PORTFOLIO MANAGEMENT

ASSET/LIABILITY MANAGEMENT

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Table of contents

1. Introduction

1.1 Background of ALM

Over the last 30 years, we have seen a strong increase of funded pension assets in most developed countries. These assets are meant to serve as source of income for future pensioners, in particular when demographic developments can be expected to avoid or hinder future taxpayers to bear the burden of pension payments to a perhaps even larger generation of pensioners. The accumulation, allocation and distribution of those pension assets as well as the resulting pay-outs are forecasted/calculated/simulated using a variety of different models. Nevertheless, it is possible to characterize the vast majority of these models and their differences with a set of typological criteria describing the most significant features of their inherent rules.

- **Social governance:**

The organization is either collective with the state (state pension), or collective with a larger private entity like the employer company or a group of companies (occupational pensions), or simply with individuals (private pensions).

- **Distribution of risks / Nature of benefits**[1]

The risks associated with the management of pension assets and liabilities can evenly be distributed across the parties involved in the governance structure. In principle, there are three major sources of risk with retirement plans:

- Investment risk is due to fluctuating returns of pension assets.
- Inflation risk is associated with the erosion of the pension benefits in real terms.
- Longevity risk is uncertainty attached to the life expectancy of individuals.

In private pensions, the total risk remains with the pensioners. In collectively organized pensions, the risk can be allocated to the organization itself, i.e. the employer company. In the last case, the organization is described as a Defined Benefit (DB) pension arrangement, in which the company promises to make pension payments to the beneficiary throughout the course of his retirement. The pension payments depend on age, years of service, and salary. Alternatively, in a pension plan of Defined Contribution (DC) type, the employer contributes a specific amount to the pension plan. The plan does not promise any benefit pay-outs. Therefore, the risk of the level of future pension benefits is allocated with the beneficiary. Combinations of Defined Benefit and Defined Contribution are described as hybrid arrangements.

- **Outsourcing arrangements:**

The organization structure can be such that risks and the management thereof can be outsourced to third parties. In the case of occupational pensions, this would typically be pension funds external to companies, whereas for private pensions it is common to outsource the risks to financial intermediaries like insurance companies or other pooling vehicles.

1 See the document “Investment policy” in “Investment strategy”.

Other rules which strongly impact the pension management in more detail are accounting rules, tax treatment, labor laws, and regulation of the respective industry and country. These features set the basic rules for the management of assets and liabilities in pensions. In contrast to the Asset/Liability Management in firms, the liabilities have a rather long duration. Even compared with the long-term prospects of a firm, they can be forecasted quite accurately. On the asset side, investments are structured in order to match this long duration and are typically diversified through the capital markets.

Asset/Liability Management or ALM is then a technique to optimize the structure of assets and liabilities within a framework defined by many rules. ALM is together with the general business plan the core tool for the management of a pension fund.

In one of their seminal publications on the topic, Keith Ambachtsheer and Don Ezra[2] describe as the very first step for a pensions' management to identify "What is the Pension Deal?" In order to answer this question it is necessary to investigate the structure of benefits and contributions and the possible distributions of deficits and surpluses over time and employers, pension funds and plan participants. The resulting structure will impact the present and future welfare situation of the employees while at the same time change the financial situation of the employing company. ALM is the tool to use for this kind of investigations.

From the viewpoint of the employer, the pension fund venture can be regarded as a part of its corporate financing activities. In that respect, ALM builds on themes explored in Corporate Finance and Financial Accounting, respectively. Furthermore, Asset/Liability analyses and management techniques have been applied to other business fields, e.g. to long-term liabilities stemming from long-term ventures like power plants, or from credit risks on banks' balance sheets. Insurance companies will also apply these techniques when managing pension risks for individuals or acquiring those risks from other companies or pension funds. The increased use of ALM techniques in those fields can be related to more stringent requirements imposed by regulators (i.e., Basle II in Banking) and by accounting rules (i.e., IFRS).

In general, it could be argued that Asset/Liability management is an integral part of the balance sheet optimization in any type of firm, or even of every individual's planning. In this chapter, we will neglect these broader applications of ALM. We will also neglect those forms of plans where the management of risk is fully left to individuals as in the case of private pensions or employer pension of the Defined Contribution type. In this chapter, we will concentrate on the classic application of ALM - the management of a Defined Benefit plan for the pension fund of an employer.

2 Ambachtsheer, K., and D. Ezra (1998): "Pension Fund Excellence", Wiley, New York.

1.2 ALM with Pension Funds

It is widely recognized that a pension plan refers to a long-term contract between the employer and the employees. That is, employees set aside current salaries in exchange for future retirement benefits. These contributions are either direct as salary deferral (also named 'Deferred Compensation'), or indirect through foregone earnings.[3] In a DB-plan the employer traditionally agrees to make lifelong pension payments to its employees according to a defined benefit formula. The benefit formula is established in advance and typically depends on the average or final salary during the working life of the employee, his age at retirement, and his years of service. The amount of lifelong pension payments after retirement can be fixed in nominal terms, or are adjusted in accordance with some standards (e.g. to price inflation or average wages).

The present value of that promise will thus depend on the future development of the salary of the employees, corresponding adjustments (e.g. for inflation) after retirement, and the duration of the payments in concordance to the life expectancy of the individuals. Discounting the expected future pension payments with an appropriate discount rate yields the actuarial present value of the employer's pension liabilities.

In order to backup the pension liabilities the employer can accumulate internal reserves by making an appropriate balance-sheet allowance (internal financing) and pay the future pension benefits from corporate revenues. Alternatively, the firm can set aside segregated assets with a trust or pension fund vehicle (external financing). The pension fund vehicle is an independent stand-alone legal entity, which generates assets from contributions by the employer and employees plus the returns on invested assets. These assets are available to pay the pension liabilities when the beneficiaries retire.

The difference between the value of assets and liabilities determines the reserves or the surplus of the pension fund at a given point in time.

$$\text{SURPLUS}_t = \text{ASSET}_t - \text{LIABILITIES}_t \quad (1)$$

It is then the task of the management of the firm or of the pension fund to optimize the relative development of assets and liabilities.

The key method to analyze the relations between the different variables is to use ALM. It essentially uses a simulation of all important variables and factors driving the pension fund. The management of the pension fund will thus be required to provide input to the simulation by following three different categories:

- **Investment policy:**

The investment policy of the fund will be based on its ability to neutralize liability risks by identifying asset classes with matching aspects. Then the fund may choose to develop capabilities to search for added value returns (Alpha) while lowering funding costs.

3 See Hustead, E.S. and O.S. Mitchell (2001): "Pensions in the Public Sector", University of Pennsylvania Press, Philadelphia, p. 6.

- **Liability policy:**

In reality, the possibility to adjust the benefit design to existing plans is very much restricted by labor laws and other regulations. Depending on the respective pension fund regulation and accounting rules setting appropriate discount rates for given liabilities may be an important parameter. It can vary from very high rates reflecting high (expected) asset returns to rather low rates explicitly or implicitly derived from safe or guaranteed absolute yields.

- **Funding & contribution policy:**

The most crucial input parameters to an ALM-system are often found in this category. Depending on regulations and accounting rules a pension fund can decide to target different funding ratios which on the one hand will have an impact on its investment policy but on the other be directly related to the contributions required and to the distribution of reserves and surpluses over time and across different generations of beneficiaries.

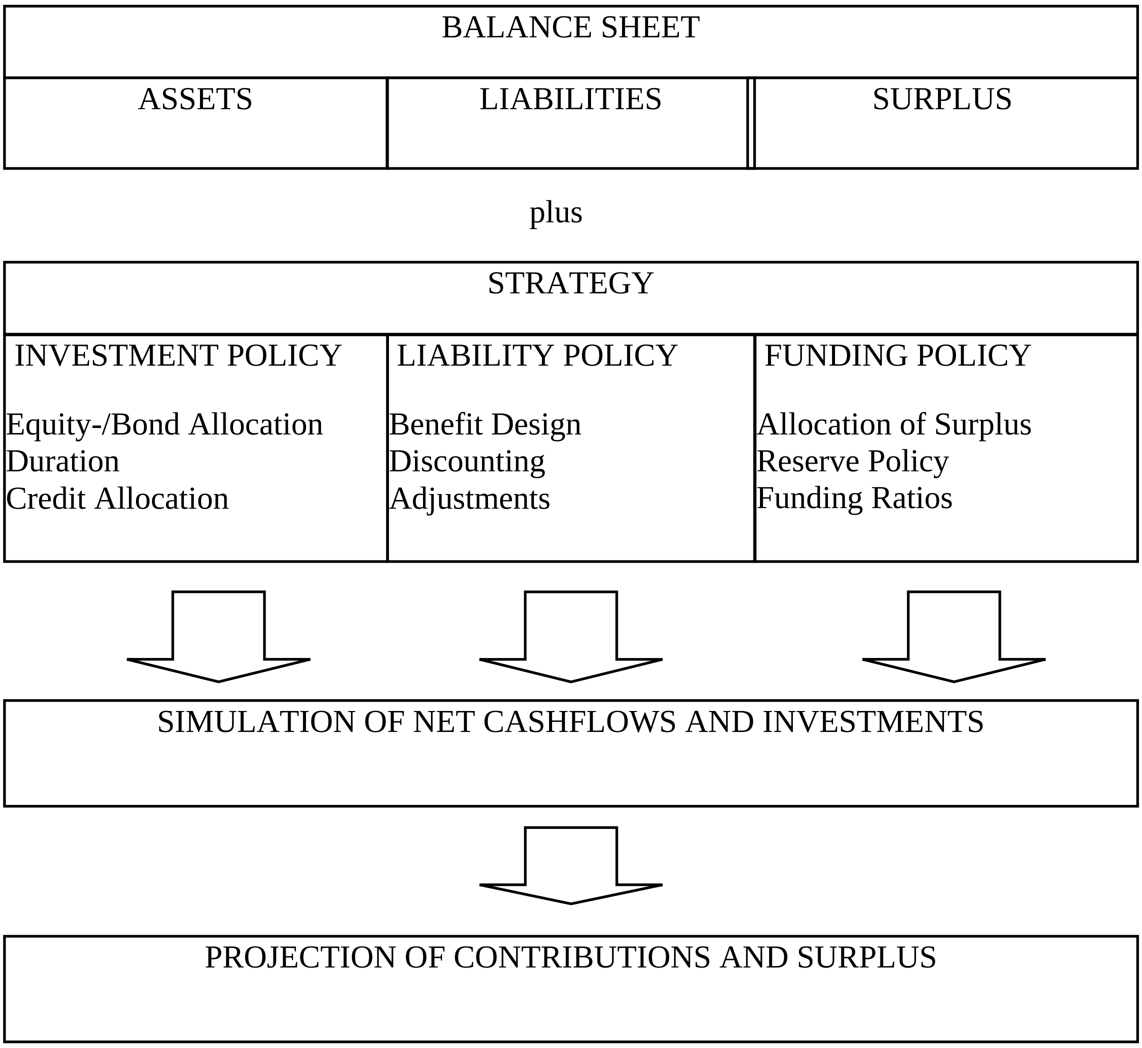

Figure 1-1: Balance Sheet of a Pension Fund

The most important input to ALM, though, arises from the parameters or functions selected for the optimization of the policy targets. Even in a deterministic world, conflicting targets will require a determination of tradeoff functions: While employees or beneficiaries of a pension fund will ask for contributions as high as possible to backup the liabilities, the employer will try to minimize funding costs for a given level of benefits. A policy that allocates surpluses to reserves or to sponsor contribution holidays will reflect a tradeoff between interests of beneficiaries and those of the firm. In a world of uncertainty about future developments, tradeoffs between risks and returns add another dimension to that picture: Investment policies with higher risks to pension payments are accompanied by lower expected pension costs.

In practice, not only the liability side of a pension fund but also many policy parameters are fixed, at least for the short-term. In a typical setup for an ALM modeling exercise the investment policy will be optimized given a set of policy variables. To some extent, this can be justified given that the asset side can indeed be efficiently adjusted in the case of an investment in liquid capital market instruments.

ALM can be regarded as an extension of basic Markowitz-type portfolio optimization[4]. The new aspect is that optimization or efficiency is derived not based on absolute risks and correlations of and between asset classes, but relative to the characteristics of the liabilities. The result will thus not be a (set of) market portfolio(s) in the traditional definition, but a Strategic Asset Allocation (SAA) with a ‘liability-neutral’ portfolio for the individual pension fund.[5]

1.3 Types of ALM Models

Every ALM model simulates the impact of exogenous variables on the target variables, i.e. the development of the balance sheet of the pension fund over time. The link between exogenous and target variables is given by many policy variables or functions. In simple sequential models, the target variables are optimized by altering a single policy variable until optimal results will be achieved. A typical procedure with a simple model would be to adjust the investment policy, i.e. the asset class allocation, to a path of balance sheets respectively cash flows deemed optimal.

In integrated models, however, different policy variables would be linked to each other. Contributions and/or funding policies may relate to specific investment policies. For instance, a policy striving for higher funding ratios may be tied to a more risky investment policy, leading at the end to a policy with contribution holidays. In a further step, it may turn out that respective combinations of investment and funding policies may all yield results considered sub-optimal. As a consequence, alternative policies on the liability side may be taken into account, such as closing the pension fund for new entrants and thus controlling the population development.

4 See “Modern portfolio theory I”.

5 See the document “Asset allocation” in “Investment strategy”.

Dynamic integrated models will use optimizing functions in which policy variables for the next period will be set depending on the state of one or more other policy variables or balance sheet components at the end of the previous period. In those models, dynamic asset allocation policies can be made dependent on the level of funding ratios, leading to pro- or counter-cyclical investment behavior. Similarly the reserve policy of the fund can be made dependent on the past funding situation, which may also impact the contribution policy of the employer.

These more sophisticated models will ultimately provide insights into the optimal overall policy of the pension fund. More restricted sequential versions can be used to analyze very specific relationships like specific investment policy changes, but will always remain sub-optimal by definition.

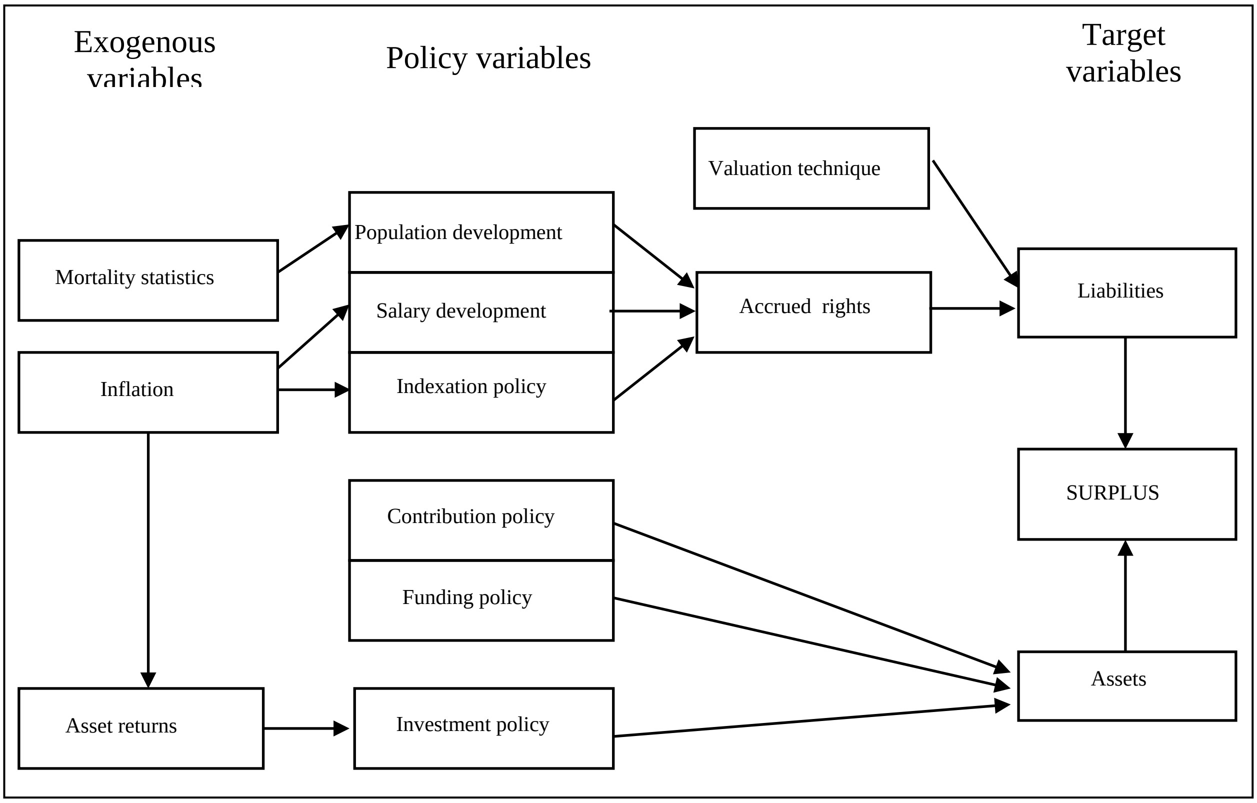

Figure 1-2: Structure of ALM Models

2. Modeling Liabilities

2.1 Types of Liabilities

A major task for the management of a pension fund is the determination of the level of contributions and how to invest plan assets. In principle, contributions must be specified based on plan liabilities which are in turn determined by actuaries who perform valuations of the obligations under the terms of the plan. In this sense, Asset/Liability-Management starts with a projection of the amount and the timing of benefits that will eventually be paid to the beneficiaries over a particular time period, e.g. 10, 20 or 30 years. To do so, information about the various benefits provided by the plan must be combined with demographic and economic assumptions. These factors are summarized in the following table and will be discussed in following:

Benefits	Demographic Assumptions	Economic Assumptions
- Retirement Benefits - Death Benefits - Disability Benefits	- Mortality - Disability - Retirement Age - Population Dynamics	- Inflation - Wage Increase - Discount Factor - Investment Returns

Table 2-1: Factors of Pension Liabilities

The primary purpose of a defined benefit plan is to provide life-long pension payments at retirement according to a well-defined benefit formula (retirement benefits). There are many benefit formulas available that could be used. The simplest version is the flat benefit formula, whereby the employer promises to pay at the age of retirement a fixed amount of money for each year of service. However, in most DB-plans a salary-related benefit formula is applied where the benefits are related to the final or average salary earned by the employee. Next to old age benefits many retirement plans provide benefits for other purposes: The most important are income benefits in the event the participant dies (death benefits) or becomes disabled (disability benefits).

Demographic assumptions are necessary to project how and when participants will leave active association with the plan (population dynamics) and how long benefits will be paid after retirement (mortality pattern). These assumptions must include anticipated mortality and disability rates, probabilities of retirement or plan exit due to changing jobs. Both, population dynamics as well as mortality patterns can be summarized in a transition model (see Table 2-2). The transition model takes into account the change of age and status for each member at the end of the period (i.e., active, former, disabled, pensioner, widow, and death).

To / From	Active	Former	Disabled	Pensioner	Widow(er)	Dead
Outside	X					
Active	X	X	X	X	X	X
Former		X			X	X
Disabled	X		X	X	X	X
Pensioner				X	X	X
Widow(er)					X	X

Table 2-2: Population Dynamics and Transition Matrix

The change between the various living conditions of the employee is described by transition probabilities.[6]

The major economic variables that have impact on the valuation of a pension plan are price inflation, salary increases, and investment returns. Usually, these variables are interrelated and should be specified simultaneously.

Assumptions on investment returns are crucial since they influence the rate at which pension liabilities should be discounted. Due to their long-term character, pension liabilities have a long duration and therefore are very sensitive to the discount rate selected. How to select the discount rate is an ongoing debate with respect to an actuarial versus an economic valuation of pension liabilities. Traditionally, actuaries choose the discount rate that achieves a reasonable projected expected return of the asset while covering the pension liabilities. Sometimes these returns are related to the past performance of the pension plan assets. If the pension assets are partly invested in equities, the discount rate also includes an equity risk premium, which - from an ex-ante perspective – is not yet realized. On the opposite, many economists argue that the relevant number for discounting future pension payments is the risk-free interest rate reflecting the financing cost of the plan sponsor.

2.2 Valuation of Pension Liabilities

We now assume a final salary DB arrangement. At time t, the expected discounted value of the liabilities for a member in terms of retirement benefits depend on the expected final salary, the likelihood that he will stay in the scheme until retirement at time T and finally the amount of years that he will receive pension payments. The value of the liabilities can be calculated with the following formula[7]

$$L_t = \alpha_t \cdot W_t \cdot RF_{t,T} \cdot \Pi_{t,T} \cdot a_{t,T} \cdot d_{t,T} \qquad (2)$$

W_t is the current pensioner's salary at age t and $\alpha(t)$ is the pension benefit accrual factor for years of service at time t. For example, if the members accrual rate for each year of service is 2% of the pensioners' earnings and he/she has worked for the company ten years the accrual factor is equal to $\alpha_t = 10 \cdot 2\% = 0.2$.

6 From a mathematical perspective, such a transition matrix defines an inhomogeneous Markov chain. Markov chains are also used to model credit ratings of fixed income instruments.

7 See Blake, D. (2006): "Pension Finance", Wiley, New York, pp. 194-196.

$RF_{t,T}$ is a revaluation factor for earnings up to the time of retirement. In the case of the so-called accumulated benefit obligation (ABO) method of pension evaluation $RF_{t,T} = 1$, i.e. there is no revaluation of salary until retirement. In the case of the projected benefit obligation (PBO) method, often a constant growth rate g of yearly earnings until retirement is used, i.e. $RF_{t,T} = (1+g)^{T-t}$. The ABO method is an appropriate measure of plan liabilities under the assumption that the plan is terminated at the calculation date. In this sense, the ABO approximates the termination liability of a pension plan. The PBO method assumes that the plan is ongoing and will not be terminated. Future influences in the ABO valuation are mainly of demographic nature, like mortality and retirement. In addition to that, the PBO method reflects future economic events like salary increase.

The so-called retention factor $\Pi_{t,T}$ describes the probability that the member remains in the scheme until retirement. There are many reasons why participants do not stay in the pension scheme up to retirement being the most important death and staff turnover.

The annuity factor is denoted by $a_{t,T}$ and represents the present value at the time of retirement of a life annuity paying each year EUR 1 as long as the retiree is alive. So, the annuity factor can be interpreted as the time that the pensioner is expected to receive benefits from the pension scheme. The annuity factor uses survival probabilities based on mortality statistics and an assumed interest rate to discount future benefit payments. In section 2.3, we will discuss the annuity factor in more detail.

Finally, $d_{t,T}$ is the discount factor between time t and the retirement age T. If the interest rate is constant at r, the discount factor is equal to $d_{t,T} = 1/(1+r)^{T-t}$.

Example 1:
To illustrate Formula (2), assume a single female member of a final salary DB pension plan is aged 40. The plan pays retirement benefits according to an accrual rate of 2% for each year of service (no death and disability benefits). She has been working for the company for 10 years and her current salary is EUR 25.000 per year. It is assumed that her current earnings grow by 1% p.a. in real terms over the next 25 years until retirement. The benefits from the pension plans are adjusted year by year according to the rate of inflation, i.e. the retirement benefits in real terms are flat. The real discount rate is assumed to be r = 3% p.a. Using the same interest rate the annuity factor for a 65-year old women with respect to a mortality table is given by a_{65} = 18.73. Based on past experience, there is a 95% probability that she will be alive by her normal retirement age at 65.
Using the above information the actuarial present value of the pension liability for this member is equal to:

$$\text{Liability} = 0.2 \cdot 25,000 \cdot (1.01)^{25} \cdot 95\% \cdot 18.73 \cdot (1.03)^{-25} = 54,491$$

The member can then expect a final salary of 25,000 · $(1.01)^{25}$ = 32,061. Based on years of employment, she can expect a life-long pension of 20% · 32,061 = 6,412 per year. At retirement, the pension fund needs 6,412 · 18.73 = 120,097 to finance the life-long benefit payments. The present value of this amount is equal to 120,097 · $(1.03)^{-25}$ = 57,359. If the pension fund invests this amount over the next 25 years in real rate with a return of 3% p.a. then at retirement the pension fund would be able to pay a life annuity of 6,412 per year for as long as she will live. Since she will reach the age of retirement only with a probability of 95%, we have to multiply 6,412 by 0.95. This equals to 6,091. Therefore, only an amount of 54,491 is necessary today to fully fund the pension promise in future.

If the pension plan covers a population of i = 1, 2, ….N(t) participants then the total actuarial present value of liabilities of a pension plan at a certain point in time t is given by the sum of the pension liability of each member:

$$TL_t = \sum_{i=1}^{N(t)} L_{i,t} \qquad (3)$$

Note, at current time t = 0 the population covered by the plan is known and given by the number of pensioners. However, within an ALM-study pension plan managers would be interested in projecting the development of the future liability structure over a horizon of t = 1, 2, … years for a varying population. This means that N(t) is no longer a deterministic, but a stochastic variable since the number of pensioners will vary with time. In such a case the uncertain number of plan participants can be modeled using a population model described in Table 2-2 of the previous section.

Adding to the dynamics and the stochastic nature of the future plan population other sources of liability risk are the volatility in the revaluation, retention, and the annuity factor.[8] Volatility in the revaluation factor results from salary inflation or productivity shocks regarding the member's earnings between now and retirement. The volatility of the retention factor depends on the changing statistics of death-in-service rates as well as staff turnover. Finally, the volatility of the annuity factor results from the uncertainty of future survival probabilities (also called mortality or longevity risk) and fluctuating interest rates.

2.3 Annuity Factors and Discount Rates

Within most of the Defined Benefit (DB) plans the employer promises to pay life-long benefits to the retired employee. From the viewpoint of financial economics such a promise can be characterized as a life annuity. It is a financial agreement between an individual (annuitant) and the annuity provider "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium".[9] The payments may be fixed in nominal terms (fixed annuity), they might rise at a pre-specified fixed nominal escalation rate (graded annuity), they could be indexed to inflation (real annuity) keeping the retiree's standard of living constant, they can depend on the performance of a specific asset portfolio (variable annuity), or they can reflect the annuity provider's experience with mortality, investment returns, and expenses (participating annuity).

In the case of an immediate annuity the benefit payments to the annuitant start with the purchase of the annuity. In the case of a deferred annuity, periodic payments to the annuitant start at some pre-specified future date. With respect to coverage a single life annuity pays only as long as the annuitant is alive. In the case of a joint and survivor annuity the surviving spouse continues to receive income as long as he or she lives.

8 See Blake D. (2006): "Pension Finance", Wiley, New York, pp. 256-258.

9 See Brown, J.R., Mitchell, O.S., and J. Poterba (2001): "The Role of Annuity Markets in Financing Retirement", MIT Press, Boston, p. 1.

As Mitchell et al. (1999)[10] point out the essential attraction of a life annuity is that the individual is protected against the risk of outliving his own assets, given uncertainty about his remaining lifetime. Usually, life annuities are sold by insurance companies or are provided by large corporations within a DB occupational pension scheme. The key technique of the annuity provider to manage its own risk position is to pool longevity risk across a sufficiently large group of annuity purchasers. The capital of those who die is allocated across surviving members of the cohort.

In order to value the benefit payments of an annuity actuaries use the principle of equivalence by calculating the present value of expected benefits paid to the annuitant. Hereby, explicit assumptions must be made about mortality risk, the annuitant's age, and the interest rate used to discount expected benefit payments. The actuarial present value of a life annuity (PVA) is then given by

$$PVA_t = R \times \sum_{t=0}^{w-x} \frac{p_{x,t}}{(1+r)^t} \qquad (4)$$

In the above equation, r denotes the (deterministic) assumed interest rate used to discount future expected cash flows, R is the annuity and $p_{x,t} = p_x \cdot \ldots \cdot p_{x+t-1}$ is the cumulative probability that a man aged x will survive to age x+t. The $p_{x,t}$-probability is thus the conditional year-to-year survival probability for a person with respect to a mortality table with ending age w. The survival probabilities depend on the sex of the person.

In the case of joint and survivor annuities, the survival probability can be calculated as joint probability of the survival of person 1 aged x and person 2 aged y, i.e., $p_{x,y,t} = p_{x,t} + p_{y,t} - (p_{x,t} \text{ x } p_{y,t})$ should be used. It includes the cases that at least one member of the couple is alive t years after retirement.

Example 2:
Using the mortality rates given in appendix A, the probabilities that a men aged 65 will survive until age 66, 67, and 68 respectively are calculated by

$$p_{x=65,t=1} = (1-0.007412) = 0.992588$$
$$p_{x=65,t=2} = (1-0.007412)\cdot(1-0.008046) = 0.992588\cdot(1-0.008046) = 0.984602$$
$$p_{x=65,t=3} = (1-0.007412)\cdot(1-0.008046)\cdot(1-0.008833) = 0.984602\cdot(1-0.008833) = 0.975905$$

Please note, that (1 - mortality rate of age x) gives the probability of not dying at age x.

Example 3:
Using the mortality rates given in appendix A, the probability that a least one member of a couple (men aged 85, women aged 65) will survive over the next 2 years is calculated as follows:

$$p_{x=85,t=2} = (1-0.049647)\cdot(1-0.055969) = 0.89716$$
$$p_{x=65,t=2} = (1-0.004056)\cdot(1-0.004309) = 0.99165$$
$$p_{x=85,y=65,t=2} = 0.89716 + 0.99165 - 0.89716\cdot 0.99165 = 0.99914$$

The second factor of Equation (4), which combines information on discount factors as well as survival probabilities, is referred to as life annuity factor ax. Formally, it is defined as

10 Mitchell, O.S., Poterba, J.R., Warshawsky, M., and J. Brown (1999): "New Evidence on the Money Worth of Individual Annuities", *American Economic Review*, 89, pp. 1299-1318.

$$a_{x,w} = \sum_{t=0}^{w-x} \frac{p_{x,t}}{(1+r)^t}. \quad (5)$$

Example 4:
Using the mortality rates given in appendix A, we calculate the annuity factor for a female aged 119 with an assumed interest rate of 3%. We assume that the payments are paid at the beginning of each year, and that the last payment is possible at the age of 121.

Step 1: Calculation of survival probabilities

$p_{x=119,t=0} = 1$

$p_{x=119,t=1} = (1-0.258115) = 0.741885$

$p_{x=119,t=2} = 0.741885 \cdot (1-0.264504) = 0.54565$

Step 2: Calculation of annuity factor

$$a_{x,w} = \frac{1}{(1.03)^0} + \frac{0.741885}{(1.03)^1} + \frac{0.54565}{(1.03)^2} = 2.2346$$

Life annuity factors are of great importance for projecting pension liabilities. They depend on age, sex, assumed interest rate, and mortality patterns. Using the mortality table given in the appendix, Table 2-3 displays the life annuity factors for three different interest rates and ages and thus illustrates the sensitivity of the annuity factor concerning interest rates.

Interest Rate	**1.50%**	**3.00%**	**5.00%**
Age		**Male**	
65	20.13	16.91	13.80
75	14.45	12.72	10.92
85	9.11	8.35	7.52
		Female	
65	22.72	18.73	14.99
75	16.73	14.48	12.20
85	10.80	9.77	8.65
		Joint & Survivor	
65	25.51	20.73	16.30
75	19.41	16.58	13.75
85	13.22	11.83	10.35

Source: Authors' own calculations

Table 2-3: Life Annuity Factors

3. Modeling Assets

3.1 Types of Asset Classes

In some ALM models, the asset side of the balance sheet is calculated in great detail in order to be able to compare the recommendations of the model with the actual investment portfolio of the fund. It seems questionable, though, if reliable long-term forecasts can be provided as input to the simulation. It is rather more appropriate to model the asset side only for those asset classes, for which it is reasonable to assume stable input parameters for a longer time horizon. Of course, the modeled asset classes should be those available to the pension fund to invest in. Equally well, the chosen input parameters should actually have an impact on the various investment and allocation policies of the fund.

Looking from the liability side of the fund, a typical plan with fixed defined benefits can be viewed as a credit given by the employees to the pension fund. This behavior would correspond to a corporate bond issued by that company with a duration matching that of the liabilities. Therefore, corporate bonds of the credit rating of the company are natural assets to be modeled. On the asset side of most funds, bonds with high credit rating in the local currency, i.e., government bonds, are added. The slope of the yield curve and its development over time is also needed to provide information for a dynamic matching of cash-flows. Risks and risk premia on the bond side will be given by the duration of the assets in terms of the government bond yield curve, and by the corporate spreads in those durations or maturities.

Equities constitute the next broad asset class. For DB plans with final salary designs, equities can be assumed to correlate with future salary developments, since they should broadly replicate the overall development of companies. Furthermore, equities can be invested to increase the duration of assets towards the duration of liabilities, when bond investments in the markets are not available with such long durations. Next to these broadly risk-neutral characteristics, equities will serve as the main asset class underlying an active investment strategy of a pension fund based on asset allocation exposures. Assumptions on the future development of the equity risk premium will have to be made, but it is reasonable to expect those to remain fairly stable over longer time horizons.

Another common asset class are alternative investments. These are often included in asset/liability models due to their expected low correlations with the traditional asset classes of equities and bonds, and due to the resulting favorable diversification characteristics. Traditionally, alternatives comprise real estate investments. Nowadays, private equity, hedge funds, and commodities are also viewed as constituents of this asset class. Unfortunately, forecasting stable risk premia for these types of assets seems to be rather difficult. Furthermore, in practice it may not be an easy task to find appropriate investments with stable relationships with managers and sufficient liquidity, at least for larger pension funds. Therefore, while alternative investments may turn out to improve the simulation results of asset/liability models, it may seem appropriate to limit those investments to smaller portions of the investment portfolio, e.g. to 10% or even 5% of overall assets. Table 3-1 gives an overview on the main asset classes, their characteristics and typical modeling assumptions.

Asset Class	Characteristics	Duration	Modeling
Government Bonds	Safe cash-flows	0 to 30+	Development of yield curve over time
Corporate Bonds	Relatively safe (refinancing position)	0 to 30+	Yield curve plus corporate spread
Equities	Salary and productivity matching, return enhancing	long	Yield curve plus equity risk premium
Alternatives	Diversification, safe cash-flows (real estate)	0 to 30+	Yield curve plus risk premium, alpha, low correlations

Table 3-1: Asset Classes, Basic Characteristics

3.2 Risk and Return Characteristics

The asset classes are subject to a number of risk factors, e.g. market risks, counter-party risks, interest rate risks. The starting point for the forecasting assumptions is the bond market yield curve in combination with inflation assumptions. Corporate bonds have a spread over government bonds but are typically closest to the refinancing position of the DB pension promise in relative terms. Equities are assumed to have a (time-varying) risk premium of 2 to 3% over long-term government bonds. The assumptions for alternative investments can vary depending on the sub-class, i.e. hedge funds would be modeled around the short-term yield. Real estate could rather follow the long-term yield but with a constant cash-flow when viewed as a bond-type investment. Alternatively, real estate could be modeled using equity-like behavior, as could be private equity. Table 3-2 gives an overview on typical risk/return assumptions used in an ALM model.

Asset Class	Return Assumptions	Std. Dev. Assumptions
Short-term Yield	2 to 3% yield	0 to 1 % p.a.
Government Bonds	4 to 5% yield	4 to 6 % p.a.
Corporate Bonds	0.5 to 1 % spread over government bonds	1 to 2% p.a. for spread
Equities	2 to 5% risk premium over government bonds	10 to 20% p.a. over long-term bonds
Alternatives	Depends on type, e.g. 0.5 to 1 % premium over short-term yield for Hedge Funds	Depends on type and vehicle

Table 3-2: Typical Risk and Return Assumptions for Asset Classes

4. Surplus and Funding Ratios

The difference between the value of the sum of assets A_t and the sum of liabilities L_t determines the surplus SP_t of the pension fund at a given point in time and is defined as:

$$SP_t = A_t - L_t. \quad (6)$$

One can distinguish between different surplus measures depending on how the value of the asset and liabilities are calculated. If accepted accounting principles are applied, the surplus is called an accounting surplus. If a pension fund has to report its financial situation to a regulator with special regulatory accounting principles it is called a regulatory surplus. In the case that the sum of assets and liabilities are measured according to their market value the surplus is called an economic surplus. Given the value of asset and liabilities fluctuate over time the surplus of a pension fund is uncertain and changes from year to year. This variability is also called the surplus risk.

Closely related to the surplus is the funding ratio FRt, which is given by the ratio of the plan's assets to its liabilities:

$$FR_t = \frac{A_t}{L_t} \quad (7)$$

If the funding ratio is greater than 100% the plan is called over-funded; if the funding ratio is less the 100% the plan is under-funded.

In principle, a pension fund should show a funding ratio close to 100 percent. The assets are then sufficient to cover the liabilities. In practice, some fluctuation around this requirement is allowed, i.e. there is some lower and upper limit $FR_t^{min} \le FR_t \le FR_t^{max}$ where no action is required. However, if the pension fund shows a systematic under-funding situation, actuaries and regulators can require higher contributions following a solvency plan. This can be achieved by increasing the regular contributions of employers and employees or by employees' supplements. If for example the funding ratio is below 90 percent or lower than 95 percent for more than three consecutive years, this can define a systematic under-funding situation. With respect to the regulation, the implementations of the solvency standards for pension funds differ between the various countries.

If the pension fund shows a systematic over-funding situation, i.e. the funding ratio is much higher than 100 percent, actuaries, plan sponsors and/or participants would be asking for ways in which the surplus can be reduced. Contribution holidays for employers and employees are the most common way for solving this problem. Other ways to reduce a surplus are the increase of benefits or to sell assets and allow the plan sponsor to withdraw excess funds. For example, in the case of the funding ratio exceeding 120 percent, the rate of regular employer contribution can be reduced by 50 percent. The plan sponsor will benefit from full contribution holiday when the funding ratio rises above 150 percent, and would be able to withdraw excess funds when the funding ratio exceeds 180 percent.

5. Integrated Optimization

5.1 Target Functions and Tradeoffs

The development of assets and liabilities and of the exogenous factors included in them (e.g. inflation, interest rates, salaries, mortality assumptions), provide the inputs to the target functions for the pension fund. In a real world, pension fund managers and their sponsors will have to pursue more than one single well-defined goal. Rather, they must respect a sometimes complex set of multiple objectives. Table 5-1 gives an overview on a number of different targets that can be pursued by a pension fund.

Asset Objectives	Liability Objectives
Maximize investment returns	Minimize risk on pension payment shortfall
Minimize volatility of investment returns	Minimize revaluation risks
Asset/Liability Objectives	
Minimize funding ratio volatility	
Minimize funding ratio shortfall probability	
Minimize company/employee contributions	

Table 5-1: Overview of Different Targets[11]

A typical combination of goals would be that the pension fund should follow an investment policy in order to achieve a pre-defined benefit level at minimum costs considering at the same time the highest possible returns of the investments. As a further goal or restriction, this should be achieved only investing in portfolios considered efficient, i.e. every portfolio selected would have to have the highest possible return for a given level of volatility or standard deviation of returns. Another goal would be to require the fund to keep the funding ratio sufficiently high over the next years.

11 See also Muralidhar, A. (2001): "Innovations in Pension Fund Management", Stanford Economics and Finance, Stanford, p. 50.

5.2 Surplus Risk Management

The liabilities of a pension plan are financed in two ways: by contributions and by asset returns. Because both are interrelated, it is important to understand the implications of an investment strategy on the funding situation and vice versa. It would certainly be one important goal for the pension fund to maintain a reasonable funding ratio over time.[12] Subsequently, we illustrate these relationships between funding ratio and asset return with a simple one period model.[13] Table 5-2 lists the basic variables of the model.

Variable	Initial Value	Future Value	Return	Mean	Volatility
Assets	A_0	A_1	R_A	μ_A	σ_A
Liabilities	L_0	L_1	R_L	μ_L	σ_L
Surplus	SP_0	SP_1	R_{SP}	μ_{SP}	σ_{SP}

Table 5-2: Definition of Asset and Liability Variables

Furthermore, assets and liabilities are correlated by ρ_{AL}.

The change of the surplus after one year is given by

$$\begin{aligned} SP_1 - SP_0 &= R_A \cdot A_0 - R_L \cdot L_0 \\ &= L_0 \cdot (R_A \cdot FR_0 - R_L) \end{aligned} \qquad (8)$$

with $FR_0 = A_0/L_0$.

Since the initial surplus can be zero, we measure the change in surplus against the current liability.[14] This measure is also called the surplus return and is given by

$$R_{SP} = \frac{SP_1 - SP_0}{L_0} = FR_0 \cdot R_A - R_L \qquad (9)$$

The above equation shows that the surplus return is the difference between the "adjusted" asset return FR_0R_A and the liabilities return. After taking expectations of both sides the mean return of the surplus distribution can be expressed as

$$\mu_{SP} = FR_0 \cdot \mu_A - \mu_L . \qquad (10)$$

12 See Muralidhar (2001), p. 12.

13 This model is derived from Leibowitz, M.L., Bader, L.N., and S. Kogelman (1996): "Return Targets and Shortfall Risk: Studies in Strategic Asset Allocation", Irwin, Chapter 3.

14 This chapter draws on Leibowitz, M.L., Bader, L.N., and S. Kogelman (1996): "Return Targets and Shortfall Risk: Studies in Strategic Asset Allocation", Irwin, pp. 55-60.

The surplus risk is then defined as

$$\sigma_{SP} = \sqrt{(FR_0 \cdot \sigma_A)^2 - 2 \cdot (FR_0 \cdot \sigma_A) \cdot \sigma_L \cdot \rho_{AL} + \sigma_L^2} \qquad (11)$$

and measures the degree of variability of the surplus return. In general, surplus risk is a non-linear function of the volatility of asset and liabilities, the correlation of asset and liabilities and the funding ratio. The surplus risk can be reduced to zero if the volatilities of the adjusted asset and liabilities return are the same and perfectly correlated, i.e., $\rho_{AL} = 1$. In the case of a non-stochastic liability return, i.e., a liability return with risk $\sigma_L = 0$, the surplus risk is proportional to the volatility of assets and the funding ratio.

A common question in this context concerns the probability of shortfalls, i.e., the probability α that the surplus return of a strategy is below a certain minimal threshold SP_{min}. Formally, the shortfall risk α is defined as

$$P[R_{SP} \le SP_{min}] \le \alpha . \qquad (12)$$

In the case of the surplus return being normally distributed, the probability of such a shortfall is given according to the formula[15]

$$P(R_{SP} \le SP_{min}) = \Phi\left[\frac{SP_{min} - \mu_{SP}}{\sigma_{SP}}\right]. \qquad (13)$$

On the other hand, if we fix the tolerated level of shortfall risk to α, the requirement is equivalent to

$$\mu_{SP} \ge SP_{min} + z_\alpha \cdot \sigma_{SP} . \qquad (14)$$

Here, z_α is the percentile of the standard normal distribution. For example, if the shortfall risk tolerance is 10%, then z_α = 1.28. Equation (14) shows a linear relationship between the expected surplus return, the surplus risk, the threshold, and the level of risk tolerance. Substituting the mean and standard deviation of the surplus return according to Equations (10) and (11), the shortfall constraint (14) can be rewritten as

$$\mu_A \ge \frac{SP_{min} + \mu_L + z_\alpha \sqrt{(FR_0 \cdot \sigma_A)^2 - 2 \cdot (FR_0 \cdot \sigma_A) \cdot \sigma_L \cdot \rho_{AL} + \sigma_L^2}}{FR_0} \qquad (15)$$

In order to fulfill the above shortfall constraint, the expected asset return must be equal or greater than the right-hand side of the equation. Equation (15) expresses the shortfall constraint in terms of asset variables. It allows drawing the shortfall constraint together with the risk/return correlations of the asset portfolios in the (μ_A, σ_A)-diagram known from Markowitz theory. To illustrate this procedure, we assume a pension fund with only two assets, an equity and a bond. If w is the relative investment weight in equities, μ_E the expected return of stocks, and μ_B the expected bonds return, then the mean return of the assets of the pension fund can be calculated as

$$\mu_A = w \cdot \mu_E + (1 - w) \cdot \mu_B \qquad (16)$$

15 The formula uses the z-transform. In case of a standard normal distribution, we could look up $\Phi^{-1}(\alpha)$ as so-called z-value. The same percentile for another normal distribution is then given by $y = \mu + z\sigma$. Here, SP_{min} takes the role of y.

Correspondingly, the standard deviation of the asset return is given by

$$\sigma_A = \sqrt{w^2 \cdot \sigma_E^2 + 2 \cdot w \cdot (1-w) \cdot \sigma_E \cdot \sigma_B \cdot \rho_{EB} + (1-w)^2 \cdot \sigma_B^2} \quad (17)$$

Here σ_E is the volatility of equities, σ_B denotes the volatility of bond returns and ρ_{EB} is the coefficient of correlations between both asset classes. Equation (16) and (17) allow us to graph the return of the various asset portfolios in the usual risk-return diagram.

To calculate surplus risk, we have to specify the correlation of assets and liabilities, which is given by the following equation:

$$\rho_{AL} = \frac{w \cdot \sigma_S \cdot \rho_{EL} + (1-w) \cdot \sigma_B \cdot \rho_{BL}}{\sigma_A} \quad (18)$$

The relationship of assets and liabilities is a weighted and normalized sum of the correlation of stocks with liabilities, and bonds with liabilities. By substituting Equations (18) and (17) into Equation (11) the surplus risk can be expressed as a function of the asset parameters. This relation could be used to draw the shortfall constraint together with the efficient frontier in a (μ_A, σ_A)-diagram.

5.3 Case Study Pension Fund Management

Case Study Part 1: Calculation of Surplus Risk

In this case study we illustrate the previous formulae using a numerical example. Assume the risk and return profile of the assets and liabilities of a pension fund as shown in Table 5-3. The required threshold for the surplus return is SP_{min} = -10% and the level of risk tolerance is equal to $\alpha = 10\%$.

Variable	Initial Value (Mio)	Expected asset return	STD asset return	Correlation with bonds	Correlation with equities
Equities	60	10.0%	18.0%	0.3	1.0
Bonds	60	5.0%	6.0%	1.0	0.3
Liabilities	100	5.0%	8.0%	0.8	0.2

Table 5-3: Input Parameters for Case Study Part 1

The initial funding ratio of the pension fund is FR_0 = 120 / 100 = 120%.

Since equities and bonds have the same initial value, the weight of each position is 0.5. The mean return of the total assets of the pension fund can then be calculated as

$$\mu_A = 0.5 \cdot 10\% + 0.5 \cdot 5\% = 7.50\%$$

This results in an expected surplus return of

$$\mu_{SP} = 1.2 \cdot 7.5\% - 5\% = 4.00\%$$

The standard deviation of the asset return is given by

$$\sigma_A = \sqrt{0.5^2 \cdot 18\%^2 + 2 \cdot 0.5 \cdot (1-0.5) \cdot 18\% \cdot 6\% \cdot 0.3 + (1-0.5)^2 \cdot 6\%^2} = 10.31\%$$

and the correlation between assets and liabilities is

$$\rho_{AL} = \frac{0.5 \cdot 18\% \cdot 0.2 + (1-0.5) \cdot 6\% \cdot 0.8}{10.31\%} = 0.41$$

Finally, the surplus risk can be calculated as

$$\sigma_{SP} = \sqrt{(1.2 \cdot 10.31\%)^2 - 2 \cdot (1.2 \cdot 10.31\%) \cdot 8\% \cdot 0.41 + 8\%^2} = 11.66\%$$

Now we are ready to check if the pension fund fulfils the shortfall constraint. Using Equation (14), we see that

$$\mu_{SP} = 4.0\% \leq -0.1 + 1.28 \cdot 11.66\% = 4.92\%$$

This means that the current asset portfolio of the pension funds is not consistent with the shortfall constraint. Alternatively, we can calculate the right-hand side of Equation (15) and obtain

$$\frac{-0.1 + 5\% + 1.28 \times \sqrt{(1.2 \cdot 10.31\%)^2 - 2 \cdot (1.2 \cdot 10.31\%) \cdot 8\% \cdot 0.41 + 8\%^2}}{1.2} = 8.3\%$$

As this clearly exceeds the mean asset return μ_A of 7.5%, the shortfall constraint in Equation (15) is also violated.

Using Equation (13), we can finally determine the actual shortfall probability

$$P(R_{SP} \leq SP_{min}) = \Phi\left[\frac{-0.1 - 4\%}{11.66\%}\right] = 11.5\%.$$

If we repeat the above calculations for the different equity weights we can draw the risk-return correlation of the assets and the shortfall-constraint in the following (μ_A, σ_A)-diagram (see Figure 5-1). The risk and return profiles of the possible portfolios consisting of equity and bonds (with a short selling restriction) are given by the solid line. The surplus shortfall constrained curve is given by the dashed line. All risk and return profiles on or above the shortfall curve are consistent with required minimum surplus return of -10% at the 90% -level of confidence.

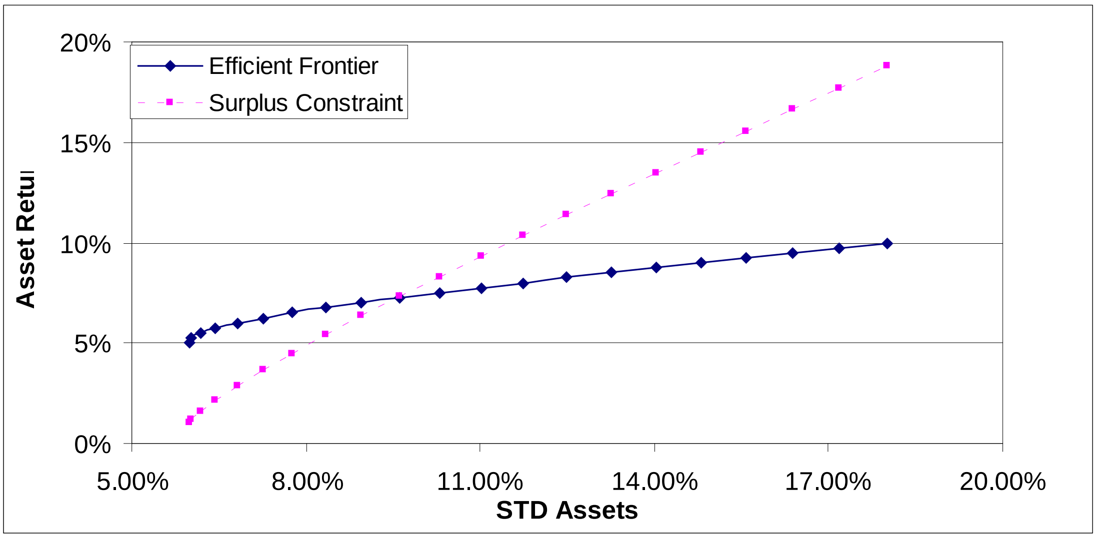

Figure 5-1: Efficient Frontier and Surplus Shortfall Constraint

In Table 5-4 we calculate for various equity investment weights the mean return of the asset portfolio, the required return to satisfy the shortfall constraint (target -10%, tolerated shortfall risk level 10%), and the effective shortfall probability. We see that equity weights of more than 45% do not satisfy the surplus constraint.

Weight equities	Mean asset return	STD asset return	Required asset return*	Shortfall probability
0%	5.00%	6.00%	1.0%	1.2%
5%	5.25%	6.03%	1.2%	1.2%
10%	5.50%	6.18%	1.6%	1.6%
15%	5.75%	6.45%	2.2%	2.2%
20%	6.00%	6.81%	2.9%	3.2%
25%	6.25%	7.26%	3.6%	4.4%
30%	6.50%	7.77%	4.5%	5.7%
35%	6.75%	8.35%	5.4%	7.2%
40%	7.00%	8.96%	6.3%	8.7%
45%	7.25%	9.62%	7.3%	10.1%
50%	7.50%	10.31%	8.3%	11.5%
55%	7.75%	11.02%	9.3%	12.9%
60%	8.00%	11.75%	10.3%	14.1%
65%	8.25%	12.49%	11.4%	15.3%
70%	8.50%	13.25%	12.4%	16.4%
75%	8.75%	14.02%	13.5%	17.4%
80%	9.00%	14.80%	14.5%	18.3%
85%	9.25%	15.59%	15.6%	19.2%
90%	9.50%	16.39%	16.6%	20.0%
95%	9.75%	17.19%	17.7%	20.7%
100%	10.00%	18.00%	18.8%	21.4%

* Required mean return of assets to satisfy the surplus shortfall constraint acc. to Equation (15). Minimum target surplus return -10%, risk tolerance 10%, current funding ratio 120%

Table 5-4: Equity Weights, Asset Returns and Shortfall Probability

Case Study Part 2: Multiple Targets

In this case, we discuss various options for target using a setting slightly different from the base case given in the previous section. The parameters are specified in Table 5-5.

	Expected asset return	STD asset return	Correlation with bonds	Correlation with equities
Stocks	8.0%	20.0%	0	1.0
Bonds	5.0%	7.0%	1.0	0
Liabilities	5.0%	7.0%	0.9	0

Table 5-5: Input Parameters for Case Study Part 2

Risk tolerance is 10%, the initial funding ratio is FR_0 = 100%, and the shortfall target is –5%. Using the various formulas from above, Table 5-6 shows the simulation results for different portfolios in terms of the average expected investment return for the pension fund, the corresponding standard deviations of those returns, as well as the shortfall surplus risk constraint according to Equation (15).

The numbers in Table 5-6 show that a portfolio comprising 100% bonds would yield a return of 5% with a standard deviation of returns at 7%. Increasing the equity allocation to 20% would exploit diversification effects between stocks and bonds. The expected returns would go up to 5.6% while at the same time the standard deviation of the investment portfolio could be lowered to 6.9%. If we only look at assets, the portfolio with 20% invested in stocks is clearly superior to the strategy with bonds only.

Weight equities	**Return assets**	**STD asset return**	**Shortfall probability***
0%	5.00%	7.00%	5.5%
5%	5.15%	6.72%	5.5%
10%	5.30%	6.61%	7.3%
15%	5.45%	6.66%	10.2%
20%	5.60%	6.88%	13.5%
25%	5.75%	7.25%	16.7%
30%	5.90%	7.75%	19.5%

* Minimum target surplus return -5%, current funding ratio 100%

Table 5-6: Equity Weights, Asset Returns and Shortfall Probability

The numbers in Table 5-6 show however that the shortfall probability increases from 5.5% in the bonds only portfolio to 13.5% for the portfolio with 20% equities. In other words, the risk of a shortfall would not just be 1 within the next 20 years but rather 1 in the next 6 or 7 years. In this case, the pension managers would have to decide on the tradeoffs and choose a portfolio with between 0% and 20% invested in equity, under further evaluations and further simulations of the outcomes using adjustments of the risk aversion parameters.

Case Study Part 3: Portfolio Selection

In order to select the final portfolio for the pension fund the fund manager must specify tradeoff functions for the various goals followed by the fund. One version could be to select the minimum risk portfolio given that it satisfies the shortfall constraint on the surplus. This implies that the volatility of the assets is weighted relatively high compared with the asset returns. For reasons of illustration, we use again the input numbers from Part 1. As can be seen in Figure 5-1 and Table 5-7 the selected portfolio would then be invested 100% in bonds.

Alternatively, the fund may specify the tradeoff such that they select the portfolio with the highest return given that it satisfies the shortfall constraint. In Table 5-7 this would be the portfolio with a 45% investment in equities. In practice, the fund may rather choose to use a function that considers the relative tradeoff of portfolio return and volatility while at the same time respecting the shortfall restriction. A simple example taking the numbers from Table 5-7 shows how relevant the decision function is. If in Table 5-7 we would take the ratio between mean return and standard deviation of assets and would try to maximize this number as the rule for portfolio selection, we would select the portfolio invested in 15% equities.

Weight equities	Mean asset return	STD asset return	Mean / STD asset return	Shortfall probability*
0%	5.0%	6.00%	0.833	1.2%
5%	5.3%	6.03%	0.870	1.2%
10%	5.5%	6.18%	0.890	1.6%
15%	5.8%	6.45%	0.892	2.2%
20%	6.0%	6.81%	0.881	3.2%
25%	6.3%	7.26%	0.861	4.4%
30%	6.5%	7.77%	0.836	5.7%
35%	6.8%	8.35%	0.809	7.2%
40%	7.0%	8.96%	0.781	8.7%
45%	7.3%	9.62%	0.754	10.1%

* Minimum target surplus return -10%, risk tolerance 10%, current funding ratio 120%

Table 5-7: Equity Weights, Asset Returns and Shortfall Probability

Case Study Part 4: Scenario Analysis and Stress Testing

Regulators often require pension funds (or insurance companies) to stress test their investment portfolios. Stress would be expressed by a simulated downturn in equity markets by, say, 20%, by an increase in interest rates of 1%, similar impacts on other asset classes, and combinations of those. Within an ALM framework the procedure would rather be to take alternative assumptions of input parameters. These could be the long-term yield, the slope of the yield curve, or the equity risk premia. Equally, one could simulate changes in the population dynamics, future wage developments or the inflation adjustment of pension payments. One would then integrate those in the whole simulation process and compare the results of these extreme scenarios with those of the mean scenario.

One advantage of scenario-building in ALM over traditional stress testing is the integral view. A drop in interest rates, for instance, may lead to a positive performance of the fixed income investments but also to an increase in discounted liabilities. A simulation of portfolio developments for this case could then be misleading. Therefore, working with different and possibly extreme scenarios on future developments should be a part of the ALM-exercise. Another purpose of these tests is that while everyone tends to agree on the average or median strategy offered by an ALM-study, extreme results provide insights into sensitivities.

One may assume that an equity risk premium of 5% over bonds as taken in Part 1 could be too optimistic. If we lower the equity risk premium from 5% to 2.5%, i.e., we reduce the return for equities to 7.5%, while keeping everything else equal, the resulting optimal portfolio under shortfall constraints would have 35% invested in equities instead of 45% (see Table 5-8). If the pension fund decides to keep the scenario with 5% risk premium as their base assumption the lower equity return with a 2.5% premium could be used as a trigger point after the implementation. Should the actual development at a future point in time be close to the more negative scenario then this would signal the management that the investment plan needs to be adjusted and the equity portion to be lowered. In the case of an under-funding situation, it would suggest to the plan management to execute a recovery plan.

Weight Equities	Mean asset return	STD asset return	Required asset return*	Shortfall Probability
0%	5.0%	6.00%	1.0%	1.2%
5%	5.1%	6.03%	1.2%	1.3%
10%	5.3%	6.18%	1.6%	1.8%
15%	5.4%	6.45%	2.2%	2.7%
20%	5.5%	6.81%	2.8%	3.9%
25%	5.6%	7.26%	3.6%	5.4%
30%	5.8%	7.77%	4.5%	7.1%
35%	5.9%	8.35%	5.4%	8.9%
40%	6.0%	8.96%	6.3%	10.7%
45%	6.1%	9.62%	7.3%	12.5%
50%	6.3%	10.31%	8.3%	14.2%

* Required mean return of assets to satisfy the surplus shortfall constraint according to Equation (15).
Minimum target surplus return -10%, risk tolerance 10%, current funding ratio 120%

Table 5-8: Equity Weights, Asset Returns and Shortfall Probability with equity risk premium over bonds equal to 2.5%

6. Implementation of Strategies

6.1 Stochastic Simulations

In this kind of simulations the model simulates dynamically into the future using the assumptions described above on assets and liabilities and on other variables like demographics and inflation (see Figure 1-2). In order to capture the long-term nature of the liabilities, the time horizon of the study will typically be 20 or 30 years (this does not preclude, in practice, to assume that a study will not need to be updated every 3 to 5 years). The input parameters will be randomized according to their standard deviations using Monte Carlo simulation techniques. The results will be time paths for the future development of these variables.

6.2 Active versus Passive ALM Strategies

The outcome of the ALM analysis comprises a final assessment of the target function for the pension fund, the expected development of liabilities and assets respectively its funding ratio and cash-flows in terms of contributions and pay-outs. All these key variables should then be varied for risk influence studies as generated by their actual developments versus expectations. For some funds like small company-financed book reserve systems risks may result from a change in liabilities and resulting pressures on company liquidity constraints. For the majority of pension funds, though, the volatility of the investment portfolio will be the major source of risk.

The first version of the investment portfolio should be risk-neutral relative to liabilities, the liability-neutral portfolio. A passive strategy would then be to invest exactly in that portfolio. For most Defined Benefit plans it would be a corporate bond portfolio with a long maturity. In cases where such an investment is not available in the capital markets pension funds may choose to engage in a swap arrangement with an investment firm. The resulting total investment exposure of the fund would exactly replicate the liability position. This strategy is called Liability Driven Investing, or LDI.

In practice, a true immunization against liability risks may not be possible or feasible since liabilities have been projected using a number of assumptions like future development of employees, wages etc. So, even with an LDI-swap, the risks embedded in these deviations need to be controlled by the pension fund. In absence of full LDI, optimal investment portfolios may be exposed to market risks as we have shown above in the examples on optimization given different tradeoffs for target functions. As a result, even with an investment strategy deemed largely passive, the pension fund will have to deal with (relative) market risks in the course of time.

In addition, the fund may wish to take some active risk relative to the liability-neutral portfolio in order to enhance returns compared to lower costs. This can be accomplished by the pension fund by establishing an internal investment team. This may be appropriate anyway in the presence of markets risks even with the presence of a liability-neutral portfolio. Alternatively or in addition, the pension fund can outsource the alpha creation and risk management to external portfolio managers. The selection of managers then may also be outsourced to investment consultants[16]. But the ongoing control of these managers will at last remain as fiduciary responsibility with the pension fund. It must be staffed with adequate personnel internally. In case of internal portfolio management, the pension fund has to provide staffing comparable to the setup in a portfolio management company in order to achieve alpha while at the same time having control and proper risk management plans in place.

Independent of the outsourcing question and of different levels of operational efficiency it can be assumed that a pension fund with ambitious targets will need substantial staffing while using minimizing strategies to keep operational resources lean. As a result, overall funding and investment strategies on one side and the corporate and staffing strategy of the pension fund on the other, are closely related. This is one of the fundamental bylaws of Asset/Liability Management.

6.3 Dynamic Adjustment of Assets and Liabilities[17]

A hybrid strategy with active and with passive elements may be an investment strategy that takes into account the development of the plan over time and that is based on more or less stringent rules. Different from classic portfolio insurance strategies within an ALM context this would entail strategies that relate the asset allocation of the fund to a set relative target like a specific funding ratio. The result could be a pro-cyclical investment policy in cases where the equity exposure for the next period is increased with a higher funding ratio for the current period. The higher funding ratio could result from either a high past performance of equities or from an increased discount rate for liabilities. Conversely, the strategy could be counter-cyclical if the fund is asked to invest more conservatively when the funding ratio exceeds a certain threshold.

Asset Dynamics
Change in equity shares by +a% for each 1% increase in FR Change in equity shares by -b% for each 1% increase in FR if FR > 110%
Contribution Dynamics
Change in contribution rates by -c% for each 1% increase in FR if FR > 120% Change in contribution rates by +d% for each 1% decrease in FR if FR < 90%
Liability Dynamics
Change in pension benefits by +e% for each 1% increase in FR if FR > 120% Change in pension benefits by -f% for each 1% decrease in FR if FR < 90%

Table 6-1: Examples for Dynamic Rules

16 See “Performance measurement and evaluation”.

17 See the document “Asset allocation” in “Investment strategy”, and Muralidhar, A. (2001): “Innovations in Pension Fund Management”, Stanford Economics and Finance, Stanford, p. 61.

Dynamic strategies can also be applied to contributions and/or to the liability side of the pension fund. In cases where the funding ratio of a fund becomes rather low due to a drop in the investment portfolio or to increases in liabilities the fund may not find appropriate investment strategies at acceptable levels of risk. As a consequence, arrangements to either increase contributions or to lower the level of liabilities compared to benefits may be the only choice to keep sufficient solvency for the long-term development of the fund. If the funding ratio becomes rather high dynamic rules may help to establish contribution holidays or increase the actual benefit payments to the pensioners or the accrual of rights.

Table 6-1 shows examples for such dynamic strategies. These rules, when specified ex-ante, can be integrated in the ALM simulation exercise. In most cases they will be path-dependent with the capital markets development and thus cannot yield distinguished results or scenarios deemed clearly superior to others. Such pre-determined rules often help in practice to have a more disciplined (investment) process in place.

Whether or not to use dynamic rule-based strategies in fund management, pension fund management or in the management of other types of businesses may thus be up for a fundamental debate. As shown by previous examples, however, ALM modeling is more than just a technique to optimize investments. With its ability to simulate balance sheets of pension funds it is rather the core element of the business plan of a fund.

7. Glossary

$a_{t,T}$	annuity factor; represents the present value at the time of retirement of a life annuity paying each year Euro 1 as long as the retiree is alive; time that the pensioner is expected to receive benefits from the pension scheme
A_t	sum of assets
α_t	pension benefit accrual factor for years of service at time t
ABO	accumulated benefit obligation
ALM	Asset/Liability Management
$d_{t,T}$	discount factor between time t and the retirement age T
DB	Defined Benefit
DC	Defined Contribution
FR_t	funding ratio
$Il_{t,T}$	retention factor; describes the probability that the member remains in the scheme until retirement
L_t	sum of liabilities
LDI	Liability Driven Investing
$p_{x,t}$	cumulative probability that a man aged x will survive to age x+t
PBO	projected benefit obligation
PVA	the actuarial present value of a life annuity
R	annuity
$RF_{t,T}$	revaluation factor for earnings up to the time of retirement
SAA	Strategic Asset Allocation
SP_t	surplus on the pension fund at a given point in time
Wt	current pensioner's salary at age t
z_α	percentile of the standard normal distribution
σ_A	standard deviation of the asset return

Appendix: Mortality Table for Calculating Life Annuity Factors

Age	Male	Female	Age	Male	Female
65	0.007412	0.004056	93	0.113725	0.085471
66	0.008046	0.004309	94	0.122365	0.092725
67	0.008833	0.004681	95	0.130701	0.099800
68	0.009645	0.005132	96	0.138588	0.106581
69	0.010472	0.005624	97	0.145893	0.112940
70	0.011318	0.006149	98	0.152884	0.118778
71	0.012192	0.006756	99	0.159730	0.123991
72	0.013128	0.007407	100	0.164390	0.135454
73	0.014137	0.008070	101	0.171002	0.141686
74	0.015245	0.008814	102	0.177574	0.147946
75	0.016546	0.009660	103	0.184105	0.154237
76	0.018029	0.010571	104	0.190592	0.160558
77	0.019772	0.011504	105	0.197033	0.166911
78	0.021819	0.012609	106	0.203425	0.173296
79	0.024243	0.013980	107	0.209766	0.179713
80	0.027132	0.015730	108	0.216051	0.186162
81	0.030503	0.017961	109	0.222275	0.192642
82	0.034415	0.020756	110	0.228433	0.199150
83	0.038881	0.024113	111	0.234518	0.205684
84	0.043946	0.027961	112	0.240524	0.212241
85	0.049647	0.032334	113	0.246442	0.218815
86	0.055969	0.037330	114	0.252263	0.225400
87	0.062945	0.042996	115	0.257976	0.231987
88	0.070548	0.049310	116	0.263568	0.238567
89	0.078743	0.056213	117	0.269026	0.245125
90	0.087317	0.063464	118	0.274334	0.251649
91	0.096087	0.070785	119	0.279475	0.258115
92	0.104922	0.078137	120	0.284428	0.264504

Source: German Society of Actuaries