor A experienced a higher return than investor B. Note also that both investors had a higher return than if they had invested the total amount at the beginning of April.

This example illustrates also that in general the return experienced by investors is based on two types of decisions:

- **Timing decisions**, normally by the investor, on contributions or withdrawals and here to buy and sell shares of the equity index fund.
- **Management decisions**, normally by the portfolio manager, to allocate the assets and to select individual securities within the respective investment portfolio and here within the equity index fund.

To separate the impact of these two decisions and to be able to measure the respective value added of the decision makers, we need to consider two different return measures:

- **Money-weighted rate of return (MWR)** to measure the return experienced by an investor, reflecting the comprehensive or total return including the timing effect of external cash flows.
- **Time-weighted rate of return (TWR)** to measure (only) the return produced by the portfolio manager, not affected by the timing of external cash flows (implicitly assuming that the portfolio manager has no discretion over the timing of external cash flows).

The rest of **chapter 1.2.1.2** describes different methodologies used in the industry to calculate a MWR. **Chapter 1.2.1.3** explains the concept of how to calculate a TWR. Finally, c**hapter 1.2.1.4** compares and describes the relationship between MWR and TWR.

1.2.1.2.2 Internal rate of return methodology

The internal rate of return (IRR) is a MWR. It is called a "true" MWR because it is a precise method for calculating a MWR and therefore no approximation of the MWR. For a specific measurement period, the IRR is the return or interest rate that causes the ending market value and the interim external cash flows to be discounted to the beginning market value. Due to its specific compounding characteristics, the IRR is also the return or interest rate that causes the beginning market value and interim external cash flows to grow to the ending market value. Here the IRR makes some implicit assumptions about the reinvestment rate and the financing rate, namely that:

- External cash inflows are financed at an interest rate (finance rate) that is identical to the IRR.
- External cash outflows are reinvested at an interest rate (reinvestment rate) that is identical to the IRR.

When discounting the ending market value and the interim external cash flows to equal the beginning market value, the formula for deriving the annualized IRR for a single period t is as follows:[3]

3 Here, discounted external cash inflows (outflows) must be subtracted from (added to) the discounted ending market value.

$$MV_{begin,t} = \frac{MV_{end,t}}{(1+IRR)^{Y_T}} - \sum \frac{\tilde{C}_d}{(1+IRR)^{Y_{t-0}}} + \sum \frac{\tilde{W}_d}{(1+IRR)^{Y_{t-0}}}.$$

Where: IRR = Internal rate of return (annualized) for a single period t,
Y_T = Length of measurement period (measured in years – 365 days),
Y_{t-0} = Length of period between the beginning of the measurement period and the date of the external cash flow (measured in years – 365 days).

When compounding the beginning market value and the interim external cash flows to equal the ending market value, the formula for deriving the annualized IRR for a single period t is as follows:[4]

$$MV_{end,t} = MV_{begin,t} \times (1+IRR)^{Y_T} + \sum \tilde{C}_d \times (1+IRR)^{Y_{T-t}} - \sum \tilde{W}_d \times (1+IRR)^{Y_{T-t}}.$$

Where: Y_{T-t} = Length of period between the date of the external cash flow and the end of the measurement period (measured in years – 365 days).

In practice, the calculation of an IRR is not trivial as the solution cannot be derived using algebra. The IRR is calculated using an interpolation technique, meaning an iteration or trial and error process that starts with an initial guess and then iteratively tries out successive values until the formula is solved.

Exhibit 1-7: IRR

Same data as in Exhibit 1-1: discounting the ending market value and the interim external cash flows to equal the beginning market value, the derived annualized (not annualized) IRR is +141.79% (+7.53%).

31.03.2014	10.04.2014	30.04.2014
-100.00	**-50.00**	**160.00**
-48.81		
148.81		
0.00		

Table 1-1: Discounting cash flows to equal beginning market value

$$=> 100.00 = \frac{160.00}{(1+141.79\%)^{\frac{30}{365}}} - \frac{50}{(1+141.79\%)^{\frac{10}{365}}} = 148.81 - 48.81\,.$$

4 Here, compounded external cash inflows (outflows) must be added to (subtracted from) the compounded beginning market value.

When compounding the beginning market value and the interim external cash flows to equal the ending market value, the derived annualized (not annualized) IRR is also +141.79% (+7.53%).

31.03.2014	10.04.2014	30.04.2014
-100.00	**-50.00**	160.00
		-107.52
		-52.48
		0.00

Table 1-2: Compounding cash flows to cause ending market value

$$=> 160.00 = 100 \times (1 + 141.79\%)^{30/365} + 50 \times (1 + 141.79)^{20/365} = 107.52 + 52.48\,.$$

Considering the two investors A and B mentioned in **chapter 1.2.1.2.1**, Exhibit 1-7 reflects the situation of investor A. For investor B, the ending market value of the investment portfolio on April 30 was 153.00 EUR what leads to an annualized (not annualized) IRR of +28.26% (+2.07%). By contrast, the annualized (not annualized) IRR of the equity index fund, simulated by an investment portfolio consisting of one share of the fund, is +19.65% (+1.49%), proving the positive timing effect of investor A (+7.53% versus +1.49%) and of investor B (+2.07% versus +1.49%).

Besides the advantage of the IRR taking into account both the timing and the amount of external cash flows, the IRR has two major drawbacks:

- IRR implies an unrealistic reinvestment and financing assumption, namely that all external cash inflows (outflows) are financed (reinvested) at an interest rate equal to the IRR.
- IRR may not have a unique solution but instead multiple solutions or in the other extreme may even have no solution.

1.2.1.2.3 Modified internal rate of return methodology

The modified internal rate of return (MIRR) is a modification of the IRR. It addresses the two major drawbacks of the IRR by using explicit instead of implicit reinvestment and financing assumptions, which leads to a unique solution for the MIRR. Similar to the IRR, the MIRR is also a MWR. It is called a "true" MWR because it is a precise method for calculating a MWR and therefore no approximation of the MWR.

For a specific measurement period, the MIRR is the return or interest rate that causes the sum of the beginning market value and the discounted interim external cash inflows to grow to the sum of the compounded interim external cash outflows and the ending market value. With this in mind, the MIRR is the rate of return that was earned assuming that external cash flows are financed or reinvested using the following assumptions:

- External cash inflows are financed at a specific interest rate (finance rate) that does not have to be identical to the IRR.
- External cash outflows are reinvested at a specific interest rate (reinvestment rate) that does not have to be identical to the IRR.

The formula for calculating the annualized MIRR for a single period t is as follows:

$$\text{MIRR} = \left(\frac{\text{Future value of all external cash outflows}}{\text{Present value of all external cash inflows}}\right)^{\left(\frac{1}{Y_T}\right)} - 1\,,\text{and}$$

$$\text{MIRR} = \left(\frac{\text{MV}_{\text{end,t}} + \sum\left(\widetilde{W}_d \times (1 + \text{rir}_{T-t})^{Y_{T-t}}\right)}{\text{MV}_{\text{begin,t}} + \sum \frac{\tilde{C}_d}{(1 + \text{fr}_{t-0})^{Y_{t-0}}}}\right)^{\left(\frac{1}{Y_T}\right)} - 1\,.$$

Where: MIRR = Modified internal rate of return (annualized) for a single period t,
rir_{T-t} = Reinvestment rate for the period from the date of the external cash outflow t to the end of the measurement period T (measured in years – 365 days),
fr_{t-0} = Finance rate for the period starting at the beginning of the measurement period to the date of the external cash inflow t (measured in years – 365 days).

Exhibit 1-8: MIRR

Same data as in Exhibit 1-7: assuming that the finance rate is an annualized +5.00%, the annualized (not annualized) MIRR is +120.56% (+6.72%).

31.03.2014	10.04.2014	30.04.2014
-100.00	**-50.00**	**160.00**
-49.93		
-149.93		**160.00**

Table 1-3: Discounting cash inflows to cause compounded cash outflows

$$\Rightarrow \text{MIRR} = \left(\frac{160.00}{100.00 + \frac{50.00}{(1 + 5.00\%)^{\left(\frac{10}{365}\right)}}}\right)^{\left(\frac{1}{\frac{30}{365}}\right)} - 1\,\text{, and}$$

$$\text{MIRR} = \left(\frac{160.00}{100.00 + 49.93}\right)^{\left(\frac{1}{0.0822}\right)} - 1 = +120.56\%\,.$$

Considering the two investors A and B mentioned in **chapter 1.2.1.2.1**, Exhibit 1-8 reflects the situation of investor A. In contrast, the annualized (not annualized) MIRR of investor B is +27.40% (+2.01%) and such of the equity index fund, simulated by an investment portfolio consisting of one share of the fund, is again +19.65% (+1.49%).

Besides, the difference between the returns – if comparing the MIRR of Exhibit 1-8 with the IRR of Exhibit 1-7 – is here because of the different assumptions on the finance rate. The MIRR uses an annualized finance rate of 5.00% while the IRR uses an annual rate of 141.79% – equal to the IRR. This means that the present value of the external cash flow on April 10 using the MIRR is higher than would be the case if using the IRR. The higher present value causes a higher average invested capital and with this a lower return. Furthermore, MIRR equals IRR if the reinvestment and finance rate for calculating the MIRR are identical to the IRR.

1.2.1.2.4 Original Dietz methodology

The Original Dietz Method (ODM) is a method of approximation of the IRR, whereby the external cash flows are weighted with 0.5 (implicitly assuming that external cash flows always occur at the middle of the measurement period). The ODM addresses the two major drawbacks of the IRR by using an explicit assumption on the average invested capital, which leads to a unique solution for the MWR.

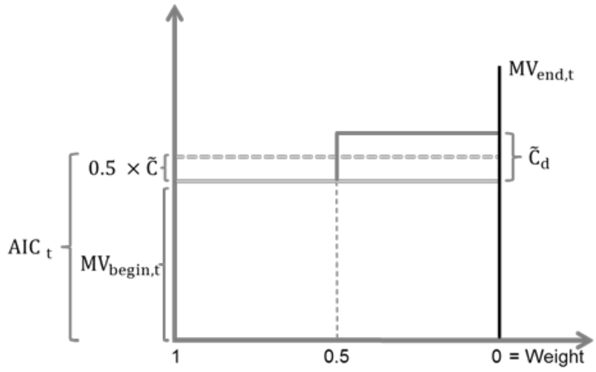

Figure 1-5: Illustration of the cash flow weighting according to ODM

Figure 1-5 illustrates the weighting assumption of the ODM, namely that all external cash flows occur at the middle of the measurement period. Interpreted as a time-weight, ODM uses a time-weight of 0.5 for all external cash flows. This leads to the following formula for the average invested capital according to ODM:

$$AIC_t = MV_{begin,t} + wNCF_t = MV_{begin,t} + \Sigma 0.5 \times \tilde{C}_d - \Sigma 0.5 \times \tilde{W}_d \, .$$

Based on this and using the general formula in **chapter 1.2.1.1.6**, the formula for calculating the (not annualized) MWR according to ODM for a single period t is as follows:

$$MWR_{ODM} = \frac{MV_{end,t} - MV_{begin,t} - \Sigma\tilde{C}_d + \Sigma\tilde{W}_d}{MV_{begin,t} + \Sigma 0.5 \times \tilde{C}_d - \Sigma 0.5 \times \tilde{W}_d} \, .$$

Where: MWR_{ODM}= MWR according to Original Dietz Method (not annualized) for a single period t.

Exhibit 1-9: MWR according to ODM

Using the same data as in Exhibit 1-8 and assuming that the external cash flows always occur at the middle of the measurement period, the annualized (not annualized) MWR according to ODM is +155.18% (+8.00%).

$$=> MWR_{ODM}(\text{not annualized}) = \frac{160.00 - 100.00 - 50.00}{100.00 + 0.5 \times 50.00} = \frac{10.00}{125.00} = +8.00\% \, .$$

Considering the two investors A and B mentioned in **chapter 1.2.1.2.1**, Exhibit 1-9 reflects the situation of investor A. By contrast, the annualized (not annualized) MWR according to ODM of investor B is +33.39% (+2.40%) and that of the equity index fund, simulated by an investment portfolio consisting of one share of the fund, is again +19.65% (+1.49%).

The differences between the MWR according to ODM and the IRR indicate that the weighting assumption of the ODM may lead to significant errors – especially for significant external cash flows, in volatile markets and for long measurement periods. Therefore, in practice, ODM is not much used.

1.2.1.2.5 Modified Dietz methodology

The Modified Dietz Method (MDM) is a method to approximate the IRR, where the external cash flows are time-weighted. The weight of the external cash flow depends on the point of time in the relevant measurement period when the external cash flow occurred (assumption: external cash flows always occur at the end of the day). The MDM addresses the two major drawbacks of the IRR by using an explicit assumption on the average invested capital, which leads to a unique solution for the MWR.

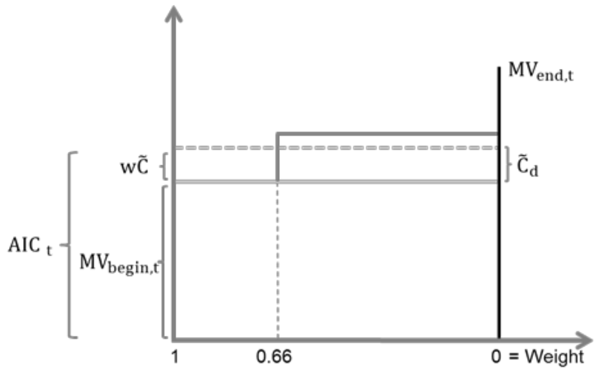

Figure 1-6: Illustration of the cash flow weighting according to MDM

Figure 1-6 illustrates the weighting assumption of the MDM, i.e. that all external cash flows are weighted with the relative period the cash was available for investment. Interpreted as a time-weight, MDM uses a day-weight between 0 and 1 for all external cash flows. For example, if the cash inflow occurred on April 10, then it was available for investment for 20 days or 2/3rds of the measurement period.

Using the general formula in **chapter 1.2.1.1.6**, the formula for calculating the (not annualized) MWR according to MDM for a single period t is as follows:

$$\mathrm{MWR_{MDM}} = \frac{\mathrm{MV_{end,t}} - \mathrm{MV_{begin,t}} - \Sigma\tilde{C}_d + \Sigma\tilde{W}_d}{\mathrm{MV_{begin,t}} + \Sigma w\tilde{C}_d - \Sigma w\tilde{W}_d}.$$

With: $w\tilde{C}_d = w_d \times \tilde{C}_d$, $w\tilde{W}_d = w_d \times \tilde{W}_d$ and $w_d = \frac{D_T - D_d}{D_T}$.

Where: $\mathrm{MWR_{MDM}}$ = MWR according to Modified Dietz Method (not annualized) for a single period t,

w_d = Time-weight for date d,

D_T = Length of measurement period (measured in days),

D_d = Length of period between the beginning of the measurement period and the date of the external cash flow (measured in days).

Exhibit 1-10: MWR according to MDM

Using the same data as in Exhibit 1-9 and assuming that the external cash flow occurred on the 10th of April, the annualized (not annualized) MWR according to MDM is +141.16% (+7.50%).

$$\Rightarrow \mathrm{MWR_{MDM}}(\text{not annualized}) = \frac{160.00 - 100.00 - 50.00}{100.00 + \frac{30-10}{30} \times 50.00} = \frac{10.00}{133.33} = +7.50\%.$$

Considering the two investors A and B mentioned in **chapter 1.2.1.2.1**, Exhibit 1-10 reflects the situation of investor A. By contrast, the annualized (not annualized) MWR according to MDM of investor B is +28.25% (+2.07%) and that of the equity index fund, simulated by an investment portfolio consisting of one share of fund, is again +19.65% (+1.49%).

The differences between the IRR and MWR according to MDM are not as big as when using MWR according to ODM. Nevertheless, the difference can be significant especially in volatile markets, for small average invested capitals, for significant external cash flows or for long measurement periods. Therefore, in practice, MDM is used only for shorter periods like a month or day – where industry best practice requires a measurement period of a day.

When calculating the MWR for a single day, it is difficult if not impossible to determine the exact time when the external cash flow occurred. Therefore in practice it is assumed that external cash flows occur at the end of the day (not available for investment during the day) or at the beginning of the day (available for investment during the day). The respective simplified formula for calculating the daily MWR are:

$$\mathrm{MWR}_{\mathrm{D,end\ of\ day}} = \frac{\mathrm{MV}_{\mathrm{end},t} - \mathrm{MV}_{\mathrm{begin},t} - \Sigma\tilde{\mathrm{C}}_{\mathrm{d}} + \Sigma\widetilde{\mathrm{W}}_{\mathrm{d}}}{\mathrm{MV}_{\mathrm{begin},t}}, \text{ and}$$

$$\mathrm{MWR}_{\mathrm{D,begin\ of\ day}} = \frac{\mathrm{MV}_{\mathrm{end},t} - \mathrm{MV}_{\mathrm{begin},t} - \Sigma\tilde{\mathrm{C}}_{\mathrm{d}} + \Sigma\widetilde{\mathrm{W}}_{\mathrm{d}}}{\mathrm{MV}_{\mathrm{begin},t} + \Sigma\tilde{\mathrm{C}}_{\mathrm{d}} - \Sigma\widetilde{\mathrm{W}}_{\mathrm{d}}}.$$

Where: $\mathrm{MWR}_{\mathrm{D,end\ of\ day}}$ = Daily MWR assuming end of day cash flows,
$\mathrm{MWR}_{\mathrm{D,begin\ of\ day}}$ = Daily MWR assuming beginning of day cash flows.

There is no rule as to which weighting assumption one should use and when, but it is common practice to use the end of day weighting. Depending on the amount of the external cash flow and the respective investments, the use of a specific weighting assumption can lead to not intuitive returns. In that case, one can adjust the weights on a case-by-case basis to reflect the actual transactions better.

1.2.1.3 Time-weighted rate of return measurement

1.2.1.3.1 True time-weighted rate of return

As discussed above, the MWR depends on two types of investment decisions: timing of cash flows and management. To evaluate the effect of all management decisions, it is therefore necessary to calculate a return that is completely unaffected by the timing of external cash flows. This type of return is called a "true" time-weighted rate of return (TWR). The TWR is often also called the portfolio manager return and is useful to evaluate the portfolio manager's skill, since it reflects only portfolio management decisions and will not be affected (either positively or negatively) by timing decisions that are not under the control of the portfolio manager. Therefore, TWRs are often used to evaluate the quality of the portfolio management process, to present performance information to prospective clients and to determine performance-based management fees. Nevertheless, in cases when the portfolio manager controls the external cash flows, as is the case for private equity portfolios, the MWR would be the appropriate return measure.

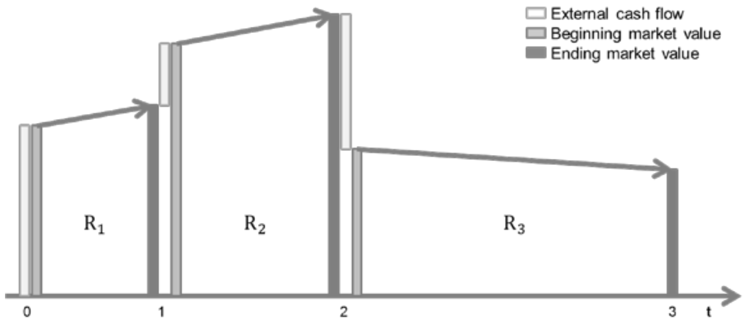

Figure 1-7: TWR calculation technique if interim external cash flows occur

The TWR is a return that eliminates the timing effect of all interim external cash flows. Figure 1-7 shows that during the measurement period (from t=0 to 3) – for an investment portfolio A – there are two interim external cash flows to be considered: a) an external cash inflow at t=1 and b) an external cash outflow at t=2. By eliminating the timing effect, the resulting TWR is identical to the TWR of another investment portfolio B (with identical sub-period returns) illustrated in Figure 1-8 without any interim external cash flows. In practice, the identity of these two TWRs is only possible assuming no costs and – more importantly – identical investments.

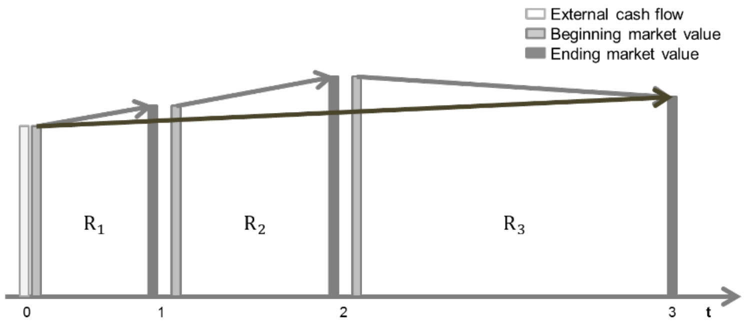

Figure 1-8: TWR calculation technique without interim external cash flows

The cumulative TWR of investment portfolio B, illustrated in Figure 1-8 by the dark brown line, is calculated as follows – see also **chapter 1.2.1.1.8**:

$$R_{p,tot} = (1 + R_1) \times (1 + R_2) \times (1 - R_3) - 1 .$$

Since in the example above, there are no interim external cash flows, the simplified formula for the calculation of the sub-period returns (R_1, R_2 and R_3) is:

$$R_{p,t} = \frac{MV_{end,t} - MV_{begin,t}}{MV_{begin,t}} .$$

To calculate the TWR of investment portfolio A, illustrated in Figure 1-7, one first has to split up the entire measurement period in sub-periods, where the time boundaries are determined by the dates of the external cash flows. Thereafter, one calculates the returns for each sub-period and then geometrically links these returns to derive the cumulative TWR for the entire measurement period.

Unlike investment portfolio B, which did not have any interim external cash flows, investment portfolio A does have some, so we need to consider these. Assuming that external cash flows occur at the end of each sub-period, the formula for calculating the sub-period returns reduces to:

$$R_{p,t} = \frac{MV_{end,t} - MV_{begin,t} - \Sigma\tilde{C}_d + \Sigma\tilde{W}_d}{MV_{begin,t}}.$$

Exhibit 1-11: "True" TWR

Using the same data as in Exhibit 1-10 and assuming that the interim external cash inflow was identical invested at no cost, the (not annualized) TWR for investment portfolio is +1.49%.

Date	Interim external cash flow	Market value investment portfolio	TWR of investment portfolio	Market value equity index fund	TWR of equity index fund
31.03.		100.00		100.00	
10.04.	50.00	136.71	–13.29%	86.71	-13.29%
30.04.		160.00	+17.04%	101.49	+17.04%
April			+1.49%	+1.49	+1.49%

Table 1-4: TWR calculation

$$\Rightarrow R_{p,tot} = \left(1 + \frac{136.71 - 100.00 - 50.00}{100.00}\right) \times \left(1 + \frac{160.00 - 136.71}{136.71}\right) - 1\text{, and}$$

$$R_{p,tot} = (1 - 13.29\%) \times (1 + 17.04\%) - 1 = +1.49\%\,.$$

The (not annualized) TWR of the equity index fund is identical to the (not annualized) return of one share of the fund and equals the percentage change of the share price (assuming no corporate actions) – here the +1.49% change from 100.00 to 101.49.

Considering the two investors A and B mentioned in **chapter 1.2.1.2.1**, Exhibit 1-11 reflects the situation of investor A. By contrast, the realized not annualized "true" TWR of investor B is also +1.49%. This shows that eliminating the timing effect of external cash flows leads, under the above-mentioned assumptions, to identical TWRs.

1.2.1.3.2 Approximations to time-weighted rate of return

If one wants to calculate the "true" TWR, one has to calculate a return for each sub-period as determined by the dates of external cash flows. This implies that there must be a valuation of the investment portfolio available for each external cash flow date. If one needs to calculate TWRs not only for investment portfolios but also for sub-portfolios, asset classes or even individual investments, we might need to calculate an extreme number of portfolio valuations, which makes the return calculation very complex and costly. That is why one often uses approximations to TWR instead of measuring "true" TWRs.

A common approximation practice, also called linked MWR method, is to calculate a MWR for each of the sub-periods, often a month or a day, and then chain-linking all sub-period MWRs to receive the TWR for the entire measurement period. The approximation error is normally quite small but may increase depending on the number and amount of the external cash flows, the volatility in the financial markets and the lengths of the periods. In the past, it was industry practice to calculate quarterly or monthly MWRs, but nowadays often daily MWRs are used.[5]

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5 Please see **chapter 1.2.1.2.5** for details on daily return calculation.

Table 1-5 contains information on an investment portfolio investing in an equity index fund for April to June 2014. The "true" TWR of the investment portfolio equals the return of the equity index fund (assuming identical investments and no costs) and is +5.10%. Instead of using the "true" TWRs, one could also approximate the monthly returns with MDM and chain-link theses sub-period MWRs to the cumulative TWR.

In the above example that would produce a cumulative TWR of +5.07% by using an April MWR of +1.99%, a May MWR of +1.00% and a June MWR of +1.99%. The very small difference between the "true" and the approximated TWR may imply that the use of the approximation method always results in small approximation errors, for instance the -0.03% we have here. However, this is a fallacy because if significant external cash flows happen or if returns are high or volatile, the approximation error may be significant. For example, if the index returns increase for all periods from +1.00% to +5.00% then the error would increase to -0.19% and if in addition the two cash inflows increase from 5.00 to 50.00 then the error would increase to +1.31%.

Date	Equity index fund return	Market value before cash flow	Cash flow	Market value after cash flow
31.03.		100.00		100.00
10.04.	1.00%	101.00	5.00	106.00
30.04.	1.00%	107.06	0.00	107.06
31.05.	1.00%	108.13	0.00	108.13
10.06.	1.00%	109.21	5.00	114.21
30.06.	1.00%	115.35	0.00	115.35

Table 1-5: Relevant portfolio valuations for TWR calculation

1.2.1.4 Money-weighted versus time-weighted rate of return

It is important to consider the characteristics of the two main concepts for calculating returns when evaluating investment performance. As a rule, before one chooses a specific calculation methodology, one needs to define the purpose of the analysis. Indeed, the intended use or analysis will determine whether one should use the MWR or the TWR – or any of the approximations. The characteristics of the two methodologies help to decide which measure to use.

The MWR is the compound rate of growth of the beginning market value and the interim external cash flows over the measurement period that produces the ending market value. The main characteristics of the MWR are:

- The MWR is a return measure that is affected by the timing, order, and size of external cash flows and therefore depends on changes in the invested capital.
- The MWR measures the return from the investor's perspective, assuming that the investor has control over external cash flows.
- The MWR uses implicit reinvestment and financing assumptions, except for the MIRR where explicit assumptions are used.
- The MWR allows no comparison with peer groups and competitors, but, if using adjustments, allows a comparison to a benchmark.[6]
- The MWR is not designed for dispersion analysis because, as a single period return, it always covers the entire measurement period, meaning there is no dispersion.[7]

6 Please see **chapter 1.2.2.3** for details on to adjust the calculation of the benchmark return.

7 Chain-linking of sub-period MWRs produces an approximation for the cumulative "true" TWR and not the cumulative MWR. The MWR always just refers to a single but entire measurement period.

- The MWR implies a specific average invested capital which is observable as part of the calculation, where variations exist depending on the underlying assumptions.
- The MWR is consistent with the gain and loss, meaning no mismatch of the algebraic signs, and allowing us to derive the gain and loss from the average invested capital.
- The MWR requires "only" a valuation of the investment portfolio at the beginning and the end of the measurement period, meaning interim valuations are not needed.
- The calculation, the decomposition and reporting of MWR is currently not common practice in the portfolio management industry.
- The MWR is not addressed by the GIPS Standards in detail, except for asset classes where normally the portfolio manager has the control over external cash flows.[8]

The TWR is the compound rate of growth of one unit of money invested in the investment portfolio at the beginning of the measurement period. The main characteristics of the TWR are:

- The TWR is a return measure that is not affected by the timing, order, and size of external cash flows and therefore does not depend on changes in the invested capital.
- The TWR measures the return from the portfolio manager's perspective, assuming that the portfolio manager has no control over external cash flows.
- The TWR allows a comparison to a benchmark as well as with peer groups and competitors.
- The TWR does not depend on reinvestment or financing assumptions because the effects of changes in the capital invested are neutralized.
- The TWR qualifies for dispersion analysis but only if the periods for calculating the sub-period returns are of the same length.
- The TWR does not imply a specific average invested capital because the effects of changing the capital invested are neutralized.
- The TWR may not be consistent with the gain and loss, implying possible mismatches of the algebraic signs, and normally does not allow us to derive the gain and loss from the approximated average invested capital.[9]
- The TWR requires a valuation of the investment portfolio for at least all dates when external cash flows happened.
- The calculation, the decomposition and the reporting of a TWR is common practice in the portfolio management industry.
- The presentation of a TWR is one of the key principles of the GIPS Standards.

MWR and TWR are often seen as two distinct and unrelated return measures. Considering that the MWR covers the effects of both the timing decisions of the investor and the management decisions of the portfolio manager, it is clear that the MWR includes the TWR.

Figure 1-9 shows the relationship between MWR and TWR. The MWR can be decomposed into three constituencies or return contributions to reflect the main investment management decisions:

- **The TWR benchmark effect**, which is the return contribution due to the decision to invest the initial money into a specific benchmark or investment strategy. It is equal to the TWR benchmark return for the measurement period. The decision maker responsible for the choice of the investment strategy or benchmark of the investment portfolio, normally the investor, produces this effect.

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8 See **chapter 1.4.3.1** for an introduction to the Global Investment Performance Standards (GIPS Standards).

9 In case of non-intuitive return figures, for instance in a case with mismatching algebraic signs, often this is linked to external cash flows happening during the measurement period.

- **The TWR management effect**, which is the return contribution due to the decisions to change the asset allocation and security selection of the investment portfolio relative to the benchmark during the measurement period. The decision maker responsible for implementing the investment strategy, normally the portfolio manager, produces this effect.
- **The MWR timing effect**, which is the return contribution due to the decisions to change the money invested in the benchmark strategy and in the active asset allocation or security selection of the investment portfolio during the measurement period. The decision maker who has the control over the external cash flows, normally the investor, produces this effect.

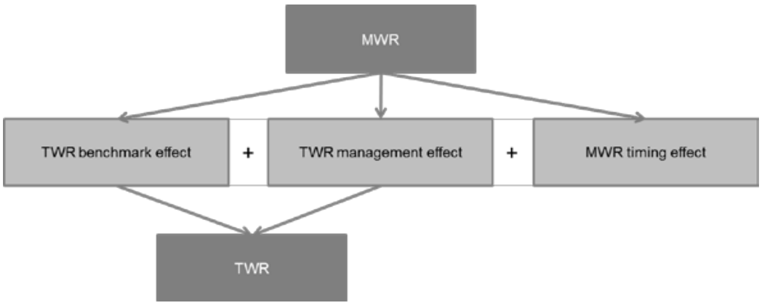

Figure 1-9: Relationship between MWR and TWR

In order to isolate the MWR timing effect completely, it is necessary to calculate not only a "true" TWR but also a "true" MWR. A "true" TWR is not affected by any external cash flow. It is best practice to calculate the "true" TWR on a daily basis and then to link the daily returns geometrically over the entire measurement period. Conversely, the "true" MWR covers the total timing effect of all external cash flows and is calculated using the internal rate of return methodology or any derivative of the IRR – as a precise method for calculating a MWR – over the entire measurement period. If instead one uses approximation methods, these can often result in a residual return component relative to the fictitious "true" return (TWR and MWR), whose missed evidence may lead to misleading feedback into the portfolio management process.

If one considers the investor A mentioned in **chapter 1.2.1.2.1** and assuming a completely indexed equity fund with no costs, the MWR is +7.53% (using the IRR), the TWR is +1.49%, and the resulting MWR timing effect is +6.04%. The TWR could then be split up into a TWR benchmark effect identical to the benchmark return (+1.49%) and a TWR management effect of +0.00%, because of missing active management and costs.

1.2.2 Benchmarks

1.2.2.1 Benchmark attributes

Performance evaluation gives feedback about the effectiveness of the portfolio management process in meeting investment targets. To allow quantitative analytics and conclusions, these targets are often transformed into a point of reference or benchmark against which the investment portfolio's performance can be compared. Depending on the purpose of the analytics or presentation, one can use various types of benchmarks in portfolio management, for example an investment strategy, index, portfolio, investment, or any other reference (e.g. inflation rate or target return).

To quantify the quality and the effectiveness of the portfolio management process and its individual steps of decision-making, it is important to use effective benchmarks. For example, an inflation rate may act as a benchmark to determine whether the investment portfolio produced a real return but is not appropriate to measure the security selection abilities of a portfolio manager. One can characterize effective benchmarks as follows:

- The benchmark is specified and known to all stakeholders in advance.
- The benchmark is representative of the investment mandate and objectives.
- The benchmark is appropriate to the relevant investment strategy.
- The benchmark is investable and can be fully replicated as a passive alternative to active portfolio management.
- The benchmark is consistent with pre-defined investment guidelines and restrictions.
- The benchmark is transparent and based on publicly available information.
- The benchmark returns are calculated by an independent third party.

Often, it is not possible to define and set up a benchmark fulfilling all of the above-mentioned characteristics. This is important because one normally expects that the difference between the portfolio and benchmark return to be only due to active portfolio management decisions. Users of performance information should thus be aware of the level of inappropriateness of the benchmark and, additionally, of "explainable" return differences. Reasons for “explainable” return differences are, for example, not considering transaction costs or taxes when rebalancing a benchmark or the settlement of corporate action at the ex date instead of the pay date, which is normally the relevant date for a portfolio manager.

1.2.2.2 Types of benchmarks

In practice, four types of benchmarks are used, with their uses being driven by the user and the intended analysis:

- Target or expected return for an investment portfolio.
- Peer group consisting of investment portfolios with a comparable investment mandate or strategy.
- Index representing a specific asset class or investment style.
- Composite of weighted indices representing specific investment strategies.

A target or expected return is a simple benchmark. Examples for such a benchmark are the aspired absolute return an investor wants to achieve over a specific time horizon or the minimum return a pension fund needs to fulfill its future obligations. The use of this type of benchmark is straightforward but is not appropriate to measure the quality and the effectiveness of the portfolio management process.

The second type of benchmark is used if the performance of an investment portfolio should be compared to other investment portfolios with a comparable investment mandate or strategy. Examples for such a benchmark are peer groups or portfolio manager universes, constructed and maintained by specialized data providers, reflecting the performance range or the weighted performance of a group of comparable mutual funds or portfolio managers. A peer group comparison provides information as to how portfolio managers performed against each other or against the considered investment portfolio. Similar to the target return, the use of this type of benchmark is straightforward but also not appropriate to measure the quality and the effectiveness of the portfolio management process. Nevertheless, a peer group or a manager universe gives an indication on the range and distribution of the achieved performance of actual managed portfolios following specific investment strategies.

Portfolio manager	Benchmark	Gross return for last		
		12 months	3 years	5 years
ABC	Europe equity index	5.5%	13.1%	20.2%
DEF	Europe equity index	6.2%	12.3%	n/a
GHI	EU equity index	4.8%	10.9%	18.6%
Peer group	**Europe equity index**	**5.5%**	**12.1%**	**19.4%**

Table 1-6: Peer group performance comparison

Table 1-6 illustrates a peer group performance comparison for different portfolio managers managing European equity portfolios. The informative value is limited to the differences in returns for the periods shown. If one only has the information shown in Table 1-6, it is not possible to assess whether portfolio manager ABC is really performing better in the long term than portfolio manager GHI. The absolute return over the last 5 years is indeed higher but the respective benchmarks indicate that the actual investment mandates could have been slightly different, possibly explaining the return differences.

The third type of benchmark is often used for single asset class portfolios such as European equity or US government bond portfolios. An index, in general, represents the average price level of a particular asset class or market. The return of an index describes how the (weighted) prices of the different constituencies of the asset class or the market evolve over time. To evaluate the performance of an investment portfolio against a specific index, it is important to understand how the respective index is constructed and what index methodology is used. Assuming that the index is an effective benchmark, the use of this type of benchmark is appropriate to measure the quality and the effectiveness of the portfolio management process.

Internet pages of index providers often contain details on the specific index construction and the index methodologies used. These details may cover the following aspects:

- The universe of securities or investments suitable for inclusion into the index, for example European equities or US government bonds.
- The rules for selecting the constituent securities or investments from the relevant universe as well as the policy and rules for adding or deleting constituents from the index.
- The transaction costs and taxes for rebalancing the index, normally assumed to be zero.
- The actual selected constituent securities or investments.
- The weighting scheme of the securities or investments within the index, for example price-weighted, equally-weighted, GDP-weighted or capital-weighted, including the policy and rules for free float adjustments.
- The handling of income earned on the securities or investments, i.e. price only index or total return index, with the best practice being to use total return indices.
- The handling of income, capital gains or other taxes, and information as to whether the index is gross or net of taxes and if net, what the assumed tax domicile and applicable tax rates are.
- The handling and reinvestment of dividend or coupon payments.
- The handling of other corporate actions like optional dividends, changes in capitalization or splits.
- The pricing methodology and the price source for the constituent securities or investments.
- The reference currency of the index, the exchange rates used as well as the policy and rules for currency conversion or currency hedging.

The fourth type of benchmark, a composite of weighted indices (also called customized or blended benchmark), is often used for portfolios investing in multiple (sub-) asset classes. An example is the benchmark of a global equity portfolio consisting of three capital-weighted regional equities indices (North America, Europe and Asia) or a multi-asset class benchmark consisting of four equally-weighted asset class indices (fixed income, equity, real estate and hedge funds). To evaluate the performance of an investment portfolio managed against a customized benchmark, it is important to understand how the respective benchmark is constructed as well as what calculation methodology and rebalancing rule are used. Assuming that the customized benchmark is an effective benchmark, the use of this type of benchmark is appropriate to measure the quality and the effectiveness of the portfolio management process.

To best reflect the investment mandate and strategy, customized benchmarks are often tailor-made and defined by the individual investor or portfolio manager. When constructing a customized benchmark, the following aspects need to be addressed and defined:

- The actual constituent indices and the respective characteristics or aspects mentioned above.
- The weighting scheme of the constituent indices, for example equally-weighted, capital-weighted, risk-weighted or weighted following a specific portfolio optimization.
- The historical and current neutral or passive weights of the constituent indices.
- The rebalancing rules, if applicable, and the frequency of adjusting the constituent weights to the neutral or passive weights, for example daily, monthly or ad-hoc.
- The handling of income earned on the securities, i.e. whether price only or total return indices are used, with the best practice being to use total return indices.
- The handling of income, capital gains or other taxes, and whether the indices are gross or net of taxes and if net, what the assumed tax domicile and applicable tax rates are.
- The transaction costs and taxes for rebalancing, normally assumed to be zero.
- The reference currency of the benchmark, the exchange rates used as well as the policy and rules for currency conversion or currency hedging.

1.2.2.3 Customized benchmark return measurement

It is quite straightforward to calculate the return of a benchmark consisting of a single index because the actual return equals the percentage change of the index values for a specific measurement period. Assuming for example a total return index, the return of the benchmark for a single period t is calculated using the generic formula for calculating returns, see **chapter 1.2.1.1.1**:

$$R_{b,t} = \frac{IV_{end,t} - IV_{begin,t}}{IV_{begin,t}}.$$

Where: $R_{b,t}$ = Benchmark return for a single period t,
$IV_{end,t}$ = Index value at the end of period t,
$IV_{begin,t}$ = Index value at the beginning of period t.

It is common practice to chain-link sub-period benchmark or index returns to produce multi-period returns, as described in **chapter 1.2.1.1.8**. This implicitly assumes that external cash flows are not relevant and therefore that a respective timing effect needs to be eliminated. Thus, in practice, benchmark returns are normally time-weighted rate of returns.

For specific asset classes, like private equity, and in instances where a MWR needs to be calculated, it might be necessary to calculate a money-weighted index or benchmark return. An often used methodology is the so called **public market equivalent (PME)** methodology, where the benchmark return is translated from a time-weighted basis into a MWR, for example an IRR, using the actual external cash flows of the relevant investment portfolio. To derive the MWR using the cash flow stream of a PME benchmark, one has to calculate the ending PME benchmark value first taking into consideration the actual external cash flows of the relevant investment portfolio. The PME benchmark value at the end of a single period t is calculated using the following formula:[10]

$$MV_{end,t}^{PME} = MV_{begin,t}^{PME} \times \left(1 + R_{b,t}\right) + \tilde{C}_{end,t}^{P} - \widetilde{W}_{end,t}^{P} \, .$$

Where:
- $MV_{end,t}^{PME}$ = PME benchmark value at the end of period t,
- $MV_{begin,t}^{PME}$ = PME benchmark value at the beginning of period t,
- $\tilde{C}_{end,t}^{P}$ = Portfolio external cash inflow at the end of period t,
- $\widetilde{W}_{end,t}^{P}$ = Portfolio external cash outflow at the end of period t.

Exhibit 1-12: Public market equivalent benchmark return

Same data as in Exhibit 1-11: assuming that the interim external cash flows of the relevant investment portfolio are simulated as respective external cash flows (buy and sell transactions) for the benchmark at no cost, and that the index is identical to the aforementioned equity index fund, the annualized (not annualized) IRR for the benchmark is +141.79% (+7.53%).

Date	TWR of equity index	Index value equity index	Portfolio interim external cash flow	PME benchmark value	PME cash flow stream
31.03.		100.00	100.00	100.00	-100.00
10.04.	-13.29%	86.71	50.00	136.71	-50.00
30.04.	+17.04%	101.49		160.00	160.00
April	+1.49%				

Table 1-7: Public market equivalent methodology

Here, the benchmark return (IRR) for April is identical to the IRR of the investment portfolio because it was assumed that the equity index fund the portfolio manager is investing in, is 100% identical with the respective equity index, implying no tracking error.

Calculating the return of a customized benchmark, built as a composite of weighted indices, is a bit more complex. The benchmark return for a single period t equals the sum of the weighted returns of the constituent indices and is calculated as follows:

$$R_{b,t} = \sum_{\text{for all indices i}} w_{b,i,t} \times R_{b,i,t} \, .$$

With: $$\sum_{\text{for all indices i}} w_{b,i,t} = 1 \, .$$

Where:
- $w_{b,i,t}$ = Weight of a constituent index i of a benchmark at the beginning of period t,
- $R_{b,i,t}$ = Return of a constituent index i of a benchmark for period t.

10 In **this chapter**, if not otherwise stated, benchmark return refers to time-weighted benchmark returns.

Exhibit 1-13: Customized benchmark return for a single period

Let us consider a customized benchmark consisting of an equity index and a bond index, assuming neutral weights of 30.0% and 70.0% and sub-period returns of +5.00% and +1.00%. In this case, the benchmark return will be +2.20%.

$$=> R_{b,t} = 30.0\% \times 5.00\% + 70.0\% \times 1.00\% = 1.50\% + 0.70\% = +2.20\%\,.$$

To calculate a multi-period return for a customized benchmark, one has to define how the weights of the constituent indices, which are relevant for the return calculation, change over time.[11] There are three rebalancing rules used in portfolio management:

- **No rebalancing**: This buy-and-hold strategy assumes that throughout the entire measurement period, the index weights are floating and not rebalanced to the neutral weights, set at the beginning of the measurement period. The relative weights between the different constituent indices change over time depending on the relative returns of the individual indices.
- **Regular rebalancing**: This rebalancing strategy assumes that throughout the entire measurement period, the index weights are rebalanced to the neutral weights after a certain period, e.g. a day, month, or quarter, set at the beginning of the measurement period. During sub-periods, the relative weights of the different constituent indices may change depending on the relative returns of the individual indices but are reset to the neutral index weights at the beginning of each new sub-period. In practice, monthly rebalancing is often used for multi-asset class benchmarks.
- **Ad-hoc rebalancing**: This rebalancing strategy is a combination of the two previous ones, where the index weights are reset not at a fixed date or after a fixed period of time but on an ad-hoc basis or after the respective decision maker has decided to do so.

The calculation of a multi-period return of a customized benchmark without any rebalancing is straightforward and is the weighted sum of the cumulative returns of the constituent indices, where the weights equal the neutral or passive weights of the constituent indices at the beginning of the measurement period:

$$R_{b,\text{tot with no rebalancing}} = \sum_{\text{for all indices } i} w_{b,i,0} \times R_{b,i,tot}\,.$$

With: $\sum_{\text{index } i} w_{b,i,0} = 1\,.$

Where: $R_{b,\text{tot with no rebalancing}}$ = Cumulative return of a buy-and-hold customized benchmark,

$w_{b,i,0}$ = Weight of a constituent index i of a benchmark at the beginning of the measurement period.

$R_{b,i,tot}$ = Cumulative return of a constituent index i of a benchmark.

The calculation of a multi-period return of a customized benchmark with regular rebalancing is a bit more complex and equals the geometrically linked product of the individual sub-period benchmark returns. The following formula is used:

11 Multi-period benchmark returns are only relevant for time-weighted benchmark returns. Money-weighted benchmark returns are always calculated for a single period like a day, month, and a year or since inception.

$$R_{b,tot\ with\ regular\ rebalancing} = \prod_{for\ all\ sub-periods\ t} (1 + R_{b,t}) - 1$$
$$= \prod_{for\ all\ sub-periods\ t} \left(1 + \sum_{for\ all\ indices\ i} w_{b,i,t} \times R_{b,i,t}\right) - 1 .$$

With: $\sum_{index\ i} w_{b,i,t} = 1$.

Where: $R_{b,tot\ with\ regular\ rebalancing}$ = Cumulative return of a customized benchmark with regular rebalancing.

Exhibit 1-14: Customized benchmark return for multiple periods

Considering the data shown in Table 1-8, the 1st quarter return for the customized benchmark with monthly rebalancing is +0.67% and without any rebalancing is +0.44%.[12]

Date	Monthly returns		Buy and hold			Monthly rebalancing		
			Beg. period weights		Return	Beg. period weights		Return
	Equity index	Bond index	Equity index	Bond index	Benchmark	Equity index	Bond index	Benchmark
January	+5.00%	-2.00%	30.00%	70.00%	+0.10%	30.00%	70.00%	+0.10%
February	-10.00%	+2.00%	31.47%	68.53%	-1.78%	30.00%	70.00%	-1.60%
March	+5.00%	+1.00%	28.83%	71.17%	+2.15%	30.00%	70.00%	+2.20%
1st quarter	**-0.77%**	**+0.96%**			**+0.44%**			**+0.67%**

Table 1-8: Customized benchmark return calculation with different rebalancing rules

The difference in returns of +0.23% indicates that the (monthly) rebalancing rule is a kind of automatic asset allocation decision to sell (buy) good (bad) performing asset classes. Therefore, defining the rebalancing rule of the customized benchmark is also a management decision affecting the overall performance of an investment portfolio.

1.2.2.4 Value added return measurement

After measuring the returns of an investment portfolio and its benchmark, the next step is to calculate the value added produced by all portfolio management decisions during the measurement period. The value added can be measured in two ways: a) as an arithmetic value added or b) as a geometric value added.

The arithmetic value added is the absolute difference between the returns of the investment portfolio and the respective benchmark. The arithmetic value added expresses the absolute excess return of the investment portfolio versus the respective benchmark. For a single period t, the arithmetic value added is calculated as follows:

$$VA_{A,t} = R_{p,t} - R_{b,t} .$$

Where: $VA_{A,t}$ = Arithmetic value added of a single period t.

By contrast, the geometric value added is the relative difference between the returns of the investment portfolio and the respective benchmark. For a single period t, the geometric value added is calculated as follows:[13]

12 Exemplary for the buy and hold rebalancing, the weight of the equity index for the February period is 31.47% that equals (30.00% * (1 + 5.00%)) / (30.00% * (1 + 5.00%) + 70.00% * (1 – 2.00%)).

$$VA_{G,t} = \frac{1 + R_{p,t}}{1 + R_{b,t}} - 1 .$$

Where: $VA_{G,t}$ = Geometric value added of a single period t.

When using TWRs, a multi-period value added for a specific measurement period is derived from the value added of the individual sub-measurement periods (not necessarily of the same length) considering compounding.[14] The multi-period value added is calculated on an arithmetic basis as follows:

$$VA_{A,tot} = R_{p,tot} - R_{b,tot} .$$

Where: $VA_{A,tot}$ = Multi-period or cumulative arithmetic value added,
$R_{b,tot}$ = Multi-period cumulative benchmark return.

Here, it is important to note that because of the mathematical characteristics, especially since the compounding effects are not considered, the chain-linked arithmetic value added of the sub-periods does not add up to the cumulative arithmetic value added for the entire measurement period:[15]

$$VA_{A,tot} \neq \prod_{\text{for all sub-periods } t} (1 + VA_{A,t}) - 1 .$$

The multi-period value added is calculated on a geometric basis as follows:

$$VA_{G,tot} = \frac{1 + R_{p,tot}}{1 + R_{b,tot}} - 1 .$$

Where: $VA_{G,tot}$ = Multi-period or cumulative geometric value added.

In contrast to the arithmetic value added, chain-linking of the geometric value added of the sub-periods does add up to the cumulative geometric value added for the entire measurement period:

$$VA_{G,tot} = \prod_{\text{for all sub-periods } t} (1 + VA_{G,t}) - 1 .$$

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13 In **this chapter**, if not otherwise stated, value added always refers to the arithmetic value added. This follows the common practice in the industry considering that the geometric value added is not initiative for the investors.

14 The calculation of a multi-period value added is only relevant if using TWRs. Money-weighted value added is always calculated just for a single period like a day, month, and year or since inception.

15 Therefore, mathematical help is needed to make them fit by using a linking algorithm, which is discussed in **chapter 1.3.2.1.3**.

Exhibit 1-15: Value added

Let us consider the data shown in Table 1-9. In this case, the 1st quarter arithmetic (geometric) value added is equal to +9.64% (+9.09%).

Date	Portfolio return	Benchmark return	Arithmetic value added	Geometric value added
January	+5.00%	+2.00%	+3.00%	+2.94%
February	+5.00%	+2.00%	+3.00%	+2.94%
March	+5.00%	+2.00%	+3.00%	+2.94%
1st quarter	**+15.76%**	**+6.12%**	**+9.64%**	**+9.09%**

Table 1-9: Arithmetic and geometric value added

As mentioned above, the cumulative arithmetic value added does not equal the chain-linked arithmetic value added of the sub-periods:

$$=> \mathrm{VA}_{\mathrm{A,tot}} = +9.64\% \neq +9.27\% = (1 + 3.00\%) \times (1 + 3.00\%) \times (1 + 3.00\%) - 1\,.$$

1.2.3 Risk measurement

1.2.3.1 Introduction to risk measurement

1.2.3.1.1 Definition of investment risk

Performance evaluation considering only returns neglects the second, but no less important dimension of investment performance. In order to reach meaningful conclusions when assessing the quality of a portfolio management process, one must also consider and analyze the investment risk taken to produce the returns of investment portfolios.

Risk is in general defined as exposure to uncertainty. In portfolio management companies, several different types of risk are of concern.[16] Investment risk is one of these and is defined as the uncertainty of the expected outcome or of meeting the investment target or investor's expectations. The complexity of risk makes it difficult to measure or estimate investment risk. Following the fundamental concept of modern portfolio theory, namely that the investor considers expected return as desirable and expected variability of returns as undesirable, statistical risk metrics are often used as proxies to quantify investment risk. The use of such risk metrics has the advantage that it implicitly estimates "objective" probabilities what allows rational decision making.

The bars in Figure 1-10 illustrate the frequency of monthly returns of an investment portfolio A for a specific measurement period. The returns seem to follow a normal distribution , which is supported by the area chart reflecting a normal distribution of returns, described by the mean return and the average variability of the returns of the investment portfolio. Considering that the area covers 100% of all cases, the area can be split into different sub-areas. Figure 1-10 shows three sub-areas covering the returns up to –2%, between –2% an +6% and above +6%. Assuming that expected returns follow the same pattern as in the past, the probability density can be interpreted as the expected probability that actual returns will be in specific ranges of returns. Using the data illustrated in Figure 1-10, the expected probability that the actual monthly return will be below –2% or higher than +6% is 15.865% in each case and the probability that the actual monthly return will be around the mean return or between –2% and +6% is 68.27%.

16 Besides investment risk, other types of risk relevant for portfolio management companies are but not limited to operational risk, regulatory and compliance risk, liquidity risk or counterparty and credit risk.

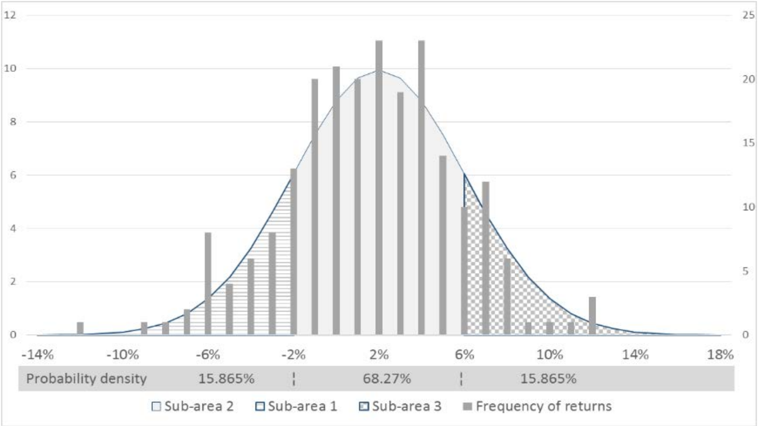

Figure 1-10: Probability density function of investment portfolio A

Investment risk takes many forms and the relevant definition as well as the respective measure of investment risk depend on the actual case and question in mind. As a rule, the purpose of the analysis needs to be defined before a specific risk measure is chosen. For example, if the investment target is to save a certain amount of money over a specific period so as to be able to amortize a mortgage, the investor might be concerned with the possibility of not having saved enough money to pay back the mortgage. Another example for a case specific type of risk is active risk. It measures the expected probability and extent of deviation from the benchmark return and is used if an investor wants to get an indication of the extent of active management pursued by a specific portfolio manager.

1.2.3.1.2 Absolute, downside and relative risk

Depending on the analysis in mind, three main types of risk measures are used: absolute risk measures, downside risk measures and relative risk measures.

Absolute risk is the actual or market risk of an investment portfolio measured by the variability of returns, including the complete return distribution. Absolute risk is often stated without any context or without an explicit comparison, and therefore is often considered on a stand-alone basis. For example, assuming a normal distribution of returns, there is a 50 percent or a 50 out of 100 chance that the actual return of an investment portfolio will be lower (higher) than the expected mean return. In practice, different measures are used to describe absolute risk. **Chapter 1.2.3.2** discusses the following absolute risk measures: variance, standard deviation, value at risk, skewness, and kurtosis.

Downside risk is a kind of absolute risk but in contrast to absolute risk, considers only a part of the return distribution. For downside risk measures, only returns below a certain reference or threshold return are considered risky. By contrast, absolute risk does not differentiate between "negative" risk (return below a certain threshold) and "positive" risk (return above a certain threshold).[17] Like absolute risk, downside risk is often stated without any context or without an explicit comparison, and therefore is often considered on a stand-alone basis. For example, considering the normal distribution of returns illustrated in Figure 1-10, there is a 15.865% percent or about 16 out of 100 chance that the actual monthly return of the investment portfolio will be below a threshold return of -2%. In practice, different measures are used to describe downside risk. **Chapter 1.2.3.2** discusses the following downside risk measures: shortfall probability, downside variance, and downside standard deviation.

17 Contrary to downside risk, upside risk measures consider only returns as "risky" which are above a certain reference or threshold return.

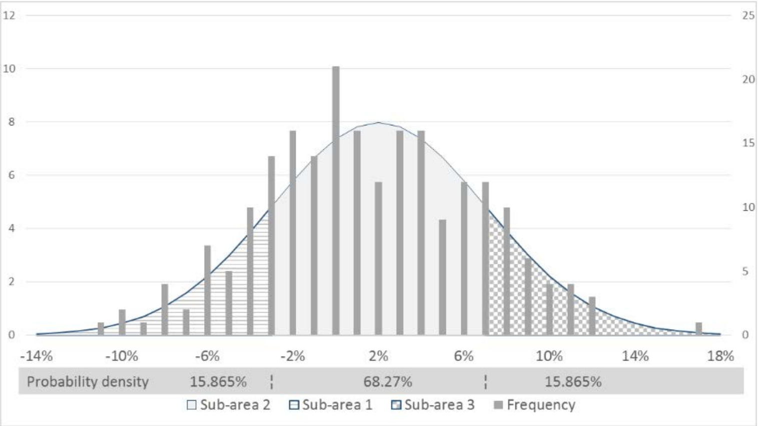

Figure 1-11: Probability density function of investment portfolio B

In contrast to absolute risk, the **relative risk** of an investment portfolio is a comparison of different risk levels and does not have to be related to return distributions. Relative risk measures the relevant risk in context or with an explicit comparison, and therefore requires a reference to which one can compare the relevant risk.

For example, considering the normal distribution of returns illustrated in Figure 1-10 there is a 68.27% percent or about 68 out of 100 chance that the actual monthly return of the investment portfolio will be between -2% and +6%. Comparing this investment portfolio to another investment portfolio B with a different return distribution, illustrated in Figure 1-11, the relative risk may be the ratio of the probabilities that the actual monthly return will be between -2% and +6%.[18] Considering the respective probability of investment portfolio B (57.62%) the relative risk would be the ratio of 68.27% and 57.62%. The relative risk of about 1.18 means that the actual monthly return of investment portfolio A will be between -2% and +6% in 18% more cases than for investment portfolio B.[19] Figure 1-12 compares the two return distributions, showing the bigger dispersion of returns of investment portfolio B.

In portfolio management, relative risk measures are not often used explicitly, but sometimes implicitly, for example when comparing two alternative equity mutual funds with the same expected mean return but with different levels of absolute risk. A risk averse investor would prefer the investment portfolio with the lower level of absolute risk or with a respective relative risk of lower than 1. Nevertheless, in portfolio management "relative" risk measures are often used, though the term "relative" does not refer to a ratio of risk levels but to the excess returns of an investment portfolio against a benchmark.[20] This interpretation of relative risk refers to the risk of deviating from a benchmark or any other reference, and therefore often acts as a measure for the level of active portfolio management. In practice, different measures are used to describe this kind of relative risk. **Chapter 1.2.3.2** discusses the following relative risk measures: covariance, correlation, tracking error variance, and tracking error standard deviation.

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18 Instead of using a ratio, one could also calculate the absolute difference between two risk levels, what may be called absolute relative risk.

19 It is important to note that relative risk gives no indication about the actual risk.

20 In **this chapter**, if not otherwise stated, relative risk always refers to the variability of excess returns and not to a ratio of risk levels, and may be called excess or active risk. Depending on the analysis, excess returns refer to the difference between the returns of an investment portfolio and the returns of a risk free investment or of other investment portfolios with similar level of risk, or to the difference between the returns of an investment portfolio and a benchmark. The latter excess return is also called value added or active return. In **this chapter**, we use the term excess return, active return and value added interchangeably.

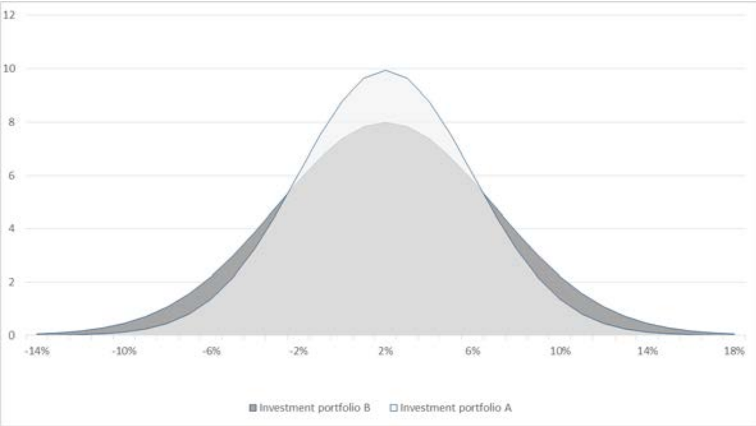


Figure 1-12: Comparing return distributions

In practice, when evaluating the performance of investment portfolios, one often uses not one single risk measure but different types of risk measures in combination to get a better view on the actual return distribution and the expected investment risk. For example, an investor may use an absolute risk measure to define a short list of investment products out of a market universe and, in addition, a relative risk measure to pick the final investment product to invest in. Furthermore, when comparing the performance of different investment portfolios, often the two dimensions of investment performance, return and risk, are considered in combination by using performance measures or risk-adjusted return measures. Performance measures are used to compare the returns of comparable investment portfolios with similar levels of risk. **Chapter 1.2.3.3** discusses the following performance measures: Sharpe ratio, Treynor ratio, Jensen's alpha, appraisal ratio, Graham & Harvey measure 1 and 2, Sortino ratio, and information ratio.

1.2.3.1.3 Ex post and ex ante risk

Investment risk can be considered and measured in two different ways: ex post (backward looking) and ex ante (forward looking). Ex post risk refers to the risk after the event, meaning the risk actually experienced by the investor or by the portfolio manager. By contrast, ex ante risk refers to the risk before the event, i.e. the expected risk by the investor or the portfolio manager.

Using risk measures based on return time series, ex post risk analyzes the dispersion of the historical returns of an investment portfolio over a specific measurement period. In comparison, ex ante risk forecasts the risk of an investment portfolio by using a specific portfolio structure, for example a list of current securities and instruments as of a specific measurement date, and estimated characteristics for the return time series of the holdings or the investment portfolio.[21]

Depending on the investment portfolio or product, the calculation of ex post and ex ante risk may lead to substantially different figures and interpretations. This discrepancy is due to the underlying assumptions and the set of sample data used. However, comparing ex ante risk with the actual experienced ex post risk gives additional information and especially provides more insights on the quality or appropriateness of the forecasted ex ante risk.

21 The characteristics are often forecasted by using historical returns of and relationships between the different securities and instruments, what may be problematic especially for extreme events.

Performance evaluation often concentrates on past investment performance and is therefore more concerned with ex post risk, monitoring whether investment targets were met and whether the participants in the portfolio management process adhered to what they promised in the first place. The focus when using ex ante risk figures is more to analyze the quality and appropriateness of the portfolio management process, the investment style or the investment portfolio to meet the investor's expectations and investment targets in future.[22]

1.2.3.2 Risk measures

1.2.3.2.1 Variance and standard deviation

If one considers investment risk to be the variability or dispersion of the returns of an investment portfolio from the mean return, the statistical measures variance and standard deviation (i.e. the square root of variance) are often used as a measure of risk.[23] As an ex post risk measure, the variance is the second central moment about the mean and measures the dispersion of the returns or the average squared deviation of the returns from the mean return.[24] Interpreted as a risk measure and everything else being equal, a high dispersion of returns reflects high investment risk for a risk averse investor. Using continuously compounded returns and assuming that the data represents a sample of the population, the historical variance, respectively the standard deviation, of periodic returns is calculated as follows:[25]

$$\mathrm{Var}_p = \sigma_p^2 = \frac{1}{N-1} \times \sum_{t=1}^{N} \left(r_{p,t} - \bar{r}_p\right)^2 \quad \text{and} \quad \sigma_p = \sqrt{\frac{1}{N-1} \times \sum_{t=1}^{N} \left(r_{p,t} - \bar{r}_p\right)^2} \,.$$

With: $$\bar{r}_p = \frac{1}{N} \times \sum_{t=1}^{N} r_{p,t} \,.$$

Where:
- Var_p = Variance of the returns of a portfolio,
- N = Number of returns in the sample,
- $\bar{r}_p$ = Mean return,
- σ_p = Standard deviation of the returns of a portfolio.

The formula above assumes a certain periodicity of the returns: daily, weekly, monthly, etc. In practice and for comparison purposes, risk measures are often presented in an annualized form. Because of its proportionality to time (or to the square root of time for the standard deviation), the annualized variance, respectively the annualized standard deviation, is calculated as follows:

22 In **this chapter**, if not otherwise stated, risk always refers to ex post risk.

23 In practice, standard deviation is often called volatility and therefore used interchangeably.

24 Using squared deviations avoids the problem of negative deviations netting positive deviations.

25 In practice, it is common practice to use *n* in the denominator, assuming the use of the entire population, which is an appropriate assumption for large sample sizes.

$$Var_{p,annualized} = t_o \times Var_{p,not\ annualized} \quad \text{and} \quad \sigma_{p,annualized} = \sqrt[2]{t_o} \times \sigma_{p,not\ annualized}$$

Where:

$Var_{p,annualized}$ = Annualized variance of the returns of a portfolio,

$Var_{p,not\ annualized}$ = Not annualized variance of the returns of a portfolio,

$\sigma_{p,annualized}$ = Annualized standard deviation of the returns of a portfolio,

$\sigma_{p,not\ annualized}$ = Not annualized standard deviation of the returns of a portfolio,

t_o = Number of observations in a year (monthly = 12, weekly = 52 or daily = 250).

In contrast to the variance of returns, the standard deviation of returns is measured in units of return, and is therefore easy to interpret and is often presented to investors. Assuming that the normal return distribution illustrated in Figure 1-11 is interpreted as expected risk, standard deviation implies that under normal market conditions one can expect the actual return to be:

- Within the range of the expected mean return plus / minus one standard deviation in about 68.27% of all cases.
- Within the range of the expected mean return plus / minus two standard deviations in about 95.45% of all cases.

If one refers to the return distribution illustrated in Figure 1-13, whose monthly mean return μ is 2% and whose monthly standard deviation σ is 4%, the following interpretation is valid:

- There is a probability of about 68.27% that the actual return over one month will fall within the range of –2.00% and +6.00% under normal market conditions.
- There is a probability of about 95.45% that the actual return over one month will fall within the range of –6.00% and +10.00% under normal market conditions.

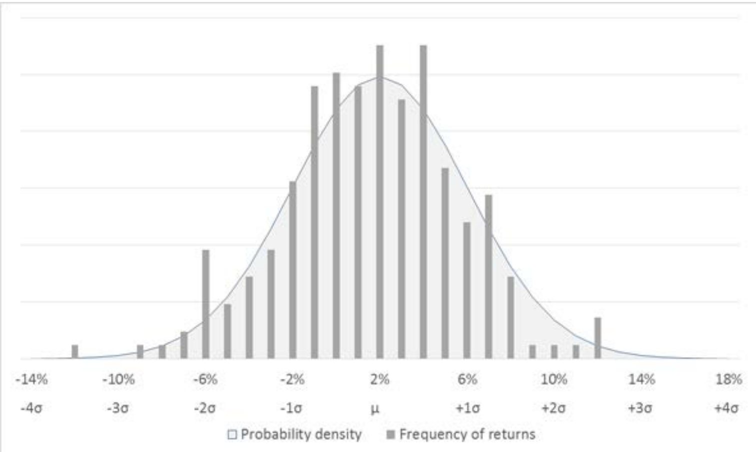


Figure 1-13: Interpretation of standard deviation of returns

Exhibit 1-16: Standard deviation

If one considers the monthly continuously compounded returns of an investment portfolio and its benchmark shown in Table 1-10, the annualized standard deviation (return) for the investment portfolio is 2.28% (+2.52%) and 2.75% (+4.07%) for its benchmark.[26]

Month	Portfolio return	Benchmark return	Month	Portfolio return	Benchmark return
1	+0.19%	+0.25%	8	+1.31%	+1.75%
2	+0.56%	+0.75%	9	-0.75%	-0.75%
3	+0.19%	+0.25%	10	-0.50%	-0.50%
4	-0.25%	-0.25%	11	-1.00%	-1.00%
5	+0.56%	+0.75%	12	+0.00%	+0.00%
6	+0.56%	+0.75%	13	+1.31%	+1.75%
7	+0.19%	+0.25%	14	+0.56%	+0.75%

Table 1-10: Monthly returns used for calculation of standard deviation

1.2.3.2.2 Value at risk

If one considers investment risk to be the probability of having a loss greater than a certain amount of money, the statistical measure value at risk (VaR) is often used as a measure of risk. VaR is the expected maximum loss, in percentage or in absolute terms, one can expect to experience under normal market conditions, over a given time horizon, for a given return distribution as well as with a stated level of confidence. In other words, VaR measures, for a specific probability, the expected absolute or percentage loss, which is not expected to be exceeded for a given time horizon and return distribution. Interpreted as a risk measure and everything else being equal, a high VaR reflects high investment risk for a risk averse investor.

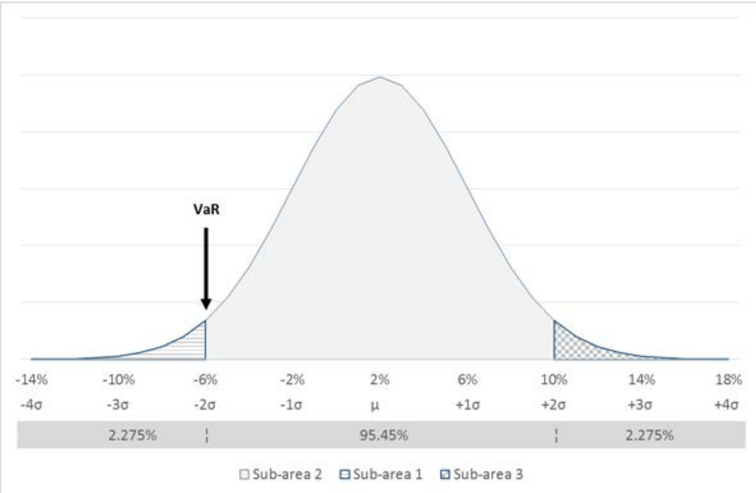


Figure 1-14: Value at risk

26 It is common practice to use at least 36 observations to calculate dispersion statistics. For illustrative purposes, 14 observations are used. Furthermore, *n* is used in the denominator, assuming the use of the entire population.

Figure 1-14 illustrates the concept of VaR and shows that in terms of statistics VaR is a percentile of a return distribution. Sub-area 1 covers all expected monthly returns below the threshold return of –6.00%, which account for 2.275% of all cases. This means that in 97.725% (equals 95.45% + 2.275% or 100.00% – 2.275%) of all cases the expected monthly return is higher than –6.00%. Interpreting this as VaR means that in 98 cases out of 100, the maximum expected loss will not exceed –6.00% over a one month period. In other words, there is a 98% probability, assuming normal market conditions, that the maximum expected loss will not exceed –6.00% over a one month period. However, VaR gives no indication about the size of the loss in the other +2.00%.

With normally distributed returns, VaR can be directly estimated as a multiple of the standard deviation of the returns, like in Figure 1-14, where the respective multiple for the VaR of –6.00% is 2. This nature helps to transform the required probability into the multiple using the z-score. In practice, fixed probabilities like 95% and 99% are often used, where the respective z-scores are 1.645 and 2.326.[27]

Typically, VaR is calculated by first modeling the entire return distribution for an investment portfolio, then calculating the VaR at the percentile corresponding to the desired confidence level. The calculation of VaR is straightforward if the necessary inputs are defined and available. Assuming normally distributed continuously compounded returns, the so-called parametric or analytic VaR is calculated as follows in percentage terms:[28]

$$\text{VaR}_{\text{C,H,per}}(\tilde{\mu},\tilde{\sigma}) = \tilde{\mu} - \tilde{\sigma} \times z\,.$$

And in absolute terms:

$$\text{VaR}_{\text{C,H,abs}}(\tilde{\mu},\tilde{\sigma}) = \text{MV} \times (\tilde{\mu} - \tilde{\sigma} \times z)\,.$$

Where:
- $\text{VaR}_{\text{C,H,per}}(\tilde{\mu},\tilde{\sigma})$ = Value at risk in percentage terms for a specific confidence level C, a specific time horizon H, an expected return of $\tilde{\mu}$ and an expected standard deviation of $\tilde{\sigma}$,
- $\tilde{\mu}$ = Expected continuously compounded return,
- $\tilde{\sigma}$ = Expected standard deviation of continuously compounded returns,
- z = Z-score for a specific confidence level C,
- $\text{VaR}_{\text{C,H,abs}}(\tilde{\mu},\tilde{\sigma})$ = Value at risk in absolute terms for a specific confidence level C, a specific time horizon H, an expected return of $\tilde{\mu}$ and an expected standard deviation of $\tilde{\sigma}$,
- MV = Market value of a portfolio.

27 A z-score of 1.645 (2.326) indicates that 90% (98%) of the returns lie within 1.645 (2.326) standard deviations of the mean return. This implies that 10% (2 %) of the time, the returns will lie outside this range (either below or above). As VaR is only concerned about values below the VaR, the right tail is not relevant. Given the symmetrical properties of the distribution, this means that the probability of the returns being below the VaR are half of 10% (2%), i.e. 5% (1%). This is equivalent to looking at the values for a confidence level of 95% (99%). For Figure 1-14, a z-score of 2.000 corresponds to a confidence level 97.725%, i.e. a confidence level of 95.45% (including both tails) + 2.275%, corresponding to the observations of the right tail ((1 – 95.45%)/2).

28 Besides the parametric approach, there are two other commonly used methodologies in the industry for modeling return distributions: the Historical Simulation (or non-parametric) and the Monte Carlo Simulation.

Exhibit 1-17: Parametric value at risk

Using the historical return and standard deviation of the investment portfolio and its benchmark mentioned in Exhibit 1-16 and assuming a market value of the investment portfolio of 100.00 EUR, the respective VaR for the next 12 months and with a confidence level of 97.725% is for the investment portfolio:

=> $\mathrm{VaR}_{97.725\%,12\text{ months},\mathrm{per}}(2.52\%, 2.28\%) = 2.52\% - 2.28\% \times 2.000 = -2.04\%$, and

$\mathrm{VaR}_{97.725\%,12\text{ months},\mathrm{abs}}(2.52\%, 2.28\%) = (2.52\% - 2.28\% \times 2.000) \times 100.00 = -2.04$.

And for its benchmark:

=> $\mathrm{VaR}_{97.725\%,12\text{ months},\mathrm{per}}(4.07\%, 2.75\%) = 4.07\% - 2.75\% \times 2.000 = -1.43\%$, and

$\mathrm{VaR}_{97.725\%,12\text{ months},\mathrm{abs}}(4.07\%, 2.75\%) = (4.07\% - 2.75\% \times 2.000) \times 100.00 = -1.43$.

1.2.3.2.3 Skewness and kurtosis

So far, we have assumed that the continuously compounded returns are normally distributed. A normal return distribution like the one in Figure 1-14 is a bell-shaped curve and is characterized by a high density of returns close to the mean return and a low density of returns far away from the mean return or in the tails of the distribution. The normal distribution peaks at the mean return and is symmetrical around the mean return. If returns are normally distributed, the mean return and the standard deviation of returns can be used to describe the distribution of returns.

However, the assumption of normally distributed returns is often not fulfilled or even not appropriate. In practice, certain types of financial instruments and investment strategies are specifically designed to produce asymmetric return distributions. Examples for such investment strategies are those that use derivative instruments, like options or futures, to produced non-normal return distributions.

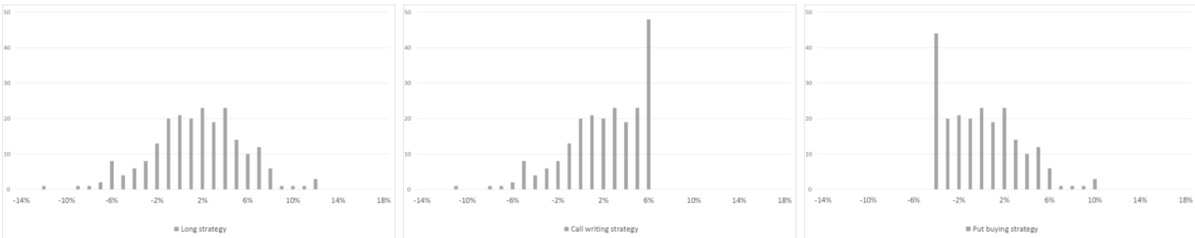


Figure 1-15: Comparison of return distributions

Figure 1-15 compares the return distributions of three investment strategies (from left to right): long strategy, long strategy plus call writing, and long strategy plus put buying. As illustrated, the use of the derivatives changes the form of the return distribution to be non-normal. In the case of asymmetric distributions, using standard deviation to describe the form of the distribution is not appropriate and we should turn to other statistics to better describe the form of the return distribution.

Skewness is one often-used statistic to assess normality and to describe the form of a return distribution. It is the third central moment about the mean, measuring how skewed the distribution is to the right or to the left. It is calculated as follows:

$$\mathrm{Skewness} = \frac{1}{N} \times \sum_{t=1}^{N} \left(\frac{r_{p,t} - \bar{r}_p}{\sigma_p} \right)^3.$$

A positively skewed distribution, like the one of the put buying strategy shown in Figure 1-15, has a skewness of greater than 0, has a mean return higher than the median return and contains more returns extending to the right tail than the normal distribution. A negatively skewed distribution, like the one of the call writing strategy shown in Figure 1-15, has a skewness of less than 0, has a mean return lower than the median return and contains more returns extending to the left tail than the normal distribution. The higher the absolute value of skewness, the more the returns are biased to the left or right tail of the distribution. In addition, a normal distribution has a skewness of 0.

Kurtosis is another often-used statistic to assess normality and to describe the form of a return distribution. It is the fourth central moment about the mean, measuring whether the distribution is peaked or flat. It is calculated as follows:

$$\text{Kurtosis} = \frac{1}{N} \times \sum_{t=1}^{N} \left(\frac{r_{p,t} - \bar{r}_p}{\sigma_p} \right)^4 .$$

The kurtosis for a standard normal distribution is 3. Often, the above definition is modified by deducting 3 from the resulting kurtosis so that the kurtosis of a normal distribution is 0. In this case, positive kurtosis (or in the case of our definition, a kurtosis with a value of greater than 3) indicates a more peaked distribution, with more returns close to the mean and more frequent large negative or positive returns than a normal return distribution. By contrast, negative kurtosis or in the case of our definition, a kurtosis with a value of less than 3, indicates a flatter distribution, which has less returns close to the mean and less frequent large negative or positive returns than a normal return distribution.

1.2.3.2.4 Shortfall probability

Shortfall probability is often used as an alternative to VaR. Under normal market conditions and for a given time period and confidence interval, shortfall probability measures the probability of exceeding an expected absolute or percentage loss for a given time horizon and return distribution. In mathematical terms, shortfall probability computes the confidence level for a certain threshold return or VaR, or the percentage of returns that fall below the threshold return or VaR. Interpreted as a risk measure and everything else being equal, a high shortfall probability for a given threshold return reflects high investment risk for a risk averse investor. It is calculated as follows:

$$\text{Shortfall probability} = \frac{\text{Number of returns} < \text{threshold return}}{N} .$$

Assuming normally distributed continuously compounded returns, the so-called parametric or analytic shortfall probability for a given VaR or threshold return in percentage terms is calculated as follows:

$$SP(VaR_{per}) = \Phi(z, VaR_{per}) .$$

With: $z = \dfrac{VaR_{per} - \tilde{\mu}}{\tilde{\sigma}}$.

Where: $SP(VaR_{per})$ = Shortfall probability for a given VaR in percentage terms.

Exhibit 1-18: Shortfall probability

Using the same data as in Exhibit 1-16 and considering the monthly continuously compounded returns for an investment portfolio and its benchmark shown in Table 1-11, we calculate the shortfall probability for a threshold return of 0.00% for the investment portfolio and for its benchmark. For the calculation, we use only the returns below the threshold return of 0.00%, highlighted in the table. This gives us a shortfall probability of 28.57% for the investment portfolio (i.e. 4 observations out of 14) and also 28.57% (=4/14) for its benchmark.

Month	Portfolio return	Benchmark return	Month	Portfolio return	Benchmark return
1	+0.19%	+0.25%	8	+1.31%	+1.75%
2	+0.56%	+0.75%	9	-0.75%	-0.75%
3	+0.19%	+0.25%	10	-0.50%	-0.50%
4	-0.25%	-0.25%	11	-1.00%	-1.00%
5	+0.56%	+0.75%	12	+0.00%	+0.00%
6	+0.56%	+0.75%	13	+1.31%	+1.75%
7	+0.19%	+0.25%	14	+0.56%	+0.75%

Table 1-11: Monthly returns used for calculation of shortfall probability

Let us assume now for illustrative purposes that the above returns follow a normal distribution. Using the respective annualized mean return and annualized standard deviation for the investment portfolio of +2.52% and 2.28% and for its benchmark +4.07% and 2.75% (see Exhibit 1-16), the shortfall probability for an annualized threshold return for 0.00% would be 13.45% for the investment portfolio and 6.94% for its benchmark.

1.2.3.2.5 Downside variance and downside standard deviation

Variance and standard deviation are not very good descriptive statistics for non-normally distributed returns. In the case of non-normal distributions, one expects a different number of returns at a particular point of the distribution than indicated by the normal distribution. Using standard deviation as a risk measure implies that the investor is interested in the dispersion of the returns around the mean return and that returns below and above have the same probability of occurrence. However, in practice, investors are often interested in the dispersion of the returns around a certain reference or threshold return, and then only in the number and extent of unfavorable returns, for example returns below a threshold return.

Non-normal return distributions and a risk attitude that is not reflected by the standard deviation lead to the use of downside risk measures. A downside risk measure considers only as risky those returns that fall below a certain reference or threshold return.

If one considers investment risk to be the variability or dispersion of the returns of an investment portfolio below a certain threshold return, the statistical measures downside variance and downside standard deviation are often used as a measure of risk. As an ex post risk measure, the downside variance (downside standard deviation) is the second lower partial moment about a certain threshold return and measures the dispersion of the returns or the average squared deviation of the returns below the threshold return.[29] Interpreted as a risk measure and everything else being equal, a high dispersion of returns below the threshold reflects high investment risk for a risk averse investor. Using continuously compounded returns and assuming that the data represents a sample of the population, the historical downside variance (downside standard deviation) of periodic returns is calculated as follows:[30]

29 When using the mean return for the threshold return, this leads to a specific downside variance (standard deviation) called semi-variance (semi-standard deviation).

30 Like variance and standard deviation, downside variance and downside standard deviation are also proportional to time and the same adjustments as mentioned in **chapter 1.2.3.2.1** can be applied to produce annualized figures.

$$\mathrm{Var}_{D,p}(r_T) = \sigma_{D,p}^2(r_T) = \frac{1}{N-1} \times \sum_{t=1}^{N} \min[(r_{p,t} - r_T), 0]^2 \text{ , and}$$

$$\sigma_{D,p}(r_T) = \sqrt{\frac{1}{N-1} \times \sum_{t=1}^{N} \min[(r_{p,t} - r_T), 0]^2} \, .$$

Where: $\mathrm{Var}_{D,p}(r_T)$ = Downside variance of the returns of a portfolio below a threshold return,

$\sigma_{D,p}(r_T)$ = Downside standard deviation of the returns of a portfolio below a threshold return,

r_T = Threshold return.

Exhibit 1-19: Downside standard deviation

Following Exhibit 1-18 and considering the monthly continuously compounded returns for an investment portfolio and its benchmark shown in Table 1-11, the annualized downside standard deviation for a threshold return of 0.00% is 1.11% for the investment portfolio and also 1.11% for its benchmark. For the calculation, we have only used the returns below the threshold return of 0.00%.

1.2.3.2.6 Covariance and correlation

Covariance and correlation are descriptive statistics that measure the association between returns of different investment portfolios. As relative risk measures, covariance and correlation measure the co-variability of returns from different investment portfolios and how closely their periodic returns move together.

Covariance measures the direction and degree of the association of the periodic returns of two investment portfolios as well as the magnitude of the variability of their returns.[31] Using continuously compounded returns and assuming that the data represents a sample of the population, the historical covariance of the periodic returns of an investment portfolios and its benchmark is calculated as follows:[32]

$$\mathrm{Cov}(r_{p,t}, r_{b,t}) = \frac{1}{N-1} \times \sum_{t=1}^{N} [(r_{p,t} - \bar{r}_p) \times (r_{b,t} - \bar{r}_b)].$$

With: $\bar{r}_b = \frac{1}{N} \times \sum_{t=1}^{N} r_{b,t}$.

Where: $\mathrm{Cov}(r_{p,t}, r_{b,t})$ = Covariance of the returns of a portfolio and the returns of its benchmark,

$r_{b,t}$ = Return of a benchmark for a single period t,

$\bar{r}_b$ = Mean benchmark return.

31 Here and in the following, an investment portfolio is compared to its benchmark.

32 Like variance, covariance is also proportional to time and the same adjustments as mentioned in **chapter 1.2.3.2.1** can be applied to produce annualized figures.

Positive covariance means that the returns of an investment portfolio and its benchmark move in a similar direction, i.e. higher (lower) returns of the investment portfolio mainly correspond with the higher (lower) returns of its benchmark. By contrast, for negative covariance, the returns of an investment portfolio and its benchmark tend to move in opposite directions, i.e. higher (lower) returns of the investment portfolio mainly correspond with the lower (higher) returns of the benchmark. The sign of the covariance therefore shows the tendency in the linear relationship between the returns of an investment portfolio and its benchmark. Except for a near zero covariance, which indicates no relationship between the returns, the magnitude of the covariance is not easy to interpret which is why the normalized version of the covariance, the correlation coefficient, is often used.

The correlation coefficient or, more simply, correlation, measures the direction and degree of the association of the periodic returns of two investment portfolios. Using continuously compounded returns and assuming that the data represents a sample of the population, the historical correlation of the periodic returns of two investment portfolios is calculated as follows:[33]

$$\mathrm{Corr}\left(r_{p,t}, r_{b,t}\right) = \frac{\mathrm{Cov}\left(r_{p,t}, r_{b,t}\right)}{\sigma_p \times \sigma_b}.$$

Where: $\mathrm{Corr}(r_{p,t}, r_{b,t})$ = Correlation of the returns of a portfolio and the returns of its benchmark,

σ_b = Standard deviation of the returns of a benchmark.

Correlation indicates the degree of linear dependence between the returns and shows by its magnitude the strength of the linear relation. It takes values between -1, i.e. perfect decreasing linear relationship (anti-correlated), and +1, i.e. perfect direct linear relationship (correlated). As correlation approaches zero there is less of a linear relationship, with a zero correlation indicating independent returns (uncorrelated).

Exhibit 1-20: Covariance and correlation

Following Exhibit 1-19 and considering the monthly continuously compounded returns for an investment portfolio and its benchmark shown in Table 1-11, the annualized covariance between the returns of the investment portfolio and its benchmark is +0.000626 and the correlation is +0.9970. In contrast to covariance, correlation is easier to interpret and here indicates nearly perfect correlation.[34]

1.2.3.2.7 Tracking error variance and tracking error standard deviation

In general, tracking risk or tracking error is the risk of deviation from a reference. Tracking error is measured using statistical measures that describe the deviation from a reference like the difference between the cumulated returns or the correlation of the periodic returns of two investment portfolios.

In practice, tracking error is often measured by the tracking error variance or the tracking error standard deviation. Both are descriptive statistics that measure the variability of excess returns of different investment portfolios, normally the excess returns of an investment portfolio against its benchmark. In other words, tracking error variance is the variance of the

33 Please note that correlation is independent of the used periodicity of the returns.

34 As shown by the return series used, perfect correlation does not indicate that the return series are identical.

portfolio's excess (sometimes also called active) returns while tracking error standard deviation is the standard deviation of the portfolio's excess returns.

As an ex post risk measure, the tracking error variance (tracking error standard deviation) is the second central moment about the mean excess return and measures the dispersion of the excess returns or the average squared deviation of the excess returns from the mean excess return. Interpreted as a risk measure and everything else being equal, a high dispersion of excess returns reflects high relative or active investment risk for a risk averse investor. Using continuously compounded returns and assuming that the data represents a sample of the population, the historical tracking error variance (tracking error standard deviation) of periodic excess returns is calculated as follows:[35]

$$\mathrm{TEVar}(r_{p,t}, r_{b,t}) = \frac{1}{N-1} \times \sum_{t=1}^{N} \left[(r_{p,t} - r_{b,t}) - (\bar{r}_p - \bar{r}_b)\right]^2 \text{ , and}$$

$$\mathrm{TESD}(r_{p,t}, r_{b,t}) = \sqrt{\frac{1}{N-1} \times \sum_{t=1}^{N} \left[(r_{p,t} - r_{b,t}) - (\bar{r}_p - \bar{r}_b)\right]^2}.$$

Where: $\mathrm{TEVar}(r_{p,t}, r_{b,t})$ = Tracking error variance of the excess returns of a portfolio against its benchmark,

$\mathrm{TESD}(r_{p,t}, r_{b,t})$ = Tracking error standard deviation of the excess returns of a portfolio against its benchmark.

Exhibit 1-21: Tracking error variance and tracking error standard deviation

Following Exhibit 1-20 and considering the monthly continuously compounded returns for an investment portfolio and its benchmark shown in Table 1-11, the annualized tracking error variance (tracking error standard deviation) of the excess returns of the investment portfolio against its benchmark is +0.000026 (0.51%).

1.2.3.3 Performance measures

1.2.3.3.1 Sharpe ratio

The Sharpe ratio, also called the reward-to-variability ratio, is based on the CAPM and is an absolute risk-adjusted return measure. It is the ratio of the excess return of an investment portfolio over the risk free rate compared to the standard deviation of the returns of the investment portfolio, i.e. the total risk of the portfolio. The risk free rate is subtracted from the return of the investment portfolio and reflects the compensation that should be earned over the risk free rate for bearing additional investment risk compared to the risk free asset.

The Sharpe ratio can be described as the excess return (over the risk free rate) per unit of total risk taken. Using annualized continuously compounded returns and the annualized standard deviation of the returns, the Sharpe ratio is calculated as follows:

35 Like variance and standard deviation, tracking error variance and tracking error standard deviation are also proportional to time and the same adjustments as mentioned in **chapter 1.2.3.2.1** can be applied to produce annualized figures.

$$RVAR_p = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p}.$$

Where: $RVAR_p$ = Sharpe ratio of a portfolio,
$\bar{r}_f$ = Mean risk free rate.

Exhibit 1-22: Sharpe ratio

For illustrative purposes in **chapter 1.2.3.3,** we consider and compare different investment alternatives: a risk free investment, four investment portfolios (A, B, C and D), and the respective benchmark.[36] Table 1-12 contains the monthly continuously compounded returns needed for the calculation of the different performance measures discussed.

Month	Risk free RF	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
1	0.17%	0.25%	0.38%	0.23%	0.25%	0.19%
2	0.17%	0.75%	1.13%	0.68%	0.75%	0.56%
3	0.17%	0.25%	0.38%	0.23%	0.25%	0.19%
4	0.17%	-0.25%	-0.13%	-0.31%	-0.06%	-0.25%
5	0.17%	0.75%	1.13%	0.68%	0.75%	0.56%
6	0.17%	0.75%	1.13%	0.68%	0.75%	0.56%
7	0.17%	0.25%	0.38%	0.23%	0.25%	0.19%
8	0.17%	1.75%	2.63%	1.58%	1.75%	1.31%
9	0.17%	-0.75%	-0.38%	-0.94%	-0.19%	-0.75%
10	0.17%	-0.50%	-0.25%	-0.63%	-0.13%	-0.50%
11	0.17%	-1.00%	-0.50%	-1.25%	-0.25%	-1.00%
12	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%
13	0.17%	1.75%	2.63%	1.58%	1.75%	1.31%
14	0.17%	0.75%	1.13%	0.68%	0.75%	0.56%

Table 1-12: Monthly returns used for calculation of performance measures

Using the returns shown in Table 1-12, Table 1-13 contains the annualized returns and standard deviation of the investment alternatives and the respective Sharpe ratios. If total risk is the only relevant criteria for performance evaluation, then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	+2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
Annualized standard deviation	0.00%	2.75%	3.35%	2.78%	2.18%	2.28%
Sharpe ratio	n/a	+0.74	**+1.85**	+0.31	+1.67	+0.21

Table 1-13: Sharpe ratio

Using the data of Table 1-13, Figure 1-16 shows the different investment alternatives in a return / risk diagram and illustrates that the Sharpe ratio corresponds to the slope of a line between the risk free investment and the respective investment portfolio. The greater the Sharpe ratio, the steeper the slope of the line and the more excess return is earned per unit of total risk.[37] As an absolute risk-adjusted return measure, the Sharpe ratio considers the total risk of investment portfolios and is therefore appropriate to use if the portfolios are compared as being total investment portfolios not considered to be merged with other investments.

36 Here and in the following, we use a benchmark as a proxy for the market portfolio. This follows industry practice when evaluating the performance of investment portfolios managed against a specific benchmark.

37 Please note that for negative values the Sharpe ratio is difficult to interpret because in such cases – everything else being equal – higher risk leads to a higher Sharpe ratio.

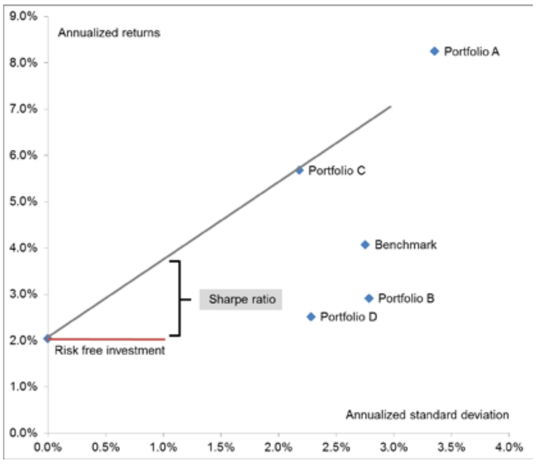


Figure 1-16: Sharpe ratio

1.2.3.3.2 Treynor ratio

The Treynor ratio, also called the reward-to-volatility ratio, is based on the CAPM and is an absolute risk-adjusted return measure. It measures the ratio of the excess return of an investment portfolio over the risk free rate and the beta of the investment portfolio. The risk free rate is subtracted from the return of the investment portfolio because of the belief that the return over risk free rate should be the compensation earned for bearing additional investment risk. The beta, which reflects systematic risk, is used instead of the total risk of the investment portfolio on the basis that the specific risk will be diversified away when using the investment portfolio in combination with other investment portfolios or investments. The Treynor ratio is thus appropriate for well-diversified portfolios where the relevant risk is the systematic risk.

The Treynor ratio can be described as the excess return (over the risk free rate) per unit of market or systematic risk taken. Using annualized continuously compounded returns and the beta of the investment portfolio in comparison to the market portfolio, in practice often the benchmark, the Treynor ratio is calculated as follows:

$$\mathrm{RVOL_p} = \frac{\bar{r}_p - \bar{r}_f}{\beta_p}.$$

With: $$\beta_p = \frac{\mathrm{Cov}(r_p, r_m)}{\mathrm{Var_m}}.$$

Where: $\mathrm{RVOL_p}$ = Treynor ratio of a portfolio,
β_p = Beta or systematic risk of a portfolio,
r_m = Return of the market portfolio,
$\mathrm{Var_m}$ = Variance of the returns of the market portfolio.

Exhibit 1-23: Treynor ratio

Using the returns shown in Table 1-12, Table 1-14 contains the annualized returns, the beta of the investment alternatives in comparison to the benchmark, used here as a proxy for the market portfolio, and the respective Treynor ratios. If market risk or beta is the only relevant criteria for performance evaluation then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	+2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
Beta	n/a	1.00	1.19	1.01	0.77	0.83
Treynor ratio	n/a	+2.03%	**+5.21%**	+0.87%	+4.73%	+0.58%

Table 1-14: Treynor ratio

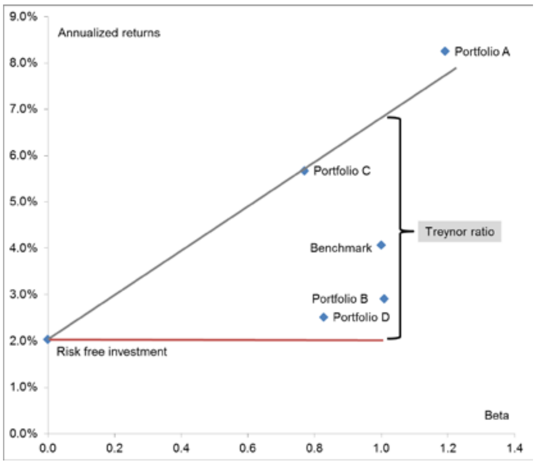


Figure 1-17: Treynor ratio

Using the data of Table 1-14, Figure 1-17 shows the different investment alternatives in a return / risk diagram and illustrates that the Treynor ratio corresponds to the slope of a line between the risk free investment and the respective investment portfolio. The greater the Treynor ratio, the steeper the slope of the line and the more excess return is earned per unit of beta or market risk.[38] As an absolute risk-adjusted return measure, the Treynor ratio considers the market risk of investment portfolios and is therefore appropriate to use if market risk of investment portfolios is compared.[39] Evaluating investment portfolios using the beta implicitly assumes that the specific risk of the individual portfolios is not relevant because it has been diversified away with the inclusion of the individual portfolio in a bigger, well-diversified investment portfolio.

1.2.3.3.3 Jensen's alpha

Jensen's alpha is based on the CAPM and is an absolute risk-adjusted return measure. It measures the excess return of an investment portfolio over the risk free rate versus the portfolio risk-adjusted excess return of the market portfolio over the risk free rate. Jensen's alpha is a measure to identify security selection abilities by decomposing the excess return into the contribution due to selectivity and the one due to taking market risk.

Jensen's alpha can be described as the market-risk-adjusted excess return (over the risk free rate). The risk free rate is subtracted from the return of the investment and market portfolio

38 Please note that for negative values, the Treynor ratio is difficult to interpret because in such cases – everything else being equal – higher beta leads to a higher Treynor ratio.

39 A meaningful comparison is only possible if the alternative investment portfolios are managed against the same benchmark.

because of the belief that the return over risk free rate should be the compensation earned for bearing additional investment risk. The beta is used instead of the total risk of the investment portfolio to identify security selection abilities. Using annualized continuously compounded returns and the beta of the investment portfolio in comparison to the market portfolio, in practice often to the benchmark, Jensen's alpha is calculated as follows:

$$\alpha_p = (\bar{r}_p - \bar{r}_f) - \beta_p \times (\bar{r}_m - \bar{r}_f) \quad \text{or} \quad (\bar{r}_p - \bar{r}_f) = \alpha_p + \beta_p \times (\bar{r}_m - \bar{r}_f) \,.$$

Where: α_p = Jensen's alpha of a portfolio,
$\bar{r}_m$ = Mean return of market portfolio.

Exhibit 1-24: Jensen's alpha

Using the returns shown in Table 1-12, Table 1-15 contains the annualized returns, the beta of the investment alternatives in comparison to the benchmark, used here as a proxy for the market portfolio, and the respective Jensen's alphas. If market-risk-adjusted excess return is the only relevant criteria for performance evaluation then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	+2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
Beta	n/a	1.00	1.19	1.01	0.77	0.83
Jensen's alpha	n/a	0.00%	**+3.79%**	-1.17%	+2.08%	-1.20%

Table 1-15: Jensen's alpha

Using the data of Table 1-15, Figure 1-18 shows the different investment alternatives in a return / risk diagram and illustrates that Jensen's alpha corresponds at the portfolio beta level to the difference between the line from the risk free investment through the respective investment portfolio and the one from the risk free investment through the market (benchmark) portfolio. The greater Jensen's alpha, the steeper the line in comparison to the market line and the more excess return is earned per unit of beta or market risk. As an absolute risk-adjusted return measure, Jensen's alpha considers the market-risk-adjusted excess return of investment portfolios and is therefore appropriate to use if one is comparing security selection abilities of investment portfolios.[40]

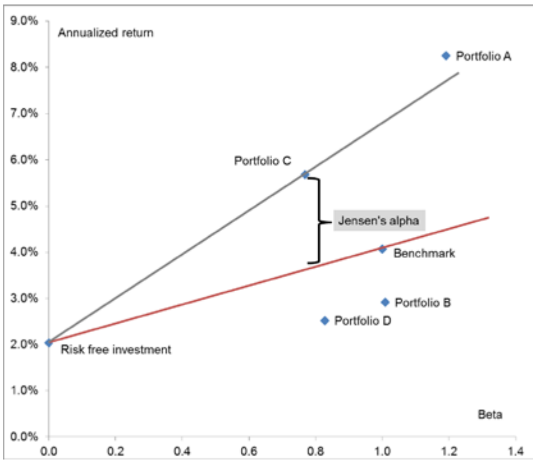


Figure 1-18: Jensen's alpha

40 A meaningful comparison is only possible for alternative investment portfolios with similar beta.

1.2.3.3.4 Appraisal ratio

The appraisal ratio, also called the Treynor/Black ratio, is CAPM based and is an absolute risk-adjusted return measure. It measures the ratio of Jensen's alpha (excess return adjusted for market risk) and the specific risk of an investment portfolio.

The appraisal ratio considers the level of specific risk and can be described as the excess return due to selectivity or the systematic risk-adjusted excess return per unit of specific risk taken. Using annualized continuously compounded returns and the Jensen's alpha of the investment portfolio in comparison to the market portfolio, in practice often to the benchmark, the appraisal ratio is calculated as follows:

$$AR_p = \frac{\alpha_p}{\sigma_\varepsilon}.$$

With: $\varepsilon_t = r_{p,t} - r_{m,t} \times \beta_p - \alpha_p$.

Where: AR_p = Appraisal ratio of a portfolio,

σ_ε = Annualized specific risk of a portfolio in comparison to its benchmark,

ε_t = Regression residual or error term of a portfolio for a single period t.

Exhibit 1-25: Appraisal ratio

Using the returns shown in Table 1-12, Table 1-16 contains the Jensen's alpha, the annualized specific risk of the investment alternatives in comparison to the benchmark, as a proxy for the market portfolio, and the respective appraisal ratios. If the specific risk is the only relevant criteria for performance evaluation then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Jensen's alpha	n/a	0.00%	+3.79%	-1.17%	+2.08%	-1.20%
Annualized specific risk	n/a	0.00%	1.05%	0.25%	0.64%	0.42%
Appraisal ratio	n/a	0.00	**+3.60**	-4.67	+3.26	-2.86

Table 1-16: Appraisal ratio

Using the data of Table 1-16, Figure 1-19 shows the different investment alternatives in a return / risk diagram and illustrates that the appraisal ratio corresponds to the slope of a line between the benchmark and the respective investment portfolio. The greater the appraisal ratio, the steeper the slope of the line and the more (Jensen's) alpha is earned per unit of specific risk.[41] The appraisal ratio penalizes investment portfolios for exposure to diversifiable risk. As an absolute risk-adjusted return measure, the appraisal ratio considers the specific risk of investment portfolios and is therefore appropriate to use if one is comparing security selection abilities of investment portfolios.[42]

41 Please note that for negative values, the appraisal ratio is difficult to interpret because in such cases – everything else being equal – higher risk leads to a higher appraisal ratio.

42 A meaningful comparison is only possible for alternative investment portfolios with the same benchmark and like for the Jensen's alpha with similar beta.

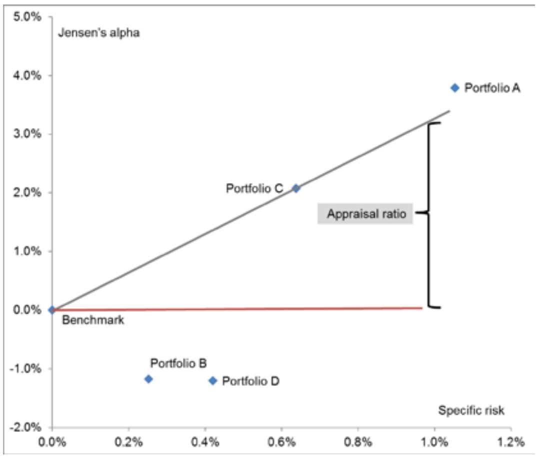


Figure 1-19: Appraisal ratio

1.2.3.3.5 Graham & Harvey 1 and 2

The Graham and Harvey measures 1 and 2 are absolute risk-adjusted return measures and equal the excess return of an investment portfolio versus its portfolio risk-adjusted benchmark (GH1) or the excess return of a benchmark risk-adjusted investment portfolio versus its benchmark (GH2).

The portfolio risk-adjusted benchmark, used for GH1, is a combination of the leveraged or unleveraged benchmark and the risk free investment so that its risk equals the portfolio risk – allowing the risk free investment to have positive standard deviation and non-zero correlations with the risky benchmark. In other words, GH1 adjusts the volatility of the benchmark so that it equals the volatility of the portfolio. The return of the benchmark for that level of volatility is then compared to the return of the portfolio.

By contrast, the benchmark risk-adjusted portfolio, used for GH2, is a combination of the leveraged or unleveraged portfolio and the risk free investment so that its risk equals the benchmark risk, i.e. the volatility of the portfolio is matched to that of the benchmark – allowing the risk free investment to have positive standard deviation and non-zero correlations with the risky portfolio. The GH1 and GH2 measures are calculated as follows:

$$\mathrm{GH1_p} = \bar{r}_p - \bar{r}_b(\sigma_b = \sigma_p) \quad \text{and} \quad \mathrm{GH2_p} = \bar{r}_p(\sigma_p = \sigma_b) - \bar{r}_b.$$

Where: $\mathrm{GH1_p}$ = Graham & Harvey measure 1 of a portfolio,
$\mathrm{GH2_p}$ = Graham & Harvey measure 2 of a portfolio.

Exhibit 1-26: Graham & Harvey 1 and 2

Using the returns shown in Table 1-12, Table 1-17 contains the annualized returns, the portfolio risk-adjusted benchmark return, the benchmark risk-adjusted portfolio return, and the respective Graham & Harvey measures 1 and 2. If the GH1 or GH2 measure is the only relevant criteria for performance evaluation then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
$\bar{r}_b(\sigma_b = \sigma_p)$	n/a	+4.07%	+4.52%	+4.10%	+3.65%	+3.73%
GH1	n/a	0.00%	**+3.73%**	-1.18%	+2.03%	-1.21%
$\bar{r}_p(\sigma_p = \sigma_b)$	n/a	+4.07%	+7.13%	+2.90%	+6.63%	+2.62%
GH2	n/a	0.00%	**+3.06%**	-1.17%	+2.56%	-1.46%

Table 1-17: Graham & Harvey 1 and 2

Using the data of Table 1-17, Figure 1-20 shows the different investment alternatives in a return / risk diagram and illustrates the Graham & Harvey measures 1 and 2 for the investment portfolio C. In addition to the straight lines, there are efficient portfolio curves between the risk free investment and the benchmark or investment portfolios illustrating the case where the risk free investment has non-zero correlations with the returns of the different investment alternatives. The greater the Graham & Harvey measure 1 and 2 the more risk-adjusted excess return is earned in comparison to the benchmark. As absolute risk-adjusted return measures, the Graham & Harvey measures consider the total risk of investment portfolios and is therefore appropriate to use if the portfolios are compared as being total investment portfolios not considered to be merged with other investments.

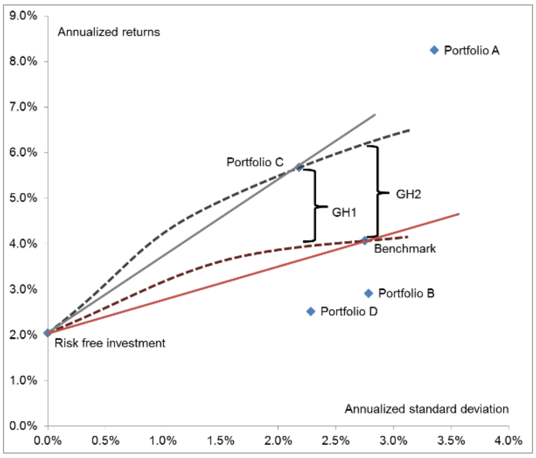


Figure 1-20: Graham & Harvey 1 and 2

1.2.3.3.6 Sortino ratio

The Sortino ratio is a downside risk-adjusted return measure and the ratio of the excess return of an investment portfolio over a certain threshold return compared to the downside standard deviation of the returns of the investment portfolio.

The Sortino ratio can be described as the excess return (over threshold return) per unit of downside risk taken. Using annualized continuously compounded returns and the annualized downside standard deviation of the returns, the Sortino ratio is calculated as follows:

$$SOR_p = \frac{\bar{r}_p - r_T}{\sigma_{D,p}(r_T)}.$$

Where: SOR_p = Sortino ratio of a portfolio.

Exhibit 1-27: Sortino ratio

Using the returns shown in Table 1-12, Table 1-18 contains the annualized returns, the annualized downside standard deviation (for a threshold return of 0.00%) of the investment alternatives and the respective Sortino ratios. If downside standard deviation is the only relevant criteria for performance evaluation then investment portfolio C is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	+2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
Annualized downside standard deviation	0.00%	1.11%	0.55%	1.38%	0.28%	1.11%
Sortino ratio	n/a	+3.68	+14.91	+2.11	**+20.53**	+2.28

Table 1-18: Sortino ratio

Using the data of Table 1-18, Figure 1-21 shows the different investment alternatives in a return / risk diagram and illustrates that the Sortino ratio corresponds to the slope of a line between the threshold return and the respective investment portfolio. The greater the Sortino ratio, the steeper the slope of the line and the more excess return over the threshold return is earned per unit of downside risk.[43] As a downside risk-adjusted return measure, the Sortino ratio considers the downside risk of investment portfolios and is therefore appropriate to use if the portfolios are compared as being total investment portfolios not considered to be merged with other investments.

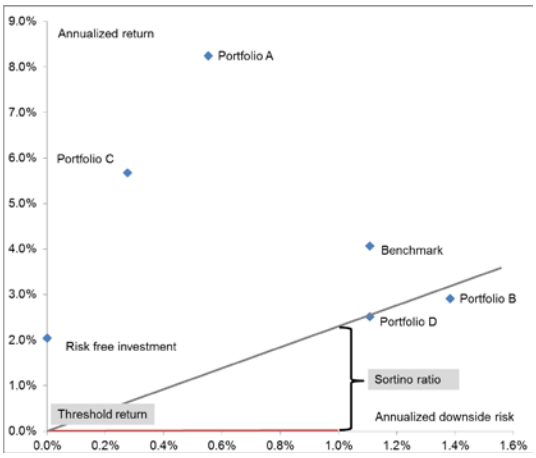


Figure 1-21: Sortino ratio

1.2.3.3.7 Information ratio

The information ratio is a relative risk-adjusted return measure. It is the ratio of the value added of an investment portfolio in comparison to its benchmark and the tracking error standard deviation of the returns of the investment portfolio.

The information ratio can be described as the excess return (over benchmark return) per unit of tracking error risk taken. Using annualized continuously compounded returns and the annualized tracking error standard deviation of the returns, the information ratio is calculated as follows:

43 Please note that for negative values, the Sortino ratio is difficult to interpret because in such cases – everything else being equal – higher risk leads to a higher Sortino ratio.

$$IR_p = \frac{\bar{r}_p - \bar{r}_b}{TESD(r_{p,t}, r_{b,t})}.$$

Where: IR_p = Information ratio of a portfolio.

Exhibit 1-28: Information ratio

Using the returns shown in Table 1-12, Table 1-19 contains the annualized returns, the annualized tracking error standard deviation of the investment alternatives and the respective information ratios. If tracking error standard deviation is the only relevant criteria for performance evaluation then investment portfolio A is the preferred portfolio.

	Risk free investment	Benchmark	Portfolio A	Portfolio B	Portfolio C	Portfolio D
Annualized return	+2.04%	+4.07%	+8.25%	+2.91%	+5.68%	+2.52%
Annualized tracking error standard deviation	n/a	0.00%	0.88%	0.25%	0.83%	0.51%
Information ratio	n/a	0.00	**+4.73**	-4.64	+1.94	-3.06

Table 1-19: Information ratio

Using the data of Table 1-19, Figure 1-22 shows the different investment alternatives in a return / risk diagram and illustrates that the information ratio corresponds to the slope of a line between the benchmark and the respective investment portfolio. The greater the information ratio, the steeper the slope of the line and the more value added over the benchmark return is earned per unit of tracking error risk.[44] The information ratio is expected to be positive if the portfolio manager had "information" available that was not priced into the market. As a relative risk-adjusted return measure, the information ratio considers the tracking error risk of investment portfolios and is therefore appropriate to use if one is comparing the active portfolio management abilities of investment portfolios.[45]

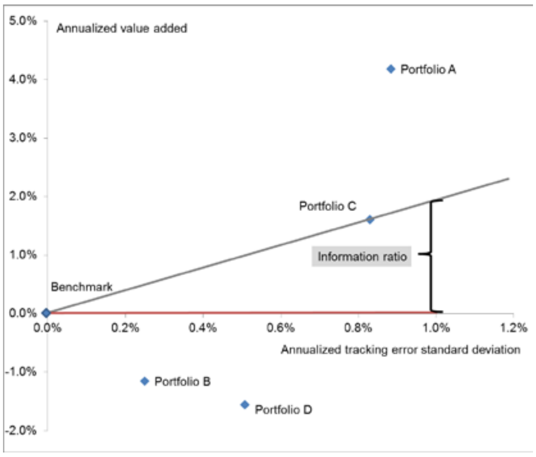


Figure 1-22: Information ratio

44 Please note that for negative values, the information ratio is difficult to interpret because in such cases – everything else being equal – higher risk leads to a higher information ratio.

45 A meaningful comparison is only possible for alternative investment portfolios with similar benchmarks.

1.3 Performance attribution*

1.3.1 Introduction to performance attribution*

*1.3.1.1.1 Definition of performance attribution**

Performance measurement on a total portfolio level measures the performance of an investment portfolio on an aggregated level. If one is interested in knowing where the performance of the portfolio is coming from, one needs to analyze the performance of the individual components of the portfolio. Based on the broad definition of performance, **chapters 1.3.2** and **1.3.3** discuss the fundamentals of return attribution and risk attribution.

In general, performance attribution is defined as the measurement of the sources of the performance of an investment portfolio and its benchmark as well as of the performance added. Applied to portfolio management, performance attribution is the measurement of the historical as well as expected return and risk contributions of the individual steps of the portfolio management process as well as of the applied financial instruments. The broad definition of performance attribution includes different calculations such as portfolio security and segment performance, contribution analysis and the decomposition of benchmark relative performance into management effects.

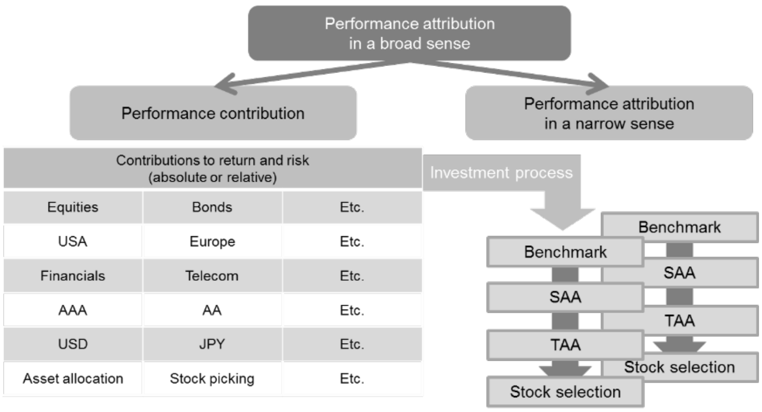


Figure 1-23: Performance contribution versus attribution

As illustrated in Figure 1-23, we distinguish between return and risk contribution as well as between return and risk attribution, whereby performance contribution is a more or less arbitrary breakdown of the performance using a given breakdown of an investment portfolio and performance attribution is a decision-oriented decomposition of the performance.

Performance attribution as a central component of the performance evaluation process is defined as a process that determines the return and risk contributions of the individual decision-making steps within a portfolio management process. Thus, performance attribution is concerned not only with the past but also with the future, and determines which return and risk contributions are due to which decisions (regarding investment category and instruments) and to which decision makers, on an ex post as well as ex ante basis.

Figure 1-24 illustrates the various levels of analysis of performance attribution as well as possible allocation criteria of return and risk contributions. It is evident that performance attribution can be carried out in a variety of different ways. On the one hand, return and risk contributions may be calculated on an absolute basis, i.e. isolated for an investment portfolio or for a specific benchmark, or on a relative basis, i.e. for an investment portfolio in comparison to its benchmark. On the other hand, the performance attribution may be focused on the past (ex post) or the future (ex ante). In summary, performance attribution is defined as the decomposition of the historical or expected absolute or relative return and/or the historical and expected absolute or relative risk.

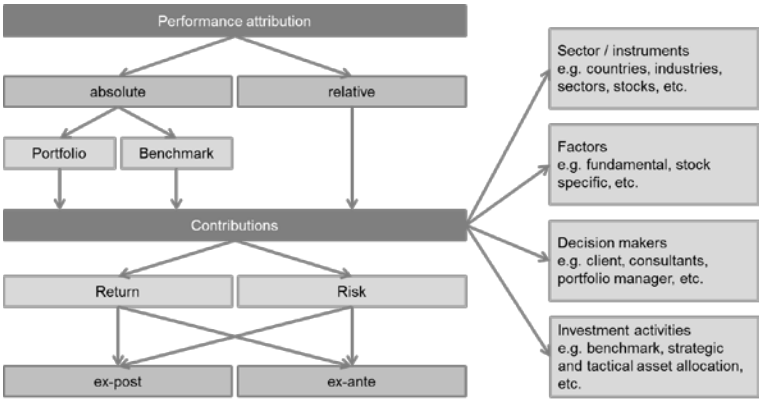


Figure 1-24: Levels of analysis and allocation criteria of performance attribution

As shown in Figure 1-24, in general, performance attribution can be used to measure the return and risk contributions of:

- Sectors and instruments (e.g. asset categories, countries, currencies or securities),
- Factors (e.g. fundamental and stock specific characteristics like P/E ratio or dividend yield),
- Decision makers (e.g. client, portfolio manager or consultant), and finally
- Investment activities (e.g. definition of the benchmark, definition of the strategic or tactical asset allocation, or the security selection).

*1.3.1.1.2 Types of performance attribution**

Following Figure 1-24 and considering the two dimensions of investment performance, we distinguish between return and risk attribution.[46]

Return attribution is the measurement and quantification of the historical as well as expected return contributions of the individual steps of the portfolio management process as well as of the applied financial instruments. We distinguish between return contribution and attribution, whereby **return contribution** is a more or less arbitrary breakdown of the return using a given breakdown of the investment portfolio and, by contrast, return attribution is a decision-oriented decomposition of the return.

46 Similar to what was done in **chapter 1.2.3** performance could be analyzed in a risk-adjusted way. Therefore, one could also apply performance attribution to risk-adjusted returns which would offer additional information useful for performance evaluation. Although it has interpretive value, in practice, risk-adjusted performance attribution is not often used. In **chapter 1.3**, we concentrate on return and risk attribution.

Figure 1-25 gives an overview of the different ways of running a return attribution. We distinguish between single factor or algebraic-based return attribution and multi-factor or regression-based return attribution, whereby the former approach is mainly used for equity and multi-asset class portfolios and the latter is used mainly for fixed income portfolios.[47]

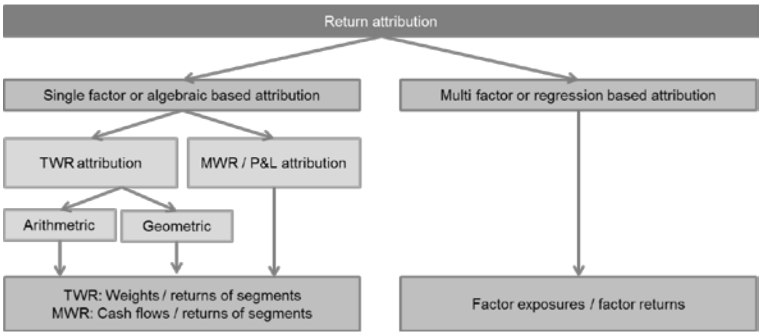


Figure 1-25: Overview of return attribution

The single factor or algebraic-based return attribution describes the return and the value added using a single factor, like a sector or asset class return. By contrast, a multi-factor or regression-based return attribution describes the return and the value added as a function of different factors, for instance sector, currency, country or stock specific fundamentals.

Furthermore, we distinguish between TWR and MWR / P&L attribution where the TWR attribution explains and decomposes the TWR excess return as the difference between the TWR of the portfolio and its benchmark, and by contrast, MWR attribution explains and decomposes the MWR excess return as the difference between the MWR of the portfolio and its benchmark. P&L attribution is similar to the MWR attribution but decomposes absolute instead of percentage numbers. In addition, we distinguish between arithmetic and geometric TWR attribution. The former explains the value added as absolute profit expressed as a percentage of the initial amount invested or as an absolute return difference and the latter explains the value added as absolute profit expressed as a percentage of the final value of the benchmark or as a relative return difference. Furthermore, Figure 1-25 shows what data is needed to produce a return attribution.

Risk attribution is the measurement and quantification of the historical as well as expected risk contributions of the individual steps of the investment process as well as of the applied financial instruments. We distinguish between risk contribution and attribution, whereby **risk contribution** is a more or less arbitrary breakdown of the risk using a given breakdown of the investment portfolio, and, by contrast, risk attribution is a decision-oriented decomposition of the risk.

47 Single factor or algebraic-based return attribution is mainly used for equity and multi-asset class portfolios because here the explanatory accuracy of multi-factor or regression-based return attribution methods is limited, the investment process is more hierarchical and the individual steps of the investment process are more independent from each other. Multi-factor or regression-based return attribution is mainly used for fixed income portfolios because of the high explanatory accuracy and because the dependencies between the different factors may be very relevant for the findings of the performance analysis.

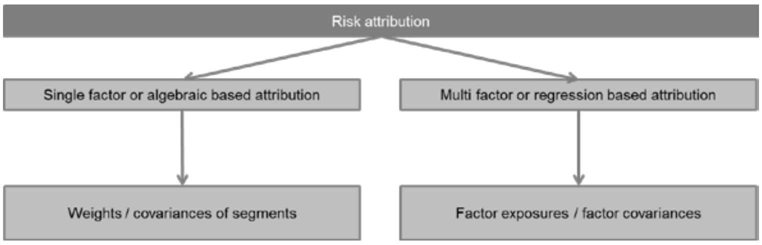


Figure 1-26: Overview of risk attribution

Figure 1-26 gives an overview of the different types of risk attribution. We distinguish between single factor or algebraic-based risk attribution and multi-factor or regression-based risk attribution, whereby the former approach is mainly used for multi-asset class portfolios and the latter is used mainly for equity and fixed income portfolios. Furthermore, Figure 1-26 shows what data is needed to produce a risk attribution. Following the types of risk measures discussed in **chapter 1.2.3.2**, risk attribution may analyze different types of risk measures, whereby the intended analysis will determine the choice of the relevant risk measure to decompose.

1.3.2 Return attribution*

1.3.2.1 Introduction to return attribution*

*1.3.2.1.1 Single period contribution to return measurement**

Contribution measurement provides information on how the weights and the returns of portfolio components contribute to the return of the investment portfolio. The return of an investment portfolio is the sum of the return contributions of the individual portfolio components, such as individual investments or any aggregation of those. Assuming simple returns, the return of an investment portfolio for a single period is calculated as follows:[48]

$$R_{p,t}^{BC} = \sum_{i=1}^{N} CR_{p,i,t}^{BC} = \sum_{i=1}^{N} w_{p,i,t} \times R_{p,i,t}^{BC} \quad \text{with} \sum_{i=1}^{N} w_{p,i,t} = 1 \,.$$

Where:

$CR_{p,i,t}^{BC}$ = Contribution to return for a portfolio component i to the return of a portfolio for period t in base currency,

$w_{p,i,t}$ = Weight for a portfolio component i at the beginning of period t,[49]

$R_{p,i,t}^{BC}$ = Return for a portfolio component i for period t in base currency.

A similar formula can be applied to a benchmark to calculate the return contributions of the individual benchmark components for a single period:

48 In the following section, we analyze and decompose returns in base currency. Currency effects are discussed in **chapter 1.3.2.2.3**.

49 We use the weight of the portfolio components at the beginning of the period so as not to overstate the return contribution of portfolio components with a higher than average return and not to understate the return contribution of portfolio components with a lower than average return. This is because the weight of the portfolio component at the end of the period is impacted by the return earned by that portfolio component.

$$R_{b,t}^{BC} = \sum_{i=1}^{N} CR_{b,i,t}^{BC} = \sum_{i=1}^{N} w_{b,i,t} \times R_{b,i,t}^{BC} \quad \text{with} \quad \sum_{i=1}^{N} w_{b,i,t} = 1 \, .$$

Where: $CR_{b,i,t}^{BC}$ = Contribution to return for a benchmark component i to the return of a benchmark for period t in base currency,

$w_{b,i,t}$ = Weight for a benchmark component i at the beginning of period t,

$R_{b,i,t}^{BC}$ = Return for a benchmark component i for period t in base currency.

Exhibit 1-29: Single period contribution to return

Let us consider an investment portfolio and its benchmark invested in three domestic asset classes. Table 1-20 contains the different return contributions as well as the data used.

Asset class	Portfolio			Benchmark		
	Weight	Return	Contribution	Weight	Return	Contribution
Cash EUR	10.00%	0.00%	0.00%	10.00%	0.00%	0.00%
Bonds EUR	80.00%	1.00%	0.80%	70.00%	0.50%	0.35%
Equities EUR	10.00%	5.00%	0.50%	20.00%	6.00%	1.20%
Total	**100.00%**	**1.30%**	**1.30%**	**100.00%**	**1.55%**	**1.55%**

Table 1-20: Single period contribution to return

*1.3.2.1.2 Multi-period contribution to return measurement**

In the previous chapter, we discussed the measurement of the return contributions over a single period. Normally, we analyze returns over multiple (sub-) periods. Considering the discussion in **chapter 1.2.1.1.8** and that here we use simple returns, we need to consider compounding effects to calculate multi-period return contributions.

To extend the return contribution measurement to multiple periods, we need to compound the return contributions using a linking algorithm. In the following section, we use a simple linking method whereby we take the prior cumulative return contribution of a portfolio component, compound it by the return of the investment portfolio of the current period, and finally add the return contribution of the portfolio component of the current period.[50] To take a simple example, if we have two periods (period 1 and period 2), we would calculate the contribution to return in Period 1 for each component in the portfolio. We would then multiply (compound) each component's contribution in period 1 by the return of the total portfolio in the subsequent period (period 2). Finally, we would add to this figure the contribution to return from each component in the portfolio for that same subsequent period (i.e. period 2).

Assuming simple returns, the return of an investment portfolio for a multi-period is calculated as follows:

$$RM_{P}^{BC} = \sum_{i=1}^{N} CRM_{p,i}^{BC} = \sum_{i=1}^{N} CRM_{p,i,t-1}^{BC} \times \left(1 + R_{p,t}^{BC}\right) + CR_{p,i,t}^{BC} \, .$$

Where: RM_{P}^{BC} = Cumulative return for a portfolio for the entire measurement period in base currency,

$CRM_{p,i}^{BC}$ = Cumulative contribution to return for a portfolio component i to the cumulative return of a portfolio for the entire measurement period in base currency,

50 See "Investment Performance Measurement" (2003) by Bruce J. Feibel.

$CRM^{BC}_{p,i,t-1}$ = Cumulative contribution to return for a portfolio component i to the cumulative return of a portfolio from the beginning of the measurement period until beginning of period t in base currency.

A similar formula can be applied to a benchmark to calculate the return contributions of the individual benchmark components for a multi-period:

$$RM^{BC}_{B} = \sum_{i=1}^{N} CRM^{BC}_{b,i} = \sum_{i=1}^{N} CRM^{BC}_{b,i,t-1} \times \left(1 + R^{BC}_{b,t}\right) + CR^{BC}_{b,i,t}\,.$$

Where: RM^{BC}_{B} = Cumulative return for a benchmark for the entire measurement period in base currency,

$CRM^{BC}_{b,i}$ = Cumulative contribution to return for a benchmark component i to the cumulative return of a benchmark for the entire measurement period in base currency,

$CRM^{BC}_{b,i,t-1}$ = Cumulative contribution to return for a benchmark component i to the cumulative return of a benchmark from the beginning of the measurement period until beginning of period t in base currency.

Exhibit 1-30: Multi-period contribution to return

Considering the single period returns of an investment portfolio and its benchmark shown in Table 1-20 and the respective returns for a second period in Table 1-21, Table 1-22 contains the different return contributions. Note that the returns and the weights are the same in both periods. As an example, let us calculate the cumulative contribution of bonds EUR to the cumulative return of the investment portfolio. We thus take the contribution to return in the first period (+0.80%) and compound it with the return of the total portfolio in the second period (+1.30%). Finally, we add the contribution to return in the second period (+0.80%).The cumulative contribution of bonds EUR to the cumulative return of the investment portfolio is thus +1.61% (rounded), which equals 0.80% * (1 + 1.30%) + 0.80%.

Asset class	Portfolio			Benchmark		
	Weight	Return	Contribution	Weight	Return	Contribution
Cash EUR	10.00%	0.00%	0.00%	10.00%	0.00%	0.00%
Bonds EUR	80.00%	1.00%	0.80%	70.00%	0.50%	0.35%
Equities EUR	10.00%	5.00%	0.50%	20.00%	6.00%	1.20%
Total	**100.00%**	**1.30%**	**1.30%**	**100.00%**	**1.55%**	**1.55%**

Table 1-21: Single period contribution to return for second period

Asset class	Portfolio		Benchmark	
	Return	Contribution	Return	Contribution
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	2.01%	1.61%	1.00%	0.71%
Equities EUR	10.25%	1.01%	12.36%	2.42%
Total	**2.62%**	**2.62%**	**3.12%**	**3.12%**

Table 1-22: Multi-period contribution to return

*1.3.2.1.3 Contribution to value added measurement**

The value added of an investment portfolio is the sum of the value added of the portfolio components, such as individual investments or any aggregation of those. Assuming simple returns, the arithmetic value added of an investment portfolio against its benchmark for a single period is calculated as follows:

$$VA_{p,t}^{BC} = \sum_{i=1}^{N} CVA_{i,t}^{BC} = \sum_{i=1}^{N} CR_{p,i,t}^{BC} - \sum_{i=1}^{N} CR_{b,i,t}^{BC} = \sum_{i=1}^{N} \left(CR_{p,i,t}^{BC} - CR_{b,i,t}^{BC}\right).$$

Where: $VA_{p,t}^{BC}$ = Value added of a portfolio for period t in base currency,

$CVA_{i,t}^{BC}$ = Contribution to value added for a portfolio component i to the value added of a portfolio for period t in base currency.

The multi-period value added is calculated as the sum of the compounded single period contributions to value added, whereby the single period contributions to value added are compounded by the cumulative return of the benchmark for the subsequent periods as well as by the prior cumulative return of the investment portfolio:[51]

$$VAM_{p}^{BC} = \sum_{i=1}^{N} CVAM_{i}^{BC} = \sum_{i=1}^{N} \sum_{t=1}^{T} CVA_{i,t}^{BC} \times \left(1 + R_{b,T-t}^{BC}\right) \times \left(1 + RM_{p,t-1}^{BC}\right).$$

Where: VAM_{p}^{BC} = Cumulative value added of a portfolio for the entire measurement period in base currency,

$CVAM_{i}^{BC}$ = Cumulative contribution to value added for a portfolio component i to the cumulative value added of a portfolio for the entire measurement period in base currency.

$R_{b,T-t}^{BC}$ = Cumulative return for a benchmark from the end of the current period t until the end of the measurement period in base currency,

$RM_{p,t-1}^{BC}$ = Cumulative return for a portfolio from the beginning of the measurement period until beginning of period t in base currency.

Exhibit 1-31: Multi-period contribution to value added

Considering the single period returns of an investment portfolio and its benchmark shown in Table 1-20 and in Table 1-21, Table 1-23 and Table 1-24 contain in addition the value added per portfolio component for the respective period. In contrast to Table 1-22, Table 1-25 contains the multi-period value added of the different portfolio components. As an example, let us calculate the cumulative contribution of bonds EUR to the cumulative value added of the investment portfolio. For each period (in our case, period 1 and period 2) we calculate the compounded single period contributions to value added. These consist of the single period contribution to value added multiplied (or compounded) by the total return of **the benchmark** for the **following** periods as well as by the total return of **the investment portfolio** for the **prior** periods.

In our example, for period 1 the contribution to value added of the bonds EUR is +0.45% and the total return of the benchmark for the following period is +1.55%. Since there is no prior period, we cannot compound by the total return of the investment portfolio for the prior period. For period 2, the contribution to value added of the bonds EUR is again +0.45%. Here, we have a prior period (period 1), so we can compound by the total return of the investment portfolio for the prior period, i.e. +1.30%, but we have no subsequent period, so cannot compound by the total return of the benchmark for the following period. The cumulative value added of bonds EUR is thus +0.91% (rounded), which equals 0.45% * (1 + 1.55%) + 0.45% * (1 + 1.30%).

51 See "Investment Performance Measurement" (2003) by Bruce J. Feibel.

Asset class	Portfolio			Benchmark			Contribution to value added
	Weight	Return	Contribution	Weight	Return	Contribution	
Cash EUR	10.00%	0.00%	0.00%	10.00%	0.00%	0.00%	0.00%
Bonds EUR	80.00%	1.00%	0.80%	70.00%	0.50%	0.35%	0.45%
Equities EUR	10.00%	5.00%	0.50%	20.00%	6.00%	1.20%	-0.70%
Total	**100.00%**	**1.30%**	**1.30%**	**100.00%**	**1.55%**	**1.55%**	**-0.25%**

Table 1-23: Single period contribution to value added for first period

Asset class	Portfolio			Benchmark			Contribution to value added
	Weight	Return	Contribution	Weight	Return	Contribution	
Cash EUR	10.00%	0.00%	0.00%	10.00%	0.00%	0.00%	0.00%
Bonds EUR	80.00%	1.00%	0.80%	70.00%	0.50%	0.35%	0.45%
Equities EUR	10.00%	5.00%	0.50%	20.00%	6.00%	1.20%	-0.70%
Total	**100.00%**	**1.30%**	**1.30%**	**100.00%**	**1.55%**	**1.55%**	**-0.25%**

Table 1-24: Single period contribution to value added for second period

Asset class	Portfolio		Benchmark		Contribution to value added
	Return	Contribution	Return	Contribution	
Cash EUR	0.00%	0.00%	0.00%	0.00%	0.00%
Bonds EUR	2.01%	1.61%	1.00%	0.71%	0.91%
Equities EUR	10.25%	1.01%	12.36%	2.42%	-1.42%
Total	**2.62%**	**2.62%**	**3.12%**	**3.12%**	**-0.51%**

Table 1-25: Multi-period contribution to value added[52]

1.3.2.2 Single factor or algebraic-based return attribution*

*1.3.2.2.1 Single period return attribution**

After calculating the returns and the contributions to return as well as to the respective value added, we are also interested in getting some insight on what the sources of the value added are. As a rule, for effective performance evaluation the decomposition of the return and the value added should follow the portfolio management process. This is an important condition if one wants to produce valid feedback into the portfolio management process. In practice, it is common to assume a three step decision-making process:[53]

- **Step 1 – benchmark selection**. This is the decision to invest the initial money into a specific benchmark or investment strategy. Benchmark selection encompasses decisions on the benchmark or the customized benchmark relevant to measuring the quality or the value added of the portfolio management process. In practice, customized benchmarks are often used, whereby benchmark selection determines the weights of the relevant benchmark components and the indices reflecting the different benchmark components.[54]
- **Step 2 – asset allocation**. This is the decision to change the asset allocation of an investment portfolio relative to the benchmark during the measurement period. Asset allocation encompasses decisions on the portfolio structure, as implemented by the

52 Please note that the multi-period contribution to value added on a portfolio component level does not equal the difference of the contributions to return between the respective component of the portfolio and the benchmark. The difference is due to the underlying compounding assumptions of the linking method used.

53 Please note that the actual portfolio management process may be more complex which makes respective adjustments necessary.

54 A benchmark component is a group of securities or investments. We can split the benchmark into different components of interest. The components have some factor in common and this factor is of interest when evaluating the performance of a benchmark or an investment portfolio. These factors are manifold and reach from country, currency, or sector to duration or P/E ratio.

portfolio manager, versus the benchmark structure. In practice, these decisions form the basis of measuring the value added of the portfolio manager due to over- and underweighting portfolio components versus the respective passive weights of the benchmark.

- **Step 3 – security selection**. This is the decision to select and weight securities of an investment portfolio relative to the benchmark during the measurement period. Security selection encompasses decisions on the portfolio structure within portfolio components, as implemented by the portfolio manager, versus the benchmark structure of the respective benchmark component. In practice, these decisions form the basis of measuring the value added of the portfolio manager due to over- and underweighting individual securities versus the respective passive weights of these securities within components of the benchmark.

The return attribution determines the return contributions to the value added due to specific management decisions, the so-called management effects. Similar to the three steps decision-making process explained above, in practice, we often decompose the value added or the total management effect into three different management effects:

- **Asset allocation effect**: The contribution to value added due to the over- and underweighting of portfolio components versus the benchmark.
- **Security selection effect**: The contribution to value added due to the over- and underweighting of individual securities within portfolio components versus the benchmark.
- **Interaction effect**: The contribution to value added due to the over- and underweighting of out- or underperforming portfolio components.[55]

To calculate the different management effects, different methodologies for single factor or algebraic-based return attribution are used in the industry. These methodologies differ in how the total value added is decomposed. The main methodologies used are:

- Brinson and Fachler (BF-Method):[56] Takes into consideration opportunity costs in the amount of the total benchmark return when calculating the asset allocation effect.
- Brinson, Hood, and Beebower (BHB-Method):[57] Takes into consideration opportunity costs in the amount of 0% when calculating the asset allocation effect, a special case of BF-Method.
- Karnosky and Singer (KS-Method) or Ankrim and Hensel (AH-Method):[58] Takes into consideration the impact of currency management decisions on the value added.

In general, the concept of return attribution is flexible enough to be adjusted to the specific portfolio management process and the favored methodology. As a rule, the way of decomposing the value added and measuring the individual management effects should reflect the way the different stakeholders take their decisions. In the following section, we explain the return decomposition using the BHB-Method as well as the BF-Method.

55 In practice, the interaction effect is often not separated and instead covered by the security selection effect. This is a reasonable practice if the interaction effect is not a separate management decision. In case of doubts, it is best to show the interaction effect, especially since the sign of the security effect may change if the interaction effect has an opposite sign and is bigger than the security effect.

56 Please see: “Measuring non-US equity portfolio performance” (1985) by G. Brinson and N. Fachler.

57 Please see: “Determinants of portfolio performance” (1986) by G. Brinson, R. Hood, and G. Beebower.

58 Please see: “Global Asset Management and Performance Attribution” (1994) by D. Karnosky and B. Singer or "Multi-currency performance attribution" (1992) by E. Ankrim and C. Hensel.

Assuming simple returns, the arithmetic management effects for a single period using the **BHB-Method** are calculated as follows:

$$VA_{p,t}^{BC} = \sum_{i=1}^{N} VA_{i,t}^{BC} = \sum_{i=1}^{N} AAE_{i,t}^{BC} + \sum_{i=1}^{N} SSE_{i,t}^{BC} + \sum_{i=1}^{N} IAE_{i,t}^{BC} .$$

With: $AAE_{i,t}^{BC} = \left(w_{p,i,t} - w_{b,i,t}\right) \times R_{b,i,t}^{BC}$.

And: $SSE_{i,t}^{BC} = \left(R_{p,i,t}^{BC} - R_{b,i,t}^{BC}\right) \times w_{b,i,t}$.

And: $IAE_{i,t}^{BC} = \left(w_{p,i,t} - w_{b,i,t}\right) \times \left(R_{p,i,t}^{BC} - R_{b,i,t}^{BC}\right)$.

Where:
- $AAE_{i,t}^{BC}$ = **Asset allocation effect** of a portfolio component i for period t in base currency,
- $SSE_{i,t}^{BC}$ = **Security selection effect** of a portfolio component i for period t in base currency,
- $IAE_{i,t}^{BC}$ = **Interaction effect** of a portfolio component i for period t in base currency.

Exhibit 1-32: Single period return attribution using BHB-Method

Considering the single period returns of an investment portfolio and its benchmark shown in Table 1-20 and in Table 1-21, Table 1-26 and Table 1-27 contain the three single period management effects calculated using the BHB-Method.

Asset class	Management effects			Contribution to value added
	Asset allocation effect	Security selection effect	Interaction effect	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	0.05%	0.35%	0.05%	0.45%
Equities EUR	-0.60%	-0.20%	0.10%	-0.70%
Total	**-0.55%**	**0.15%**	**0.15%**	**-0.25%**

Table 1-26: Single period return attribution for first period using BHB-Method

Asset class	Management effects			Contribution to value added
	Asset allocation effect	Security selection effect	Interaction effect	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	0.05%	0.35%	0.05%	0.45%
Equities EUR	-0.60%	-0.20%	0.10%	-0.70%
Total	**-0.55%**	**0.15%**	**0.15%**	**-0.25%**

Table 1-27: Single period return attribution for second period using BHB-Method

Table 1-26 and Table 1-27 show that the portfolio manager added value because of active asset allocation and security selection decisions for bonds EUR and lost value because of active asset allocation and security selection decisions for equities EUR. Furthermore, the interaction effect is positive because the portfolio manager over-weighted the out-performing bonds EUR and underweighted the under-performing equities EUR.

Assuming simple returns, the arithmetic management effects for a single period using the **BF-Method** are calculated as follows:[59]

59 If you compare the formula of the BHB-Method with that of the BF-Method, you see that the difference comes from the different calculation of the asset allocation effect. BHB-Method subtracts opportunity costs in the amount of 0% (and therefore not shown in the formula) while the BF-Method subtracts opportunity costs in the amount of the return of the total benchmark.

$$VA^{BC}_{p,t} = \sum_{i=1}^{N} VA^{BC}_{i,t} = \sum_{i=1}^{N} AAE^{BC}_{i,t} + \sum_{i=1}^{N} SSE^{BC}_{i,t} + \sum_{i=1}^{N} IAE^{BC}_{i,t} \,.$$

With: $AAE^{BC}_{i,t} = \left(w_{p,i,t} - w_{b,i,t}\right) \times \left(R^{BC}_{b,i,t} - R^{BC}_{b,t}\right)$.

And: $SSE^{BC}_{i,t} = \left(R^{BC}_{p,i,t} - R^{BC}_{b,i,t}\right) \times w_{b,i,t}$.

And: $IAE^{BC}_{i,t} = \left(w_{p,i,t} - w_{b,i,t}\right) \times \left(R^{BC}_{p,i,t} - R^{BC}_{b,i,t}\right)$.

Exhibit 1-33: Single period return attribution using BF-Method

Taking into consideration the single period returns of an investment portfolio and its benchmark shown in Table 1-20 and in Table 1-21, Table 1-28 and Table 1-27 contain the three single period management effects calculated using the BF-Method.

Asset class	Management effects			Contribution to value added
	Asset allocation effect	Security selection effect	Interaction effect	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	-0.11%	0.35%	0.05%	0.30%
Equities EUR	-0.45%	-0.20%	0.10%	-0.55%
Total	**-0.55%**	**0.15%**	**0.15%**	**-0.25%**

Table 1-28: Single period return attribution for first period using BF-Method

Asset class	Management effects			Contribution to value added
	Asset allocation effect	Security selection effect	Interaction effect	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	-0.11%	0.35%	0.05%	0.30%
Equities EUR	-0.45%	-0.20%	0.10%	-0.55%
Total	**-0.55%**	**0.15%**	**0.15%**	**-0.25%**

Table 1-29: Single period return attribution for second period using BF-Method

Comparing Table 1-28 and Table 1-29 with Table 1-26 and Table 1-27 shows that the differences are only in the asset allocation effect on a portfolio component level. Here, the asset allocation effect for the bonds EUR is negative (–0.11% versus +0.05%) because an under-performing asset class (versus the total benchmark return) was over-weighted and the asset allocation effect for equities EUR is a bit higher (–0.45% versus –0.60%) because an out-performing asset class (versus the total benchmark return) was underweighted.

*1.3.2.2.2 Multi-period return attribution**

In the previous chapter, we discussed the measurement of the return contributions to value added over a single period. Normally, we analyze returns and value added over multiple periods. Considering the discussion in **chapter 1.2.2.4** and that here we use simple returns, we need to consider compounding effects to calculate multi-period return attributions.

To extend the return attribution to multiple periods, we use the same linking algorithm as discussed for the multi-period contribution to value added, see **chapter 1.3.2.1.3**. Assuming simple returns, the management effects of an investment portfolio for a multi-period are calculated as follows:

$$\mathrm{VAM}_p^{BC} = \sum_{i=1}^{N} \mathrm{CVAM}_i^{BC} = \sum_{i=1}^{N} \mathrm{AAEM}_i^{BC} + \sum_{i=1}^{N} \mathrm{SSEM}_i^{BC} + \sum_{i=1}^{N} \mathrm{IAEM}_i^{BC} \,.$$

With: $$\mathrm{AAEM}_i^{BC} = \sum_{i=1}^{N} \sum_{t=1}^{T} \mathrm{AAE}_{i,t}^{BC} \times \left(1 + \mathrm{R}_{b,T-t}^{BC}\right) \times \left(1 + \mathrm{RM}_{p,t-1}^{BC}\right).$$

And: $$\mathrm{SSEM}_i^{BC} = \sum_{i=1}^{N} \sum_{t=1}^{T} \mathrm{SSE}_{i,t}^{BC} \times \left(1 + \mathrm{R}_{b,T-t}^{BC}\right) \times \left(1 + \mathrm{RM}_{p,t-1}^{BC}\right).$$

And: $$\mathrm{IAEM}_i^{BC} = \sum_{i=1}^{N} \sum_{t=1}^{T} \mathrm{IAE}_{i,t}^{BC} \times \left(1 + \mathrm{R}_{b,T-t}^{BC}\right) \times \left(1 + \mathrm{RM}_{p,t-1}^{BC}\right).$$

Where: AAEM_i^{BC} = Cumulative asset allocation effect of a portfolio component i for the entire measurement period in base currency,

SSEM_i^{BC} = Cumulative security selection effect of a portfolio component i for the entire measurement period in base currency,

IAEM_i^{BC} = Cumulative interaction effect of a portfolio component i for the entire measurement period in base currency.

Exhibit 1-34: Multi-period return attribution using BHB- and BF-Method

Considering the periodic management effects calculated using the BHB-Method (Table 1-26 and Table 1-27) and using the BF-Method (Table 1-28 and Table 1-29), Table 1-30 and Table 1-31 contain the respective multi-period management effects.

Asset class	Management effects			Contribution to value added
	Asset allocation	Security selection	Interaction	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	0.10%	0.71%	0.10%	0.91%
Equities EUR	-1.22%	-0.41%	0.20%	-1.42%
Total	**-1.12%**	**0.30%**	**0.30%**	**-0.51%**

Table 1-30: Multi-period return attribution using BHB-Method

Asset class	Management effects			Contribution to value added
	Asset allocation	Security selection	Interaction	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	-0.21%	0.71%	0.10%	0.60%
Equities EUR	-0.90%	-0.41%	0.20%	-1.11%
Total	**-1.12%**	**0.30%**	**0.30%**	**-0.51%**

Table 1-31: Multi-period return attribution using BF-Method

*1.3.2.2.3 Multi-currency return attribution**

So far, we have discussed single currency return attribution. In practice, investment portfolios normally also invest in international investments and therefore in foreign currencies. Performance attribution needs to be adjusted to cover also the effect of currency management. If there is no currency hedging and if the effect of currency management is not relevant for the performance analysis then international investments can easily be included by analyzing their returns on a base currency basis. Table 1-32 illustrates for a single period the contribution to value added for the EUR multi-asset class portfolio discussed before, where we added an amount of 30% USD investments to the investment portfolio and its benchmark. Table 1-33 shows the respective management effects using the BHB-Method.

Asset class	Portfolio			Benchmark			Contribution to value added
	Weight	Return	Contribution	Weight	Return	Contribution	
Cash EUR	7.00%	0.00%	0.00%	7.00%	0.00%	0.00%	0.00%
Bonds EUR	56.00%	1.00%	0.56%	49.00%	0.50%	0.25%	0.32%
Bonds USD	10.00%	7.73%	0.77%	15.00%	7.20%	1.08%	-0.31%
Equities EUR	7.00%	5.00%	0.35%	14.00%	6.00%	0.84%	-0.49%
Equities USD	20.00%	12.00%	2.40%	15.00%	13.07%	1.96%	0.44%
Total	**100.00%**	**4.08%**	**4.08%**	**100.00%**	**4.13%**	**4.13%**	**-0.04%**

Table 1-32: Single period contribution to value added for the first period

Asset class	Management effects			Contribution to value added
	Asset allocation	Security selection	Interaction	
Cash EUR	0.00%	0.00%	0.00%	0.00%
Bonds EUR	0.04%	0.25%	0.04%	0.32%
Bonds USD	-0.36%	0.08%	-0.03%	-0.31%
Equities EUR	-0.42%	-0.14%	0.07%	-0.49%
Equities USD	0.65%	-0.16%	-0.05%	0.44%
Total	**-0.09%**	**0.03%**	**0.02%**	**-0.04%**

Table 1-33: Single period return attribution for first period using BHB-Method

By contrast, if one is interested in the effects of foreign currencies and currency management, the return attribution needs to be adjusted. The intuitive approach is to decompose the base currency returns into the return contributions on local currency basis and the return contribution of the foreign currencies. The main critique to this approach is that the return on a local currency basis cannot be achieved because this would imply costless currency hedging, what is normally not the case.[60]

In the following section, we discuss a methodology based on the methodologies developed by Karnosky and Singer (KS-Method) and Ankrim and Hensel (AH-Method). This method recognizes that the currency return is the sum of the actual interest-rate differential or "forward premium" between the relevant currencies and the unexpected "currency surprise" return:

$$C_{i,t}^{BC/LC} = \frac{SP_{i,\text{end of t}}^{BC/LC}}{SP_{i,\text{begin of t}}^{BC/LC}} - 1 = (1 + FP_{i,t}^{BC/LC}) \times (1 + E_{p,i,t}^{BC/LC}) - 1 \,.$$

With: $$FP_{i,t}^{BC/LC} = \frac{(1 + RI_{BC,t})}{(1 + RI_{i,t})} - 1 \,.$$

And: $$E_{p,i,t}^{BC/LC} = \frac{SP_{i,\text{end of t}}^{BC/LC}}{FR_{i,\text{begin of t}}^{BC/LC}} - 1 = \frac{SP_{i,\text{end of t}}^{BC/LC}}{SP_{i,\text{begin of t}}^{BC/LC} \times (1 + FP_{i,t}^{BC/LC})} - 1 \,.$$

Where:
- $SP_{i,\text{end of t}}^{BC/LC}$ = Spot rate in currency i at the end of period t,
- $SP_{i,\text{begin of t}}^{BC/LC}$ = Spot rate in currency i at the beginning of t,
- $FP_{i,t}^{BC/LC}$ = Forward premium in currency i for period t,
- $E_{p,i,t}^{BC/LC}$ = Currency surprise return for a currency i of a portfolio for period t,
- $FR_{i,\text{begin of t}}^{BC/LC}$ = Forward rate in currency i at the beginning of period t,
- $RI_{BC,t}$ = Cumulative interest rate in base currency for period t,
- $RI_{i,t}$ = Cumulative interest rate in currency i for period t.

60 However, in practice, such a contribution analysis is often part of investment reporting to illustrate the contribution of the investments on a local basis and that of foreign currencies.

Exhibit 1-35: Currency surprise and forward premium

Considering an investment portfolio with the base currency EUR and the following spot rates and interest rates, the forward premium is –1.94% (negative as USD interest rate is higher than that of EUR) and the currency surprise is +8.78%.

31st of March: Spot rate EUR/USD = 0.7500, $RI_{EUR,t}$ = +1.00% and $RI_{USD,t}$ = +3.00%,

30th of April: Spot rate EUR/USD = 0.8000.

$$=> c_t^{BC/LC} = \frac{0.8000 - 0.7500}{0.7500} = +6.66\%\,,$$

$$FP_t^{BC/LC} = \frac{(1 + 1.00\%)}{(1 + 3.00\%)} - 1 = -1.94\%\,, \text{and}$$

$$E_t^{BC/LC} = \frac{0.8000}{0.7500 \times (1 - 1.94\%)} - 1 = \frac{0.8000}{0.7354} - 1 = +8.78\%\,.$$

The actual calculations necessary for a multi-currency return attribution are complex because of additional compounding and cross products to be considered. For that reason, we illustrate the calculations only for a single period and use continuously compounded instead of simple returns.[61] Furthermore, in the following section, we assign the return contributions due to forward premiums to the asset allocation and not to the currency management decisions because forward premiums are not seen as active currency management decisions.

The **CCR** of an investment portfolio **without currency hedging** is defined as:

$$r_{p,t}^{BC} = \sum_{i=1}^{N} w_{p,i,t} \times r_{p,i,t}^{BC} = \sum_{i=1}^{N} w_{p,i,t} \times \left(r_{p,i,t}^{LC} + c_{i,t}^{BC/LC}\right)$$

$$= \sum_{i=1}^{N} w_{p,i,t} \times \left(r_{p,i,t}^{LC} + fp_{i,t}^{BC/LC} + e_{p,i,t}^{BC/LC}\right)$$

$$= \sum_{i=1}^{N} w_{p,i,t} \times \left(r_{p,i,t}^{LC} + fp_{i,t}^{BC/LC}\right) + \sum_{i=1}^{N} w_{p,i,t} \times e_{p,i,t}^{BC/LC}\,.$$

And for its benchmark: $r_{b,t}^{BC} = \sum_{i=1}^{N} w_{b,i,t} \times \left(r_{b,i,t}^{LC} + fp_{i,t}^{BC/LC}\right) + \sum_{i=1}^{N} w_{b,i,t} \times e_{b,i,t}^{BC/LC}\,.$

Where:
- $r_{p,t}^{BC}$ = Return of a portfolio for period t in base currency,
- $r_{p,i,t}^{BC}$ = Return for a portfolio component i for period t in base currency,
- $r_{p,i,t}^{LC}$ = Return for a portfolio component i for period t in local currency,
- $c_{i,t}^{BC/LC}$ = Currency return for currency i for period t,
- $fp_{p,i,t}^{BC/LC}$ = Forward premium in currency i of a portfolio for period t,
- $e_{p,i,t}^{BC/LC}$ = Currency surprise return for a currency i of a portfolio for period t,
- $r_{b,t}^{BC}$ = Return of a benchmark for period t in base currency,
- $r_{b,i,t}^{LC}$ = Return for a benchmark component i for period t in local currency,
- $e_{b,i,t}^{BC/LC}$ = Currency surprise return for a currency i of a benchmark for period t,

61 Please note that the use of CCRs leads to slightly different return and value added figures than if using simple returns.

$fp_{b,i,t}^{BC/LC}$ = Forward premium in currency i of a benchmark for period t.

Aggregating the forward premium to the local currency returns leads to a **hedged or adjusted CCR** for the portfolio and benchmark components, which going forward can be defined as:

$r_{adj,pi,t}^{BC} = r_{p,i,t}^{LC} + fp_{i,t}^{BC/LC}$, and $r_{adj,bi,t}^{BC} = r_{b,i,t}^{LC} + fp_{i,t}^{BC/LC}$.

Where: $r_{adj,p,i,t}^{BC}$ = Return for a portfolio component i for period t hedged into base currency,

$r_{adj,b,i,t}^{BC}$ = Return for a benchmark component i for period t hedged into base currency.

Exhibit 1-36: Single period asset and currency contribution

Considering the returns of the different asset classes for an investment portfolio and its benchmark shown in Table 1-32 and the currency exchange rates EUR/USD as well as interest rates for EUR and USD used in Exhibit 1-35, Table 1-34 and Table 1-35 contain the single period asset and currency contribution calculated using the above formula.[62] For example, if we look at the contribution of bonds USD to the return of the investment portfolio, we can calculate the asset contribution effect by taking the weight (10%) and multiplying this by the sum of the return in local currencies (+1.00%) and the forward premium (–1.96%). This gives us 10% x (1.00%–1.96%) = –0.10%. The currency contribution is the weight of the bonds USD in the portfolio (10%) multiplied by the currency surprise (+8.41%), i.e. 10% x 8.41% = +0.84%. Adding up the two components gives us the contribution to total return of +0.74%.

Portfolio	Portfolio								
	Weight	$r_{p,i,t}^{BC/LC}$	$r_{p,i,t}^{LC}$	$c_{i,t}^{BC/LC}$	$fp_{i,t}^{BC/LC}$	$e_{p,i,t}^{BC/LC}$	Asset Contribution	Currency Contribution	Contribution to total return
Cash EUR	7.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Bonds EUR	56.00%	1.00%	1.00%	0.00%	0.00%	0.00%	0.56%	0.00%	0.56%
Bonds USD	10.00%	7.45%	1.00%	6.45%	-1.96%	8.41%	-0.10%	0.84%	0.74%
Equities EUR	7.00%	4.88%	4.88%	0.00%	0.00%	0.00%	0.34%	0.00%	0.34%
Equities USD	20.00%	11.33%	4.88%	6.45%	-1.96%	8.41%	0.58%	1.68%	2.27%
Total	**100.00%**	**3.91%**	**1.97%**	**1.94%**	**-0.59%**	**2.52%**	**1.39%**	**2.52%**	**3.91%**

Table 1-34: Single period contribution to return for an investment portfolio

Portfolio	Benchmark								
	Weight	$r_{b,i,t}^{BC/LC}$	$r_{b,i,t}^{LC}$	$c_{i,t}^{BC/LC}$	$fp_{i,t}^{BC/LC}$	$e_{b,i,t}^{BC/LC}$	Asset Contribution	Currency Contribution	Contribution to total return
Cash EUR	7.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Bonds EUR	49.00%	0.50%	0.50%	0.00%	0.00%	0.00%	0.24%	0.00%	0.24%
Bonds USD	15.00%	6.95%	0.50%	6.45%	-1.96%	8.41%	-0.22%	1.26%	1.04%
Equities EUR	14.00%	5.83%	5.83%	0.00%	0.00%	0.00%	0.82%	0.00%	0.82%
Equities USD	15.00%	12.28%	5.83%	6.45%	-1.96%	8.41%	0.58%	1.26%	1.84%
Total	**100.00%**	**3.95%**	**2.01%**	**1.94%**	**-0.59%**	**2.52%**	**1.42%**	**2.52%**	**3.95%**

Table 1-35: Single period contribution to return for a benchmark

The **value added** of a multi-currency investment portfolio **without currency hedging** against its multi-currency benchmark is defined as:

62 For simplicity, we assume here that the currency surprise return for the portfolio and that for the benchmark are identical.

$$va_{p,t}^{BC} = r_{p,t}^{BC} - r_{b,t}^{BC}$$
$$= \left[\sum_{i=1}^{N} w_{p,i,t} \times r_{adj,p,i,t}^{BC} - \sum_{i=1}^{N} w_{b,i,t} \times r_{adj,b,i,t}^{BC}\right]$$
$$+ \left[\sum_{i=1}^{N} w_{p,i,t} \times e_{p,i,t}^{BC/LC} - \sum_{i=1}^{N} w_{b,i,t} \times e_{b,i,t}^{BC/LC}\right].$$

Where: $va_{p,t}^{BC}$ = Value added for a portfolio for period t in base currency.

The first part of the formula measures the value added due to hedged asset allocation and hedged security selection and the second part measures the value added due to currency management. The latter management effect can be split into different components, normally currency selection and hedge selection.

The **CCR** of an investment portfolio **with currency hedging** is defined as:[63]

$$r_{p,t}^{BC} = \sum_{i=1}^{N} w_{p,i,t} \times r_{adj,p,i,t}^{BC} + \sum_{i=1}^{N} w_{p,i,t} \times e_{p,i,t}^{BC/LC} + \sum_{i=1}^{N} h_{p,i,t} \times f_{p,i,t}^{BC/LC} \text{ , with } \sum_{i=1}^{N} h_{p,i,t} = 0 \text{ .}$$

With: $f_{p,i,t}^{BC/LC} = e_{p,i,t}^{BC/LC}$.

And for its benchmark:

$$r_{b,t}^{BC} = \sum_{i=1}^{N} w_{b,i,t} \times r_{adj,b,i,t}^{BC} + \sum_{i=1}^{N} w_{b,i,t} \times e_{b,i,t}^{BC/LC} + \sum_{i=1}^{N} h_{b,i,t} \times f_{b,i,t}^{BC/LC} \text{ , with } \sum_{i=1}^{N} h_{b,i,t} = 0 \text{ .}$$

With: $f_{b,i,t}^{BC/LC} = e_{b,i,t}^{BC/LC}$.

Where: $h_{p,i,t}$ = Hedge weight of currency i for a portfolio at the beginning of period t,
$f_{p,i,t}^{BC/LC}$ = Hedge return of currency i for a portfolio for period t,
$h_{b,i,t}$ = Hedge weight of currency i for a benchmark at the beginning of period t,
$f_{b,i,t}^{BC/LC}$ = Hedge return of currency i for a benchmark for period t.

The **management effects** for an investment portfolio **with currency hedging** through currency forwards using the concept of the BHB-Method are calculated as follows:

$$va_{p,t}^{BC} = \sum_{i=1}^{N} AAE_{i,t}^{Adj} + \sum_{i=1}^{N} SSE_{i,t}^{Adj} + \sum_{i=1}^{N} IAE_{i,t}^{Adj} + \sum_{i=1}^{N} CAE_{i,t}^{BC} + \sum_{i=1}^{N} HSE_{i,t}^{BC} + \sum_{i=1}^{N} CIAE_{i,t}^{BC} \text{ .}$$

With: $AAE_{i,t}^{Adj} = \left(w_{p,i,t} - w_{b,i,t}\right) \times r_{adj,b,i,t}^{BC}$.

And: $SSE_{i,t}^{Adj} = \left(r_{adj,p,i,t}^{BC} - r_{adj,b,i,t}^{BC}\right) \times w_{b,i,t}$.

And: $IAE_{i,t}^{Adj} = \left(w_{p,i,t} - w_{b,i,t}\right) \times \left(r_{adj,p,i,t}^{BC} - r_{adj,b,i,t}^{BC}\right)$.

And: $CAE_{i,t}^{BC} = \left(w_{p,i,t} - w_{b,i,t}\right) \times e_{b,i,t}^{BC/LC} + \left(h_{p,i,t} - h_{b,i,t}\right) \times f_{b,i,t}^{BC/LC}$.

63 The hedge weight is less than 0.00% if a currency is sold forward (hedged) and greater than 0.00% if a currency is bought forward.

And: $CHSE_{i,t}^{BC} = \left(e_{p,i,t}^{BC/LC} - e_{b,i,t}^{BC/LC}\right) \times \left(w_{b,i,t}\right) + \left(f_{p,i,t}^{BC/LC} - f_{b,i,t}^{BC/LC}\right) \times \left(h_{b,i,t}\right)$.

And: $CHIAE_{i,t}^{BC}$

$$= \left(w_{p,i,t} - w_{b,i,t}\right) \times \left(e_{p,i,t}^{BC/LC} - e_{b,i,t}^{BC/LC}\right) + \left(h_{p,i,t} - h_{b,i,t}\right) \times \left(f_{p,i,t}^{BC/LC} - f_{b,i,t}^{BC/LC}\right).$$

Where: $AAE_{i,t}^{Adj}$ = Asset allocation effect of a portfolio component i for period t hedged into base currency,

$SSE_{i,t}^{Adj}$ = Security selection effect of a portfolio component i for period t hedged into base currency,

$IAE_{i,t}^{Adj}$ = Interaction effect of a portfolio component i for period t hedged into base currency,

$CAE_{i,t}^{BC}$ = Currency asset allocation effect of a currency i of a portfolio for period t in base currency,

$CHSE_{i,t}^{BC}$ = Currency and hedge selection effect of a currency i of a portfolio for period t in base currency,

$CIAE_{i,t}^{BC}$ = Currency interaction effect of a currency i of a portfolio for period t in base currency.

Exhibit 1-37: Multi-currency return attribution using BHB-Method

Considering the data for the investment portfolio and its benchmark shown in Table 1-34 and Table 1-35, Table 1-36 and Table 1-37 contain the necessary input data and the different asset management and currency management effects.[64]

Asset class	Portfolio				Benchmark			
	Weight	$r_{adj,p,i,t}^{BC}$	$e_{p,i,t}^{BC/LC}$ / $f_{p,i,t}^{BC/LC}$	$r_{p,i,t}^{BC}$	Weight	$r_{adj,b,i,t}^{BC}$	$e_{b,i,t}^{BC/LC}$ / $f_{b,i,t}^{BC/LC}$	$r_{b,i,t}^{BC}$
Cash EUR	7.00%	0.00%	0.00%	0.00%	7.00%	0.00%	0.00%	0.00%
Bonds EUR	56.00%	1.00%	0.00%	1.00%	49.00%	0.50%	0.00%	0.50%
Bonds USD	10.00%	-0.97%	8.41%	7.45%	15.00%	-1.46%	8.41%	6.95%
Equities EUR	7.00%	4.88%	0.00%	4.88%	14.00%	5.83%	0.00%	5.83%
Equities USD	20.00%	2.92%	8.41%	11.33%	15.00%	3.87%	8.41%	12.28%
EUR Forward	10.00%		0.00%	0.00%	20.00%		0.00%	0.00%
USD Forward	-10.00%		8.25%	8.25%	-20.00%		8.41%	8.41%
Total	**100.00%**	**1.39%**	**1.70%**	**3.09%**	**100.00%**	**1.42%**	**0.84%**	**2.26%**

Table 1-36: Input data for multi-currency return attribution

Asset class	Asset management effects			Currency management effects			Contribution to value added
	Asset allocation	Security selection	Interaction	Currency allocation	Hedge selection	Interaction	
Cash EUR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Bonds EUR	0.03%	0.24%	0.03%	0.00%	0.00%	0.00%	0.31%
Bonds USD	0.07%	0.07%	-0.02%	-0.42%	0.00%	0.00%	-0.30%
Equities EUR	-0.41%	-0.13%	0.07%	0.00%	0.00%	0.00%	-0.47%
Equities USD	0.19%	-0.14%	-0.05%	0.42%	0.00%	0.00%	0.42%
EUR Forward				0.00%	0.00%	0.00%	0.00%
USD Forward				0.84%	0.03%	-0.02%	0.86%
Total	**-0.11%**	**0.04%**	**0.03%**	**0.84%**	**0.03%**	**-0.02%**	**0.82%**

Table 1-37: Multi-currency return attribution based on BHB-Method

64 Please note that here the currency surprise return for USD for the portfolio is different to the currency surprise return for USD for the benchmark because of different forward rates used. This hedge selection results in a positive contribution as the portfolio forward rate was EUR/USD 0.7390 instead of EUR/USD 0.7354.

1.3.2.3 Multi-factor or regression-based return attribution*

A multi-factor or regression-based return attribution describes the return and the value added as a function of different factors. In practice, different types of regression analysis are used, namely those that use factors according to the CAPM or to the APT and those that are based on simple or on complex, multiple linear or quadratic, regression methods.

Methods using simple and multiple linear or quadratic regression require a large set of observations to provide statistically significant results. Therefore, it is not easy to implement them in practice. However, when portfolio accounting data is missing, there is no alternative. The methods based on regression have a theoretical justification since they are based on modern portfolio theory. They lead to theoretically interesting results and many of the research papers published are based on regression analysis.

*1.3.2.3.1 Methods based on simple linear regression**

Methods based on simple linear regression looks at the ex post characteristic line of an investment portfolio, explaining the excess return by the market excess return and determines the ex post Jensen's α_P and beta β_P with a simple regression:

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p \times \left(r_{m,t} - r_{f,t}\right) + \varepsilon_t \,.$$

Where: $r_{f,t}$ = Risk free rate for period t,
$r_{m,t}$ = Return of the market portfolio for period t.

The assessment of statistical significance is measured with t-test statistic for the regression parameters and with Fisher's F-test statistic for the variance:[65]

$$t_\alpha = \frac{\alpha_p}{\sigma_\alpha}, t_\beta = \frac{\beta_p}{\sigma_\beta}, F = (n-2) \times \frac{R^2}{1-R^2}.$$

Where: σ_α = Standard deviation of Jensen's α_P of a portfolio,
σ_β = Standard deviation of the beta β_P of a portfolio,
n = Number of observations,
R^2 = Coefficient of determination.

Based on Jensen's α_p and beta β_p, we can proceed with the attribution analysis using the ex-post SML. In the following example, we analyze the contributions to the selectivity effect measured by Jensen's α_p:

- The net selectivity effect, i.e. the contribution to excess return due to the portfolio manager's ability to select securities,
- The diversification effect, i.e. the contribution to excess return due to the portfolio manager's ability to determine the level of diversification, i.e. incomplete diversification.

Figure 1-27 illustrates Jensen's α_P as the difference between the return r_P of an investment portfolio P and the return r_e of the respective equilibrium portfolio E on the SML. We can construct a well-diversified investment portfolio B on the SML with the same level of total risk as investment portfolio P and a return r_b. The net selectivity is determined by subtracting the effect of incomplete diversification from the total selectivity effect and equals the difference between r_p and r_b. The so-called **Fama's decomposition of Jensen's α_P** is calculated as follows:

65 The null hypothesis are $\alpha_p = 0$ and $\beta_p = 0$. In the following, we always assume that the analysis has statistical significance.

$$\alpha_p = r_p - r_e = \text{net selectivity effect} + \text{diversification effect} = (r_p - r_b) + (r_b - r_e)$$

With: $r_b = r_f + \frac{\sigma_P}{\sigma_m} \times (r_m - r_f)$.

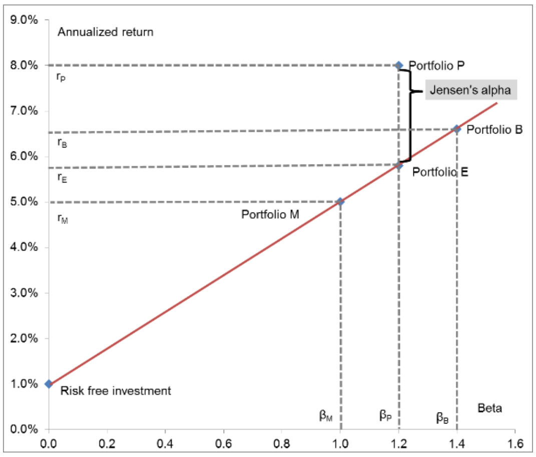


Figure 1-27: Decomposition of Jensen's alpha

*1.3.2.3.2 Methods based on complex regressions**

To increase the statistical significance of the regression, we may use more sophisticated statistical models. In Figure 1-28, the observed scatterplots obviously show a non-linear structure, which is fitted with a quadratic characteristic line (Treynor-Mazuy) and a broken characteristic line (Merton-Henriksson). Both approaches assume that a portfolio manager with market timing skills increases the beta β_p of an investment portfolio when he expects a bullish market and reduces the beta β_p when he expects a bearish market.

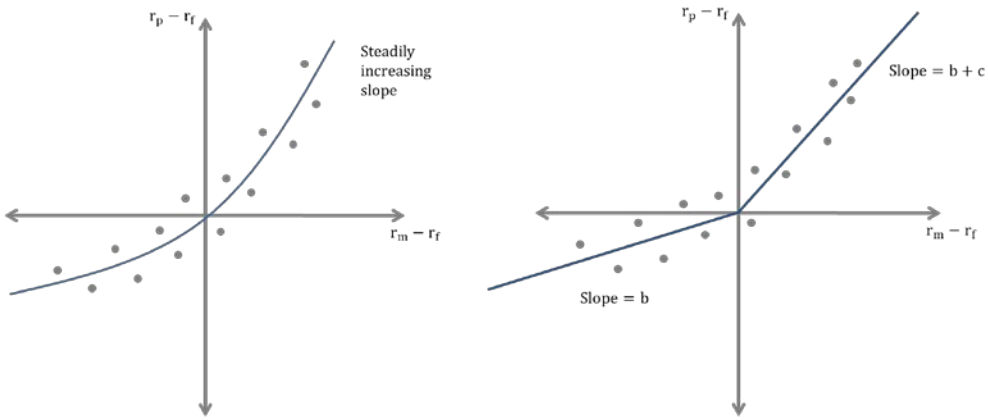


Figure 1-28: Ex post characteristic curve (Treynor-Mazuy) and broken characteristic line (Merton-Henriksson)

Treynor and Mazuy proposed a quadratic regression to analyze both market timing and security selection.[66] This approach assumes that a portfolio manager with market timing skills increases the beta β_p of an investment portfolio when the market excess return is expected to increase, where this effect is measured by the convexity γ_p of the parabolic curve. Furthermore, it assumes that a portfolio manager with security selection skills has a positive constant bias, measured by the constant α_P:

66 Please see: "Can mutual funds outguess the market?" (1966) by J. Treynor and K. Mazuy, Harvard Business Review 44, July-August, pages 131-136.

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p \times (r_{m,t} - r_{f,t}) + \gamma_p \times (r_{m,t} - r_{f,t})^2 + \varepsilon_t \quad \text{for every observation } t$$

Market timing ability is measured by the convexity impact of the average market excess return and security selection ability is measured by α_P:

$$\text{Market timing} = \gamma_p \times (r_m - r_f)^2 \quad \text{and security selection} = \alpha_p .$$

Merton and Henriksson proposed an attribution analysis based on linear regression, which can assess both market timing and security selection in one analysis.[67] This approach assumes that a portfolio manager with market timing skills increases the beta β_p of an investment portfolio to a positive market return beta β_+ when he expects a bullish market and reduces it to a negative market return beta β_- when he expects a bearish market (see Figure 1-28). Technically, this is measured by introducing two beta dummy variables in the linear regression:

$$r_{p,t} - r_{f,t} = \alpha_p + \delta_+ \times \beta_+ \times (r_{m,t} - r_{f,t}) + \delta_- \times \beta_- \times (r_{m,t} - r_{f,t}) + \varepsilon_t .$$

In case of a bearish market, the dummy variable δ_- is 1 and δ_+ is 0, and in case of a bullish market, the dummy variable δ_- is 0 and δ_+ is 1.

The market timing ability is measured by the difference between the positive and the negative market return beta multiplied with the average market excess return:

$$\text{Market timing} = (\beta_+ - \beta_-) \times (r_m - r_f) .$$

The higher the difference between β_+ and β_-, the better the timing ability. The security selection is evaluated by the α_P:

$$\text{Security selection} = \alpha_p .$$

Another form of regression analysis is **style analysis**. Analyzing the style of equity, bond, or multi-asset class portfolio managers will largely depend on the type of investment portfolio one analyses and on the type of determinants (factors) of value added one wishes to consider. Commonly, one analyses investment style using asset class factors, but economic multi-factor models are also commercially available. Style analysis is often applied to characterize the management style quantitatively and to be used for manager search by consultants.

To illustrate the possible differences of asset class factor analyses, let us look at some reasonable contribution break-ups. Note that the choice of factors should be made considering the availability of the appropriate indices:

- For domestic equities portfolio:

$$r = \alpha + \beta_{sc} \times r_{\text{small cap}} + \beta_{mc} \times r_{\text{medium cap}} + \beta_{lc} \times r_{\text{large cap}} .$$

- For international equities portfolio:

$$r = \alpha + \beta_{US} \times r_{US} + \beta_{Europe} \times r_{Europe} + \beta_{\text{Far East}} \times r_{\text{Far East}} .$$

- For an international multi-asset class portfolio:

$$r = \alpha + \sum \beta_i \times r_i .$$

The difficulty to interpret the contribution break-up is notably reduced with asset class factors compared to economic or technical factors, provided that the asset classes:

67 Please see: "On market timing and investment performance. II. Statistical procedures for evaluating forecasting skills" (1981) by R. Henriksson and R. Merton, Journal of Business 54 (3), pages 363-406.

- Are mutually exclusive,
- Are exhaustive, and
- Have different returns.

The analysis process is also largely simplified because it "only" requires common security classification versus tremendous security level databases with the accordingly high number of quantitative analysts to feed and to maintain them.

The problems usually appear when choosing the benchmark for each asset class: in a comparative perspective, the choice of the benchmark sets the reference against which portfolio managers are going to be measured and subjective preferences that any single manager has for his actual benchmark will inevitably appear.

The analysis of management style, or in other words the exposures, managers have taken in the various possible components over a long period, provides investors with an insight to the type of investment strategy managers follow and their success. Style analysis with high R^2 values indicate that the investment style is close to that characterized by the benchmark. Small R^2 values can on the other hand have various reasons like important rotation among or within the asset classes, inappropriate benchmark indices, and so on.

Style analysis can help to understand one's strengths and weaknesses better, but as performance measurement in general, it is a statistical analysis that can in the best case be as good as the models it is based upon and the quality of the collected data.

1.3.3 Risk attribution*

1.3.3.1 Introduction to risk attribution*

*1.3.3.1.1 Contribution to risk measurement**

Contribution to risk measurement provides information on how the weights or exposures, and the risks and correlations of portfolio components combine to the risk of an investment portfolio.[68] The risk of an investment portfolio is the sum of the risk contributions of the individual portfolio components, such as individual investments or any aggregation of those. Using variance as the measure for risk and assuming constant weights for the portfolio components over time, the variance of an investment portfolio is calculated as follows:[69]

$$\mathrm{Var}_p = \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_{p,i} \times w_{p,j} \times \mathrm{Cov}\left(r_{p,i}, r_{p,j}\right) = \sum_{i=1}^{n} w_{p,i} \times \mathrm{Cov}\left(r_{p,i}, r_p\right)$$
$$= \sum_{i=1}^{n} w_{p,i} \times \mathrm{Corr}\left(r_{p,i}, r_p\right) \times \sigma_{p,i} \times \sigma_p.$$

Where: $\mathrm{Cov}\left(r_{p,i}, r_{p,j}\right)$ = Covariance of the returns of a portfolio component i and the returns of a portfolio component j,

$\mathrm{Cov}\left(r_{p,i}, r_p\right)$ = Covariance between the returns of portfolio component i and the returns of a portfolio,

68 **Chapter 1.3.3** is based on the article "Risk contribution is exposure times volatility times correlation" (January 2010) by B. Davis and J. Menchero, available on the internet.

69 Contribution to risk measurement can also be applied to other risk measures.

$\text{Corr}(r_{p,i}, r_p)$ = Correlation between the returns of portfolio component i and the returns of a portfolio.

The contribution to variance of a portfolio component is calculated as follows:

$$\text{CVar}_{p,i} = w_{p,i} \times \text{Corr}(r_{p,i}, r_p) \times \sigma_{p,i} \times \sigma_p \,.$$

Where: $\text{CVar}_{p,i}$ = Contribution of portfolio component i to the variance of a portfolio.

Dividing variance by the standard deviation yields the standard deviation of the investment portfolio:

$$\sigma_p = \frac{\text{Var}_p}{\sigma_p} = \sum_{i=1}^{n} w_{p,i} \times \text{Corr}(r_{p,i}, r_p) \times \sigma_{p,i} \,.$$

The contribution to standard deviation of a portfolio component is calculated as follows:

$$\text{CSD}_{p,i} = w_{p,i} \times \text{Corr}(r_{p,i}, r_p) \times \sigma_{p,i} \,.$$

Where: $\text{CSD}_{p,i}$ = Contribution of portfolio component i to the standard deviation of a portfolio.

Exhibit 1-38: Contribution to risk

Let us consider the monthly continuously compounded returns for an investment portfolio and its two portfolio components, bonds and equities, as shown in Table 1-38, and let us further assume monthly rebalancing. The historical annualized variance (return) for the investment portfolio is +0.000147 (+1.75%), +0.000041 (–0.17%) for bonds and +0.005173 (+9.43%) for equities. The correlation between the returns for the bonds and the portfolio returns is -0.272975 and +0.939649 between the returns for the equities and the portfolio returns. The contributions to variance are –0.000017 for bonds and +0.000164 for equities. The annualized standard deviation for the investment portfolio is +1.212%, and the contributions to standard deviation are –0.140% for bonds and +1.352% for equities.

Month	$w_{p,Bonds}$	$w_{p,Equities}$	$r_{p,Bonds}$	$r_{p,Equities}$	$CR_{p,Bonds}$	$CR_{p,Equities}$	r_p
1	80.00%	20.00%	0.10%	-1.00%	0.08%	-0.20%	-0.12%
2	80.00%	20.00%	0.20%	-3.00%	0.16%	-0.60%	-0.44%
3	80.00%	20.00%	0.00%	1.00%	0.00%	0.20%	0.20%
4	80.00%	20.00%	-0.20%	2.00%	-0.16%	0.40%	0.24%
5	80.00%	20.00%	-0.10%	3.00%	-0.08%	0.60%	0.52%
6	80.00%	20.00%	-0.30%	-1.00%	-0.24%	-0.20%	-0.44%
7	80.00%	20.00%	0.10%	0.00%	0.08%	0.00%	0.08%
8	80.00%	20.00%	0.00%	1.00%	0.00%	0.20%	0.20%
9	80.00%	20.00%	0.20%	-1.00%	0.16%	-0.20%	-0.04%
10	80.00%	20.00%	0.10%	0.00%	0.08%	0.00%	0.08%
11	80.00%	20.00%	-0.30%	5.00%	-0.24%	1.00%	0.76%
12	80.00%	20.00%	-0.20%	4.00%	-0.16%	0.80%	0.64%
13	80.00%	20.00%	0.30%	1.00%	0.24%	0.20%	0.44%
14	80.00%	20.00%	-0.10%	0.00%	-0.08%	0.00%	-0.08%
Cumulative			-0.20%	11.00%	-0.16%	2.20%	2.04%

Table 1-38: Portfolio data used for calculation of contribution to risk

Using historical covariance as a proxy for expected covariance, the expected annualized variance and risk contributions of an investment portfolio would be identical to the historical ones. However, this is only the case if we assume identical asset allocation or constant weights for the portfolio components. In practice, asset allocation is not constant over time.

To reflect the effect of the changing asset allocation over time, the calculation of the historical risk contributions needs to consider these changes. The adjustment requires the use of return contributions instead of unweighted returns of the portfolio components:[70]

$$\mathrm{CVar}_{p,i} = \mathrm{Cov}(w_{p,i} \times r_{p,i}, r_p) = \mathrm{Cov}(CR_{p,i}, r_p) = \mathrm{Corr}(CR_{p,i}, r_p) \times \sigma(CR_{p,i}) \times \sigma_p \,.$$

Where: $\mathrm{Cov}(CR_{p,i}, r_p)$ = Covariance between the contributions of portfolio component i to the returns of a portfolio and the returns of the portfolio,

$\sigma(CR_{p,i})$ = Standard deviation of the contributions of portfolio component i to the returns of a portfolio.

Dividing contribution to variance by the standard deviation yields the contribution to standard deviation of the investment portfolio:

$$\mathrm{CSD}_{p,i} = \frac{\mathrm{CVar}_{p,i}}{\sigma_p} = \mathrm{Corr}(CR_{p,i}, r_p) \times \sigma(CR_{p,i}) \,.$$

*1.3.3.1.2 Contribution to active risk measurement**

Active risk is a type of relative or excess risk that refers to the risk of an investment portfolio that is due to active portfolio management decisions. Contribution to active risk measurement provides information on how the weights or exposures and the risks and correlations of portfolio components versus those of the respective benchmark components combine to the active risk of an investment portfolio against its benchmark. The active risk of an investment portfolio is the sum of the active risk contributions of the individual portfolio components, such as individual investments or any aggregation of those. Using tracking error variance as the measure for active risk and return contributions instead of unweighted returns, the tracking error variance of an investment portfolio against its benchmark is calculated as follows:[71]

$$\begin{aligned}\mathrm{TEVar}_p = \sigma^2(r_p - r_b) &= \sum_{i=1}^{n} \mathrm{Cov}(CR_{p,i} - CR_{b,i}, r_p - r_b) \\ &= \sum_{i=1}^{n} \mathrm{Corr}(CR_{p,i} - CR_{b,i}, r_p - r_b) \times \sigma(CR_{p,i} - CR_{b,i}) \times \sigma(r_p - r_b) \,.\end{aligned}$$

Where: TEVar_p = Tracking error variance of a portfolio,

$\mathrm{Cov}(CR_{p,i} - CR_{b,i}, r_p - r_b)$ = Covariance of the excess return contributions of a portfolio component i and the excess returns of a portfolio against its benchmark.

The contribution to tracking error variance of an individual portfolio component is calculated as follows:

70 Please note: $w_{p,i}$ is implicit covered by $CR_{p,i}$.

71 The concept of contribution to active risk measurement could also be applied to other excess risk measures.

Furthermore, please note that tracking error can be calculated in different ways with different interpretations. An alternative interpretation is: Tracking error is the tracking error of a long/short investment portfolio (long: original investment portfolio and short benchmark). This interpretation leads to the same total tracking error but with different contributions.

$$CTEVar_{p,i} = Corr\left(CR_{p,i} - CR_{b,i}, r_p - r_b\right) \times \sigma\left(CR_{p,i} - CR_{b,i}\right) \times \sigma\left(r_p - r_b\right).$$

Where: $CTEVar_{p,i}$ = Contribution of portfolio component i to the tracking error variance of a portfolio.

Dividing tracking error variance by the tracking error standard deviation yields the tracking error standard deviation of the investment portfolio:

$$TESD_p = \frac{TEVar_p}{\sigma(r_p - r_b)} = \sum_{i=1}^{n} Corr\left(CR_{p,i} - CR_{b,i}, r_p - r_b\right) \times \sigma(CR_{p,i} - CR_{b,i}).$$

The contribution to tracking error standard deviation of a portfolio component is calculated as follows:

$$CTESD_{p,i} = Corr\left(CR_{p,i} - CR_{b,i}, r_p - r_b\right) \times \sigma(CR_{p,i} - CR_{b,i}).$$

Where: $CTESD_{p,i}$ = Contribution of portfolio component i to the tracking error standard deviation of a portfolio.

Exhibit 1-39: Contribution to active risk

Let us consider the data for the investment portfolio in Exhibit 1-38 and the monthly continuously compounded returns for a benchmark and its two benchmark components, bonds and equities as shown in Table 1-39 and let us further assume monthly rebalancing. The annualized variance (return) is +0.000049 (+1.01%) for the benchmark, +0.000026 (–0.14%) for bonds and +0.007450 (+11.31%) for equities, and the contributions to variance are –0.000002 for bonds and +0.000051 for equities. The annualized standard deviation is +0.701% for the benchmark, and the contributions to standard deviation are –0.030% for bonds and +0.731% for equities.

Month	$w_{b,Bonds}$	$w_{b,Equities}$	$r_{b,Bonds}$	$r_{b,Equities}$	$CR_{b,Bonds}$	$CR_{b,Equities}$	r_b
1	90.00%	10.00%	0.08%	-1.20%	0.07%	-0.12%	-0.05%
2	90.00%	10.00%	0.16%	-3.60%	0.14%	-0.36%	-0.22%
3	90.00%	10.00%	0.00%	1.20%	0.00%	0.12%	0.12%
4	90.00%	10.00%	-0.16%	2.40%	-0.14%	0.24%	0.10%
5	90.00%	10.00%	-0.08%	3.60%	-0.07%	0.36%	0.29%
6	90.00%	10.00%	-0.24%	-1.20%	-0.22%	-0.12%	-0.34%
7	90.00%	10.00%	0.08%	0.00%	0.07%	0.00%	0.07%
8	90.00%	10.00%	0.00%	1.20%	0.00%	0.12%	0.12%
9	90.00%	10.00%	0.16%	-1.20%	0.14%	-0.12%	0.02%
10	90.00%	10.00%	0.08%	0.00%	0.07%	0.00%	0.07%
11	90.00%	10.00%	-0.24%	6.00%	-0.22%	0.60%	0.38%
12	90.00%	10.00%	-0.16%	4.80%	-0.14%	0.48%	0.34%
13	90.00%	10.00%	0.24%	1.20%	0.22%	0.12%	0.34%
14	90.00%	10.00%	-0.08%	0.00%	-0.07%	0.00%	-0.07%
Cumulative			-0.16%	13.20%	-0.14%	1.32%	1.18%

Table 1-39: Benchmark data used for calculation of contribution to active risk

The annualized tracking error variance of the investment portfolio against its benchmark is +0.000030, and the contributions to tracking error variance are –0.000001 for bonds and +0.000031 for equities. The annualized tracking error standard deviation of the investment portfolio against its benchmark is +0.547%, and the contributions to tracking error standard deviation are –0.027% for bonds and +0.574% for equities.

Using historical tracking error covariance as a proxy for expected tracking error covariance, the expected annualized tracking error variance and risk contributions of an investment portfolio would be identical to the historical ones. However, this is only valid if one assumes identical asset allocation and security selection. In practice, asset allocation and security selection are not constant over time.

1.3.3.2 Single factor or algebraic-based risk attribution*

After calculating the contributions to risk and the contributions to active risk, we are also interested in getting some insight on the impact of the active management decisions on the active risk. Similar to the return attribution, the risk attribution should follow the portfolio management process in order to produce valid and meaningful feedback into the portfolio management process. In the following section, we decompose the active risk of an investment portfolio using the framework of the BHB-Method:[72]

$$\begin{aligned}\mathrm{TEVar}_p &= \sum_{i=1}^{n} \mathrm{CTEVar}_{p,i} = \sum_{i=1}^{N} \mathrm{CTEVar}_{p,i}^{AAE} + \sum_{i=1}^{N} \mathrm{CTEVar}_{p,i}^{SPE} + \sum_{i=1}^{N} \mathrm{CTEVar}_{p,i}^{IAE} \\ &= \sum_{i=1}^{N} \mathrm{Cov}\left(\mathrm{AAE}_{p,i}, \mathrm{VA}_p\right) + \sum_{i=1}^{N} \mathrm{Cov}\left(\mathrm{SPE}_{p,i}, \mathrm{VA}_p\right) + \sum_{i=1}^{N} \mathrm{Cov}\left(\mathrm{IAE}_{p,i}, \mathrm{VA}_p\right)\end{aligned}$$

$$\begin{aligned}\mathrm{CTEVar}_{p,i} &= \mathrm{CTEVar}_{p,i}^{AAE} + \mathrm{CTEVar}_{p,i}^{SPE} + \mathrm{CTEVar}_{p,i}^{IAE} \\ &= \mathrm{Cov}\left(\mathrm{AAE}_{p,i}, \mathrm{VA}_p\right) + \mathrm{Cov}\left(\mathrm{SPE}_{p,i}, \mathrm{VA}_p\right) + \mathrm{Cov}\left(\mathrm{IAE}_{p,i}, \mathrm{VA}_p\right).\end{aligned}$$

Where:
- $\mathrm{CTEVar}_{p,i}^{AAE}$ = Contribution to tracking error variance of a portfolio due to the asset allocation effect for portfolio component i,
- $\mathrm{CTEVar}_{p,i}^{SPE}$ = Contribution to tracking error variance of a portfolio due to the security selection effect for portfolio component i,
- $\mathrm{CTEVar}_{p,i}^{IAE}$ = Contribution to tracking error variance of a portfolio due to the interaction effect for portfolio component i.

Dividing contribution to tracking error variance by the tracking error standard deviation yields the contribution to tracking error standard deviation of the investment portfolio:

$$\begin{aligned}\mathrm{CTESD}_{p,i} &= \mathrm{CTESD}_{p,i}^{AAE} + \mathrm{CTESD}_{p,i}^{SPE} + \mathrm{CTESD}_{p,i}^{IAE} \\ &= \frac{\mathrm{CTEVar}_{p,i}^{AAE}}{\mathrm{TESD}_p} + \frac{\mathrm{CTEVar}_{p,i}^{SPE}}{\mathrm{TESD}_p} + \frac{\mathrm{CTEVar}_{p,i}^{IAE}}{\mathrm{TESD}_p}.\end{aligned}$$

Where:
- $\mathrm{CTESD}_{p,i}^{AAE}$ = Contribution to tracking error standard deviation of a portfolio due to the asset allocation effect for portfolio component i,
- $\mathrm{CTESD}_{p,i}^{SPE}$ = Contribution to tracking error standard deviation of a portfolio due to the security selection effect for portfolio component i,
- $\mathrm{CTESD}_{p,i}^{IAE}$ = Contribution to tracking error standard deviation of a portfolio due to the interaction effect for portfolio component i.

72 Please see **chapter 1.3.2.2.1** for details to the BHB-Method. The risk attribution framework can also be applied to other performance attribution methodologies. Furthermore, please note that $\mathrm{VA}_p = r_p - r_b$.

Exhibit 1-40: Single factor or algebraic risk attribution using BHB-Method

Here, we use the weights and continuously compounded returns of the investment portfolio presented in Table 1-40 and the respective data for the benchmark in Table 1-39.

Month	$w_{p,Bonds}$	$w_{p,Equities}$	$r_{p,Bonds}$	$r_{p,Equities}$	$CR_{p,Bonds}$	$CR_{p,Equities}$	r_p
1	80.00%	20.00%	0.10%	-1.00%	0.08%	-0.20%	-0.12%
2	80.00%	20.00%	0.20%	-3.00%	0.16%	-0.60%	-0.44%
3	80.00%	20.00%	0.00%	1.00%	0.00%	0.20%	0.20%
4	80.00%	20.00%	-0.20%	2.00%	-0.16%	0.40%	0.24%
5	80.00%	20.00%	-0.10%	3.00%	-0.08%	0.60%	0.52%
6	80.00%	20.00%	-0.30%	-1.00%	-0.24%	-0.20%	-0.44%
7	80.00%	20.00%	0.10%	0.00%	0.08%	0.00%	0.08%
8	60.00%	40.00%	0.00%	1.00%	0.00%	0.40%	0.40%
9	60.00%	40.00%	0.20%	-1.00%	0.12%	-0.40%	-0.28%
10	60.00%	40.00%	0.10%	0.00%	0.06%	0.00%	0.06%
11	60.00%	40.00%	-0.30%	5.00%	-0.18%	2.00%	1.82%
12	60.00%	40.00%	-0.20%	4.00%	-0.12%	1.60%	1.48%
13	60.00%	40.00%	0.30%	1.00%	0.18%	0.40%	0.58%
14	60.00%	40.00%	-0.10%	0.00%	-0.06%	0.00%	-0.06%
Cumulative			-0.20%	11.00%	-0.16%	4.20%	4.04%

Table 1-40: Portfolio data used for risk attribution using BHB-Method

Table 1-41 contains the necessary return contributions to calculate the contributions to risk and active risk.

Month	$CR_{p,Bonds}$	$CR_{p,Equities}$	$CR_{b,Bonds}$	$CR_{b,Equities}$	$r_p - r_b$
1	0.08%	-0.20%	0.07%	-0.12%	-0.07%
2	0.16%	-0.60%	0.14%	-0.36%	-0.22%
3	0.00%	0.20%	0.00%	0.12%	0.08%
4	-0.16%	0.40%	-0.14%	0.24%	0.14%
5	-0.08%	0.60%	-0.07%	0.36%	0.23%
6	-0.24%	-0.20%	-0.22%	-0.12%	-0.10%
7	0.08%	0.00%	0.07%	0.00%	0.01%
8	0.00%	0.40%	0.00%	0.12%	0.28%
9	0.12%	-0.40%	0.14%	-0.12%	-0.30%
10	0.06%	0.00%	0.07%	0.00%	-0.01%
11	-0.18%	2.00%	-0.22%	0.60%	1.44%
12	-0.12%	1.60%	-0.14%	0.48%	1.14%
13	0.18%	0.40%	0.22%	0.12%	0.24%
14	-0.06%	0.00%	-0.07%	0.00%	0.01%
Cumulative	-0.16%	4.20%	-0.14%	1.32%	2.86%

Table 1-41: Contributions to return

Table 1-42 contains the necessary management effects, using the BHB-Method, to decompose the contributions to active risk.

Month	$AAE_{p,Bonds}$	$AAE_{p,Equities}$	$SPE_{p,Bonds}$	$SPE_{p,Equities}$	$IAE_{p,Bonds}$	$IAE_{p,Equities}$	$r_p - r_b$
1	-0.01%	-0.12%	0.02%	0.02%	0.00%	0.02%	-0.07%
2	-0.02%	-0.36%	0.04%	0.06%	0.00%	0.06%	-0.22%
3	0.00%	0.12%	0.00%	-0.02%	0.00%	-0.02%	0.08%
4	0.02%	0.24%	-0.04%	-0.04%	0.00%	-0.04%	0.14%
5	0.01%	0.36%	-0.02%	-0.06%	0.00%	-0.06%	0.23%
6	0.02%	-0.12%	-0.05%	0.02%	0.01%	0.02%	-0.10%
7	-0.01%	0.00%	0.02%	0.00%	0.00%	0.00%	0.01%
8	0.00%	0.36%	0.00%	-0.02%	0.00%	-0.06%	0.28%
9	-0.05%	-0.36%	0.04%	0.02%	-0.01%	0.06%	-0.30%
10	-0.02%	0.00%	0.02%	0.00%	-0.01%	0.00%	-0.01%
11	0.07%	1.80%	-0.05%	-0.10%	0.02%	-0.30%	1.44%
12	0.05%	1.44%	-0.04%	-0.08%	0.01%	-0.24%	1.14%
13	-0.07%	0.36%	0.05%	-0.02%	-0.02%	-0.06%	0.24%
14	0.02%	0.00%	-0.02%	0.00%	0.01%	0.00%	0.01%
Cumulative	0.02%	3.72%	-0.04%	-0.22%	0.00%	-0.62%	2.86%

Table 1-42: Contribution to return and management effects using BHB-Method

Table 1-43 contains the summary data for the risk attribution decomposing the standard deviation of an investment portfolio and its benchmark as well as the contributions to tracking error standard deviation due to the different management effects.

Assets	CSD_p	CSD_b	$CTESD_p$	$CTESD_p^{AAE}$	$CTESD_p^{SPE}$	$CTESD_p^{IAE}$
Bonds	-0.166%	-0.030%	0.039%	0.082%	-0.064%	0.021%
Equities	2.376%	0.731%	1.607%	2.082%	-0.127%	-0.347%
Total assets	2.209%	0.701%	1.646%	2.164%	-0.191%	-0.326%

Table 1-43: Contribution to risk and management effects using BHB-Method

Table 1-43 shows that the investment portfolio had a higher annualized standard deviation (+2.209%) mainly due to the overweight in equities in the second half of the measurement period. Most of the tracking error standard deviation is coming from the active decisions in equities (+1.607%) which leads to a positive total cumulative asset allocation effect of +3.74% (= +0.02% + 3.72%) as shown in Table 1-42.

1.3.3.3 Multi-factor or regression-based risk attribution*

A multi-factor or regression-based risk attribution explains the risk of an absolute oriented investment portfolio by exposures to different factors, for instance market, size, and style factors and by stock specific risk. For an investment portfolio managed against a specific benchmark, risk attribution describes the active risk by factor exposures (different from the benchmark) and the stock specific risk.

Using multi-factors to explain the return of an investment portfolio leads to:

$$
\begin{aligned}
r_p &= \sum_{i=1}^{n} w_{p,i} \times r_{p,i} = \sum_{i=1}^{n} w_{p,i} \times \left(\sum_{l=1}^{m} b_{p,i,l} \times F_l + \varepsilon_{p,i} \right) \\
&= \sum_{i=1}^{n} w_{p,i} \times \sum_{l=1}^{m} b_{p,i,l} \times F_l + \sum_{i=1}^{n} w_{p,i} \times \varepsilon_{p,i} \\
&= \sum_{i=1}^{n} \sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times F_l + \sum_{i=1}^{n} w_{p,i} \times \varepsilon_{p,i} \,.
\end{aligned}
$$

With: $r_i = b_{p,i,1} \times F_1 + b_{p,i,2} \times F_2 + \cdots + b_{p,i,m} \times F_m + \varepsilon_{p,i} = \sum_{l=1}^{m} b_{p,i,l} \times F_l + \varepsilon_{p,i}$.

Where: $b_{p,i,l}$ = Sensitivity of the returns of portfolio component i to factor l,
F_l = Return of factor l,
$\varepsilon_{p,i}$ = Non-factor or specific return of a portfolio component i.

Calculating the variance of an investment portfolio by using factor returns yields:[73]

$$
\begin{aligned}
Var_p &= \sigma^2\left(\sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times F_l + \sum_{i=1}^{n} w_{p,i} \times \varepsilon_{p,i}\right) \\
&= \sigma^2\left(\sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times F_l\right) + \sigma^2\left(\sum_{i=1}^{n} w_{p,i} \times \varepsilon_{p,i}\right) \\
&= \sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times Cov\left(F_l, r_p\right) + \sum_{i=1}^{n} w_{p,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) \\
&= \sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times Corr\left(F_l, r_p\right) \times \sigma_l \times \sigma_p + \sum_{i=1}^{n} w_{p,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .
\end{aligned}
$$

Where: $Cov(F_l, r_p)$ = Covariance of the returns of a factor l and the returns of a portfolio,
$Corr(F_l, r_p)$ = Correlation of the returns of a factor l and the returns of a portfolio,
σ_l = Standard deviation of the returns of factor l.

The contribution to variance of an individual portfolio component is calculated as follows:

$$CVar_{p,i} = w_{p,i} \times \sum_{l=1}^{m} b_{p,i,l} \times Corr\left(F_l, r_p\right) \times \sigma_l \times \sigma_p + w_{p,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

Dividing variance by the standard deviation yields the standard deviation of the investment portfolio:

$$\sigma_p = \frac{Var_p}{\sigma_p} = \sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times Corr\left(F_l, r_p\right) \times \sigma_l + \frac{1}{\sigma_p} \times \sum_{i=1}^{n} w_{p,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

The contribution to standard deviation of an individual portfolio component is calculated as follows:

$$CSD_{p,i} = w_{p,i} \times \sum_{l=1}^{m} b_{p,i,l} \times Corr\left(F_l, r_p\right) \times \sigma_l + \frac{1}{\sigma_p} \times w_{p,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

If interested in the active risk of an investment portfolio managed against a specific benchmark, we use an approach similar to the one described in **chapter 1.3.3.1.2**. Using multi factors to explain the excess return of an investment portfolio (over benchmark return) leads to:[74]

73 Here we assume that the specific returns $\varepsilon_{p,i}$ are not correlated amongst each other, that the expected specific return $E(\varepsilon_{p,i})$ equals zero, and that the specific returns $\varepsilon_{p,i}$ are independent from the factor returns F_l.

74 Because of the bottom-up analysis, we implicitly assume that $b_{p,i,l} = b_{b,i,l}$ and $\varepsilon_{p,i} = \varepsilon_{b,i}$.

$$r_p - r_b = \sum_{i=1}^{n} \left(w_{p,i} \times r_{p,i} - w_{b,i} \times r_{b,i}\right) =$$
$$= \sum_{i=1}^{n}\sum_{l=1}^{m} w_{p,i} \times b_{p,i,l} \times F_l + \sum_{i=1}^{n} w_{p,i} \times \varepsilon_{p,i} - \sum_{i=1}^{n}\sum_{l=1}^{m} w_{b,i} \times b_{b,i,l} \times F_l$$
$$- \sum_{i=1}^{n} w_{b,i} \times \varepsilon_{b,i} = \sum_{i=1}^{n}\sum_{l=1}^{m} (w_{p,i} - w_{b,i}) \times b_{p,i,l} \times F_l + \sum_{i=1}^{n} (w_{p,i} - w_{b,i}) \times \varepsilon_{p,i} \,.$$

Where: $b_{b,i,l}$ = Sensitivity of the returns of benchmark component i to the factor l,
$\varepsilon_{b,i}$ = Non-factor or specific return of a benchmark component i.

Calculating the tracking error variance of an investment portfolio by using factor returns yields:

$$TEVar_p = \sigma^2\left(\sum_{i=1}^{n}\sum_{l=1}^{m} (w_{p,i} - w_{b,i}) \times b_{p,i,l} \times F_l + \sum_{i=1}^{n} (w_{p,i} - w_{b,i}) \times \varepsilon_{p,i}\right)$$
$$= \sigma^2\left(\sum_{i=1}^{n}\sum_{l=1}^{m} w_{A,i} \times b_{p,i,l} \times F_l\right) + \sigma^2\left(\sum_{i=1}^{n} w_{A,i} \times \varepsilon_{p,i}\right)$$
$$= \sum_{i=1}^{n}\sum_{l=1}^{m} w_{A,i} \times b_{p,i,l} \times Cov(F_l, r_A) + \sum_{i=1}^{n} w_{A,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right)$$
$$= \sum_{i=1}^{n}\sum_{l=1}^{m} w_{A,i} \times b_{p,i,l} \times Corr(F_l, r_A) \times \sigma_l \times TESD_p + \sum_{i=1}^{n} w_{A,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

Where: $w_{A,i}$ = Active weight of portfolio component i,
$Cov(F_l, r_A)$ = Covariance of the returns of a factor l and the excess returns of a portfolio against its benchmark,
$Corr(F_l, r_A)$ = Correlation of the returns of a factor l and the excess returns of a portfolio against its benchmark.

The contribution to tracking error variance of an individual portfolio component is calculated as follows:

$$CTEVar_{p,i} = w_{A,i} \times \sum_{l=1}^{m} b_{p,i,l} \times Corr(F_l, r_A) \times \sigma_l \times TESD_p + w_{A,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

Dividing tracking error variance by the tracking error standard deviation yields the tracking error standard deviation of the investment portfolio:

$$TESD_p = \frac{TEVar_p}{TESD_p} = \sum_{i=1}^{n}\sum_{l=1}^{m} w_{A,i} \times b_{p,i,l} \times Corr(F_l, r_A) \times \sigma_l + \frac{1}{TESD_p} \times \sum_{i=1}^{n} w_{A,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

The contribution to tracking error standard deviation of an individual portfolio component is calculated as follows:

$$CTESD_{p,i} = w_{A,i} \times \sum_{l=1}^{m} b_{p,i,l} \times Corr(F_l, r_A) \times \sigma_l + \frac{1}{TESD_p} \times w_{A,i}^2 \times \sigma^2\left(\varepsilon_{p,i}\right) .$$

1.4 Performance presentation*

1.4.1 Introduction to performance presentation*

Performance presentation is a sub-type of reporting. In general terms, it is the presentation of information and more specifically when looking at the asset management industry, it is the presentation of investment performance information. Performance presentation is also a specific type of investment reporting that focuses on the illustration of the performance of investment portfolios over some specified measurement period.

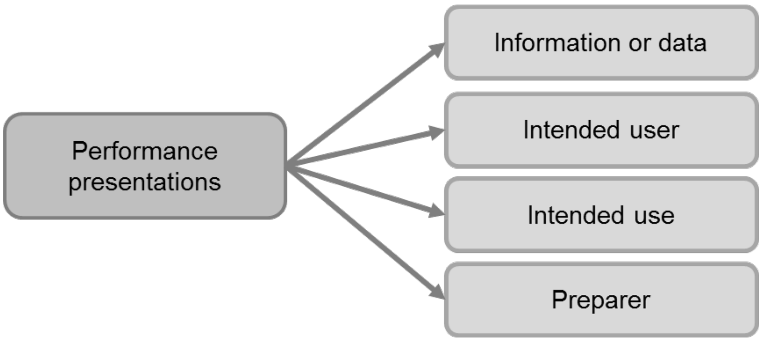


Figure 1-29: Factors that determine design and content of performance presentations

Depending on the intended use and user, performance presentations may contain different types of information and analytics. Because of the variety of investment products and information needs, there are many ways to define and design a performance presentation. Figure 1-29 illustrates the factors that determine the actual content and design of performance presentations:

- **Information or data**. The information or data presented covers all kinds of investment performance information such as absolute gain and loss figures, absolute or relative return and risk figures, ex ante and ex post performance analytics, comparative performance figures of a portfolio versus its benchmark, composite, and peer group, or management effects explaining the value added or active risk of an investment portfolio.
- **Intended user**. The users of performance presentations are manifold and encompass all participants and stakeholders of the portfolio management process, such as prospective and existing clients, risk and compliance officers, portfolio managers, members of the investment committee, senior management, investment consultants, regulators, or tax authorities.
- **Intended use**. The use and the purpose of performance presentations are manifold and drive to a great extent the content and design of the presentation. Examples are the identification of investment skills, the measurement of the sources of absolute or relative performance of an investment portfolio, the monitoring of the implementation of an investment strategy or agreed risk guidelines, the performance review of investment products, or peer group comparisons.
- **Preparer**. The preparers of performance presentations are manifold and cover for example the performance or risk management department within an asset management company, the investment reporting department within a custodian, investment consultant, investment controller, or the investor.

1.4.2 Types of performance presentation*

We can group performance presentations along the lines of their end use. Figure 1-30 illustrates the main types of these performance presentations: a) sales-oriented presentations, b) management-oriented presentations, and c) monitoring-oriented presentations.

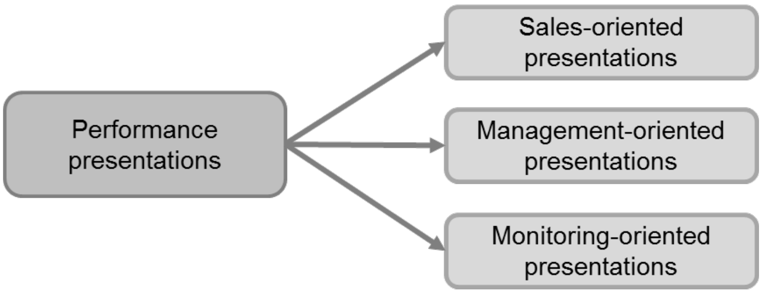


Figure 1-30: Types of performance presentation

Sales-oriented performance presentations are used in the sales process. Based on the sub-processes of the sales process, we distinguish between pre-sales, time-of-sales and after-sales performance presentations.

Pre-sales performance presentations contain general information about the asset management company and aggregated or high-level performance information about specific investment products. These reports provide performance information based on time-series analysis, such as rolling return and risk figures, as well as high-level data and information on the holdings or product. They allow the prospective client to get a better sense of the investment strategy and investment product under consideration. The purpose of pre-sales presentations is to attract prospective clients to step into a detailed discussion on the product offerings. Examples for such performance presentations are a GIPS Standards compliant presentation or a fund factsheet.

Time-of-sales performance presentations cover more detailed performance information than the pre-sales presentations. The goal is to provide the prospective client with additional performance information allowing him to get an even better insight on the investment products of interest. These reports provide detailed data and information on return and risk characteristics using a performance attribution or a risk analysis. The purpose of time-of-sales presentations is to convince the prospective client that the considered investment product fits his needs, requirements, and expectations.

After-sales performance presentations contain detailed performance information not on an investment product but on the actual client's investment portfolio. The performance presentation is often part of a more comprehensive investment reporting. These reports contain specific information needed by the client to understand in an efficient and effective way the past performance of the individual investment portfolio. Normally, investment reporting covers different data and information about the investment portfolio such as list of holdings, list of transactions, asset allocation, gain and loss overview, cash flow analysis, rolling performance time series analytics or detailed return and risk attribution. The purpose of after-sales presentations is to provide information to existing clients, so that they get a good understanding of the investments made, the results achieved, the risk taken, and the incurred costs and taxes. As a monitoring tool, the after-sales presentations are also used to check whether the proposed investment strategy was actually implemented and to illustrate the quality of the respective portfolio management decisions.

Management-oriented performance presentations are used in the different sub-processes of the portfolio as well as the overall management process. Performance analytics are used in every step of the decision making process to support management decisions by providing performance data and information on the investment product or portfolio under consideration. Within the portfolio management process, one can use different performance information depending on the actual management activity. Examples are the analysis of consequences on expected return and risk due to changes in the investment strategy or the portfolio's asset allocation, or the comparison of return and risk characteristics of an investment product with those of relevant products from competitors. The purpose of management-oriented performance presentations is to support and help the participants of the portfolio management process to assess past investment performance and to make decisions to improve future investment performance.

In addition to this type of management-oriented presentations, there are other performance presentations used in the overall management process. By contrast, the focus here is different and it is often not an individual investment portfolio or product that is of interest. From a top down perspective, senior management needs management information on the performance or quality of the whole asset management organization, individual departments (for instance fixed income, equity or multi-asset class portfolio management), individual portfolio management teams or individual portfolio managers. The performance information provided here should be useful to identify whether the organizational unit is on track to meet agreed objectives or whether corrective measures are necessary. Useful tools to get an overview of the investment performance of organizational units are composite performance reports containing time series performance analytics of groups of portfolios with similar characteristics. Examples of this are aggregated composite reports and peer group comparisons or a detailed performance analysis used as a basis for a performance review of an investment product on the watch list. The purpose of management-oriented performance presentations is to support and help senior management of asset management companies to make decisions to assess past performance and to improve future performance of the whole organization to meet agreed business targets.

Monitoring-oriented performance presentations are used in different sub-processes of the overall monitoring process. Nowadays, it is common practice to define specific investment guidelines or restrictions and certain regulations for investment portfolios and products, including those related to performance characteristics. These restrictions may refer to return and especially to risk figures like absolute return target, value-at risk, ex-ante tracking error, or ex-ante contributions to volatility. The purpose of monitoring-oriented performance presentations is to show that the investment portfolio or strategy was and is in compliance with investment policies or guidelines and regulatory requirements. Furthermore, monitoring-oriented presentations have a positive side effect in that they increase transparency on activities as well as return and risk contributions resulting from individual decisions, which may not be very apparent otherwise.

The three different types of performance presentations vary depending on the intended user and the intended use, but they all share the same overall purpose: to create or increase transparency relating to the investments made, the results achieved, the risk taken, and the costs and taxes incurred. Therefore, performance presentations are an important controlling tool for the participants and stakeholders of the portfolio management process, with the presented figures and analytics as well as their respective interpretation being the basis for discussions and decisions to improve future investment performance.

1.4.3 Best practice for performance presentations*

1.4.3.1 Global Investment Performance Standards*

The Global Investment Performance Standards (GIPS Standards) are globally accepted best practices and industry standards for the presentation of investment performance to prospective asset management clients.[75] The objectives of the GIPS Standards are to increase transparency, to provide disclosure, to ensure consistency and to enable comparability with respect to marketing of investment products and especially to investment performance presentations.

The case for the GIPS Standards is best explained by looking at Figure 1-31. Assume that an investor is doing a manager search and that, as a starting point, he asks two asset management companies to send him the performance track record for a specific investment product. Figure 1-31 illustrates what he, in the extreme, gets back from the marketing officers: just a single return number. If the investor only looks at the two annualized return figures, which are here an identical 5.0%, it will be difficult for him to decide which portfolio manager to choose. To draw proper conclusions and come up with a final decision, he needs a lot of additional data and information. If he considers only the return figures, the investor will not be able to draw meaningful conclusions. Indeed, amongst other things, it is not obvious:

- Where the return figures came from – a model portfolio, a GIPS composite, a representative portfolio, the best performing investment portfolio, or the largest investment portfolio.
- Whether the return figures belong to investment portfolios managed by the respective asset management company or to a track record of a specific portfolio manager produced while working for a former employer.
- Whether the return figures are gross- or net-of-fees returns and whether different fees are reflected within the return calculation.
- What the underlying reporting period is – since inception, last 12 months, or the best historical performance period.
- What the reference currency is.
- How the return figures were calculated – as a money-weighted rate of return, as a time-weighted rate of return and, if the latter, what approximation method was used
- What the investment strategy or the investment objective was.
- Whether the investment portfolio was managed against a specific benchmark and if so, what the definition of the benchmark is.

Portfolio manager A	Portfolio manager B
5%	5%

Figure 1-31: Simple performance track records

75 Please see the current version of the GIPS Standards, which is available on the internet.

Without answers to these questions, an investor is hardly able to compare the different asset management companies, respectively their performance track records. „Cherry Picking", i.e. the intentional selection of a specific investment portfolio or observation period, the use of sample portfolios, model portfolios or simulations, the transfer of performance track records as well as the use of not standardized methodologies for calculating performance figures can lead to the issue that performance track records of different asset management companies may not be comparable. This was the main rationale and reason for the development of a unique set of rules to guide the asset management industry how to calculate and present investment performance to prospective clients.

Let us illustrate the main issue in presenting performance to prospective clients, i.e. the determination of the performance track record. We will use a fictitious case of a sample asset management company XYZ. In the example, a prospective client asks a marketing officer to present the historical performance track record for a specific investment product ABC. As shown in Figure 1-32 the asset management company XYZ managed three portfolios in this product category for the period Year 1 until Year 3. Portfolio C was terminated at the end of Year 1 and portfolio B closed at the end of Year 2. Only portfolio A was managed for the entire period from Year 1 to Year 3. Besides the portfolio returns, Figure 1-32 also shows the assets under management as of the beginning of each year. The question is now which performance track record should be shown to the prospective client.

Year	Year 1	Year 2	Year 3
Portfolio A			
Return	6.2%	-2.0%	4.2%
Beginning assets	10.0	10.6	10.4
Portfolio B			
Return	5.0%	-3.3%	
Beginning assets	100.0	105.0	
Portfolio C			
Return	4.1%		
Beginning assets	500.0		

Figure 1-32: Data input for determination of a performance track record

There are several possibilities to determine a performance track record for an asset management company:

- Possibility 1: Sample or representative portfolio.
- Possibility 2: Portfolio with the longest performance history.
- Possibility 3: Equally weighted average of the returns for all actual portfolios.
- Possibility 4: Equally weighted average of the returns for all portfolios ever managed.
- Possibility 5: Asset-weighted average of the returns for all actual portfolios.
- Possibility 6: Asset-weighted average of the returns for all portfolios ever managed.
- Possibility 7: Model portfolio or model strategy.
- Possibility 8: Portfolio with the best performance history.

In the sample case, the marketing officer decides to follow possibility 3, which seems to be reasonable at first sight. The indexed cumulative return in Figure 1-33 illustrates this performance track record which in our case is identical to the performance track record of investment portfolio A.

The next question that arises is whether a prospective client is able to assess the quality of the asset management company by using the performance track record shown in Figure 1-33. The answer is clearly "no" because the performance of an asset management company is normally not identical with the performance of the actual managed investment portfolios but with the performance of all investment portfolios ever managed. Neglecting the terminated investment portfolios in determining the performance track record results in an effect called “survivorship bias”. Survivorship bias, i.e. the survivorship of the best, means in this context that neglecting the terminated portfolios is identical to neglecting the badly performing portfolios since badly performing portfolios are normally closed while portfolios with good performance survive.

In our example, i.e. possibility 3, the performance track record is becoming better over time because portfolios C and B, which on average performed worse, were terminated at the end of Year 1 and Year 2. The performance track record was thus improved by their omission. Incorporating the terminated portfolios, as in possibility 4, would result in a cumulative return for the period Year 1 to Year 3 of +6.61% in comparison to +8.45% for possibility 3.

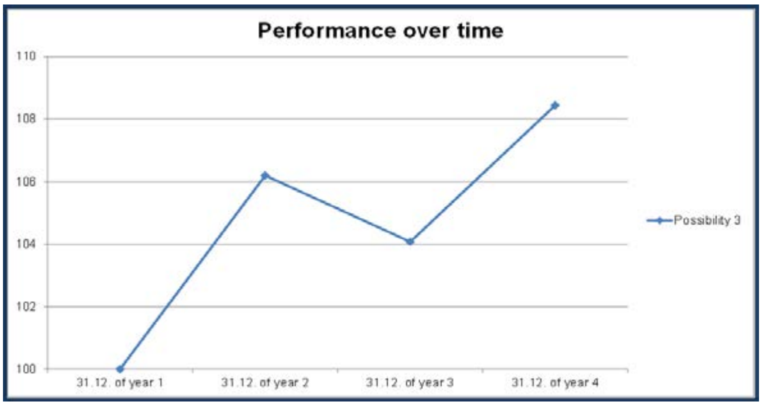


Figure 1-33: Performance track record according to possibility 3

Possibility 4 also seems to be a reasonable one to determine a performance track record. However, it does not consider the assets under management of the different underlying portfolios A, B, and C over time. The returns of the different portfolios are equally weighted which results in a bias favoring the smaller portfolios. In our example in Year 1 the performance track record benefits from the equal weighting of the large portfolio C which had a relatively bad return of 4.10%. Taking into consideration the assets under management of the different portfolios over time, as in possibility 6, would result in the performance track record shown in Figure 1-34. The performance track record of possibility 6 is not as good as that of possibility 4 because the badly performing portfolios C and B had quite a high weight and contribution to the overall performance in the first two years. The cumulative return for possibility 6 is +5.21%.

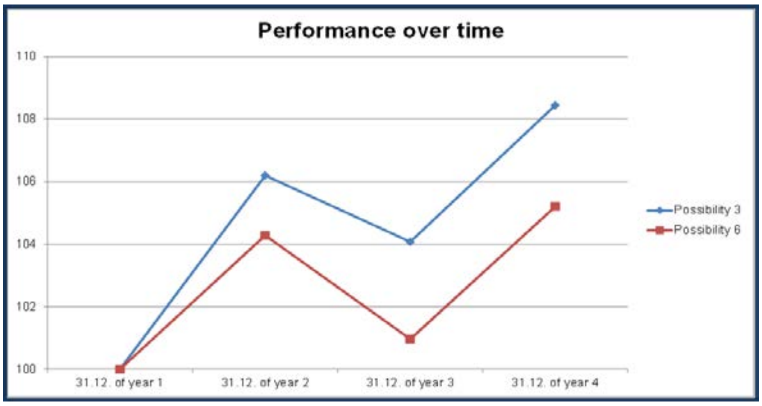


Figure 1-34: Performance track record according to possibility 6

Determining a performance track record using possibility 6 follows the principle of the GIPS Standards, namely to show the performance of an asset-weighted aggregation or composite of comparable investment portfolios. The question that arises now is, whether a performance track record, such as the one presented in Figure 1-34, is sufficient to assess the quality of a portfolio manager. The answer is again clearly "no". An observer needs more information on the portfolio manager and the investment product. This information can range from the benchmark return, the number of portfolios managed in such a product, the assets under management, the calculation method used, to the fee structure. Figure 1-35 illustrates a simple performance presentation covering some of the basic performance information needed to ensure a good starting point to evaluate the performance and quality of an asset management company.

	Composite return	Benchmark return	Number of portfolios	Assets of composite
Year 3	4.2%	4.8%	1	10.4
Year 2	-3.2%	-1.5%	2	115.6
Year 1	4.3%	3.3%	3	610.0

Figure 1-35: Simple performance presentation

By contrast, Figure 1-36 shows a sample performance presentation for a balanced growth composite, which complies with the GIPS Standards and includes the minimum information required by the GIPS Standards. This basic information on the performance for a specific product or composite enables the prospective client to concentrate on meaningful questions and to avoid unnecessary ones.

Sample 1
Investment Firm
Balanced Growth Composite
1 January 2002 through 31 December 2011

Year	Composite Gross Return (%)	Composite Net Return (%)	Custom Benchmark Return (%)	Composite 3-Yr St Dev (%)	Benchmark 3-Yr St Dev (%)	Number of Portfolios	Internal Dispersion (%)	Composite Assets ($ M)	Firm Assets ($ M)
2002	–10.5	–11.4	–11.8			31	4.5	165	236
2003	16.3	15.1	13.2			34	2.0	235	346
2004	7.5	6.4	8.9			38	5.7	344	529
2005	1.8	0.8	0.3			45	2.8	445	695
2006	11.2	10.1	12.2			48	3.1	520	839
2007	6.1	5.0	7.1			49	2.8	505	1,014
2008	–21.3	–22.1	–24.9			44	2.9	475	964
2009	16.5	15.3	14.7			47	3.1	493	983
2010	10.6	9.5	13.0			51	3.5	549	1,114
2011	2.7	1.7	0.4	7.1	7.4	54	2.5	575	1,236

Sample 1 Investment Firm claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. Sample 1 Investment Firm has been independently verified for the periods 1 January 2000 through 31 December 2010. The verification report is available upon request. Verification assesses whether (1) the firm has complied with all the composite construction requirements of the GIPS standards on a firm-wide basis and (2) the firm's policies and procedures are designed to calculate and present performance in compliance with the GIPS standards. Verification does not ensure the accuracy of any specific composite presentation.

Notes:

1. Sample 1 Investment Firm is a balanced portfolio investment manager that invests solely in U.S.-based securities. Sample 1 Investment Firm is defined as an independent investment management firm that is not affiliated with any parent organization. Policies for valuing portfolios, calculating performance, and preparing compliant presentations are available upon request.
2. The Balanced Growth Composite includes all institutional balanced portfolios that invest in large-cap U.S. equities and investment-grade bonds with the goal of providing long-term capital growth and steady income from a well-diversified strategy. Although the strategy allows for equity exposure ranging between 50–70%, the typical allocation is between 55–65%. The account minimum for the composite is $5 million.
3. The custom benchmark is 60% YYY U.S. Equity Index and 40% ZZZ U.S. Aggregate Bond Index. The benchmark is rebalanced monthly.
4. Valuations are computed and performance is reported in U.S. dollars.
5. Gross-of-fees returns are presented before management and custodial fees but after all trading expenses. Composite and benchmark returns are presented net of non-reclaimable withholding taxes. Net-of-fees returns are calculated by deducting the highest fee of 0.83% from the monthly gross composite return. The management fee schedule is as follows: 1.00% on the first $25 million; 0.60% thereafter.
6. This composite was created in February 2000. A complete list of composite descriptions is available upon request.
7. Internal dispersion is calculated using the equal-weighted standard deviation of annual gross returns of those portfolios that were included in the composite for the entire year.
8. The three-year annualized standard deviation measures the variability of the composite and the benchmark returns over the preceding 36-month period. The standard deviation is not presented for 2002 through 2010 because monthly composite and benchmark returns were not available and is not required for periods prior to 2011.

Figure 1-36: Sample GIPS compliant performance presentation

In summary, the starting point of a performance evaluation should always be a GIPS compliant performance presentation because composite presentations:

- Enhance transparency and understanding of the performance measurement methods used and the performance presentation itself.
- Avoid cherry-picking of investment portfolios or time periods.
- Increase the comparability of different investment products and asset managers.
- Improve the ability to assess the quality of asset management companies or of specific investment products or composites.
- Enable objective and fact-based discussions on the performance and therefore enable the observer to focus on the essential issues.

The advantages of analyzing a performance presentation, which complies with the GIPS Standards, are not only for the benefit of the investor but also for the benefit of the senior management within the asset management companies. The reason for this is that, if a discussion or argument about investment performance arises, a senior manager is often in the same situation as a prospective client and needs to ask many unnecessary questions. Therefore, effective performance evaluation should be based on composites maintained according to the GIPS Standards. Enabling a meaningful evaluation of the performance and the quality of asset management companies and investment products is the main benefit for the client and the asset management company arising from GIPS compliant performance presentations.

1.4.3.2 Principles for Investment Reporting*

Investment or performance reporting to existing or to internal clients, for instance senior management, is often not sufficiently transparent to allow the observer or user to understand the investment information provided.[76] Similar to the GIPS Standards, the Principles for Investment Reporting (PIR) address this lack of transparency by defining principles for designing and preparing effective investment reporting or performance presentations.

PIR defines five principles for investment reporting that must be followed. It also recommends some additional aspects to ensure effective investment reporting:[77]

- **Communication occurs between the preparer and the user as to the purpose of and need for investment reporting**. Based on this principle, effective investment reporting reflects the following qualities:
  - Active communication occurs between the preparer and the user of the investment report, and all decisions about content of the report from this communication are documented by the preparer in, or as part of, a client agreement that is reviewed periodically by both parties.
  - The purpose of the investment report and the reasons for its content and production are transparent and clearly stated.
  - The investment report provides information on changes in the investment strategy or investment style.
- **Control processes, policies, and procedures are documented and followed**. Based on this principle, effective investment reporting reflects the following qualities:
  - The investment report is timely and accurate.

76 In this chapter, we use investment reporting and performance presentation interchangeably.

77 Please see pages 6 and 7 of the "Principles for Investment Reporting", first edition, published in 2013, which is available on the internet.

- The investment report complies with all applicable laws and regulations.
- Data quality is defined and managed according to transparent criteria, and control processes are defined, documented, and made available on request.
- Methodologies used for the production of the data in the report are disclosed or made available upon request.
- The preparer consistently adheres to the processes that are documented and makes them available upon request.
- The preparer follows an error-correction policy, which is available upon request, discloses material errors affecting information included on prior investment reports, and the definition of "material" is agreed with the user.
- The preparer has a policy for handling potential conflicts of interests—especially concerning the provision of investment information to third parties to whom this information may be advantageous. Where conflicts of interest exist, they are declared. Segregation of duties is sufficient that a fair and accurate representation of the assets to be reported occurs.
- Production and control processes are periodically reviewed.

- **Client preferences are reflected in the investment report Documentation.** Based on this principle, effective investment reporting reflects the following qualities:
  - The design of the investment report reflects what the preparer and user agreed.
  - The intended user or audience of the investment report and the expected use of the information are considered when the preparer designs the report.

- **Clear and transparent presentation of investment risks and results.** Based on this principle, effective investment reporting reflects the following qualities:
  - Historical information presented in the investment report is not changed without disclosure to the user.
  - The investment report is a fair representation of the investments made, results achieved, risks taken, and costs incurred.
  - The investment report is relevant and appropriate for the purpose stated and the assets and investment strategies being presented.
  - The investment report provides the user of the report with appropriate comparative data—such as index data, a customized benchmark, peer group data, or a GIPS composite—to allow the user to assess the relative performance of the investments.
  - The investment report provides information on investment risks that have been experienced and are expected, including changes to assumptions previously adopted.
  - The impact of taxes in general and the impact of taxes on performance are, where germane, reflected in the investment report.

- **Comprehensive fee disclosure**. Based on this principle, effective investment reporting reflects the following qualities:
  - The investment report is transparent regarding the fees and remuneration (e.g., commissions, referral fees) to be received by the preparer of the investment report and by third parties, such as custodians, investment management companies, or consultants, relating to the management or administration of the assets being reported.

Performance evaluation should be effective and therefore based on investment reporting or performance presentations that fulfill the principles and recommendations of the PIR. The PIR are a set of rules that, if followed, helps the user of reports to save time and resources as well as to better understand the information presented, and therefore to better qualify the investment performance of a portfolio manager.

1.5 Investment controlling*

1.5.1 Definition and outline of investment controlling*

Nowadays, it is more and more important for an asset management company to have efficient and appropriate management information on the performance of their discretionary managed portfolios. Without decision-oriented information on the performance or quality of its products and portfolio managers, a specific asset management company will find it more and more difficult to withstand the current and future challenges of the asset management industry.[78]

Investment controlling is an area of activity that is part of the overall controlling function within the asset management company and is an important component of the recurring portfolio management process. In general, investment controlling is defined as independent monitoring of the quality of investment portfolios and products to ensure that agreed upon objectives are reached. The task of investment controlling is to gather, process, check and distribute investment information necessary to monitor and support the overall business objectives. In this respect, the objective of investment controlling is to configure and implement the infrastructure – particularly within the framework of the portfolio management process – in such a way that the processes (e.g. forecasting, decision making and implementation), the quality and the results (e.g. returns), the risks (e.g. of using derivatives) and the costs or taxes become more transparent and comprehensible. Performance evaluation is part of investment controlling because it produces, analyzes, presents, and interprets the investment performance information.[79]

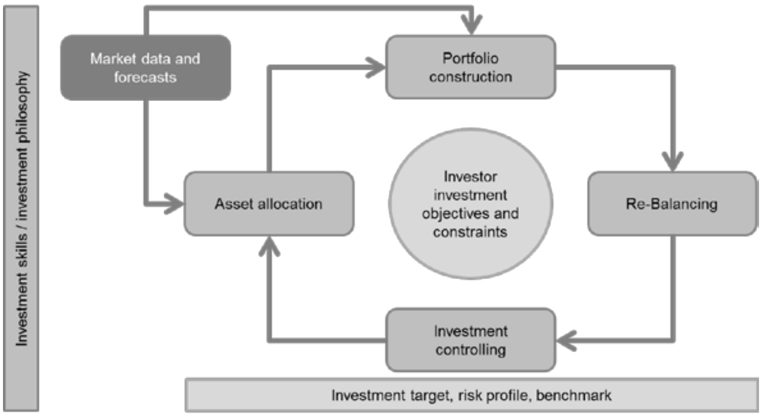


Figure 1-37: Investment controlling as a part of the portfolio management process

As presented in the Figure 1-37 investment controlling is an integrated part of the portfolio management process. It constitutes the last step of this process and analyses the result of the overall process, the investment performance, but also the decisions, activities and circumstances relevant for that result. Figure 1-38 illustrates that, besides performance evaluation, investment controlling may in addition encompass other controlling-related activities like compliance monitoring or risk management.[80]

78 In the following, we focus on asset management companies but the concept of investment controlling is of the same relevance for investors and consultants because they have similar controlling and monitoring needs.

79 Performance evaluation is often even called investment controlling or vice versa.

80 In the following, we focus on performance evaluation as a sub-process of investment controlling.

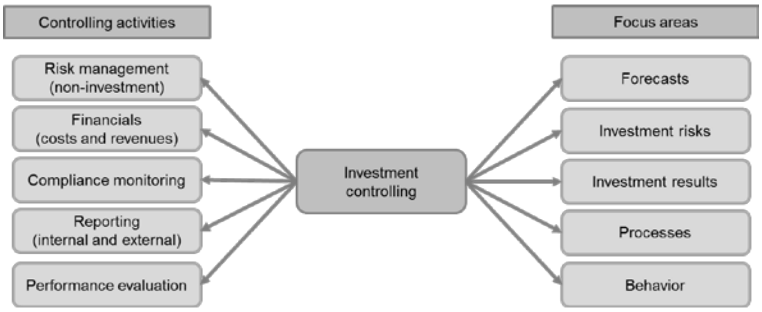


Figure 1-38: Controlling activities and focus areas of investment controlling

Furthermore, Figure 1-38 shows the different focus areas of investment controlling:

- **Forecasts that are the basis for investment decisions**: for example long-term, mid-term or short-term expectations, expected return and risk of investment strategies and instruments (absolute and relative performance figures), or expected correlations between investment alternatives.
- **Investment risks taken through implementation of investment strategies and investment in financial instruments**: for example ex ante (forward looking) and ex post (historical) investment risk, absolute and relative risk (like volatility, value at risk or tracking error), the sources of the absolute or relative risk, different types of investment risk (market risk, credit risk or liquidity risk), different types of risk factors (like share prices, currencies, interest rates or commodities), or portfolio, benchmark, composite, and peer group risk.
- **Investment results achieved**: for example ex ante (forward looking) and ex post (historical) investment results, absolute and relative return, profit and loss, time- and money-weighted rate of returns, the sources of the absolute or relative return, impact of fees and taxes, or portfolio, benchmark, composite, and peer group performance.
- **Processes and procedures concerned with portfolio management**: all kinds of internal (sub-) processes in portfolio management (as illustrated in Figure 1-37) and related aspects like policies and procedures, investment restrictions, models used for risk management or portfolio construction, best execution, investment strategies and styles, or operational issues and risks.
- **Behavior of the people involved in portfolio management**: for example, approach to and attitude towards transparency and disclosure, handling of error correction, rules and policy for conflicts of interest, attitude towards segregation of duties, approach to communication (proactive or reactive), compensation of portfolio managers (both in terms of performance-based fees and remuneration of individual employees), or adherence to industry best practices.

The objective of investment controlling is to increase transparency on the portfolio management processes, and on the different drivers and their impact on the quality of investment portfolios and products. This is of great value for the portfolio management because it is the basis for discussions on corrective measures and on how to improve future investment performance.

Therefore, the purpose and the objectives of investment controlling are manifold and include the following:

- To produce independent analysis of investment portfolios or products.
- To improve the understanding of the different activities and decisions within the portfolio management process and their contributions to investment risk and return.
- To increase the focus on relevant topics through factual and target-oriented performance analysis of the quality of the portfolio management.

- To reduce discussions that are not based on facts by using more objective and less subjective investment information during the performance review process.
- To provide client-specific or tailor-made analytics which reflect the actual portfolio management process and client needs.
- To produce in-depth analysis to identify the real drivers of the investment risk and return.
- To monitor risk and return against their designated benchmark, limits and / or objectives.
- To improve the internal management information and monitoring system.
- To identify and address potential process issues to reduce unintended business risks at an early stage.
- To create a basis not only for ongoing analyses but also for structural changes in the portfolio management process.
- To implement industry best practices like the GIPS Standards or the Principles for Investment Reporting.
- To stimulate the dialogue between participants and stakeholders of the portfolio management process, which may lead to innovation, change in practices and a strengthened brand and reputation.

1.5.2 Generic performance evaluation process*

Based on the general definition of investment controlling, performance evaluation is an integrated part of investment controlling. The actual setup of the performance evaluation process depends on the historical development and the specific circumstances of the asset management company (for instance, assets under management, type of clients and client base, level of centralization within the organization, or number and types of investment portfolios or products), but in principle follows the generic process illustrated in Figure 1-39.

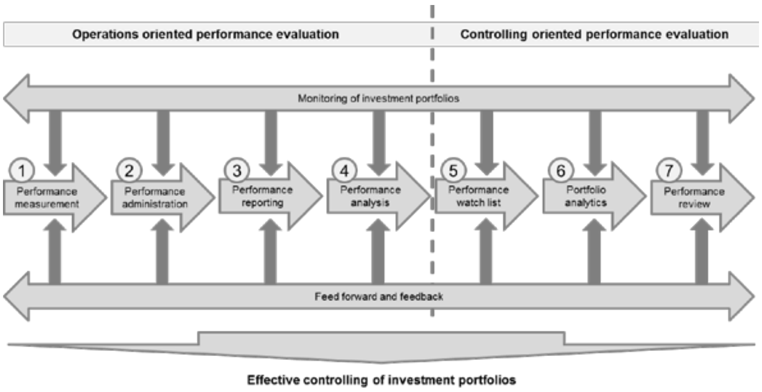


Figure 1-39: Generic performance evaluation process

The generic performance evaluation process is a recurring process, which uses input from but also produces feedback to the different participants and stakeholders of the portfolio management process. It consists of seven sub-processes, illustrated and marked from 1 to 7 in Figure 1-39. We differentiate between **operating processes**, covering performance measurement, performance administration, performance reporting, and performance analysis, and **controlling processes**, covering performance watch list, portfolio analytics and performance review.[81]

81 The controlling oriented sub-processes are often also called performance appraisal but then they focus more on the analysis of investment performance aspects and are minor considering the further aspects of investment controlling.

The first four steps are processing and data management oriented. They are used for calculating, maintaining or storing, analyzing and presenting various investment performance figures of a specific investment portfolio or product, a group of products, or even of a whole asset management company. The last three steps are less processing-oriented and instead provide qualitative statements on the portfolio management process and its results as well as how the investment performance was produced. The performance watch list identifies problematic investment portfolios or products that are subsequently analyzed by portfolio analytics and discussed in depth in the performance review. In this respect, working out proposals for improvement and pointing out possible consequences for the portfolio management process are the primary objectives of the controlling oriented analyses conducted in steps 5 to 7.

Although the arrow diagram in Figure 1-39 suggests that the sub-processes are sequential, in practice, this might not be the case because of feedback loops, interactions and variations in how organizations operate. Many performance evaluation activities are run on a monthly basis, although daily updates are becoming more common especially through the introduction of online tools (e.g., online reporting and online management information systems) and more powerful IT systems.

Furthermore, the performance evaluation process itself is part of a more comprehensive process covering additional processes and activities and, in the extreme, the whole asset management organization. Performance evaluation is dependent on processes that precede it such as data management or investment accounting, which provide the underlying data and information necessary for the individual performance evaluation activities. As a data and information provider, performance evaluation delivers investment performance information to subsequent processes, such as investment reporting to existing clients which delivers performance figures or analytics and is run by the operations department; the management fee calculation, based on absolute or excess returns of investment portfolios, which is run by the finance department, or the market research, for instance peer group comparisons, which is run by the marketing department.

In the following sections, we illustrate the performance evaluation process in order to get an overview on how the different sub-processes are linked together and to better understand, how performance evaluation fits into the general investment controlling framework.

Performance measurement

This first step of the performance evaluation process deals with all aspects of return and risk measurement, i.e. the calculation of all necessary return and risk measures or figures such as gross and net returns, time- and money-weighted rate of returns, risk figures such as volatility or tracking error and so on. Performance measurement normally focuses on the total investment portfolio level and is a time series analysis.

Performance administration

Performance administration normally covers the benchmark calculation and more importantly, the construction and maintenance of the composites. Composites are aggregates of investment portfolios with comparable investment strategies, investment objectives, and/or styles. In order to assess the asset management skills of one or more portfolio managers or even of a whole asset management organization, it is necessary to classify the different investment portfolios and group the portfolios with similar characteristics, for instance a benchmark, investment strategy, and/or style, to a composite. Constructing and maintaining composites according to the GIPS Standards may also be part of the performance administration process. In addition, complying with the GIPS Standards is often seen as the basis or even prerequisite for an effective investment controlling of an asset management company.

Performance reporting

This step of the performance evaluation process includes the presentation of different performance figures for specific time periods, normally on a total investment portfolio or product level. If an observer is interested in more detailed information on the sources of return and risk, he may start reviewing the performance figures on a total portfolio level and afterwards analyze a performance attribution provided by the next step of the performance evaluation process.

Figure 1-40 and Figure 1-41 are examples for a performance reporting presenting the performance of a global equity composite. The performance reports contain different performance figures and information useful to get a good overview of the performance history of the investment portfolio or composite under review.

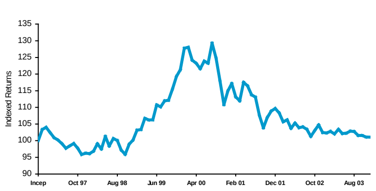
Equities World BM MSCI active Mandates direct

Benchmark	MSCI World (ri) in CHF	**Series Type**	Asset Weighted Gross Return	**Reporting Currency**	CHF
No. of A/ Cs	5	**Inception Date**	01 Jan 1997	**Market Value (m) End of Period**	84.27
Composite Code	ZU-COMP250	**Reporting Date**	31 Dec 2003		

	Periodical Returns in %		
	Composite	Benchmark	Relative
1 Month	1.67	1.69	-0.03
3 Months	6.60	7.11	-0.50
6 Months	8.43	9.58	-1.15
1 Year	17.98	19.64	-1.66
2 Years	-12.65	-10.46	-2.18
3 Years	-13.77	-11.82	-1.96
4 Years	-13.70	-11.83	-1.87
5 Years	-2.33	-2.46	0.13
Since Incep.	4.05	3.93	0.12

	Calendar Year Returns in %		
	Composite	Benchmark	Relative
YTD	17.98	19.64	-1.66
2003	17.98	19.64	-1.66
2002	-35.32	-32.99	-2.33
2001	-15.99	-14.47	-1.52
2000	-13.46	-11.84	-1.62
1999	60.21	46.07	14.14
1998	21.29	17.51	3.78
1997	22.50	26.25	-3.75

Annual Risk Figures in %	Composite	Benchmark
Volatility over 1 Year	16.90	15.80
Volatility Since Inception	22.98	20.80
Sharpe Ratio over 1 Year	1.06	1.24
Sharpe Ratio Since Inception	0.10	0.10
Tracking Error over 1 Year	2.12	N/ A
Tracking Error Since Inception	4.69	N/ A
Information Ratio over 1 Year	-0.78	N/ A
Information Ratio Since Inception	0.03	N/ A
Correlation over 1 Year	0.99	N/ A
Correlation Since Inception	0.98	N/ A



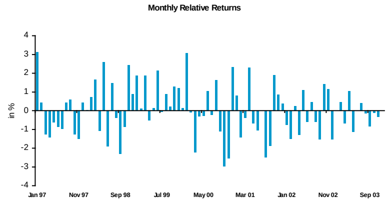


Figure 1-40: Sample performance report – part 1

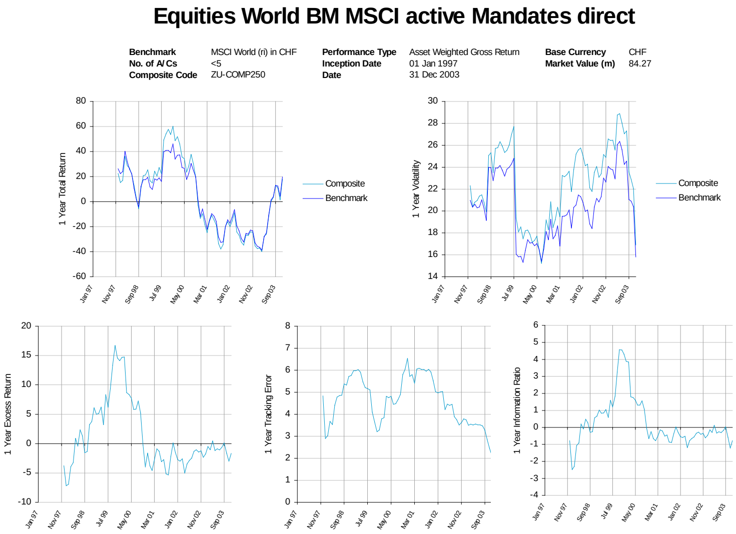


Figure 1-41: Sample performance report – part 2

Similar to the variety of investment products, there is no unique or ultimate performance report. The layout, structure, and content depend on the specific investment portfolio or product and on the intended use. Before designing and setting up a performance report, we should discuss and define different relevant aspects, like:

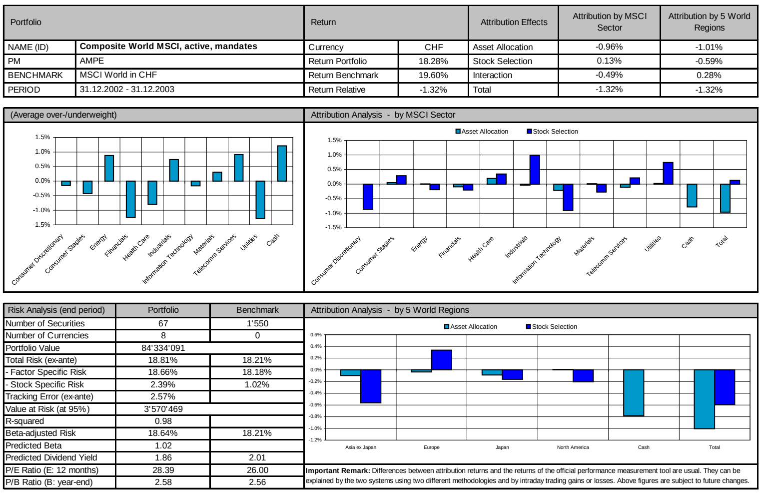
- Which product, composite or account should be analyzed?
- Which time periods should be considered?
- Whether gross or net return should be reported?
- Which return and risk measures should be presented?
- Whether rolling and/or annual performance figures should be presented?

It should be noted that it is crucial to analyze the investment performance from different angles in order to get the whole picture since performance figures are very sensitive to the methodology, time periods and input data used. This means that if one varies the time period, even for only one month forward, this can lead to a strongly underperforming investment portfolio turning into a strongly outperforming one.

Performance attribution

Performance attribution is a central component of the performance evaluation process. It is defined as a process that determines the return and risk contributions of the individual decision making steps within a portfolio management process. Thus, performance attribution is concerned not only with the past but also with the future, and determines which return and risk contributions are due to which decisions (regarding investment category and instruments) and to which decision makers on an ex-post as well as ex-ante basis.

If one considers the various levels of analysis of performance attribution as well as possible allocation criteria of return and risk contributions, as presented in Figure 1-24 in **chapter 1.3.1.1.1**, it is evident that there are various ways of running a performance attribution. Setting up and running a performance attribution, like the one illustrated in Figure 1-42, is complex and requires a lot of knowledge and experience, not only of the software being used but also on the necessary input data as well as on the portfolio management process. In contrast to portfolio analytics, the sub-process performance attribution focuses on the proper set up and production of performance attribution. The interpretation of the investment performance information provided is covered by the sub-process portfolio analytics.



Portfolio		Return		Attribution Effects	Attribution by MSCI Sector	Attribution by 5 World Regions
NAME (ID)	**Composite World MSCI, active, mandates**	Currency	CHF	Asset Allocation	-0.96%	-1.01%
PM	AMPE	Return Portfolio	18.28%	Stock Selection	0.13%	-0.59%
BENCHMARK	MSCI World in CHF	Return Benchmark	19.60%	Interaction	-0.49%	0.28%
PERIOD	31.12.2002 - 31.12.2003	Return Relative	-1.32%	Total	-1.32%	-1.32%

Risk Analysis (end period)	Portfolio	Benchmark
Number of Securities	67	1'550
Number of Currencies	8	0
Portfolio Value	84'334'091	
Total Risk (ex-ante)	18.81%	18.21%
- Factor Specific Risk	18.66%	18.18%
- Stock Specific Risk	2.39%	1.02%
Tracking Error (ex-ante)	2.57%	
Value at Risk (at 95%)	3'570'469	
R-squared	0.98	
Beta-adjusted Risk	18.64%	18.21%
Predicted Beta	1.02	
Predicted Dividend Yield	1.86	2.01
P/E Ratio (E: 12 months)	28.39	26.00
P/B Ratio (B: year-end)	2.58	2.56

Important Remark: Differences between attribution returns and the returns of the official performance measurement tool are usual. They can be explained by the two systems using two different methodologies and by intraday trading gains or losses. Above figures are subject to future changes.

Figure 1-42: Sample performance attribution report

Performance watch list

The performance watch list consists of investment portfolios or products to be monitored and maybe reviewed. The reason why accounts or composites are put on a watch list are manifold and may be of quantitative but also of qualitative nature such as an underperformance versus a benchmark or a peer group, too much or too little risk, client complaints and so on.

The determination of a performance watch list follows the process illustrated in Figure 1-43. It normally starts with a mechanical filter focusing on the historical and forward looking characteristics of the individual investment portfolios or products, ex-ante risk limits or risk budgeting constraints as well as on the clients' feedback. In the next step, the investment controlling committee decides on the investment portfolios or products that seem to be problematic and that will go onto the performance watch list. Afterwards, within the performance review meeting, each watch list portfolio or product is analyzed and discussed in detail taking into consideration all kinds of information from the investment contract and guidelines up to an ex ante risk breakdown. Because of this performance review, corrective steps may be defined and implemented. If the investment performance does not improve over longer time horizons, serious performance problems are reported to senior management via an escalation process.

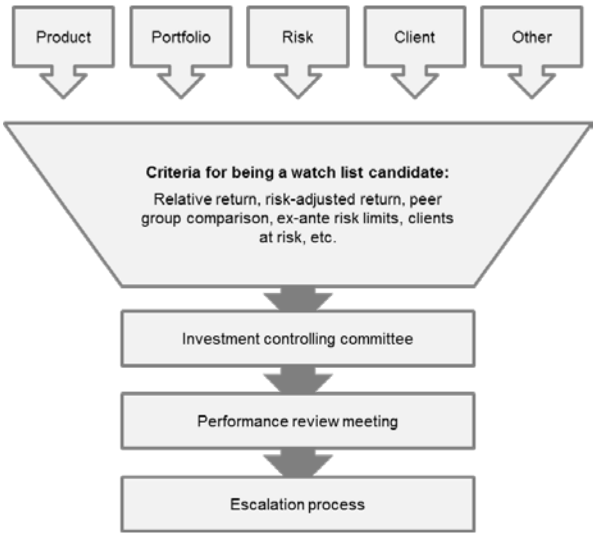


Figure 1-43: Sample performance watch list process

Portfolio analytics

Portfolio analytics provides deeper insight into the investment portfolios or products on the watch list by taking all available quantitative information, especially delivered by one of the preceding steps of the performance evaluation process, and other additional relevant information necessary in order to understand better how the investment performance was produced.

Portfolio analytics is very often also referred to as performance attribution. We distinguish between these two steps to highlight that the performance attribution analysis on its own and without any qualitative judgment is nothing else but a detailed performance reporting. Through the interpretation of performance figures and taking into consideration all relevant circumstances, performance attribution becomes a meaningful management information tool. One can contrast the two by defining performance attribution as the quantitative assessment and portfolio analytics as the qualitative assessment of the performance of an investment portfolio or product.

We will explain what problems or pitfalls may occur during this step of the performance evaluation process in the next **chapter 1.5.3**. At this point, we will only mention that performance attribution or portfolio analytics bear a high risk of misinterpretation and should be handled carefully.

Performance review

The performance review deals with investment portfolios or products that are on the performance watch list. It analyzes and discusses where the performance problems came from and whether any corrective action is necessary to bring the product back on the right track. Within this step of the performance evaluation process, the investment portfolios or products are analyzed in detail taking considering all kinds of information, from the investment contract and guidelines up to an ex ante risk breakdown. Amongst others, the following aspects may be addressed within a typical performance review:

- **Investment performance**. What are the sources of the performance and what are the decisions that produced the performance?
- **Investment restrictions**. What are the relevant investment restrictions? What was the impact on the investment performance? How was the portfolio manager limited in implementing the aspired investment strategy?
- **Investment decisions and guidelines**. What were the relevant decisions made and guidelines to be followed? What was the impact on the investment performance?
- **Portfolio management process**. Were there any changes to the portfolio management process, the investment strategy, the investment guidelines, the portfolio manager team, or to other organizational circumstances? What was the impact on the investment performance?
- **Benchmark**. Is the benchmark appropriate? Is the benchmark comparison fair? Are there any explainable and not decision-based performance differences between the investment portfolio and the benchmark?
- **Peer groups**. Is the peer group appropriate? Do the alternative investment products follow similar investment strategies?
- **Fees and taxes**. What are the relevant fees and taxes to be considered? What was the impact on the investment performance?

1.5.3 Pitfalls in performance evaluation*

Investment controlling and performance evaluation are very complex because both deal with complex matters and with a lot of information and data. This complexity as well as the lack of appropriate analytics and the lack of experience or knowledge of the people involved imply that wrong and misleading conclusions can be drawn from the information provided by performance evaluation. Misinterpretation of the investment performance may lead to erroneous or even wrong management decisions as they assess the quality of the decision makers and give feedback into the decision making process.

In this chapter we discuss different cases for setting up a performance attribution in an inappropriate and therefore misleading way. Figure 1-24, presented in **chapter 1.3.1.1.1**, illustrates the different ways of setting up a performance attribution. Following the distinction between performance contribution and attribution, performance evaluation should use a performance attribution analysis reflecting the actual portfolio management process. To set up a performance attribution, the performance analyst needs to have a good understanding of the whole portfolio management process in order to identify the obvious and the less obvious decisions and decision makers.

In the first case, illustrated in Figure 1-44, we decompose the value added of a multi-asset class portfolio in two different ways. The first decomposition (left side) assumes a three step portfolio management process covering the benchmark definition, the definition of the asset allocation and the security selection. By contrast, the other portfolio management process (right side) is a more complex decision-making process and consists of six steps: the (official) benchmark definition, the (internal) benchmark selection, the definition of the asset allocation, the definition of the fixed income asset allocation, the definition of the equity asset allocation and portfolio implementation or security selection. Taking into consideration the different return contributions, one could conclude that the portfolio manager for the three step process is a poor stock picker and that, on the contrary, one could argue that the portfolio manager for the six step process is a good stock picker. Neglecting the real investment process and not reflecting it in the performance attribution potentially causes wrong interpretation and can lead to wrong conclusions.

Three step investment process		Six step investment process	
Portfolio return	+ 6.5%	Portfolio return	+ 6.5%
- Benchmark return	+ 4.5%	- Benchmark return	+ 4.5%
= Value added	+ 2.0%	= Value added	+ 2.0%
Asset allocation	+ 3.0%	Benchmark selection	+ 0.3%
+ Security selection	- 0.7%	+ Asset allocation team	+ 1.0%
+ Interaction	- 0.3%	+ FI-specialist	- 0.5%
		+ EQ-specialist	+ 0.8%
		+ Security selection	+ 0.7%
		+ Interaction	- 0.3%

Figure 1-44: Are all steps of the decision making process reflected?

In the second case, shown in Figure 1-45, we consider a situation often observed when reviewing the performance of a mutual fund. In our example, we consider a European equity mutual fund with MSCI Europe as its official (external) benchmark. The product management of the mutual fund company positioned this mutual fund internally as a growth product with a European growth index as its internal benchmark. To measure this mutual fund against the index MSCI Europe and to decompose its value added is not appropriate if one wants to measure the quality of the portfolio manager. To get the right picture, the set up of the return attribution has to be changed in a way that the value added versus the external benchmark is split in three effects: a) benchmark selection, b) asset allocation and c) security selection. Such a return decomposition ensures that the contribution of the positioning of the product and the contribution of the asset manager are isolated and independently assessed. In our case, the signs of the asset allocation and security selection effects changed just because we changed the relevant benchmark to the European growth index.

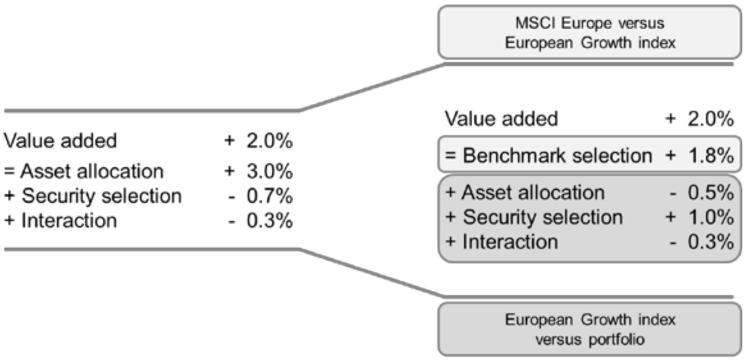


Figure 1-45: Is a product or a portfolio manager under review?

Another pitfall in using a return attribution in order to assess the quality of a portfolio manager is the averaging of the management effects over the total observation period. Normally, the return attribution calculates a total figure for the different management effects for the whole reporting period and does not show the management effects over time, for example on a monthly basis. In Figure 1-46, we compared the management effects for the whole period with the ones on a monthly basis. The total figures indicate that the portfolio manager was quite a bad stock picker but a good asset allocator during the reporting period. However, this is the wrong conclusion because the positive asset allocation effect was mainly generated in the first two months of the reporting period and the monthly asset allocation effect over the last seven months was constantly negative. With respect to the security selection effect, there is a similar situation but vice versa. The monthly security selection effect was negative over the first 8 months but constantly positive over the last seven months. It is now necessary to analyze this further to get additional insight into the figures and to come up with the “right” conclusions.

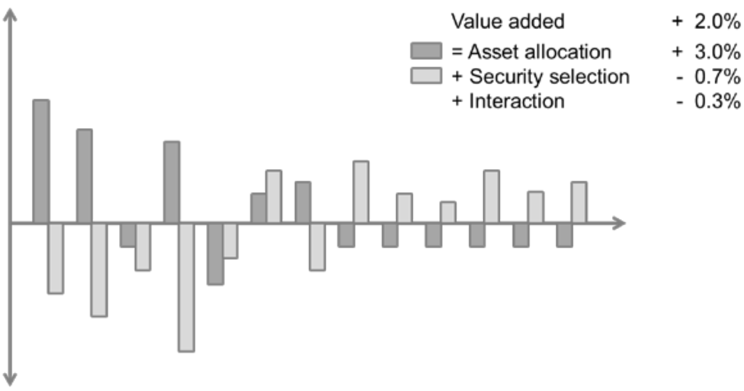


Figure 1-46: Do the management effects vary over time?

In Figure 1-47, we illustrate another common mistake made when setting up a return attribution, which may occur especially for products encompassing regional and not single countries. Return attribution software is normally quite flexible in setting up the analysis, so that the performance analysts can choose from different segments such as countries or sectors when decomposing the return. Depending on the segment chosen to decompose the return, the return attribution may come up with different management effects. In our example, by assuming that the portfolio manager of a European equity portfolio is following a) a sector approach or b) a country approach, we come up with different management effects.[82] Setting up the return attribution in a wrong way may lead to a wrong interpretation and to wrong conclusions. This again shows that setting up the return attribution according to the investment process is essential in order to get a meaningful analysis and appropriate feedback into the portfolio management process.

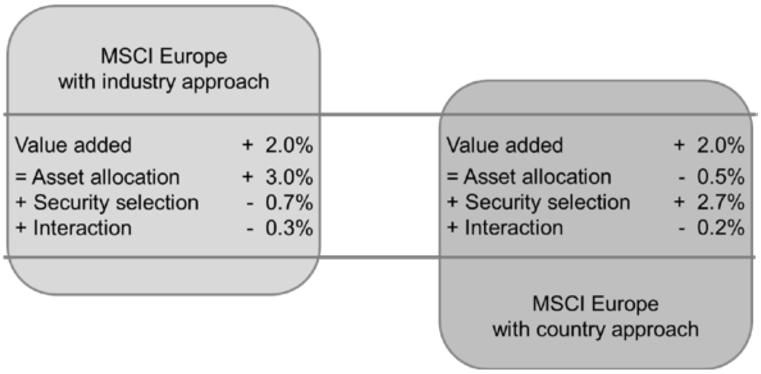


Figure 1-47: Is the investment style reflected correctly?

Furthermore, one has to consider both the investment universe and the implementation of the investment strategy when setting up the return attribution. As shown in Figure 1-48, for the cases A and B one specifically has to define whether certain decisions are security selection or asset allocation decisions. For case A, one must define whether the 20% investments in small and mid caps is a security selection or an asset allocation decision. If the performance analyst sets up the return attribution in an incorrect manner, he will get misleading figures for the management effects. In case B, the performance analyst has to specify whether the investments in (US) biotech stocks is a security selection decision versus the US equity index or whether it is an asset allocation decision versus the overall benchmark because biotech stocks are not an explicit part of the overall benchmark.

82 A single stock may be the best stock within a specific sector (indicating good security selection) but on the other hand the worst performing stock within a specific country (indicating bad security selection).

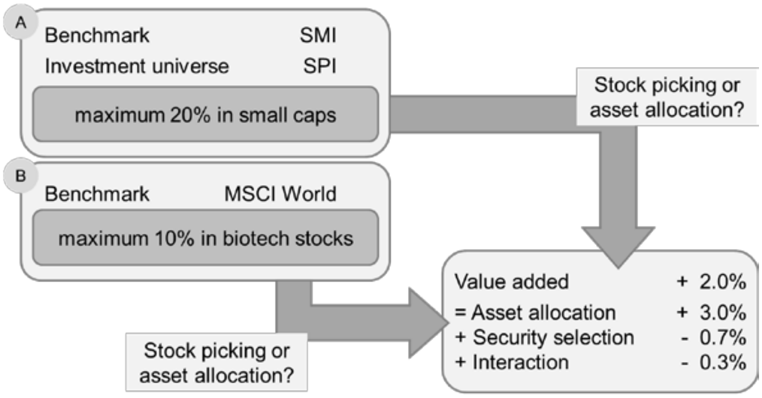


Figure 1-48: Is each decision in the investment process reflected?

Running a return attribution implicitly assumes that the portfolio manager has no guideline restrictions, for example with respect to the minimum or maximum weight of a specific security or asset class within the portfolio. If this assumption is not true and there are restrictions that do not allow the portfolio manager to invest according to the passive investment alternative, the benchmark, the calculated management effects, as illustrated in Figure 1-49, may be misleading. Indeed, limiting an investment may have a positive or negative impact on the value added. For example, some equity indices are dominated by a specific security. Let us assume for example that 30% of the index consists of one security. Imposing a maximum weight limit of 10% per security is an implicit bet on this specific security since we are predefining the minimum underweight of this security to be 20%. The question is now who is responsible for the return contribution due to this investment restriction. The same is true if there are minimum limits for specific securities.

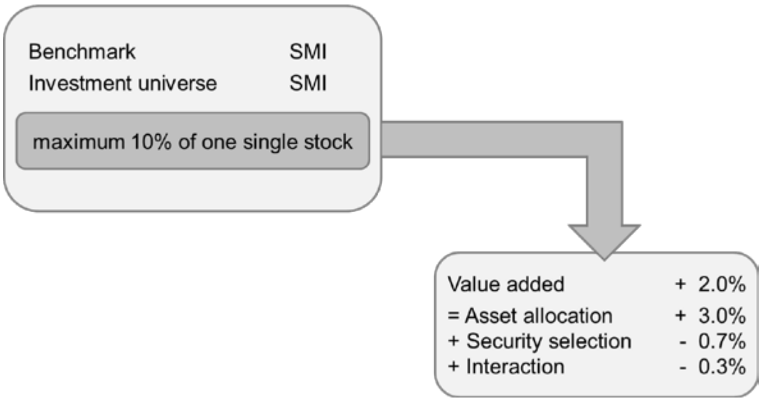


Figure 1-49: To what extend are the results influenced by the guidelines?

Until now, we have only addressed issues related to setting up a return attribution but there are similar problems when setting up a risk attribution. The issues with risk attribution are a bit bigger because the risk attribution software available is not as flexible as the return attribution software. Normally, the risk attribution software decomposes a specific absolute or relative risk figure following a specific risk model, which may not be representative of the actual investment process. Therefore, the figures of a risk attribution have to be handled with care because the different risk factors and their contributions to the overall absolute or relative risk are often not linked to the steps of the portfolio management process or the different decisions taken. Figure 1-50 illustrates this issue by indicating that in an ideal world the risk attribution would have to be linked to the return attribution.

Risk Model: Global	Portfolio	Tracking Error
Total Risk (ex-ante)	18.81%	2.57%
Factor Specific Risk	18.66%	1.50%
- Region	11.50%	0.18%
- Country	6.98%	0.83%
- Industry	2.64%	0.77%
- Fundamental	1.44%	0.78%
- Currency	8.42%	0.27%
- Covariance (+/-)	9.35%	0.52%
Stock Specific Risk	2.39%	2.08%

by MSCI Sector	Asset Allocation	Stock Selection	Interaction		Total
Consumer Discretionary	0.00%	-0.86%	-0.03%		-0.89%
Consumer Staples	0.05%	0.28%	0.03%		0.36%
Energy	0.00%	-0.19%	-0.02%		-0.21%
Financials	-0.10%	-0.21%	0.01%		-0.30%
Health Care	0.20%	0.35%	-0.03%		0.52%
Industrials	-0.04%	0.99%	0.12%		1.07%
Information Technology	-0.21%	-0.91%	0.01%		-1.11%
Materials	0.01%	-0.28%	-0.08%		-0.35%
Telecomm Services	-0.11%	0.21%	-0.08%		0.02%
Utilities	0.02%	0.74%	-0.41%		0.35%
Cash	-0.78%	0.00%	0.00%		-0.78%
Total	**-0.96%**	**0.13%**	**-0.49%**		**-1.32%**

Figure 1-50: Is the risk contribution consistent with the return attribution?

1.6 Tables

1.6.1 Notations and abbreviations

$R_{p,t}$ = Simple return of a portfolio for a single period t.

$MV_{end,t}$ = Market value at the end of period t.

$MV_{begin,t}$ = Market value at the beginning of period t.

NCF_t = Net external cash flow for period t.

$\tilde{C}_d$ = External cash inflow at date d.

$\widetilde{W}_d$ = External cash outflow at date d.

AIC_t = Average invested capital for period t.

$wNCF_t$ = Weighted net external cash flow for period t.

$w\tilde{C}_d$ = Weighted external cash inflow at date d.

$w\widetilde{W}_d$ = Weighted external cash outflow at date d.

CCR = Continuously compounded return.

$r_{p,t}$ = Continuously compounded return of a portfolio for a single period t.

$R_{p,t}^{BC}$ = Return of a portfolio for period t in base currency.

$R_{p,t}^{LC}$ = Return of a portfolio for period t in local currency.

$C_t^{BC/LC}$ = Currency return for period t.

$R_{p,tot}$ = Multi-period or cumulative simple return of a portfolio.

$r_{p,tot}$ = Multi-period or cumulative continuously compounded return of a portfolio.

MWR = Money-weighted rate of return.

TWR = Time-weighted rate of return.

IRR = Internal rate of return.

Y_T = Length of measurement period (measured in years – 365 days).

Y_{t-0} = Length of period between the beginning of the measurement period and the date of the external cash flow (measured in years – 365 days).

Y_{T-t} = Length of period between the date of the external cash flow and the end of the measurement period (measured in years – 365 days).

MIRR = Modified internal rate of return.

rir_{T-t} = Reinvestment rate for the period from the date of the external cash outflow t to the end of the measurement period T (measured in years – 365 days).

fr_{t-0} = Finance rate for the period starting at the beginning of the measurement period to the date of the external cash inflow t (measured in years – 365 days).

ODM = Original Dietz Method.

MWR_{ODM} = MWR according to the Original Dietz Method.

MDM = Modified Dietz Method.

MWR_{MDM} = MWR according to the Modified Dietz Method.

w_d = Time-weight for date d.

D_T = Length of measurement period (measured in days).

D_d = Length of period between the beginning of the measurement period and the date of the external cash flow (measured in days).

$MWR_{D,\text{end of day}}$ = Daily MWR assuming end of day cash flows.

$MWR_{D,\text{begin of day}}$ = Daily MWR assuming beginning of day cash flows.

$R_{b,t}$ = Benchmark return for a single period t.

$IV_{end,t}$ = Index value at the end of period t.

$IV_{begin,t}$ = Index value at the beginning of period t.

PME = Public market equivalent.

$MV_{end,t}^{PME}$ = PME benchmark value at the end of period t.

$MV_{begin,t}^{PME}$ = PME benchmark value at the beginning of period t.

$\tilde{C}_{end,t}^{P}$ = Portfolio external cash inflow at the end of period t.

$\widetilde{W}_{end,t}^{P}$ = Portfolio external cash outflow at the end of period t.

$w_{b,i,t}$ = Weight of a constituent index i of a benchmark at the beginning of period t.

$R_{b,i,t}$ = Return of a constituent index i of a benchmark for period t.

$R_{b,\text{tot with no rebalancing}}$ = Cumulative return of a buy-and-hold customized benchmark.

$w_{b,i,0}$ = Weight of a constituent index i of a benchmark at the beginning of the measurement period.

$R_{b,i,tot}$ = Cumulative return of a constituent index i of a benchmark.

$R_{b,\text{tot with regular rebalancing}}$ = Cumulative return of a customized benchmark with regular rebalancing.

$VA_{A,t}$ = Arithmetic value added of a single period t.

$VA_{G,t}$ = Geometric value added of a single period t.

$VA_{A,tot}$ = Multi-period or cumulative arithmetic value added.

$R_{b,tot}$ = Multi-period or cumulative return of a benchmark.

$VA_{G,tot}$ = Multi-period or cumulative geometric value added.

Var_p = Variance of the returns of a portfolio.

N = Number of returns in the sample.

$\bar{r}_p$ = Mean return.

σ_p = Standard deviation of the returns of a portfolio.

$Var_{p,annualized}$ = Annualized variance of the returns of a portfolio.

$Var_{p,\text{not annualized}}$ = Not annualized variance of the returns of a portfolio.

$\sigma_{p,\text{annualized}}$ = Annualized standard deviation of the returns of a portfolio.

$\sigma_{p,\text{not annualized}}$ = Not annualized standard deviation of the returns of a portfolio.

t_o = Number of observation in a year (quarterly = 4, monthly = 12 or daily = 250).

VaR = Value at risk.

$VaR_{C,H,per}(\tilde{\mu}, \tilde{\sigma})$ = Value at risk in percentage terms for a specific confidence level C, a specific time horizon H, an expected return $\tilde{\mu}$, and an expected standard deviation $\tilde{\sigma}$.

$\tilde{\mu}$ = Expected continuously compounded return.

$\tilde{\sigma}$ = Expected standard deviation of continuously compounded returns.

z = Z-score for a specific confidence level C.

$VaR_{C,H,abs}(\tilde{\mu}, \tilde{\sigma})$ = Value at risk in absolute terms for a specific confidence level C, a specific time horizon H, an expected return $\tilde{\mu}$ and an expected standard deviation $\tilde{\sigma}$.

MV = Market value of a portfolio.

$SP(VaR_{per})$ = Shortfall probability for a given VaR in percentage terms.

$Var_{D,p}(r_T)$ = Downside variance of the returns of a portfolio below a threshold return.

$\sigma_{D,p}(r_T)$ = Downside standard deviation of the returns of a portfolio below a threshold return.

r_T = Threshold return.

$Cov(r_{p,t}, r_{b,t})$ = Covariance of the returns of a portfolio and the returns of its benchmark.

$r_{b,t}$ = Return of a benchmark for a single period t.

$\bar{r}_b$ = Mean benchmark return.

$Corr(r_{p,t}, r_{b,t})$ = Correlation of the returns of a portfolio and the returns of its benchmark.

σ_b = Standard deviation of the returns of a benchmark.

$TEVar(r_{p,t}, r_{b,t})$ = Tracking error variance of the excess returns of a portfolio against its benchmark.

$TESD(r_{p,t}, r_{b,t})$ = Tracking error standard deviation of the excess returns of a portfolio against its benchmark.

CAPM = Capital Asset Pricing Model.

$RVAR_p$ = Sharpe ratio of a portfolio.

$\bar{r}_f$ = Mean risk free rate.

$RVOL_p$ = Treynor ratio of a portfolio.

β_p = Beta or systematic risk of a portfolio.

r_m = Return of the market portfolio.

Var_m = Variance of the returns of the market portfolio.

α_p = Jensen's alpha of a portfolio.

$\bar{r}_M$ = Mean return of market portfolio.

AR_p = Appraisal ratio of a portfolio.

σ_ε = Specific risk of a portfolio in comparison to its benchmark.

ε_t = Regression residual or error term of a portfolio for a single period t.

$GH1_p$ = Graham & Harvey measure 1 of a portfolio.

$GH2_p$ = Graham & Harvey measure 2 of a portfolio.

SOR_p = Sortino ratio of a portfolio.

IR_p = Information ratio of a portfolio.

$CR^{BC}_{p,i,t}$ = Contribution to return for a portfolio component i to the return of a portfolio for period t in base currency.

$w_{p,i,t}$ = Weight for a portfolio component i at the beginning of period t.

$R^{BC}_{p,i,t}$ = Return for a portfolio component i for period t in base currency.

$CR^{BC}_{b,i,t}$ = Contribution to return for a benchmark component i to the return of a benchmark for period t in base currency.

$w_{b,i,t}$ = Weight for a benchmark component i at the beginning of period t.

$R^{BC}_{b,i,t}$ = Return for a benchmark component i for period t in base currency.

RM^{BC}_P = Cumulative return for a portfolio for the entire measurement period in base currency.

$CRM^{BC}_{p,i}$ = Cumulative contribution to return for a portfolio component i to the cumulative return of a portfolio for the entire measurement period in base currency.

$CRM^{BC}_{p,i,t-1}$ = Cumulative contribution to return for a portfolio component i to the cumulative return of a portfolio from the beginning of the measurement period until beginning of period t in base currency.

RM^{BC}_B = Cumulative return for a benchmark for the entire measurement period in base currency.

$CRM^{BC}_{b,i}$ = Cumulative contribution to return for a benchmark component i to the cumulative return of a benchmark for the entire measurement period in base currency.

$CRM^{BC}_{b,i,t-1}$ = Cumulative contribution to return for a benchmark component i to the cumulative return of a benchmark from the beginning of the measurement period until beginning of period t in base currency.

$VA^{BC}_{p,t}$ = Value added of a portfolio for period t in base currency.

$CVA_{i,t}^{BC}$ = Contribution to value added for a portfolio component i to the value added of a portfolio for period t in base currency.

VAM_{P}^{BC} = Cumulative value added of a portfolio for the entire measurement period in base currency.

$CVAM_{i}^{BC}$ = Cumulative contribution to value added for a portfolio component i to the cumulative value added of a portfolio for the entire measurement period in base currency.

$R_{b,T-t}^{BC}$ = Cumulative return for a benchmark from the end of the current period t until the end of the measurement period in base currency.

$RM_{p,t-1}^{BC}$ = Cumulative return for a portfolio from the beginning of the measurement period until beginning of period t in base currency.

$AAE_{i,t}^{BC}$ = Asset allocation effect of a portfolio component i for period t in base currency.

$SSE_{i,t}^{BC}$ = Security selection effect of a portfolio component i for period t in base currency.

$IAE_{i,t}^{BC}$ = Interaction effect of a portfolio component i for period t in base currency.

$AAEM_{i}^{BC}$ = Cumulative asset allocation effect of a portfolio component i for the entire measurement period in base currency.

$SSEM_{i}^{BC}$ = Cumulative security selection effect of a portfolio component i for the entire measurement period in base currency.

$IAEM_{i}^{BC}$ = Cumulative interaction effect of a portfolio component i for the entire measurement period in base currency.

$SP_{i,\text{end of t}}^{BC/LC}$ = Spot rate in currency i at the end of period t.

$SP_{i,\text{begin of t}}^{BC/LC}$ = Spot rate in currency i at the beginning of t.

$FP_{i,t}^{BC/LC}$ = Forward premium in currency i for period t.

$E_{p,i,t}^{BC/LC}$ = Currency surprise return for a currency i of the portfolio for period t.

$FR_{i,\text{begin of t}}^{BC/LC}$ = Forward rate in currency i at the beginning of period t.

$RI_{BC,t}$ = Cumulative interest rate in base currency for period t.

$RI_{i,t}$ = Cumulative interest rate in currency i for period t.

$r_{p,t}^{BC}$ = Return of a portfolio for period t in base currency.

$r_{p,i,t}^{BC}$ = Return for a portfolio component i for period t in base currency.

$r_{p,i,t}^{LC}$ = Return for a portfolio component i for period t in local currency.

$c_{i,t}^{BC/LC}$ = Currency return for currency i for period t.

$fp_{p,i,t}^{BC/LC}$ = Forward premium in currency i of a portfolio for period t.

$e_{p,i,t}^{BC/LC}$ = Currency surprise return for a currency i of a portfolio for period t.

$r_{b,t}^{BC}$ = Return of a benchmark for period t in base currency.

$r_{b,i,t}^{LC}$ = Return for a benchmark component i for period t in local currency.

$r_{adj,p,i,t}^{BC}$ = Return for a portfolio component i for period t hedged into base currency.

$e_{b,i,t}^{BC/LC}$ = Currency surprise return for a currency i of a benchmark for period t.

$fp_{b,i,t}^{BC/LC}$ = Forward premium in currency i of a benchmark for period t.

$r_{adj,p,i,t}^{BC}$ = Return for a portfolio component i for period t hedged into base currency.

$r_{adj,b,i,t}^{BC}$ = Return for a benchmark component i for period t hedged into base currency.

$va_{p,t}^{BC}$ = Value added for a portfolio for period t in base currency.

$h_{p,i,t}$ = Hedge weight of currency i of a portfolio at the beginning of period t.

$f_{p,i,t}^{BC/LC}$ = Hedge return of currency i of a portfolio for period t.

$h_{b,i,t}$ = Hedge weight of currency i of a benchmark at the beginning of period t.

$f_{b,i,t}^{BC/LC}$ = Hedge return of currency i of a benchmark for period t.

$AAE_{i,t}^{Adj}$ = Asset allocation effect of a portfolio component i for period t hedged into base currency.

$SSE_{i,t}^{Adj}$ = Security selection effect of a portfolio component i for period t hedged into base currency.

$IAE_{i,t}^{Adj}$ = Interaction effect of a portfolio component i for period t hedged into base currency.

$CAE_{i,t}^{BC}$ = Currency asset allocation effect of a currency i of a portfolio for period t in base currency.

$CHSE_{i,t}^{BC}$ = Currency and hedge selection effect of a currency i of a portfolio for period t in base currency.

$CHIAE_{i,t}^{BC}$ = Currency and hedge interaction effect of a currency i of a portfolio for period t in base currency.

$r_{f,t}$ = Risk free rate for period t.

$r_{m,t}$ = Return of the market portfolio for period t.

σ_α = Standard deviation of Jensen's α_P of a portfolio.

σ_β = Standard deviation of the beta β_P of a portfolio.

n = Number of observations.

R^2 = Coefficient of determination.

$Cov(r_{p,i}, r_{p,j})$ = Covariance of the returns of a portfolio component i and the returns of a portfolio component j.

$Cov(r_{p,i}, r_p)$ = Covariance between the returns of portfolio component i and the returns of a portfolio.

$\mathrm{Corr}(r_{p,i}, r_p)$ = Correlation between the returns of portfolio component i and the returns of a portfolio.

$\mathrm{CVar}_{p,i}$ = Contribution of portfolio component i to the variance of a portfolio.

$\mathrm{CSD}_{p,i}$ = Contribution of portfolio component i to the standard deviation of a portfolio.

$\mathrm{Cov}(CR_{p,i}, r_p)$ = Covariance between the contributions of portfolio component i to the returns of a portfolio and the returns of the portfolio.

$\sigma(CR_{p,i})$ = Standard deviation of the contributions of portfolio component i to the returns of a portfolio.

TEVar_p = Tracking error variance of a portfolio.

$\mathrm{Cov}(CR_{p,i} - CR_{b,i}, r_p - r_b)$ = Covariance of the excess return contributions of a portfolio component i and the excess returns of a portfolio against its benchmark.

$\mathrm{CTEVar}_{p,i}$ = Contribution of portfolio component i to the tracking error variance of a portfolio.

$\mathrm{CTESD}_{p,i}$ = Contribution of portfolio component i to the tracking error standard deviation of a portfolio.

$\mathrm{CTEVar}_{p,i}^{AAE}$ = Contribution to tracking error variance of a portfolio due to the asset allocation effect for portfolio component i.

$\mathrm{CTEVar}_{p,i}^{SPE}$ = Contribution to tracking error variance of a portfolio due to the security selection effect for portfolio component i.

$\mathrm{CTEVar}_{p,i}^{IAE}$ = Contribution to tracking error variance of a portfolio due to the interaction effect for portfolio component i.

$\mathrm{CTESD}_{p,i}^{AAE}$ = Contribution to tracking error standard deviation of a portfolio due to the asset allocation effect for portfolio component i.

$\mathrm{CTESD}_{p,i}^{SPE}$ = Contribution to tracking error standard deviation of a portfolio due to the security selection effect for portfolio component i.

$\mathrm{CTESD}_{p,i}^{IAE}$ = Contribution to tracking error standard deviation of a portfolio due to the interaction effect for portfolio component i.

$b_{p,i,l}$ = Sensitivity of the returns of portfolio component i to factor l.

F_l = Return of factor l.

$\varepsilon_{p,i}$ = Non-factor or specific return of a portfolio component i.

$\mathrm{Cov}(F_l, r_p)$ = Covariance of the returns of a factor l and the returns of a portfolio.

$\mathrm{Corr}(F_l, r_p)$ = Correlation of the returns of a factor l and the returns of a portfolio.

σ_l = Standard deviation of the returns of factor l.

$b_{b,i,l}$ = Sensitivity of the returns of benchmark component i to the factor l.

$\varepsilon_{b,i}$ = Non-factor or specific return of a benchmark component i.

$w_{A,i}$ = Active weight of portfolio component i.

$Cov(F_l, r_A)$ = Covariance of the returns of a factor l and the excess returns of a portfolio.

$Corr(F_l, r_A)$ = Correlation of the returns of a factor l and the excess returns of a portfolio.

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1.6.3 Table of tables

1.6.4 Table of exhibits