PORTFOLIO MANAGEMENT

MODERN PORTFOLIO THEORY II

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1. Efficient markets*

To introduce the idea of market efficiency, a relatively abstract concept, it is easiest to start with an example. Consider the following stock price time series of a drug company called Firm X.

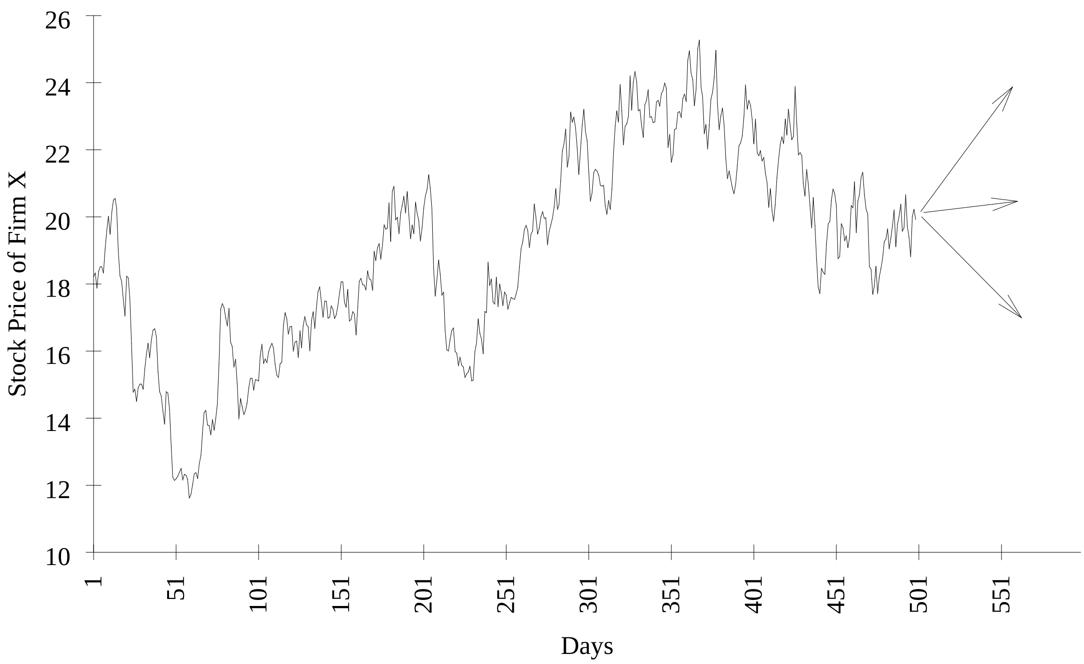

Figure 1-1: Stock price of firm X: value over time

Assume that on the 501st day the price of the stock of the firm X is EUR 20. What will be the price of the stock of the Firm X on the 502nd day? Also, assume that the news that Firm X is about to undertake a project with very poor prospects becomes known to public. Hence, some of the investors who own stock X will try to sell it as soon as possible. Others will be reluctant to buy the stock at the prevailing price. Because of these reactions, the stock price will quickly fall to a lower equilibrium level on that day.

Alternatively, what would happen if Firm X announces a breakthrough in the research for a new drug? If an assumption is made that will take another two years for the drug to become available to patients, what will rational investors do in this situation? They will try to buy the stock as soon as possible at the lowest possible price. Of course, many other investors in the market will simultaneously try to do the same. Nobody will wait two years until the higher cash flows materialize. In this case, the stock price will increase as soon as the new information becomes available.

To illustrate the process of settling down, consider the effect of bad news on the price of a five-year zero coupon bond. The bond has a face value of EUR 1'000. The relevant opportunity rate for a bond with the same maturity and the same risk is 7% (assume a flat term structure of interest rates).

Given this discount rate, today's bond price can be calculated as follows:

$$P_0 = \frac{1000}{1.07^5} = 712.99$$

We therefore discount the expected future payout of the bond with the appropriate opportunity rate of return. The discount rate of 7% represents the required rate of return any investor will get on average in the capital market for an investment with the same risk characteristics.

If we assume that the relevant time horizon of the investor is only one year, what's the expected rate of return for an investor who holds the bond from year 0 to year 1? Assume interest rates do not change.

$$\text{Expected rate of return} = \frac{\text{price}_1 - \text{price}_0}{\text{price}_0} = \frac{\frac{1000}{1.07^4} - 712.99}{712.99} = 7\%$$

What happens if investors expect an increase in the opportunity rate of return, i.e. if the expected rate of return rises to 10% p.a.? Today's bond price will drop. What will the new, lower equilibrium price of the bond be? The bond price has to fall until the expected rate of return of the bond equals 10%. So how can we determine the new bond price?

$$P_0 = \frac{1000}{1.10^5} = 620.92 \text{ CHF}$$

The new price of this bond will be EUR 620.92. This price implies an expected rate of return of 10%.

$$\text{Expected rate of return} = \frac{\frac{1000}{1.10^4} - 620.92}{620.92} = 10\%$$

Now what will happen, if the new price is higher (or lower) than EUR 620.92? We will answer this question for the case in which the price is higher, say EUR 649.93. Assume any investor can buy the bond for this price. What return, y, does he expect to receive on average for five years?

$$P_0 = \frac{1000}{(1+y)^5} = 649.93 \text{ CHF}$$

$$y = \left(\frac{1000}{649.93}\right)^{\frac{1}{5}} - 1 = 9\%$$

What will investors do if they can choose between two investments, both with the same risk characteristics, one offering a return of 9% (a bond) and the other offering 10% (an alternative investment in the capital market)? They will invest in the capital market. Will any rational investor put his wealth in the bond? Ignoring risk considerations, the answer is no, since everybody prefers a rate of return of 10% to a rate of return of 9%. What will happen to the price of the bond under these circumstances? Since no one wants to hold the bond, its price will fall. It will fall, in fact, until investors are indifferent about investing either in the bond or in the capital market. This will take place as soon as the bond offers a rate of return of 10%. Therefore, the price has to fall to exactly EUR 620.92.

We can think also about *equilibrium* expected price changes (opportunity rates of return). If we ignore cash payouts, the price of a stock increases at the same rate on average as the opportunity rate of return of an asset with the same risk. For example, assume that today's stock price of Firm X is EUR 20, and the market requires a rate of return of 20% per year for investment opportunities with a risk similar to that of Firm X. In such a case, we would expect the stock price of Firm X to increase at a rate of 20% per year. And in equilibrium, we expect the stock price to be EUR 20·1.2 = EUR 24 in one year.

What can we learn from this discussion? Any investment strategy has to cover its costs. In other words, the invested capital has to earn a return that is appropriate for the risk of the investment, the time value of money and the transaction costs of the strategy. In a competitive capital market, most of the long-term investment strategies are just covering these costs. But just as capital markets are competitive, so, too, are markets for goods and services. What income do you expect, for instance, from opening a new barber shop? Can you expect to make large profits? The answer, in general, is no, as long as you have no comparative advantage as a barber. If it were possible, in fact, to make large profits, what would happen? Other people would recognise this, open new barbershops and try to imitate the successful business strategies. Of course, in the long run this will reduce everybody's profits to a level that covers the costs of running a barbershop.

Capital markets are competitive. They produce and process information. This process continuously repeats itself for all the securities traded in the market; as in any other industry, there is entry where there are profits, and there is exit where there are losses.

News that is cheap to process is quickly reflected in prices. This includes easily-observed regularities or trends that are easy to spot. Additionally, the market cannot overreact systematically to dividend announcements. Any such patterns are self-destroying in a competitive market.

News that is more expensive to process may take longer to be reflected in prices. This is a source of potential profits. In the long run, however, and unless there are superior talents, competition will drive these profits down. This does not mean necessarily that competition makes all information production technologies obsolete. If that were the case, new information production opportunities would be created. All it means is that these technologies simply cover their costs (opportunity costs of time etc.). There are many industries that survive simply by covering production costs-the real world abounds with examples. Of course, in the short run, many investment strategies can be profitable (and many can be catastrophic).

A market in which security prices adjust rapidly to the publication of new information, and therefore in which the current prices of the securities fully reflect all information about that security is commonly said to be an **efficient market**, or more precisely an **informationally efficient market**.

This precision is noteworthy because the reason of being of a market is the allocation of capital to the most promising investment opportunities in the market. This is what we call an **allocationally efficient market**. In order for a market to be allocationally efficient it must be both externally and internally efficient. An **internally efficient market** is a market in which transaction prices are low and execution speed is high thanks to fierce competition among brokers and dealers. An **externally efficient market** is what we defined as an informationally efficient market.

Note that from now on, we will use the term efficient market in the sense of external market efficiency.

1.1 Information efficient markets*

Prices of stocks, bonds and other financial assets are determined in the capital markets. For example, the stock price P_0 of Firm X can be determined as follows:

$$P_0 = \sum_{t=1}^{\infty} \frac{E(Div_t)}{(1+R_t)^t}$$

where:

E	Expectation operator
Div_t	Dividend payment in t
R_t	Opportunity rate of return

Therefore, to price assets, market participants form their expectations of future interest rates, future risk characteristics of the firm, and future cash distributions, from various types of information. In the case of expected future distributions of a stock, this information can include the following characteristics of a firm:

- Product quality of Firm X
- Capital budgeting policy of Firm X
- Financial policy of Firm X
- Experience and abilities of Firm X's management
- Future macroeconomic perspectives
- Growth opportunities of Firm X's industry
- Main competitors of Firm X

*1.1.1 Assumptions**

An information efficient market requires a *large number of competing market participants, each of which independently analyses and values securities in order to optimise their profit.*

A second assumption is that *the competing investors attempt to adjust the price of securities immediately to all available and relevant information in order to reflect the effect of it.*

If prices are bid immediately to fair levels, it must be that they increase or decrease only in response to new information. Thus, the third assumption is that *the price changes at any moment are based solely on the random arrival of new information.* This is the essence of the argument that stock prices should follow a random walk, that is, that price changes should be random and unpredictable. Any information that could be used to predict stock performance must already be reflected in stock prices.

We have just seen that competitive pressure forces the security's price to the new level instantly. Moreover, as long as everyone is attempting to draw reasonable judgments regarding the implications of the information for the security's price, the new price level will unbiasedly represent the market's summary judgment of the value of the security. Neither systematic underreaction nor systematic overreactions are possible in an information efficient market.

Although it may not literally be true that 'all' relevant information will be uncovered, it is virtually certain that there are many investigators that may improve investment performance. Competition among these analysts ensures that, as a general rule, stock prices ought to reflect all available information.

*1.1.2 Characteristics of perfect information efficient markets**

There are some noteworthy observations about **perfect** information efficient markets.

1) *Investors should expect to make a return on their investment that just covers its costs (fair return)*

   This means that using fundamental analysis or technical analysis in order to find mispriced securities will not generate above average returns.

   Technical trading refers to the attempt to predict future prices from the pattern of past price movements. It is essentially the search for recurrent and predictable patterns in stock prices. This approach is diametrically opposed to the notion of an efficient market.

   To demonstrate this, let us examine the stock price pattern of Firm X again. In the following figure we've added three straight lines, indicating possible stock price tendencies over periods of 150 days. The example shows one possible pattern. Of course, there are many other patterns that could be detected. Note that it is always possible to identify patterns in a series of historical prices ex post.

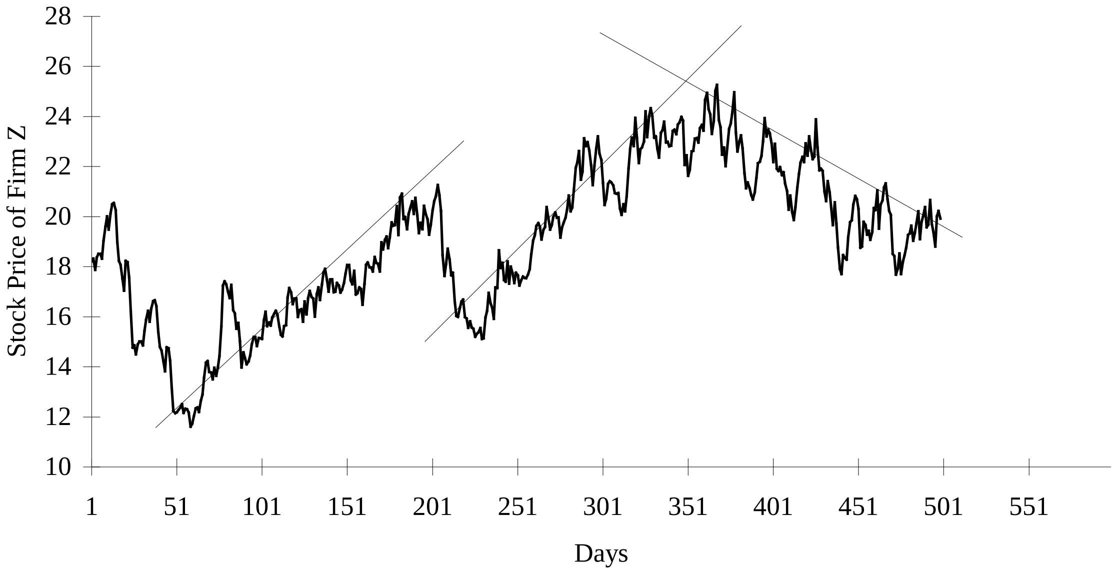

Figure 1-2: Stock price of firm X

Alternatively, in Figure 1-3 we take the same stock price data and calculated a 20-day moving average from day 1 to 100. This is a very common approach to generate buy and sell signals. One possible rule to follow would be to buy the asset as soon as the actual stock price becomes higher than the 20-day moving average, and vice versa.

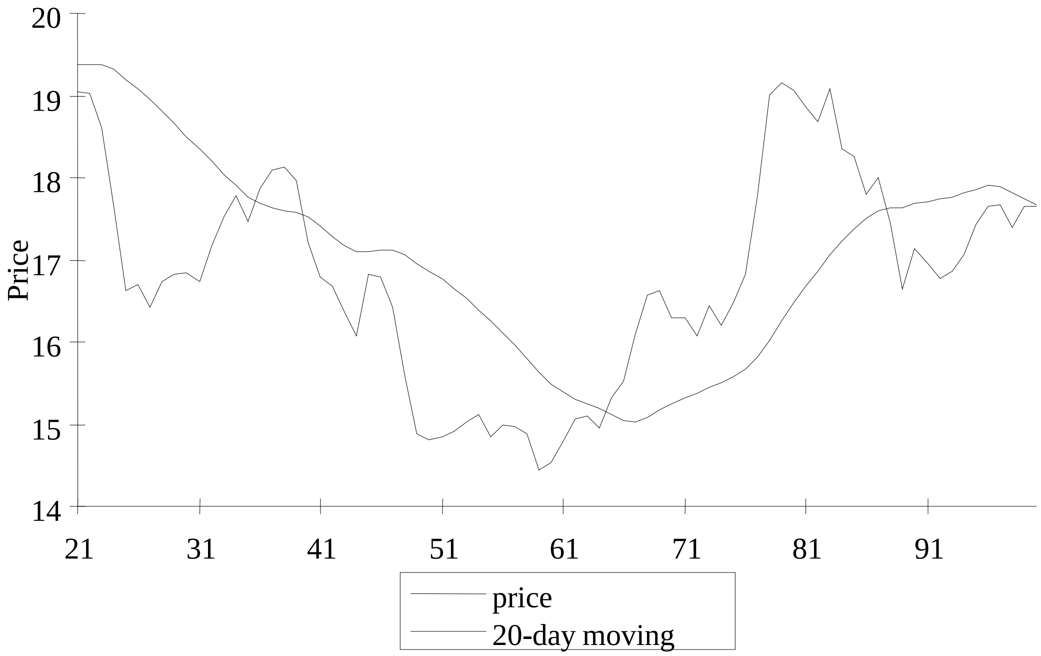

Figure 1-3: Price observations and moving average

If capital markets are information efficient, it is not possible to generate positive abnormal returns with this investment strategy. The calculation of the 20-day moving average relies exclusively on historical stock data. But since, in an efficient market, all available information is already reflected in the actual stock price, historical data have no power to predict future stock prices.

What neither Figure 1-2 nor Figure 1-3 show is that the data are not the actual prices of any real market; they were artificially computed with a random number generator in a spreadsheet program. The point is, that it is always possible, on hindsight, to discover 'patterns' in any time series. But often these patterns are merely a fiction of our imagination.

Some chartists also work with *filter rules*, mathematical rules that can be applied to produce buy and sell signals. For example, a filter rule may dictate that a stock should be purchased when it moves up in price by z percent or sold (or shorted) when it falls from its previous high by z percent. Fama and Blume[1] compared the rates of return earned from applying such a trading rule with the rates of return earned from a policy of buying and holding the common stock for each of the 30 Dow-Jones Industrial Securities. The following table shows the average rates of return of the two strategies at different filter levels.

1 FAMA Eugene F. and BLUME M., 1966, "Filter Rules and Stock Market Trading Profits", Journal of Business (Supplement)

Filter	Filter Rule Return (average)	Buy and Hold Return (average)
0.005	0.115	0.104
0.010	0.055	0.103
0.020	0.002	0.103
0.050	–0.019	0.100
0.100	0.030	0.093
0.200	0.043	0.098
0.300	–0.005	0.064

Table 1-1: Average annual rates of return applied to 30 common stocks (1956-1962)[2]

These figures show that only the 0.5% filter, with its 11.5% return is higher than the 10.4% return of the buy and hold strategy. Telling whether this difference is statistically significant requires more information than is available. In particular, these calculations ignore transaction costs. Over the period concerned, 12'514 transactions would have been necessary to follow the 0.5% filter rule. Taking the commissions of the NYSE into consideration, the average annual return from this filter strategy turns out to be negative (–103.5%).

More recent studies such as Scaillet and Bajgrowicz (2008)[3] confirm that it is almost impossible to identify technical trading rules that generate significant risk adjusted performance. They use persistence tests[4] to show that an investor would never have been able to select ex ante the future best-performing rules among a set of classical technical trading rules. Moreover, they show that even the in-sample performance is completely offset by the introduction of transaction costs. Overall, their results seriously call into question the economic value of technical trading rules.

It is also important to note that the patterns identified by simple technical trading rules can sometimes be misleading. Indeed, structure does not necessarily imply predictability. This fact is particularly well illustrated in the paper “Robust structure without predictability: ‘the compass rose’ pattern of the stock market” (Journal of Finance 1996) by T.F. Crack and O. Ledoit. This article describes a series of surprising patterns that emerge when considering successive return series. These patterns are not linked with predictability but are due to market microstructure effect, namely the tick size (the minimum amount by which a stock price is allowed to change). The figure below is obtained by plotting the returns at time on day t against the return on day t+1. It seems that it should be possible to benefit from the apparent predictability but it is in fact impossible.

2 FAMA Eugene F. and BLUME M., 1966, “Filter Rules and Stock Market Trading Profits”, Journal of Business (Supplement)

3 SCAILLET O. and BAJGROWICZ P., 2008, “Technical trading revisited: persistence tests, transaction costs, and false discoveries“ Swiss Finance Institute DP 2008.5

4 i.e. they measure the performance of a portfolio rebalanced every six months no longer in-sample, but out-of-sample. Hence, only historical available information is exploited to select the rules.

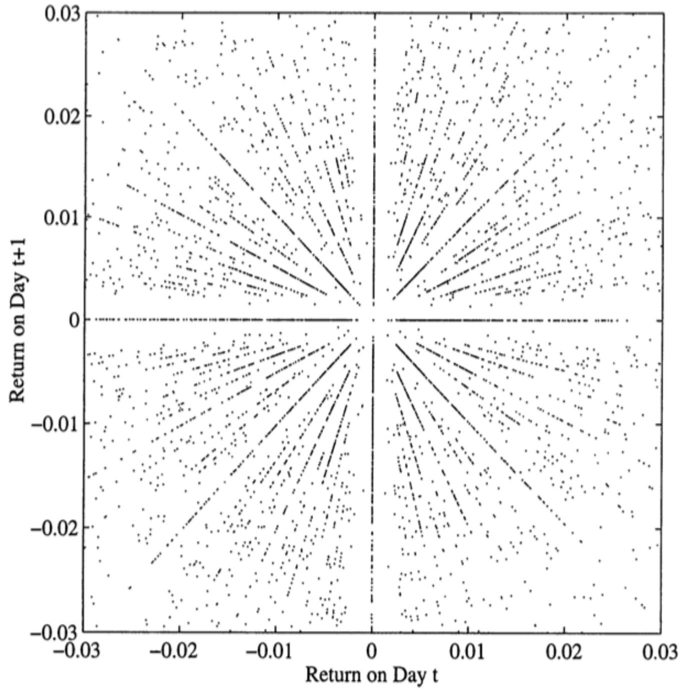

Figure 1-4: "Compass rose" structure in stock prices

2) *Future performance cannot be deduced from past performance*

Investment strategies that were successful in the past are no more likely to perform better than strategies that were not successful in the past.

3) *Markets can only be efficient if enough person believe the market is not efficient*

This statement is only in appearance a paradox. Indeed, what makes the price of securities reflecting the true value is precisely the independent analysis of securities done by of numerous investors.

If everybody believed that the markets are perfectly efficient and that it is therefore not possible to generate above average profit by searching for undervalued securities nobody would bother to analyse securities. Consequently the price of the securities could not adjust to new information and thus the security might get mispriced.

4) *Capital markets react quickly and completely to new information*

As soon as new information is available to capital markets, price reactions are very quick. And to profit to the full extent from this new information, investors trade stocks quickly according to their changing expectations. So, competitive pressure forces the security's price to the new level instantly. Moreover, as long as everyone is attempting to draw reasonable judgements regarding the implications of the information for the security's price, the new price level will unbiasedly represent the market's summary judgement of the value of the security. Neither systematic underreaction nor systematic overreaction is possible in an information efficient market.

Some empirical studies have examined the stock price reaction of a sample of firms that became targets in a merger before and after the merger announcement. In most take-overs, stockholders of the acquired firms (targets) sell their shares to the acquirer at substantial premiums over market value. The announcement of a take-over attempt is good news for shareholders of the target firm and should therefore cause stock prices to jump.

Figure 1-5 shows the typical price pattern around take-over announcements.

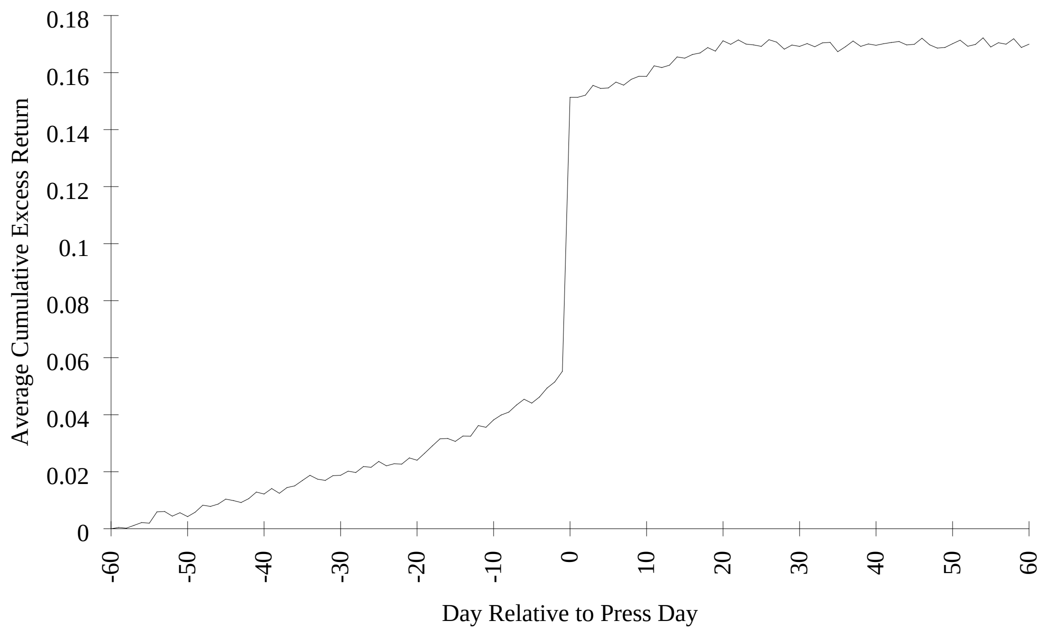

Figure 1-5: Price pattern of selling firms around take-over announcements

On the announcement day (day 0) one can observe a large, positive abnormal return (defined below). Immediately after the announcement, the stock price no longer increases or decreases abnormally. This price reaction pattern is perfectly consistent with the notion of information efficient capital markets. The price reaction occurs quickly (within one day) and completely on the day the new information becomes public (announcement day). On average, no new information becomes public on this particular sample of firms after the announcement day. Consequently, there are no apparent systematic patterns in the cumulative abnormal returns after day zero.

Abnormal returns are constructed relative to a model (for example the CAPM) which predicts a return for the security as a function of its systematic risk level and/or other characteristics. If the model is correctly specified, the abnormal return should not be statistically different from zero on average.

5) On average, capital markets participants ignore irrelevant information

If we ignore tax considerations and information signalling, stock splits per se should be irrelevant events. Assume an investor owns a share with a nominal value of EUR 100 and a market value of EUR 500. This firm wants to split the shares 2:1. This means that every investor gets two new shares in exchange for one old share. The investor now owns two shares with a face value of EUR 50 and a market value of EUR 250. In this example, one would expect the market not to react with an abnormal return on the split announcement date. This problem has been examined extensively in the empirical literature.

Figure 1-6 shows the typical reaction pattern of stock prices before and after a stock split.

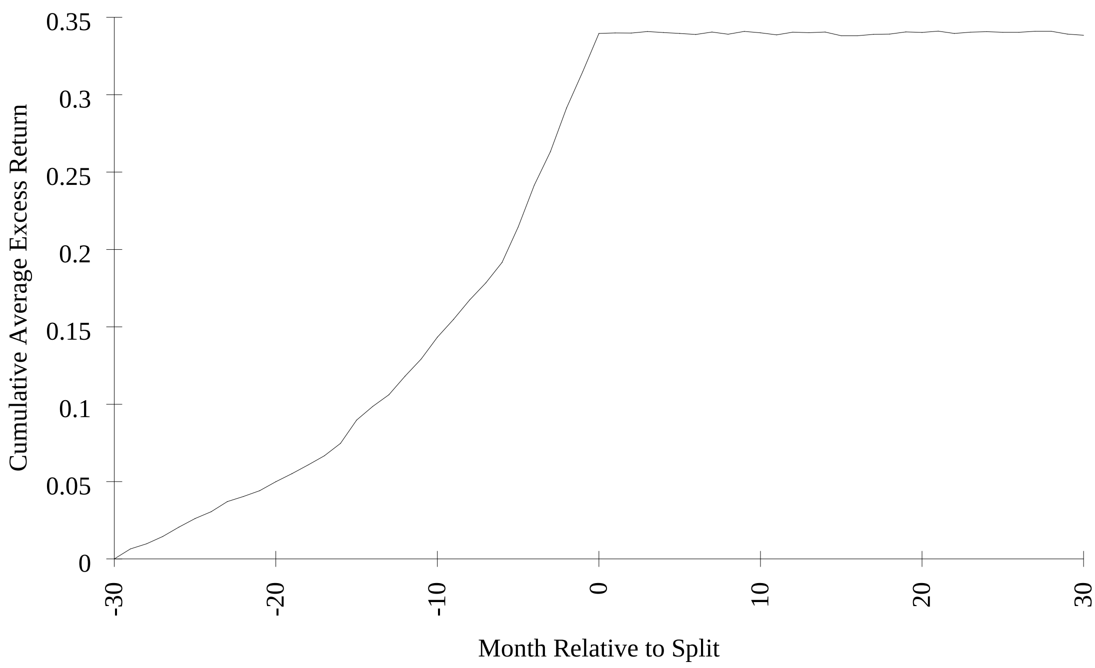

Figure 1-6: Stock price pattern surrounding stock splits

Positive abnormal returns are observed before the stock split, because firms tend to split in times when prices have gone up. In the month of the split and thereafter, however, there is no tendency for the cumulative abnormal return to increase. This is consistent with efficient capital markets. Market participants only react to new (unexpected) information, announcement of an unexpectedly high dividend, unexpectedly high earnings, etc.

*1.1.3 The importance of transparency—the Volkswagen short squeeze**

In the 2008 30th October issue of The Economist[5] an investor was quoted saying that "They [Porsche] may struggle to sell 911s to hedge-fund managers for years and years to come". The reason for this was not any sudden drop in quality of the ultimate car for the prosperous. Instead the quote referred to a short squeeze in the Volkswagen (VW) share that many hedge fund managers found themselves in during October 2008. The short squeeze was caused by some dubious disclosure procedures by Porsche over its ownership of the VW share.

Traders take short positions in a stock, i.e. sell a stock they do not own, to profit from falling stock prices. They do this by borrowing the stock from other share holders for a small fee. Short sellers cause an artificial increase in the number of outstanding shares (since they re-sell shares). Thus the biggest fear of short sellers is to be caught in a situation where a sufficient majority of share holders refuse to sell any shares, at the same time as the original owner of the shorted shares demands them back. There are consequently more shares which need to be bought back by the short sellers than sold and the rise of the share price has theoretically no limit. This situation is referred to as a short squeeze. Regulations are supposed to prevent this and markets are indeed often transparent enough. Information flows are highly efficient to keep short sellers informed on whom are majority share owners and avoid this.

Porsche however managed, through lax regulation and derivatives trading, to be at the centre of a short squeeze in the VW share during October 2008. In 2008, Porsche had for a long time had an interest to acquire VW. In October it owned 42.6% of the shares. The high demand from Porsche had pushed the share price to very high levels on a fundamental basis as compared to other car manufactures. Many hedge fund managers believed that this would not be sustained once Porsche had acquired VW and consequently shorted the share. Around mid October 2008 as much as 12.9% of the VW shares where lent to short sellers. The other majority stake holder in Volkswagen was the state of Lower Saxony which had a 20.2% stake. Due to local laws the stake gives the state significant voting power to protect local jobs and is thus a highly unlikely seller[6].

On Sunday 27th October Porsche disclosed ownership in call options on 31.5% of VW shares. In total, Porsche had a 74.1% stake. Added with the state of Lower Saxony the two share holders had claims on 94.3% of the VW shares. Additionally, about 5% of the shares were held by index tracking funds. The new disclosure by Porsche revealed that there were less than 1% shares to cover around 12.9% of short positions. Not surprisingly the Volkswagen share price sky rocketed as can be seen in Figure 1-7. At its peak on the 28th October it had soared 377% from its closing price on Friday before the Porsche announcement, making it the most valuable company by market capitalization in the world.

5 Squeezy money', The Economist, October 30th 2008

6 F. Norris, 'A Clever Move by Porsche on VW's Stock', The New York Times, October 30th 2008

Figure 1-7: Bloomberg screen of the Volkswagen share price on the 28th October 2008.

This example illustrates the need for transparent markets. In the aftermath of this event many suggested that the culprit of the event was opaque German regulations allowing undisclosed stake-building through derivative. Furthermore, the example also illustrates that markets are not always fully efficient. Indeed, the market has overreacted to information on the positions of Porsche: the course returned to a more reasonable level the following days. Finally, this example shows how quickly market participants react to new information.

1.2 Efficient market hypothesis*

The notion that stocks already reflect some type of information is referred to as the **Efficient Market Hypothesis** (EMH).

EMH implies that market price always reflects the *true* value of the asset. If markets are efficient, then purchase or sale of any asset at the prevailing market price is never a positive net present value (NPV) transaction. In other words, on average, you always receive a fair compensation for the risk effectively taken. The invested capital has to earn a return that is appropriate for the risk of the investment, the time value of money and the transaction costs of the strategy. In a competitive capital market, most of the long-term investment strategies are just covering these costs; there is no ‘free lunch’.

More formally Fama described in his article[7] how investors generate price expectation notionally as follows:

$$E\left(\tilde{P}_{j,t+1} | \Phi_t\right) = \left(1 + E\left(\tilde{r}_{j,t+1} | \Phi_t\right)\right) P_{jt}$$

where:

- E expected value operator
- $P_{j,t}$ price of security j at time t
- $\tilde{P}_{j,t+1}$ price of security j at time t+1
- $\tilde{r}_{j,t+1}$ the one period percent rate of return for security *j* during period *t*+1
- Φ_t the set of information available at time *t*

The equation above denotes that expected price of the security j at time t+1, given the information available at time t, is equal to the current price multiplied by 1 plus the expected return on security j, given the set of available information. The question is, what does the available information set Φ_t consist of? The answer depends on the particular form of the market hypothesis being considered.

*1.2.1 Forms of market efficiency**

In his original article Fama divided the general efficient market hypothesis into different sub-hypothesis, according to the kind of information that is already reflected in the asset's price. We traditionally distinguish three forms of market efficiency.

*1.2.1.1 Weak Form**

If the market is efficient with regard to *past information*, in other words if all historical information is already discounted in prices, the market is said to be **weak-form efficient**.

This hypothesis implies that there should be no gain from any trading rule that decides whether to buy or to sell a security based on past rate of returns or any other past market data.

*1.2.1.2 Semi-strong form**

If market prices incorporate all the *publicly available information,* it is said to be **semistrong-form efficient**. The semistrong-form hypothesis includes the weak-form hypothesis because all the past market data considered in the weak-form hypothesis is public. Public information does also include non-market information such as political news, news about the economy, earnings and divided announcements, publication of analyst reports, ratios and so on.

Therefore there should be no gain from decision based on new information after it has been made available to the public because the security price already reflects all such new public information.

7 Eugene F. Fama, ‘Efficient Capital Markets: A Review of Theory and Empirical Work’, Journal of Finance 25, no. 2 (May 1970): 383-417

*1.2.1.3 Strong Form**

A market in which all information, including *privately held information*, is reflected in prices is said to be **strong form efficient**. The strong-form efficiency implies the semistrong-form, which in turn implies weak-form efficiency. This means that no group of investors in the market has exclusive access to relevant information, thus no group is capable of generating consistent excess return.

*1.2.2 Testing market efficiency**

Now that we have defined what a perfect efficient market is and that we have seen the three forms of the EMH, we can ask ourselves how to determine the form of efficiency that can be attributed to a market under investigation.

The methodology used to test the validity of market efficiency depends on the efficiency form under investigation.

1. Weak-form hypothesis

To test the validity of the weak-form hypothesis *statistical test of independence* (autocorrelation and runs tests) and *trading rule test*, whereby the risk-return results derived from trading simulation are compared to simple buy-and-hold policy, are used.

2. Semistrong-form hypothesis

The studies used to test this form of market hypothesis can be divided into two sets:

1. *Event studies* are used to test how fast the stock prices reflect the arrival of new information. Defenders of the EMH would expect the price reacts so quickly that it is not possible for investors to earn excess risk-adjusted return by investing after the public release of new information.

2. *Time series analysis* of returns, *cross-section distribution* of returns or other individual stock ratios are used to predict future rate of returns using available public information beyond the market information considered in the weak-form hypothesis.

Any of theses tests examines whether a given investment strategy yields *abnormal* or *excess returns*. To perform these tests, we need to define what normal return is. In other words, we need a benchmark, a model that tells us what the required (or equilibrium, or opportunity) rate of return is for a given risk and a given investment horizon. One possible benchmark is the Capital Asset Pricing Model (CAPM) that we will discover in later. Note that all market efficiency tests are conditional on a given model of equilibrium returns; they may therefore not yield the same conclusions.

Consequently these studies on market efficiency are dual tests of the EMH and the CAPM. Abnormal returns may occur because the markets are not efficient or because the CAPM does not provide the correct estimates of the expected returns.

3. Strong-form hypothesis

Typical tests for the strong-form hypothesis do compare the returns over time of different identifiable investment groups in order to determine if one constantly earns above risk-adjusted return.

1.2.3 Market anomalies*

Like many other hypothesis formulated in finance and economics, the evidence of the EMH is mixed. A lot of studies have been published about the subject, some of which do support the EMH and thus indicate that capital markets are efficient. Other studies however have shown results that are not consistent with the hypothesis and have revealed some anomalies, raising questions about the support for them (Note that the test and the results depend on the form of the EMH tested). The following anomalies are observable, thus public. This implies that they are challenging the EMH in its semi-strong form.

1.2.3.1 Size Effect (i.e. Small Firm Effect)*

Several studies[8] have found that the risk-adjusted return for extended periods (10-15 years) indicated that the small firms (expressed in market value) consistently experienced significantly larger risk-adjusted returns than the larger firms.

Because of the use of the CAPM as model to predict normal returns, some authors argued that the observed difference in risk-adjusted return between small and large firms might actually be much smaller[9] than initially assumed, the CAPM being not the right model to predict the returns of small firms.

Other authors confirmed the first observations, but also found a strong positive relationship between average price per share and the market value, e.g. firms with small market value have low stock prices. Because transaction costs vary inversely with the price per share, they must be considered when analysing the small firm effect.

1.2.3.2 Book Value/Market Value*

Significant positive relationship between book value (BV) and market value (MV) and future stock returns have been found by Rosenberg, Reid and Lanstein[10]. They contended that this relationship was evidence against the EMH.

However Fama and French provided the strongest support for the importance of this ratio by evaluating the joint effects of market beta, size, E/P ratio, leverage, and the BV/MV ratio on the cross-section average returns on the NYSE, AMEX and NASDAQ stocks. Like Rosenberg, Reid and Lanstein, they found a significant positive relationship between the BV/MV ratio and future stock returns that persisted when other variables were included. They also found that the BV/MV ratio in combination with size dominated other ratios.

Several studies followed the publication of Fama and French, the majority of which reached the same conclusion.

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8 R.W. Banz, 'The Relationship Between Market Return and Market Value of Common Stocks', Journal of Financial Economics 9, no.1 (March 1981): 3-18; and Marc R. Reinganum, 'Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yield and Market Values', Journal of Financial Economics 9, no. 1 (March 1981): 19-46

9 Marc M. Reinganum, 'Abnormal Returns in Small Firm Portfolios', Financial Analyst Journal 37, no. 2 (March-April 1981): 52-57; and Richard Roll, ' A Possible Explanation of the Small Firm Effect', Journal of Finance 36, no. 4 (September 1981): 879-888.

10 Barr Rosenberg, Kenneth Reid, Ronald Lanstein, 'Persuasive Evidence of Market Inefficiency', Journal of Portfolio Management 11, no. 3 (Spring 1985): 9-17.

In summary, the tests of publicly available ratios that are used to predict future stock returns have provided substantial evidence against the semistrong-form EMH.

*1.2.3.3 High P/E Ratio Effect**

Some practitioners have suggested that low P/E stocks are likely to outperform high P/E stocks. This is because the price of growth companies tends to be overvalued (Overestimation of the growth), whereas low-growth firms trends to be undervalued.

A relationship between historical P/E rations and risk-adjusted return would constitute evidence against the semistrong-form EMH.

A study conducted by Basu concluded that low P/E ratio stocks experienced significantly higher rates of return relative to the market, whereas the opposite was true for high P/E ratio stocks[11].

Other studies reached the same conclusion.

*1.2.3.4 Year-End or January Effect**

It can be observed that returns in November and December tend to be below average and on the opposite the returns in the first two weeks of January tend to be above average.

Obviously those who believe in efficient markets would not expect such seasonal pattern to persist. Typically arbitrageurs should eliminate this anomaly.

The January anomaly is intriguing in that it is so persuasive and despite numerous studies, the January anomaly poses as many questions as it answers.

*1.2.3.5 Day of the Week Effect**

Besides the January effect, there are several other *calendar* effects, one of which is the day of week effect or weekend effect. French has observed that the mean return for Monday was significantly negative, whereas the mean return of the four remaining days was positive[12]. Other authors like Gibbons and Hess found similar results[13].

The following tests do challenge the strong-form of the EMH.

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11 S. Basu, ‘Investment Performance of Common Stocks in Relation to Their Price-Earning Ratios: A Test of the Efficient Market Hypothesis’, Journal of Finance 32, no. 3 (June 1977): 622-682; and S. Basu, ‘The Information Content of the Price-Earnings Ratios’, Financial Management 4, no. 2, (Summer 1975): 53-64.

12 Kenneth R. French, ‘Stock Returns and the Weekend Effect’, Journal of Financial Economics 8, no.1 (March 1980): 55-70

13 Michal R. Gibbons and Patrick Hess, ‘ Day of the Week Effect and Asset Returns’, Journal of Business 54, no. 4, (October 1981): 579-596

*1.2.3.6 The Value Line Enigma**

Several Studies have been conducted in order to establish whether it is possible to identify groups of analysts that have the ability to consistently select undervalued stocks. The idea behind this is that these investments professional have to obvious advantage to other types of investors except for their training and experience. If anybody is able to consistently select undervalued stock, then it should be these professionals. One of these tests is the Value Line enigma

The value line is a well-known advisory service that publishes several reports. One of these reports indicates the Value Line's expectations of stock performances over the coming 12-month. Several factors are used to define the ranking that goes from 1 to 5, from the best to the worst.

The Value Line indicated several years after the initial publication that the risk-adjusted return between the different rankings varied significantly, the stocks ranked as 1 significantly (20%) outperformed the one ranked as 2. Subsequently several studies have been conducted in order to analyse this phenomenon. The older studies[14] tend to support the Value Line enigma and thus to challenge the EMH in its strong form. However, more recent studies[15] do indicate the opposite. Once again, there is no clear-cut conclusion possible from these findings.

Barber et al.[16] conduct an extensive empirical study on whether investors can profit from public buy or sell recommendations from securities analysts in the US. They more specifically investigate the profitability of trading strategies which buy the most favourable consensus recommendations and sell the least favourable consensus recommendations[17]. They do indeed find this strategy to yield excess returns. Hence there seem to be some value in the information provided by the analysts. The strategy however requires significant trading activity and when accounting for trading costs the strategy no longer yields excess returns.

*1.2.3.7 The Momentum Strategy**

In 1993 Jegadeesh and Titman[18] published a seminal article claiming that a simple momentum trading strategy can generate double digit excess returns. More specifically, the momentum strategy evaluates stocks over a certain period, often in the range of 3-12 months. Given the performance over this period the strategy buys past winners and sells past losers. Note the way to define winners and losers can greatly vary across momentum strategies. Yet, the somehow classic approach, called price momentum, simply calculates "raw rates of return" or past performance. The outperformers, that is, the winners, are then bought and the underperformers, or the losers, are sold. Stocks are held over a certain investment period, often as well in the range of 3-12 months. Jegadeesh and Titman control if the excess returns are not due to systematic risk exposures. For instance, winners could be stocks with average high betas whereas losers could be of low betas. If it were the case, the double digit excess

14 Fischer Black, "Yes, Virginia, There is Hope: Tests of the Value Line Ranking System", Financial Analysts Journal 29, no.5 (September-October 73): 10-14.

15 Mark Hulbert, "Proof of Pudding", Forbs, (December 10, 1990): 316.

16 Brad Barber, Reuven Lehavy, Maureen McNichols, and Brett Trueman, 'Can Investors Profit from the Prophets? Security Analyst Recommendations and Stock Returns', Journal of Finance, 2001,Vol. 56 , No. 2:531-553

17 In the article, consensus refers to the average recommendation of several securities analysts.

18 Narasimhan Jegadeesh and Sheridan Titman, 'Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency', Journal of Finance, 1993, 48, no. 1:65-91

returns yield by momentum strategies will result from a pure net beta exposure. They can reject this hypothesis. They also check if the results are not just the expression of another size effect. Once again, they reject the hypothesis. Overall, their results imply a violation of the semi strong EMH.

Chordia and Shivakumar[19] however suggest that the excess returns can be explained by the business cycle and varying risk premia. By using a set of macroeconomic variables the authors are able to explain periods of over and under performance of the momentum strategy. After controlling these variables the momentum strategy no longer generate abnormal returns. More specifically they find the momentum strategy to over perform in expansionary periods and underperform in recessionary periods. The article hence continues to argue that the momentum strategy is not necessarily inconsistent with the EMH.

Chordia and Shivakumar acknowledge however the difficulty of interpreting their results. Any test of the EMH is a joint test. Indeed, one always has to decide what the appropriate pricing model of stock returns is. Doing so, one sets the norm and once the norm is known, excess returns can be calculated. Hence any test cannot give an absolute answer on whether the EMH holds or not, since the rejection of the null hypothesis can be due to the choice of an inappropriate pricing model.

1.3 Are markets efficient?*

We have just seen that in the academic community there is no unanimity concerning the relevance of the EMH. In summary we can say that according to most of the early empirical studies, it appeared effectively that markets were at least semistrong-form efficient. But there has recently been a renewal in the 'not-so-efficient' markets hypothesis. The predictive power of some economic and financial indicators as well as some regularities in historical data have effectively been uncovered on most of stock markets. Determining if these findings are the fruits of statistical illusions or real opportunities is, however, tricky.

It is now appropriate to briefly cover the interpretation of financial bubbles in relation to the EMH. The then US Federal Reserve Board Chairman Alan Greenspan called into question in 1996 of how to judge whether market actors where caught in 'irrational exuberance' and escalating asset values as a consequence. The phrase came to represent much of the following dot com crash, not the least due to the book *Irrational Exuberance* by academic Robert Shiller. This book was published just before the crash and to some extent predicted the illusions of the expectations of the internet revolution.

With the 2007-08 crisis in mind as well, it is easy in hindsight to argue that market participants did indeed act irrationally. There are many examples as well of clear signs of what is to come before financial bubbles burst. On the other hand is the wisdom of hindsight easily deceptive by picking out the information which best explains how events played out. This information however was not the only one available when events actually played out. Arguing that people should select which information is the best at any point is impossible as well. Hence a financial bubble in itself does not constitute a violation of the EMH.

19 Tarun Chordia and Lakshmanan Shivakumar, 'Momentum, Business Cycle, and Time-varying Expected Returns', Journal of Finance, 2002, 57, no. 2:985-1019

However there are an increasing amount of behavioural theories to explain why market participants may act irrationally[20]. There is also a strong, though not absolute, consensus in the financial industry that more consideration has to be taken to the human nature in order to better understand financial markets.

Grossman and Stiglitz[21] have written a seminal paper on the theoretical impossibility of having efficient markets in market equilibrium. In brief they observe that markets cannot constantly be in equilibrium, the reason being the cost to acquire information. The logic of their thinking is somehow hard to contradict. If acquiring and treating information are costly, prices cannot fully and freely convey information. If it were the case, no one will have the incentive to collect and to treat the information in the first place. Therefore, prices will contain no information and markets could not be said to be efficient. Stated otherwise, if the market is in equilibrium no arbitrageurs would be able to make any abnormal returns on their private (and costly) information. For the two authors, a certain degree of disequilibrium should therefore exist where prices only partially reflect the information of the most informed actors.

As previously implied, the EMH is not a very popular theory in the community of professional portfolio managers. Consequently, the EMH has never been widely accepted on Wall Street, and debate continues today on the degree to which security analysis can improve investment performance.

Ultimately, however, the issue of market efficiency boils down to whether skilled investors can make consistent abnormal trading profits. Casual evidence does not support claims that professionally managed portfolios can consistently beat the market.

1.4 Market efficiency and investment policy*

In a weak-form efficient market, the use of **technical analysis** and technical rules, i.e. the search for recurrent and predictable patterns in stock prices, does not lead to superior performance since the information used in building such strategies (the past prices) is already reflected in actual prices. For technical analysis to be successful, stock prices must respond slowly to fundamental supply-and-demand factors. This prerequisite, of course, is diametrically opposed to the notion of an efficient market.

Fundamental analysis uses financial statements, earnings and dividend prospects of the firm to determine proper stock prices. Fundamental analysis is difficult because it is not enough to do a good analysis of a firm; you can make money only if your analysis is better than that of your competitors. The goal of fundamental analysis is to attain insight into future performance of the firm that is not yet recognized by the rest of the market. If the market is semistrong-form efficient, fundamental analysis does not permit to achieve superior performances since all publicly available information is already reflected in prices.

20 Andrei Shleifer, 'Inefficient Markets: An Introduction to Behavioral Finance', 2000, Oxford University Press

21 Sanford J. Grossman and Joseph E. Stiglitz, ‘On the Impossibility of Informationally Efficient Markets’, American Economic Review, 1980, Vol. 70, No. 3:383-408

At this stage, one could cast some doubt on the utility of fundamental analysis. The situation may indeed seem to be quite paradoxical because if markets are efficient, there's no incentive for anyone to do some analysis, but if nobody does, information won't be reflected in prices and as a result, the market won't be efficient anymore, thus creating an incentive for analysis. This paradox is resolved once we are aware that information is costly. In this case the equilibrium is reached when, at the margin, the cost of acquiring information is equal to the benefits this information provides to his acquirer.

The strength of your belief in the level of the market's efficiency determines the kind of management you will adopt. Defenders of the efficient markets hypothesis will adopt a **passive strategy**, i.e. they won't try to beat the market. They will simply adopt a *buy-and-hold strategy*, holding the market (or a mix of the market and the riskless security depending on their risk tolerance) since the market is supposed to be fairly evaluated.

On the other side, those who think markets leave room for opportunities will engage in **active strategies**, trying to buy (sell) undervalued (overvalued) stocks and/or trying to forecast the evolution of the market as a whole. These two strategies are respectively known as *stock picking and market timing.*

There is, however, a role for rational portfolio management, even in perfectly efficient markets. Investors' optimal positions will vary according to factors such as age, tax bracket, risk aversion, and employment. The role of the portfolio manager in an efficient market is to tailor the portfolio to these needs, rather than to beat the market.

1.5 Lessons from market efficiency*

*1.5.1 For portfolio managers**

Stay realistic. If you try to realise a systematically higher return than the market offers for the same risk, you don't have to beat only one or two investors; your competitors are thousands of well-educated, intelligent traders and investors who are, on average, as smart as you are. You have to have some significant comparative advantages.

Always remember hamburgers, it is difficult to enter the burger market and make money. It is also difficult to be better than your competitors. This doesn't mean that selling hamburgers cannot be profitable. It simply means that it is extremely difficult to do better than covering costs in the medium to long run. You have to ask yourself under what conditions it is profitable to be in such a competitive market. If you just replicate what your competitors do, unless they are currently running profitable operations, entry will not make you any richer.

Diversify your risk. If you don't have a comparative advantage in the capital market (i.e. if you're unable to beat the market) try to diversify your risk. And don't forget taxes and transaction costs. For instance, if you engage in stock picking instead of simply following a buy-and-hold strategy, every time you trade in a security, you have to bear transaction costs. Your strategy must cover these costs.

1.5.2 For financial managers*

Financial assets are traded in competitive financial markets. This doesn't mean that banks engage in irrelevant activities. Banks offer intermediation.

1.5.3 For others*

For non-banks, financial decisions are not likely to create much value. If a firm wants to raise a specific amount of cash in the capital market, it has to promise future cash flows with a present value that is equal to the amount it desires today. It doesn't have to promise more, and it cannot promise less. But this means zero value creation on average. Taxes, bankruptcy and their transaction costs, information asymmetries and agency costs are the factors that could change this conclusion.[22]

22 The interested reader should consult WELCH Ivo, 1995, “A Primer on Capital Structure”, Finanzmarkt und Portfolio Management, pp. 232-249, or BREALEY Richard A. and MYERS Stewart C., 1996, “Principles of Corporate Finance”, 5th edition, McGraw-Hill, New York

2. Arbitrage Pricing Theory*

2.1 Assumptions Underlying the APT*

The Arbitrage Pricing Theory[23] (APT) is an alternative approach to the equilibrium determination of asset prices. The model describes expected returns and is built on two key assumptions:

- Asset returns are generated by a multi-index model.
- There is an absence of arbitrage opportunities.

*2.1.1 Return Generating Process**

Asset returns are all generated by the same linear model, which has been introduced in the chapter on "Modern portfolio theory I", the multifactor (or multi-index) model. The return of an asset i is:

$$R_i = a_i + b_{i1} \cdot F_1 + b_{i2} \cdot F_2 + \ldots\ldots + b_{ik} \cdot F_k + \varepsilon_i$$

Where R_i is the return on asset i, a_i is the return on asset i if the realization of all factors equals zero, b_{ij} are the sensitivities (or systematic risks) of asset i with respect to factor j, F_j is the realized return on factor j and ε_i is the residual component of asset i (i.e. the fraction of return not determined by the factors and which is specific to the company).

This model is based on several assumptions:

- The residual component of asset i is on average equal to zero, $E(\varepsilon_i) = 0$
- The residual component of assets i and j are not correlated, $Cov(\varepsilon_i, \varepsilon_j) = 0$, meaning that all common variation in the two returns are captured by the factors.
- The residual components of asset i is not correlated with the factors, $Cov(\varepsilon_i, F_k) = 0$.

Note that:

- It is only assumed that a k-factor model generates asset returns, but there is no indication about the number and nature of these factors.
- The risk factors may be correlated.
- Every asset i has its own set of sensitivities to the different factors b_{i1}, b_{i2}, …, b_{ik}. These sensitivities can take negative or positive values.

Example:
Assume that a 3-factor model determines the returns of French stocks where the factors are: long-term interest rate (factor 1), industrial production (factor 2), variation of the exchange rate (factor 3). The sensitivities are determined through regression analysis, and the following results have been obtained:

23 See ROSS Stephen A., 1976, "The Arbitrage Theory of Capital Asset Pricing", Journal of Economic Theory, Vol. 13, pp. 341-360.

Stock	b_{i1}	b_{i2}	b_{i3}
Carrefour	-0.20	2.35	-0.98
Sanofi-Aventis	-0.45	2.21	-1.21
EDF	0.13	-1.08	-0.42
L'Oréal	-0.28	0.56	-0.02

The multifactor model is often described with an alternative but equivalent expression in terms of factor deviation to their mean. In order to get this expression, take the expectation of the previous equation

$$E(R_i) = a_i + b_{i1} \cdot E(F_1) + b_{i2} \cdot E(F_2) + \ldots\ldots + b_{ik} \cdot E(F_k) + E(\varepsilon_i)$$

The last term of the right-hand side expression is equal to zero, as it is the average of the residual component. This expression is then subtracted from the original multifactor model to get

$$R_i = E(R_i) + b_{i1} \cdot (F_1 - E(F_1)) + b_{i2} \cdot (F_2 - E(F_2)) + \ldots\ldots + b_{ik} \cdot (F_k - E(F_k)) + \varepsilon_i$$

This expression is equivalent to the original multifactor model. Note that, by construction, the variables $(F_i - E(F_i))$ have an expected value of zero.

*2.1.2 Absence of Arbitrage Opportunities**

This assumption is one of the pillars of modern financial theory. It is also widely used in other fields of finance (e.g. option pricing theory). This assumption is derived from the very general economic principle known as the law of one price, which states that two identical items cannot sell at different prices. Translated in financial terms, this principle states that two securities with identical risks cannot have different expected returns. If such a situation does exist, arbitrageurs would buy the high return security and sell short the low return security. They would have a portfolio without an initial investment and risk, which would offer a positive profit known as arbitrage (or riskless) profit. This process, known as arbitrage, would last until the equilibrium is reached where securities with the same risk will provide the same expected return. Stated differently, this assumption states that it is impossible for an investor to earn a positive expected return from a portfolio without assuming some risk and making some net investment of funds.

*2.1.3 Other Assumptions and a Definition**

The APT also requires the following assumptions:

- Investors prefer more to less but there is no specific assumption about their risk-aversion.
- The number of available securities in financial markets is much larger than the number of factors generating the asset returns.
- Short sales are allowed and assets are infinitely divisible.

Before turning to the description of the APT, let us define a well-diversified portfolio. Such a portfolio is supposed to be diversified over a large number of securities and to have very small proportions invested in each security. This implies that the residual component of this portfolio ε_p is equal to zero (by the law of large numbers and assuming $E(\varepsilon_p) = 0$) and that its specific risk $\sigma_{\varepsilon p}$ is negligible (close to or equal to zero).

2.2 The APT and its Derivation*

*2.2.1 Development of the APT**

Under the previous set of assumptions, the APT states that the equilibrium expected return on every available asset on the market is

$$E(R_i) = \lambda_0 + b_{i1} \cdot \lambda_1 + b_{i2} \cdot \lambda_2 + \ldots\ldots + b_{ik} \cdot \lambda_k$$

Where b_{ij} is the sensitivity of the security i to the factor j and λ_j is the risk premium on the factor j.

Moreover, it can be shown that

$$\lambda_0 = R_f$$

Where R_f is the risk-free rate, and also that

$$\lambda_j = E(PF_j) - R_f$$

Where PF_j is the return on a pure factor j portfolio. A pure factor j portfolio is defined as a portfolio which has a sensitivity b_j to factor j equal to one and sensitivities b_m ($m \neq j$) to other factors all equal to zero. In the special case when the factors of the multifactor model are independent (not correlated), the pure factor portfolios are equivalent to factors themselves.

Under the above assumptions we have

$$\begin{aligned} E(PF_j) &= R_f + 0 \cdot \lambda_1 + 0 \cdot \lambda_2 + \cdots\cdots + 1 \cdot \lambda_j + 0 \cdot \lambda_k \\ &= R_f + \lambda_j \end{aligned}$$

At this point, note that:

- The APT is *different* from the original multifactor model. APT determines equilibrium expected returns and is an exact relationship. The multifactor model determines realized returns and includes a residual undetermined component. However, the assumption that realized returns are generated by the multifactor model is necessary to obtain the APT.
- The APT does not specify the number and the nature of the factors. It only presents the structure of expected returns.
- The equilibrium relationship of the APT has a structure similar to the CAPM, except that it allows multiple sources of risk and that the assumptions underlying both models are different.

Example

Using the data of the previous example, we assume that the 3 factors are uncorrelated and that their expected return are respectively $E(F_1)$=6%, $E(F_2)$=4%, $E(F_3)$=3% and that the risk-free rate is equal to 2%. The risk premia associated to the factors are equal to: λ_0=2%, λ_1=4%, λ_2=2%, λ_3=1%. The APT equilibrium expected returns on the securities are:

Carrefour	E(R)=2%-0.20 (4%)+2.35 (2%)-0.98 (1%)	=	4.92%
Sanofi-Aventis	E(R)=2%-0.45 (4%)+2.21 (2%)-1.21 (1%)	=	3.41%
EDF	E(R)=2%+0.13 (4%)-1.08 (2%)-0.42 (1%)	=	-0.06%
L'Oréal	E(R)=2%-0.28 (4%)+0.56 (2%)-0.02 (1%)	=	1.98%

*2.2.2 Formal Derivation of the APT**

We derive the equilibrium relationship of the APT by assuming a two-index return generating process. This is enough to allow generalization to any arbitrary number of factors. The two-factor model is

$$R_i = a_i + b_{i1} \cdot F_1 + b_{i2} \cdot F_2 + \varepsilon_i$$

A sufficient condition to prove the existence of the APT is that a well-diversified portfolio with the following characteristics exists:

- Its net investment is zero.
- It has no systematic risk.

The fact that this portfolio is well diversified implies that it has no specific risk. By the absence of arbitrage opportunities condition, the expected return of this portfolio has to be equal to zero.

Denoting x_i as the fraction of the portfolio invested in asset i and n as the total number of available securities, we can rewrite the condition on this portfolio more formally. Since the portfolio has zero investment, the fractions of investments in different assets must be such that

$$\sum_{i=1}^{n} x_i = 0$$

Recall that the systematic risk of a portfolio is the weighted sum of the risk of its components. A portfolio without risk due to factor 1 and 2 is such as

$$\sum_{i=1}^{n} x_i \cdot b_{i1} = 0 \quad \text{and} \quad \sum_{i=1}^{n} x_i \cdot b_{i2} = 0$$

As this portfolio is riskless and without investment, by the absence of arbitrage opportunities condition, it has to yield a zero expected return, that is

$$\sum_{i=1}^{n} x_i \cdot E(R_i) = 0$$

Assuming that we know the weights x_i, this leaves us with the question of what should be the value of the expected returns of individual stocks. As we have n securities on the market, we have 4 equations and n unknown variables. A solution to this system is when the expected returns are a linear combinations of their respective risk sensitivities, such as

$$E(R_i) = \lambda_0 + \lambda_1 \cdot b_{i1} + \lambda_2 \cdot b_{i2}$$

This is the equilibrium equation of the APT. In order to prove this result, let us take an example with (only!) 5 securities in the economy, but which can be generalized to any number of assets. We have to solve the following system:

x_1	$+x_2$	$+x_3$	$+x_4$	$+x_5$	=0	(1)
x_1b_{11}	$+x_2b_{21}$	$+x_3b_{31}$	$+x_4b_{41}$	$+x_5b_{51}$	=0	(2)
x_1b_{12}	$+x_2b_{22}$	$+x_3b_{32}$	$+x_4b_{42}$	$+x_5b_{52}$	=0	(3)
$x_1E(R_1)$	$+x_2E(R_2)$	$+x_3E(R_3)$	$+x_4E(R_4)$	$+x_5E(R_5)$	=0	(4)

Assuming that we know the weights x_i, we have 4 equations and 5 unknown variables, the $E(R_i)$. If we replace $E(R_i)$ by the APT equation in equation (4) we obtain the following equation:

$$x_1 \cdot (\lambda_0 + \lambda_1 \cdot b_{11} + \lambda_2 \cdot b_{12}) + x_2 \cdot (\lambda_0 + \lambda_1 \cdot b_{21} + \lambda_2 \cdot b_{22}) + x_3 \cdot (\lambda_0 + \lambda_1 \cdot b_{31} + \lambda_2 \cdot b_{32})$$
$$+ x_4 \cdot (\lambda_0 + \lambda_1 \cdot b_{41} + \lambda_2 \cdot b_{42}) + x_5 \cdot (\lambda_0 + \lambda_1 \cdot b_{51} + \lambda_2 \cdot b_{52})$$

We group terms in λ_0, λ_1, and λ_2 and obtain

$$\lambda_0 \cdot (x_1 + x_2 + x_3 + x_4 + x_5) + \lambda_1 \cdot (x_1 \cdot b_{11} + x_2 \cdot b_{21} + x_3 \cdot b_{31} + x_4 \cdot b_{41} + x_5 \cdot b_{51})$$
$$+ \lambda_2 \cdot (x_1 \cdot b_{12} + x_2 \cdot b_{22} + x_3 \cdot b_{32} + x_4 \cdot b_{42} + x_5 \cdot b_{52})$$

The coefficient of λ_0 is the left-hand side of equation (1), the coefficient of λ_1 is the left-hand side of equation (2) and the coefficient of λ_2 is the left-hand side of equation (3). These 3 coefficients are all equal to zero as indicated by equation (1), (2) and (3). This means that $E(R_i) = \lambda_0 + \lambda_1 \cdot b_{i1} + \lambda_2 \cdot b_{i2}$ is a solution to equation (4) and solves the system. We have shown that without arbitrage opportunities and if asset returns are driven by a linear multifactor model then expected returns on assets must be linearly related to their risks.

The only question that remains unanswered is the magnitude of the coefficients λ_0, λ_1 and λ_2. The equilibrium model produced by the APT when a two-factor model generates the returns is

$$E(R_i) = \lambda_0 + \lambda_1 \cdot b_{i1} + \lambda_2 \cdot b_{i2}$$

Intuitively λ_1 and λ_2 are the returns for bearing risks associated with factors 1 and 2. More insight can be gained by examining certain type of portfolios. Assume a portfolio that is insensitive to factors 1 and 2. Such portfolio has therefore $b_{i1}=0$ and $b_{i2}=0$ and is riskless, its expected return has to yield the riskless rate R_f. Therefore λ_0 should be equal to R_f. Let us now examine a portfolio which is only sensitive to factor 1 with a unit sensitivity. This portfolio is such that $b_{i1}=1$ and $b_{i2}=0$. From the equilibrium equation, the expected return of this portfolio is equal to $\lambda_0 + \lambda_1$. We know that and can therefore claim that λ_1 is the expected return of a portfolio only subject to risk of factor 1, having a unit measure of this risk (a pure factor 1 portfolio) minus the risk-free rate. The same type of analysis applies to λ_2.

All the analyses of this section can be generalized to show that if the securities are generated by a multifactor model with k factors, then the asset expected returns are described by a k-dimensional hyperplane

$$E(R_i) = \lambda_0 + \lambda_1 \cdot b_{i1} + \lambda_2 \cdot b_{i2} + \ldots\ldots + \lambda_k \cdot b_{ik}$$

where

$\lambda_0 = R_f$

$\lambda_j = E(PF_j) - R_f$

2.2.3 *An Illustration of the APT**

Assume that the returns are generated by a two-factor model:

$$R_i = a_i + b_{i1} \cdot F_1 + b_{i2} \cdot F_2 + \varepsilon_i$$

Let us assume that we have 3 stocks that are priced according to the APT. These securities have the following characteristics:

Security	**E(R$_i$)**	**b$_{i1}$**	**b$_{i2}$**
BNP Paribas	5%	0.4	-0.3
Sanofi-Aventis	8%	0.6	0.3
Alstom	9%	0.4	0.7

We obtain the following equations:

BNP Paribas	$5 = \lambda_0 + 0.4 \cdot \lambda_1 - 0.3 \cdot \lambda_2$
Sanofi-Aventis	$8 = \lambda_0 + 0.6 \cdot \lambda_1 + 0.3 \cdot \lambda_2$
Alstom	$9 = \lambda_0 + 0.4 \cdot \lambda_1 + 0.7 \cdot \lambda_2$

This results in the APT equilibrium equation:

$$E(R_i) = 5 + 3 \cdot b_{i1} + 4 \cdot b_{i2}$$

where:

$\lambda_0 = 5$, $\lambda_1 = 3$ and $\lambda_2 = 4$

This is also the equation of a plane in the E(R$_i$), b$_{i1}$, b$_{i2}$ space. Any linear combination of these securities must be on the same plane.

Let us now consider what happens to a security that is not priced according to the APT (i.e. that is not on that plane). Assume the L'Oréal stock has an expected return of 3.75% a b$_{i1}$ of 0.45 and a b$_{i2}$ equal to 0.35. We compare the L'Oréal stock with a portfolio p by placing 25% of the funds in the BNP Paribas stock, 25% of the funds in the Sanofi-Aventis stock and 50% in the Alstom stock. The sensitivities of this portfolio p to factor 1 and 2 are:

$$b_{p1} = 0.25 \cdot (0.40) + 0.25 \cdot (0.60) + 0.50 \cdot (0.40) = 0.45$$
$$b_{p2} = 0.25 \cdot (-0.30) + 0.25 \cdot (0.30) + 0.50 \cdot (0.70) = 0.35$$

The risks of portfolio p are therefore identical to the risks of the L'Oréal stock. The expected return on portfolio p is:

$$E(R_p) = 0.25 \cdot (5\%) + 0.25 \cdot (8\%) + 0.50 \cdot (9\%) = 7.75\%$$

Alternatively, since portfolio p must lie on the plane described above (the APT equation) we could have obtained its expected return from the equation of the plane:

$$E(R_p) = 5 + 3 \cdot (0.45) + 4 \cdot (0.35) = 7.75\%$$

By the absence of arbitrage opportunities assumption, two portfolios with the same risk cannot have different expected returns. In fact, if such a situation existed it would quickly disappear as arbitrageurs would step in and would buy portfolio p and sell short the L'Oréal stock, thereby obtaining an arbitrage profit of 4%. Let us illustrate this by assuming that an investor buys EUR 1000 of portfolio p and sells short EUR 1000 of L'Oréal stock.

Security	Initial Cash Flow	End of Period Cash Flow	b_{p1}	b_{p2}
Portfolio p	-1000	1077.50	0.45	0.35
L'Oréal stock	1000	-1037.50	-0.45	-0.35
Arbitrage portfolio	0	40.00	0.00	0.00

The arbitrage portfolio involves zero investment, no systematic risk (b_{p1}, b_{p2}) and earns EUR 40. Arbitrage would continue until the stocks L'Oréal lies on the same plane as BNP Paribas, Sanofi-Aventis and Alstom stocks.

2.3 The Link between the APT and the CAPM*

The CAPM and the APT should not be considered as mutually exclusive models but as models having different approaches to the same reality. This section describes the link between the two models and also their major differences.

Let us first review the link between the two models. The simplest case is where returns are generated by a one-factor model and that the single factor is the market portfolio. In this case, the CAPM and the APT have the same equilibrium pricing equation, despite the fact that they are based on different assumptions. This case is trivial and uninteresting since there is only one source of risk.

A more interesting case is where the security returns are generated by a k-factor model. It is then possible to relate the beta of a security to its sensitivities to the k factors. In a two-factor model and given the properties of covariance, the covariance of the return on the security i with the return of the market portfolio M is:

$$\begin{aligned}\mathrm{Cov}(R_i,R_M) &= \mathrm{Cov}((a_i + b_{i1}\cdot F_1 + b_{i2}\cdot F_2 + \varepsilon_i),\ R_M)\\ &= \left[\mathrm{Cov}(F_1,R_M)\cdot b_{i1}\right] + \left[\mathrm{Cov}(F_2,R_M)\cdot b_{i2}\right] + \mathrm{Cov}(\varepsilon_i,R_M)\end{aligned}$$

Dividing both sides of the equation by σ_M^2 and knowing that, by the CAPM definition, $\beta_i = \mathrm{Cov}(R_i, R_M)/\sigma_M^2$ we get:

$$\beta_i = \left[\frac{\mathrm{Cov}(F_1,\ R_M)}{\sigma_M^2}\cdot b_{i1}\right] + \left[\frac{\mathrm{Cov}(F_2,\ R_M)}{\sigma_M^2}\cdot b_{i2}\right] + \frac{\mathrm{Cov}(\varepsilon_i,\ R_M)}{\sigma_M^2}$$

Assuming that the error terms are not correlated with the market returns, we obtain:

$$\beta_i = \beta_{F1}\cdot b_{i1} + \beta_{F2}\cdot b_{i2}$$

Where β_{F1} and β_{F2} are the respective betas of factors 1 and 2 (with respect to the market). Since β_{F1} and β_{F2} are constant and independent of the share (they only depend on correlations between the factors and the market), the beta of a security is a function of b_{i1} and b_{i2}. Hence, if two securities have different betas, it is due to different sensitivities to the factors.

As a result of the CAPM assumptions, the expected returns of security i is related to the beta:

$$E(R_i) = R_f + (E(R_M) - R_f)\cdot\beta_i$$

Hence, in a two-factor model, the returns are linked to the two factors through the following relationship:

$$
\begin{aligned}
E(R_i) &= R_f + (E(R_M) - R_f) \cdot (\beta_{F1} \cdot b_{i1} + \beta_{F2} \cdot b_{i2}) \\
&= R_f + [(E(R_M) - R_f) \cdot \beta_{F1}] \cdot b_{i1} + [(E(R_M) - R_f) \cdot \beta_{F2}] \cdot b_{i2} \\
&= R_f + \lambda_1 \cdot b_{i1} + \lambda_2 \cdot b_{i2}
\end{aligned}
$$

where:

$$
\lambda_1 = (E(R_M) - R_f) \cdot \beta_{F1}
$$
$$
\lambda_2 = (E(R_M) - R_f) \cdot \beta_{F2}
$$

The CAPM gives another meaning to the risk premiums λ_1 and λ_2 which were defined as expected excess returns on pure factor portfolios. Assuming that factor 1 is positively correlated with market return means that $Cov(F_1,R_M)$ is positive. Consequently, β_{F1} is positive and, since $E(R_M)–R_f > 0$, λ_1 is also positive. Thus, higher the value of b_{i1}, higher would be the expected return of the security. The reverse will be true for negative betas.

This result means that the two theories are consistent. Nevertheless, the CAPM should not be considered to be a particular case of the APT since their assumptions are different.

- The CAPM makes strong assumptions on investors' behaviour, on the role of the market portfolio and on the way equilibrium is achieved.
- The APT makes less stringent assumptions on investors' behaviour, does not assign a particular role to the market portfolio and is more realistic by assuming multiple sources of risk.

The apparent superiority (or generality) of the APT from a theoretical point of view is however lost from an empirical point of view. The CAPM makes very precise predictions on the way expected returns are determined. On the other hand, the APT remains silent on the number and nature of factors generating asset returns. As we will see in the next section, despite empirical tests of the APT using sophisticated statistical techniques, no consensus and definitive conclusion on the nature and number of APT factors has been obtained so far in the academic literature.

2.4 Empirical Tests of the APT*

*2.4.1 Identifying factors**

Since the APT is based on a factor model of security returns, any test of its predictions must incorporate such a factor model. The test will be, in fact, a joint test of an equilibrium theory and of the appropriateness of the selected factor model. This means that it may be very difficult to interpret the results of the tests since we do not know if the APT does not work or if the model was only badly specified (a set of wrong factors or the wrong number of factors was used).

Another problem is that it is very difficult to determine the relevant factors to be used in the model. Hence, there are two alternative approaches that can be used to estimate an APT model:

- The risk factors and the sensitivities of the assets to those factors can be simultaneously computed using statistical techniques, such as factor analysis or principal components analysis[24]. But these methods have drawbacks. First, the factors are not identified economic variables. It is necessary to compare the factors with existing variables to identify them. Moreover, the economic interpretation of factors may change over time, so that, for example, factor 2 for a sample period will certainly differ from factor 2 in another sample period. Second, the number of relevant factors found appears to vary according to the number of stocks used in the analysis, while it should be independent of the sample used.

- Economic theory and knowledge of financial markets can be used to hypothesise and pre-specify an intuitively appealing set of factors that can be measured from available macroeconomic and financial data. But the right selection of an appropriate set of factors involves as much art as it does science.

2.5 Pre-Specifying Factors*

Several studies have used the second approach. For instance, Chen, Roll and Ross (1986)[25] have determined a large fraction of the covariance between the securities using the following macroeconomic factors:

- The spread in yield between a long-and a short-term treasury bond, as differences affect the value of payments far in the future relative to near-term payments.
- The rate of inflation, which impacts both the level of the discount rate and the size of the future cash flows.
- The spread between low-grade bonds and treasury bonds, which is a measure of market reaction to risk.
- The growth rate of industrial production, as changes in industrial production affects the opportunities facing investors and the real value of cash flows.

The problem with this method is that i) it is not necessary that there are only four factors and ii) that the above factors are the proper ones. Hence, even if these factors explain most of the variance, it does not necessarily validate the APT. The sample used in estimating the factors may not be large enough or may not be representative. It is possible that a different sample may yield entirely different factors from those found using the first sample.

Nevertheless, it was possible to find evidence that there are risk premia associated with these factors, but it must be noted that the market index often proved to be one of the (if not the only one) relevant factors. This means that it is very hard to determine which factors really drive stock returns. Chen, Roll and Ross end up with the market as not being a significant factor for portfolio returns in their study.

24 Both techniques extract from the data a set of indices that best explain the variance of the data, so that the covariance of residual returns (returns after the influence of the indices have been removed) is as small as possible.

25 CHEN Rai-Fu, ROLL Richard and ROSS Stephen A., 1983, “Economic Forces and the Stock Market”, Journal of Business, Vol. 59, pp. 386-403.

Another example of a study using pre-specified factors is the work of Fama and French (1993). They found that the following factors significantly affect securities returns:

- The difference in return on a portfolio of small stocks and a portfolio of large stocks.
- The difference in return on a portfolio of high book-to-market value stocks and a portfolio of low book-to-market value stocks.
- The difference between the monthly long-term government bond return and the one-month Treasury bill return.
- The difference in the monthly return on a portfolio of long-term corporate bonds and a portfolio of long term government bonds.

As can be seen from these factors, microeconomic variables reflecting the difference in firm size and book-to-market value ratios are also relevant in explaining the returns. This shows that the search for relevant factors is still going on, and their identification is the crucial step before the APT can really become a recognized model from an empirical point of view.

2.6 Some Applications of the APT*

In practice, the APT can be used for different purposes. This section describes two applications of the model: tracking an index and active portfolio management.

The first use of the APT is linked to the use of multifactor models in the creation of a portfolio of stocks that closely tracks an index. An obvious way to construct an index fund is to hold stocks constituting the portfolio in the same proportions as the stocks are represented in the index. This way of replicating the index can be very costly when the index is composed of a large number of securities. For instance, an investor who would like to hold the FTSE 350 with this method will have to hold more than 350 securities and frequently readjust his positions to replicate the exact composition of the index. This would incur a lot of transaction and monitoring costs. Another way to achieve the same goal is to use a multifactor model to determine the sensitivities of the index to the underlying factors. The investor could then hold a reduced set of securities in a portfolio that would have exactly the same sensitivities to the factors as the index. The index would then be perfectly tracked with a reduced number of stocks. Nowadays, several funds avoid certain stocks for ethical or other reasons (e.g. tobacco companies). With the help of a multifactor model, one could still form a portfolio avoiding those stocks but still matching the sensitivities of the index to track. The problem with this application is that the exact identities of factors generating the returns are unknown. This is where APT can help us, with the statistical tools used in its empirical tests. These methods (factor analysis and principal components analysis) allow us to determine statistically factors and sensitivities of the securities and the index, without knowing the identity of the factors. The sensitivities of the stocks are used to form a portfolio replicating the sensitivities of the index.

Example:
Assume that we have the following sensitivities to 4 factors determined with factor analysis.

Stock	b_{i1}	b_{i2}	b_{i3}	b_{i4}
Carrefour	-1.60	0.42	1.02	-4.66
Sanofi-Aventis	0.50	0.12	0.98	3.88
CAC 40 Index	-0.55	0.27	1.00	-0.39

It is possible to form a portfolio with 50% invested in Carrefour and the other 50% in Sanofi-Aventis and obtain exactly the same sensitivities to the 4 factors as the CAC 40 index.

b_{p1}	=0.5·(-1.60)	+0.5·0.50	=	-0.55
b_{p2}	=0.5·0.42	+0.5·0.12	=	0.27
b_{p3}	=0.5·1.02	+0.5·0.98	=	1.00
b_{p4}	=0.5·(-4.66)	+0.5·3.88	=	-0.39

Active portfolio management involves making bets about securities or group of securities, i.e., designing a portfolio based on the belief that one or more securities are mispriced. APT can be used to determine the equilibrium (or normal) expected return on a security and then compare it with some predictions about future return on the security. The first kind of expected return is the global return obtained through APT. If an analyst predicts a higher return than that obtained through the APT for a stock, then the investor would buy that stock. On the other hand, if the analyst predicts that a stock will have a lower return than its expected APT return, the investor should sell that security.

Another kind of active portfolio management could be designed on the basis of predictions on realization of a factor. If an analyst predicts a higher return on a pure factor portfolio than that used in the APT (λ's), then it is worthwhile increasing the investor's exposure to that factor, by investing in securities with high positive sensitivities to that factor.

Example:
We have 3 factors which are uncorrelated and whose expected returns are E (F_1) = 6%, E (F_2) = 4%, and E (F_3) = 3% respectively. If an analyst predicts that the expected return on the first factor will be 10% instead of the equilibrium 6%, then it would be profitable to invest in securities that have a high positive sensitivity to factor 1.