FIXED INCOME

FIXED INCOME PORTFOLIO MANAGEMENT STRATEGIES

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* final level

1. Passive management

Passive management is a management style where the portfolio manager follows a strategy that does not require any forecasting. Generally, the goal is to mirror the performance of a benchmark. A benchmark is an alternative “virtual portfolio” which is constructed entirely according to predetermined rules. This benchmark is often referred to as an index and is usually imposed by the entity on whose behalf the portfolio manager is working for as an agent.

1.1 Buy and hold

A simple passive strategy is to build a portfolio and hold it to maturity. The coupons and the proceeds of matured bonds are reinvested in new issues. Doing so, the only variation in total return comes from the reinvestments. Usually, there is some control on the level of risk. The most common control is to set the duration of the bond portfolio equal to the duration of the relevant index.[1]

1.2 Indexation

1.2.1 Introduction

The purpose of bond indexing is to replicate the performance of a predetermined benchmark as closely as possible. There are however, some major differences between equity and bond indexing:

The benchmarks are usually much broader in terms of the number of bonds encompassed.[2] Furthermore, the turnover on these indices is substantially larger than for equity indices, as a substantial proportion of the bonds mature each year and as significant amounts are also issued each year. The last difference is that large segments of the bond market are often very illiquid. For this reason, futures on bond indices are currently rare. They are usually based on notional bonds.

These three considerations can make the life of a bond indexer difficult, but bonds are much more interchangeable and less specific than equities.

1.2.2 Indexing technology

In this section, our focus is on both the stratified and optimisation sampling techniques, while we outline the differences with respect to the use of these techniques for equities.[3]

[1] It is a means of controlling the interest rate risk. In section 2, we show why matching duration is an important part of risk control.

[2] In the US and Europe the most common indices include several thousand bonds.

[3] See Portfolio Management module, for the description of these techniques.

Stratified sampling

The goal here is both, to limit the number of bonds included in the indexed portfolio and to avoid trading too small bond positions and being in the illiquid segment of the market.

Building an indexed portfolio requires the following steps:

- First, the universe of bonds in the benchmark is partitioned into cells based on some of the characteristics[4] of bonds. Then, the weight of each cell is measured given the weights of the bonds in the benchmark.
- For each cell with non-zero weight, we select a limited number of bonds belonging to that cell and create a price weighted portfolio which matches as closely as possible some average characteristics of the cell (i.e. its average duration and convexity) and has the appropriate weight in the indexed portfolio.

With bonds, the rationing in the number of bonds comes from the explicit limit the indexer imposes when building a bond portfolio for each cell.

Optimised sampling

The availability of bond risk models has provided the needed tool to overcome the drawbacks of the stratified sampling approach:

- By giving an ex-ante measure of the tracking error of the indexed portfolio with respect to the benchmark,
- By giving access to an optimiser, which allows a portfolio construction that takes into account the risk trade-off between factors and the transaction costs.
- By allowing the indexer to choose the level of tracking error to be achieved by restricting the number of bonds in the indexed portfolio.

1.3 Interest rate immunisation

The interest rate immunisation technique was originally proposed by Frank Redington, an English actuary, in 1952 to help life-insurance companies assuring an equilibrium between their assets and liabilities. In 1971, Fisher and Weil extended the concept to bond portfolios.[5]

Any variation of the interest rates has two opposite effects. An increase in the interest rates will lead to a lower value of the bond portfolio, but a higher return on the coupons reinvestment. The fundamental mechanism underlying the concept of portfolio immunization is a portfolio structure that balances the change in the value of the portfolio at the end of the investment horizon with the return from the reinvestment of portfolio coupons payments. Thus, immunization is the process of offsetting price risk and reinvestment risk.

[4] These characteristics include sector, debtor’s category, coupon, term to maturity, the duration and quality rating.

[5] See Fisher and Weil (1971).

Let us consider the following example: an investor holds a bond with a 5 years maturity and an 8% coupon rate at a quoted price of 108.42. The bond's yield is 6%. The investor wonders what will be its final wealth for various time horizons if there was a sudden unexpected variation in the market yield today (and if the market yield would remain stable after this variation).

The following table shows the results for 1 to 5 years time horizons, and 4% to 8% final market yield (i.e. a variation of ±2%).

		New market yield				
		4%	**5%**	**6%**	**7%**	**8%**
Time Horizon	1 year	122.52	118.64	114.93	111.39	108.00
	2 years	127.42	124.57	121.83	119.18	116.64
	3 years	132.52	130.80	129.14	127.53	125.97
	4 years	137.82	137.34	136.88	136.45	136.05
	5 years	143.33	144.21	145.10	146.01	146.93

Table 1-1: The effect of a yield variation on terminal wealth

The following figure shows graphically the results.

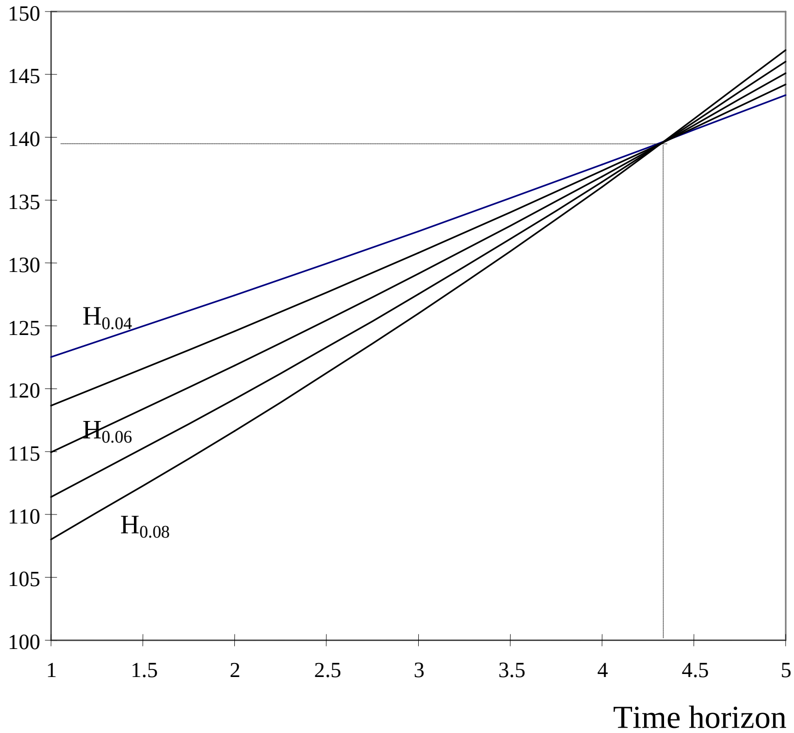

Figure 1-1: The effect of a yield variation on terminal wealth

The final wealth differs more or less from the status-quo assumption (if there was no variation, we would be on the curve $H_{0.06}$). On short time-horizon, the effect on the bond price is higher than the effect on the reinvestment of the coupons, and the investor would like to avoid an interest rate increase; for longer time-horizons, the reinvestment of coupons takes a higher part of the final wealth, and the investor wants to avoid an interest rate decrease.

More surprisingly, **there seems to be a time horizon** (4.34 years) **for which the final wealth will be the same, whatever the initial interest rate variation**. If the investor selects such a time horizon, he will be certain to end up with a 6% yield and a final wealth of:

$$W = 108.42 \cdot (1.06)^{4.34} = 139.64$$

In that case, the portfolio is said to be **immunised against interest rates variations**. One can show theoretically that this time horizon is in fact the duration of the bond under consideration.

One might think that therefore, an investor would not be affected by interest rate variations simply by creating a portfolio with duration equal to the time horizon of the investor:

$$\text{Duration} = \text{Time horizon } (= \text{TH})$$

as the portfolio duration is the weighted average of the durations of the individual issues. But this condition is in fact not sufficient because duration changes:

- Non-continuously with the lapse of time (it increases at coupon payments), whereas the investor's time horizon decreases continuously with the lapse of time.
- As soon as there is an interest rate modification.
- In a non-linear way due to coupon payments. Their reinvestment may also affect the immunization equilibrium.

For these reasons, immunization has to be a dynamic process and not a static one, the portfolio should be periodically rebalanced at each coupon payment and the condition to be satisfied by the portfolio to be immunized should be rewritten as:

$$\text{Duration}_t = \text{Time horizon}_t \text{ (for all t)}$$

Example:[6]
An investor has a 3-year time-horizon, and he wants to create a bond portfolio with a final return of 6.5%. His investment universe is made of two Japanese bonds: bond A is a 4-year 3% bond, traded at 88.01; its yield to maturity is 6.5%, and its duration 3.817 years; bond B is a 3-year 5% bond, traded at 96.03; its yield to maturity is 6.5%, and its duration 2.856 years.

As the time horizon is three years, we must have:

$$\begin{cases} D_P = TH = 3 \text{ years} \\ x_A + x_B = 1 \end{cases} \Rightarrow \begin{cases} 3.817 \cdot x_A + 2.856 \cdot x_B = 3 \\ x_A + x_B = 1 \end{cases}$$

The solutions of these equations are x_A = 0.149885 and x_B = 0.850115. Hence, the investor should buy for 149'885 JPY of bond A (i.e. 1'703.05 bonds) and 850'115 JPY of bond B (i.e. 8'852.6 bonds).

By creating such a portfolio, his final wealth in three years will be (with certainty):

$$W = 1'000'000 \cdot (1.065)^3 = 1'207'950 \text{ JPY}$$

What happens if there is immediately a parallel shift of the yield curve, so that the new market yield is 5%, whatever the maturity?

6 Source: Asner and Dumont (1985).

In that case, the bond prices would adjust to the new market yield. The portfolio would be made of 1'703.05 bond A, with a duration of 3.821 years, quoted at 92.91 (total amount of 158'227 JPY) and of 8'852.6 bonds B, with a duration of 2.859 years quoted at 100.00 (total amount of 885'260 JPY).

The portfolio contains 15.16% of bond A and 84.84% of bond B, for a total value of 1'043'487 JPY. Its new duration is

$$D_P = 0.1516 \cdot 3.821 + 0.8484 \cdot 2.859 = 3.01 \text{ years}$$

The immunization equation is not verified any more. The investor should rebalance the portfolio to remain immunized, as there could be another interest rate variation.

How often should the portfolio be rebalanced to adjust its duration? More frequent rebalancing increases transaction costs, less frequent rebalancing causes the portfolio duration to wander from the target duration; both reduce the likelihood of achieving the target return. Therefore, there is a certain trade-off between frequent rebalancing and minimising the transaction costs.

Remember that:

- immunisation protects a portfolio against the losses due to an interest rate variation, but it also eliminates any benefits that could come from an interest rate variation.
- as immunisation is based on duration, it assumes a parallel shift of the (flat) interest rate term structure. Because of this, immunization will work better on long-term strategies than on short-term ones (as short-term rates are very volatile).

both the passage of time and change in yield will leave the portfolio imperfectly immunized.

1.4 Asset-liability management

Classic portfolio management approaches focus only on the asset side of an investor’s balance sheet, but asset-liability management is a portfolio management strategy that considers both sides of the balance sheet and involves matching the cash flows of the liabilities with ones of the portfolio.

The goal is to build a bond portfolio that funds the liability stream and/or will immunise the interest rate risk on the liability side as much as possible.

Notice that this problem is similar to the problem of indexing immunisation, but here the benchmark that we try to replicate is defined as the future liabilities payments.

1.4.1 Liability immunisation

Here the focus is to fund a liability stream and to control for parallel shifts in the yield curve.[7] We will identify two separate cases depending on whether there are single or multiple liability cash-flows.

[7] Note that for the investor the liability stream is a set of negative cash-flows.

For the remainder of this discussion, we will adopt the following notation:

L is the present value of the liability stream,

A is the present value of the cash-flow stream from the bond portfolio, i.e. its current value.

1.4.1.1 Single period liability immunisation

In this case, we assume that the liability stream consists of a single one at time T:

$$L_T > 0 \text{ and } L_t = 0 \quad \text{for } t = 1,..., T\text{-}1$$

The investor knows that both the value of his liability and of the bond portfolio A used for the funding of the liability will change when there is a parallel shift of the interest rate term structure. His dual goal is to fund the liability and to avoid the risk inherent to such parallel shifts.

An easy solution for this investor would be to buy an adequate number of zero-coupon bonds maturing at time T. If the present value of the zero-coupon bonds is equal to or higher than the present value of the liability using the same discount rate (or in other words, if the nominal of the zero-coupon bonds is equal to or higher than the nominal of the future liability), the investor is sure to fund his liability at time T, subject of course to default risk.[8] Note that the duration of both the liability and the discount bonds are exactly matched. Unfortunately, such an advantageous zero-coupon bond may not exist.

In reality the investor may have to solve his problem with more ordinary bonds. In this case, he might build a bond portfolio A, which satisfies the following rules:

$$A = L \qquad \text{(I)}$$

and

$$D_A = T \qquad \text{(II)}$$

The first rule states that the present value of the bond portfolio should be equal to the present value of the future liability. This will ensure that the funding is effective. It is important to realise that both the liability and the bond portfolio should be discounted at the same rate k, i.e. the internal rate of return of the bond portfolio. The second condition requires that the duration of both, the bond portfolio and the liability are equal.[9] It will protect the investor against parallel shifts in the term structure. From ($R_i = \frac{\Delta P_i}{P_i} \approx -\frac{D_i}{(1+k)} \cdot \Delta k_i$) applied to both the asset and the liability sides, we have[10]

$$\Delta A = -A \cdot \frac{D_A}{(1+k)} \cdot \Delta k_A$$

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8 If asset A and liability L have different credit risks, there is a risk of parallel shifts in the term strucuture, due to changes in the spread between the two.

9 The duration of a single liability is its maturity.

10 See “Interest rates - Term structure and Applications”

$$\Delta L = -L \cdot \frac{T}{(1+k)} \cdot \Delta k_A$$

Using both (I) and (II), we easily get:

$$\Delta A = \Delta L$$

In other words, by matching the duration of the liability and the bond portfolio, the investor ensures that the change in value of both liability and assets caused by parallel shifts in the term structure will broadly be equal. The joint portfolio of the bonds and the liability is immunised against these parallel shifts.

This technique is not without risk as it is based on a flat term structure that only undergoes parallel shifts, which is rarely the case in reality. A risk analysis of the bond portfolio with respect to the single liability as a benchmark would give an indication of all the risks taken.

Note that the application of the two rules (I) and (II) do not lead to a single solution. There are potentially an infinite number of bond portfolios with duration equal to T. The portfolio manager can try to minimise the value of the bond portfolio. This is done by an optimisation technique known as linear programming and can be easily solved using well established algorithms. The choice of the set of bonds over which the optimisation will take place is very important. The set must be homogeneous in terms of quality rating. Otherwise, the optimised portfolio will concentrate on bonds with higher yields just because they are cheap but without taking in account the fact that they are also more risky.

However, the portfolio manager can also choose from this infinite number of portfolios taking into account that these portfolios are not equivalent in terms of the risk taken. In the literature, there is some emphasis that in the case of a single liability, the bond cash-flow should occur around the maturity of the liability. The idea is that by doing so, the bond portfolio will be less sensitive to non-parallel movements of the yield curve. A useful measure of the dispersion of the bond portfolio's cash-flow around its duration is:

$$DS_A = \frac{\sum_i (t_i^A - D_A)^2 \cdot A_i}{\sum_i A_i}$$

where:

DS_A	is the dispersion of the cash-flows of portfolio A around its duration
D_A	is the duration of portfolio A
A_i	is the present value of cash-flow i of portfolio A
t_i^A	is the maturity of cash-flow i of portfolio A

Note that this dispersion measure is also a measure of the dispersion of the bond portfolio's cash-flow around the single liability. Common sense suggests that smaller this dispersion is, the smaller the risk taken by the investor[11]. This suggests a third rule[12]:

$$DS_A \approx 0$$

[11] Notice that in the case of our first example with a zero-coupon discount bond, the dispersion is equal to zero.

[12] The sign $\approx$ means "as near as possible".

As occurs with interest rate immunisation, liability immunisation is a dynamic strategy because of the same reasons. Therefore, there is a need to readjust the bond portfolio so that the duration of the portfolio matches that of the liability, if the discrepancy is getting too large. Reinvestment of coupon income and matured principle helps to alleviate the transaction costs. If needed, the portfolio manager may have to sell the bonds with the higher duration.

1.4.1.2 Multiperiod liability immunisation

In this case the investor must fund a stream of several liabilities at different dates in the future.

$$L_t \geq 0 \quad \text{for} \quad t = 1,\ldots, T$$

One way to solve this problem is to apply the technique for a single liability to each of the several liabilities just mentioned. If there are N liabilities, we will build N bond portfolios along the lines of the previous point. Notice that if we aggregate all the liabilities and the bond portfolios, we get:

$$A = L \qquad \text{and} \qquad D_A = D_L$$

These equations can be taken as rules to be implemented to solve this multiperiod problem in building a single bond portfolio to immunise the liability stream. By following these two rules, the funding is insured and the immunisation does indeed take place in the case of parallel shifts of a flat yield curve.

But, there is a problem in implementing these two rules. In order to identify the duration of the liability stream, we must know the internal rate of return of the bond portfolio which must have the same duration as the liability stream. An iterative process is needed to solve this issue.

It is also suggested that the dispersion of the bond portfolio cash-flow should be as near as possible as that of the liabilities. The same reasons of risk control against non-parallel movements in the yield curve are the basis of this rule:

$$DS_A \approx DS_L$$

In terms of dynamic readjustment of the bond portfolio duration to the level of the duration of the liability stream, the liability side may now create large discontinuities if a large liability matures at a specific time.

1.4.1.3 Surplus immunisation

There are cases where the investor owns assets in excess of the present value of his liabilities. The amount of excess assets is called the surplus:

$$S = A - L$$

One possible goal for this investor is to immunise the surplus with respect to parallel shifts in the interest rate term structure. In order to achieve that, the rule of equalising the duration of the assets with the duration of the liabilities should be modified. We have:

$$\Delta S = \Delta A - \Delta L = (L \cdot \frac{D_L}{(1+k)} - A \cdot \frac{D_A}{(1+k)}) \cdot \Delta k$$

In order for the surplus not to change to any parallel shifts in the term structure ($\Delta S = 0$), we can easily see that we must have:

$$A \cdot D_A = L \cdot D_L$$

For a positive surplus, immunisation requires that the duration of the assets should be smaller than the duration of the liabilities, as the volatility of the term structure is applied to a larger amount on the asset side.

1.4.2 Cash flow matching

Cash flow matching is an appealing strategy. The idea of cash flow matching is to create a bond portfolio whose stream of cash flows exactly matches a stream of liabilities.[13]

The simplest way to create this portfolio would be the purchase of a zero-coupon bond for each liability and maturity. Unfortunately, this is not always possible, as most available bonds are not zero-coupons. Hence, cash flow matching operates through an iterative process: at each step, a bond is selected with a maturity that matches the longer duration liability, and an amount of principal equal to the amount of this liability is invested in this bond. The remaining elements of the liability stream are then reduced by the coupon payments on this bond. The process continues for the next longer maturity liability, going backward in durations until all liabilities have been matched by payments on the securities selected for the portfolio.

The following example helps understanding the process. Consider a company with the following liabilities:

Time (years)	**1**	**2**	**3**	**4**	**5**	**6**
Liability	L_1	L_2	L_3	L_4	L_5	L_6

We want to create a dedicated cash flow matching portfolio.

First, find a bond A, with par value P_A maturing at the end of year 6, paying a coupon C_A each year. Invest an amount in this bond, such that the cash flow paid at the end of year 6, i.e. (P_A+ C_A), is equal to L_6. To simplify, we will assume that a perfect matching is possible, that is, $P_A+C_A= L_6$.

Now, the liabilities to face out are the following:

Time (years)	**1**	**2**	**3**	**4**	**5**	**6**
Liability	L_1	L_2	L_3	L_4	L_5	L_6
Cash inflows	C_A	C_A	C_A	C_A	C_A	$P_A + C_A$
Remaining liabilities	$L_1 – C_A$	$L_2 – C_A$	$L_3 – C_A$	$L_4 – C_A$	$L_5 – C_A$	0

Now, find a bond B, with par value P_B maturing at the end of year 5, paying a coupon C_B. Invest an amount in this bond, such that the cash flow paid at the end of year 5, i.e. ($P_B + C_B$), is equal to $L_5 – C_A$. To simplify, we will assume again that a perfect matching is possible, that is, $P_B + C_B + C_A = L_5$.

13 The terminology of dedicated bond portfolio is also used.

Now, the liability cash flows to be matched for the remaining four years are as follows:

Time	1	2	3	4	5	6
Liability Cash flows	L_1 $C_A + C_B$	L_2 $C_A + C_B$	L_3 $C_A + C_B$	L_4 $C_A + C_B$	L_5 $P_B + C_A + C_B$	L_6 $P_A + C_A$
Remaining liabilities	$L_1 - C_A - C_B$	$L_2 - C_A - C_B$	$L_3 - C_A - C_B$	$L_4 - C_A - C_B$	0	0

Continue the process with year 4, 3, 2, and 1.

Here again, we can employ linear programming techniques to construct a least-cost cash flow matching portfolio from an acceptable universe of bonds.

But cash flow matching suffers from some major drawbacks: because of the difficulties of perfect date matching, funds are generally available before the exact target date. Furthermore, exact amount matching is not always possible, particularly because of rounding in the bond quantities traded. Hence, cash flow matching has to be a rather conservative strategy, which leads to an opportunity cost.

2. Active management

In an active management approach, the portfolio manager seeks to achieve a positive excess return of the portfolio over the benchmark, i.e. a positive difference between the return of the managed portfolio and the benchmark.

2.1 Forecasting and portfolio construction

Active management requires forecasts of returns for the available assets. For the bond portfolio management, the basic forecasts relate to changes in the yield curve (parallel shifts, slope, and convexity) or changes in spreads (sector, quality, etc.).[14]

All the principles about information ratio maximisation and optimisation, which are described for equity management,[15] are also valid for bond management.

There are two different ways of building bond portfolios depending on whether there are explicit forecasts of the returns or not. If there are, then a full optimisation process can be implemented. If not, the portfolio should be built by constraining the risk exposures to be consistent with the forecast scenario for the yield curve. The use of a risk model can then minimise risk given the constraints imposed by the forecast.

2.2 Active management in practice

2.2.1 Constant duration

A way to improve a buy and hold strategy is to impose a "constant" average duration for the managed portfolio during the full interest rate cycle. The idea behind this simple rule is that interest rates basically follow a mean reversion process.[16] If interest rates rise above (decrease below) their average value, they are bound to decline (increase) later.

For example, assume that initially the level of interest rates is at the middle of its range of fluctuation (i_0) and that the index duration is at an average level D_0. If, from this level, interest rates increase, the duration of the index will decline. If, at the same time, the portfolio manager matches his portfolio duration to the duration of the index, he will basically replicate the performance of the index. When interest rates reach an upper trigger limit i_U, the manager will bring the duration of his portfolio to the average level of the index duration D_0. By maintaining a constant duration D_0 for the bond portfolio until the interest rates fall back to their average level i_0, the portfolio manager takes an active positive exposure which will be rewarded, should the term structure indeed decline later on. An opposite argument can be used for a decline in interest rates from level i_0 and with a lower trigger limit i_L.

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14 With multifactor models, which are explained in section 3, the forecasts are reduced to factor returns.

15 See the “Portfolio Management” module.

16 In other words, interest rates should fluctuate in a reasonably well defined and stable range. One real danger related to this strategy is for example a shift in the medium term inflation rate. If such a shift occurs, it is quite likely that the fluctuation range of interest rates will shift accordingly.

2.2.2 Return enhancement

Use of a valuation model

Having access to a bond valuation model, the portfolio manager can build a bond portfolio from bonds which are designated as mispriced on the low side by the model. Note that the nature of a bond valuation model is much more technical than for equities. Furthermore, there is much more interchangeability (or fungibility) between bonds in terms of their risk characteristics than there is among equities.

Options overwriting

As in the case for equities, the portfolio manager can try to enhance the returns of a bond portfolios by writing interest rate related calls or puts. This technique is very similar to the one described for equities. For bonds, the forecast is on the timing of long term interest rates.

2.2.3 Yield spread strategies

The bond market can be classified into different segments on the basis of different characteristics: type of issuer (Treasury, corporate…), credit risk (risk-free, AAA …), coupon level (zero-coupon, high coupon, low coupon), maturity[17] (short, medium, long term), etc. In principle, any difference in these characteristics should command a difference in the yield (**yield spread)**.

Yield spread (or just **spread**) strategies are based on the positioning of portfolio components in order to gain from movements in spreads between the various segments of the bond market. The main technique is **bond swapping,** that is exchanging an overvalued bond in the portfolio for another bond that the portfolio manager believes is undervalued by the market. In both cases the undervaluation and the overvaluation is based on a set of forecasts over the future movements of the term structure of interest rates, and are measured in terms of the spread: in the case of the undervaluation, the spread is too wide, and in the case of the overvaluation, the spread is too narrow. Once the yield spread between the two bonds comes to a realignment (the yield of the bond that has been sold increases and the yield on the purchased bond decreases), the manager capitalizes on the difference, by reversing the bond swap.

Yield spreads can emerge from different sources. One of the most important is the **credit spread**: bonds of lower quality trade at a spread with respect to higher quality ones (and ultimately to the relevant benchmark government bond). The spread between low and high quality bonds tends to widen when the economy is going to face a recession and narrows during boom phases (lower quality issuers face more difficulties in servicing their debt when the general level of economic activity is low and thus also their income from operations tends to decrease). Knowing that, the portfolio manager can swap low quality bonds for high quality ones when the level of economic activity is approaching the peak (the so called **flight to quality**) and do the opposite when the recession is approaching its through.

[17] Differences in maturity are considered in the section 2.2.4.

Another important source of spread lies in the **call provision**. The probability that the issuer will exercise the call option is closely related to the level of interest rates (it is decreasing with the level of interest rates) and to their volatility (like any option, the value of the call provision increases with the volatility of the underlying security – in this case the interest rate). Thus when the portfolio manager expects a decrease in the level of interest rates he can swap callable for non-callable bonds, since the spread is likely to increase (since the issuer is more likely to exercise the option as interest rates decrease).

2.2.4 Yield curve strategies

Yield curve strategies use the distribution of the maturities of the bonds of the portfolio to take advantage of the forecasted movements (**shift**, **twist** and **butterfly**)[18] of the yield curve. When the yield curve changes, the effect of the maturity structure of the portfolio can have a significant impact on its total return. The yield curve strategies can be categorized in three groups: **bullet**, **barbell** and **ladder** strategies. Figure 2-1 show these strategies graphically.

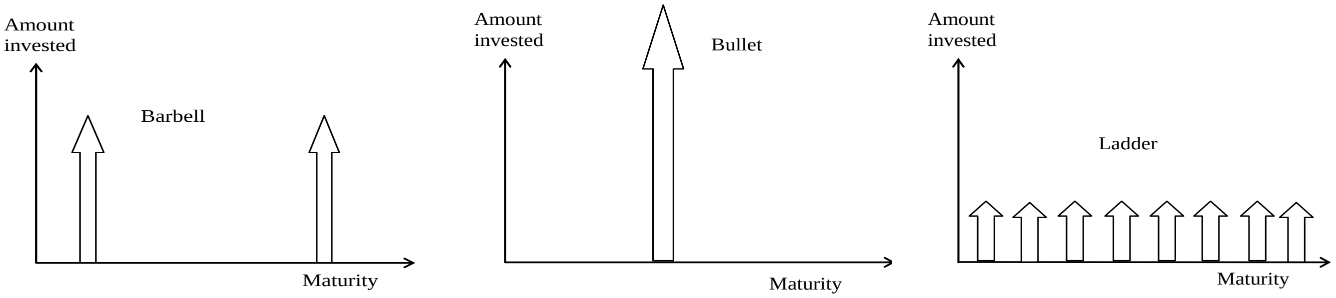

Figure 2-1: Bullet, barbell and ladder strategies

In a bullet strategy the portfolio contains bonds concentrated on a single maturity, while in a barbell strategy the bonds in the portfolio have maturities concentrated at the two extremes of the yield curve. If for example, the portfolio manager anticipates an imminent parallel shift of the yield curve, the investor would prefer a portfolio following a barbell strategy over a similar duration portfolio following a bullet strategy. As the convexity of the barbell portfolio would be greater than the bullet portfolio, and in this scenario it would improve its returns.[19]

Maturity spacing or **laddering** means spacing the maturities in a fixed income portfolio.

> **Example:**
> A client of yours has 2’000’000 USD to invest in a bond portfolio; you advise him to invest equal dollar amounts at regular intervals along the yield curve, for example purchasing ten bonds each with 200’000 USD face value (10% of 2’000’000), maturing annually for 10 consecutive years.
> As time passes and your first bond matures, you invest in another ten-year bond, and you continue this cycle.

[18] A **shift** refers to a parallel shift of the yield curve; a **twist** refers to a change in the slope of the yield curve; and a **butterfly** refers to when the short end and the long end of the yield curve move in the same direction as each other, but at a different rate of change than the middle maturities of the curve.

[19] The price of a bond with higher convexity increases more when the interest rates fall and decreases less when rates go up than a lower convexity bond. See Term Structure module for more information on convexity.

This approach means that you are never concentrated in one maturity, which reduces the re-investment risk, given the relative small amount that has to be reinvested in any period, and minimises the risk of being in the wrong maturity at the wrong stage of the interest rate cycle, but at the cost of maximizing the risk of not being in the right maturity.

3. Portfolio construction based on a factor model*

Bond management has evolved to be as sophisticated, if not more, than equity management. Part of this evolution is linked to the increased volatility of the interest rate term structure, which has been occurring since the seventies. Also, a better understanding of the determinants of default risk, of liquidity premia, and of tax advantages linked to current yield has also increased our understanding of bond valuation. Furthermore, the introduction and the wider use of more sophisticated bonds with call or put options, sinking provisions, and uncertain cash flows[20] has made the life of the bond investor more difficult. The building of multifactor models (MFM) for bonds has been a quantified answer to this increase of sophistication and has provided the necessary tools for better risk control in bond management.

A MFM is a model that relates the return on an asset (in this case, bonds) to the values of a limited number of factors to explain a phenomenon. In a generic form, it can be written for a three-factor model as: [21]

$$R_i = B_{i,1} \cdot F_1 + B_{i,2} \cdot F_2 + B_{i,3} \cdot F_3$$

where:

R_i is the return of the asset (bond) i,

$B_{i,n}$ is the factor exposure,[22]

F_n is the factor n.

For the sake of simplicity, the remainder of this section will focus only on the risk, which is the total volatility of the returns of a bond, caused by the volatility of the term structure.[23]

3.1 Model specification*

*3.1.1 The single factor duration model**

Although it was not intended as such, the duration concept can be considered as a crude but nevertheless powerful single risk factor model. In a previous chapter, it was shown that for instantaneous small changes in the level of interest rate, we can use the modified duration to calculate the asset's returns.[24] But, it can also be used as an approximation[25] for a small time period. So, with a prediction of a parallel shift of the term structure, we obtain the expected return as:

$$R_i^{t,t+1} = -D_{i,t}^{\mathrm{mod}} \cdot \Delta k_i^{t,t+1}$$

where:

[20] For example, the mortgage-backed securities in the US.

[21] In reality, not all the return is explained by these factors so an error term is added.

[22] Is the changein the return of asset i per unit change on factor n.

[23] We will deliberately ignore other risks associated with specific characteristics of the bond, such as its credit quality and the existence of call or put options.

[24] For more information on duration, see the module "Interest rates. Term structure and applications".

[25] Because we ignore the interest accrual and assume that the duration is constant through time.

$R_i^{t,t+1}$ is the return to bond i during time period t to t+1,
$D_{i,t}^{mod}$ is the modified duration of bond i at time t,
$\Delta k_i^{t,t+1}$ is the change of the yield of bond i during time period t to t+1.

This predictive model relates the volatility of a bond return to the volatility of its current yield, the proportional coefficient being the bond's modified duration. This can be interpreted as a simple single factor model where the modified duration of a bond is its factor exposure. We know that this model relies on simplistic assumptions of the term structure and its movements.[26] Nevertheless, it does explain around 75% of the variance of non-callable US Treasury bond returns.

The quality of the forecast can be improved by taking into account convexity, to predict price changes in bonds in response to parallel shifts in the term structure. We have:

$$R_i = -D_{i,t}^{mod} \cdot \Delta k_i^{t,t+1} + C_{i,t} \cdot (\Delta k_i^{t,t+1})^2$$

Where $C_{i,t}$ is the convexity of bond i at time t.

This constitutes a two factor model. The modified duration and the convexity of a bond are its factor exposures. Although the addition of convexity increases the level of explanation for the variance of bond returns to more than 80%, this model still relies on the same simplistic and restrictive assumption of parallel shifts of a flat interest rate term structure.

*3.1.2 The full term structure MFM**

In order to correct the deficiencies of the previous approaches, we need to have a better description of the movements in the term structure of interest rates. To calculate the returns of the bond, we first need its price. As already known, the price of a bond is just the present value of its future cash flows. However, the question is about which interest rate to use in order to discount the cash flows. In the previous approach, we calculated the yield to maturity and used the same interest rate for all cash flows. In contrast, in this paragraph, we deviate from the unrealistic assumption of a flat yield curve, so the discount yields do not need to be identical for different maturities. The discount rates at the respective maturity buckets are calculated with the prices of a series of risk-free discount bonds maturing at those maturities. For example, at time t the price of a risk-free discount bond maturing in j years with a yield of k would be:

$$PDB_{t+j}^{t} = \frac{1}{\left(1+k_{t+j}^{t}\right)^{j}}$$

where:

PDB_{t+j}^{t} is the price at time t of a default-risk-free discount bond maturing at time t+j,
k_{t+j}^{t} is the yield at time t of a default-risk-free discount bond maturing at time t+j.

[26] The underlying assumption is that the term structure is flat and that only small parallel shifts should be considered.

For the rest of the section, let us assume for simplicity that we have only a predetermined set of maturities (3 months, 6 months, 1 year … 10 years), that coincide with the payoffs periods. So we have a series of payoffs for **coupon** bond i at these maturities with present values at time t of $(CF_i^{t+3m}, CF_i^{t+6m}, CF_i^{t+1y}, ..., CF_i^{t+10y})$ and a series of risk-free **discount** bonds maturing also at these maturities with prices at time t $(PDB_{t+3m}^t, PDB_{t+6m}^t, PDB_{t+1y}^t, ..., PDB_{t+10y}^t)$.

Therefore, the price of a coupon bond i at time t is:

$$\mathrm{P}_i^t = \sum_{j=1}^{T} CF_i^{t+j} \cdot PDB_{t+j}^t = CF_i^{t+3m} \cdot PDB_{t+3m}^t + CF_i^{t+6m} \cdot PDB_{t+6m}^t + CF_i^{t+1y} \cdot PDB_{t+1y}^t + ... + CF_i^{t+10y} \cdot PDB_{t+10y}^t$$

where:

P_i^t is the price of a coupon bond i at time t,

CF_i^{t+j} is the cash-flow of bond i at time t+j,

From this equation, after some operations,[27] we obtain that the return of the coupon bond i in the small time period between t and t+1 is given by:

$$\mathrm{R}_i^{t,t+1} = \sum_{j=1}^{T} \mathrm{z}_i^{t+j} \cdot \mathrm{RDB}_{t+j}^{t,t+1} = \mathrm{z}_i^{t+3m} \cdot \mathrm{RDB}_{t+3m}^{t,t+1} + \mathrm{z}_i^{t+6m} \cdot \mathrm{RDB}_{t+6m}^{t,t+1} + \mathrm{z}_i^{t+1y} \cdot \mathrm{RDB}_{t+1y}^{t,t+1} + ... + \mathrm{z}_i^{t+10y} \cdot \mathrm{RDB}_{t+10y}^{t,t+1}$$

where:

$\mathrm{z}_i^{t+j} = \dfrac{\mathrm{CF}_i^{t+j} \cdot \mathrm{PDB}_{t+j}^t}{\mathrm{P}_i^t}$ is the fraction of the value of the bond i related to cash flow t+j,

$\mathrm{RDB}_{t+j}^{t,t+1} = \dfrac{\Delta \mathrm{PDB}_{t+j}^{t,t+1}}{\mathrm{PDB}_{t+j}^t}$ is the return between t and t+1 of the default-risk free discount bond maturing at t+j.

Each z_i^{t+j} represents the percentage weight of the total price of bond i that corresponds to the cash flow that matures in time t+j. So, a zero-coupon bond maturing in one year would have a $\mathrm{z}_i^{t+1y} = 1$ and the rest of the z_i^{t+j} would be equal to 0, therefore, a zero-coupon bond is only affected by movements of the term structure at the bucket corresponding to its maturity.[28] As the sum of present values of the cash flows (the numerators of the different z_i^{t+j}; i.e., $\mathrm{CF}_i^{t+j} \cdot \mathrm{PDB}_{t+j}^t$) is equal to the bond's price, the sum of the z_i^{t+j} must equal 1. The terms $\mathrm{RDB}_{t+j}^{t,t+1}$ represent the movements of each of the risk-free discount bonds. As each of the risk-free discount bonds can have a different return, this approach allows to represent all possible movements of the term structure, not only parallel shifts as with the duration approach. So the return of coupon bond i can be seen as a weighted average of the returns of the default-risk free discount bonds.

This equation can be seen as a multifactor model of the return of a coupon bond, where the terms $\mathrm{RDB}_{t+j}^{t,t+1}$ are the factors and the terms z_i^{t+j} are the factor exposures.

27 For the mathematical derivations see the Appendix of this module.

28 In reality this is not that simple, as maturities of bonds do not match exactly with the maturities of risk-free bonds.

From here, we can analyse the risk of coupon bond i by calculating the variance of its returns using the variances of the risk-free discount bonds and the respective factor exposures of bond i. Since the changes of the yields of the different discount bonds are not independent, apart from their variances, we would need to know the covariances of each pair of them.[29] Both historical estimates and forecast of these covariances can be obtained from time series analysis of the factor returns. With the resulting equation for the variance of the returns of a coupon bond, we have a fully predictive risk model, which can be used along the same lines as it was used for equities.

As will be illustrated later, this approach explains more of the variance of bond returns than the two previous approaches. Its only drawback is that this model is not convenient to use in order to implement strategies for complicated movements in the term structure of interest rates, due to the high number of factors to be estimated.

*3.1.3 The shift, twist and butterfly MFM**

This last approach tries to solve the criticism of the previous one by modelling the movements of the term structure with a smaller number of factors but without losing too much of the risk explanation.

The essence of this approach is to model the returns, and therefore movements, of each of the previous discount bonds as a response of only three possible movements, that act as factors.[30] These movements are the shift, twist and butterfly.

In order to better understand how these three factors do indeed model movements in the term structure, it is useful to see the effects of a movement for each basic factor on discount bond returns, the other two being equal to zero:

- In a shift, every zero-coupon bond will exhibit a return of the same sign. This is what one would expect from a parallel shift in the term structure.
- In a twist, one end of the term structure changes in the opposite direction of the other end, while the middle of the term structure remains unchanged. The slope of the term structure changes.
- A butterfly means that, at both ends of the term structure, discount yields move in the same direction, whereas in the middle of the term structure, they either stay unchanged or move in the opposite direction of the two maturity extremities (long and short end). In other words, the shape of the term structure changes in concavity.

Figure 3-1 show examples of these movements graphically:

[29] For the mathematical calculation of the variance of the returns see the Appendix of this module.

[30] For the mathematical derivation see the Appendix of this module.

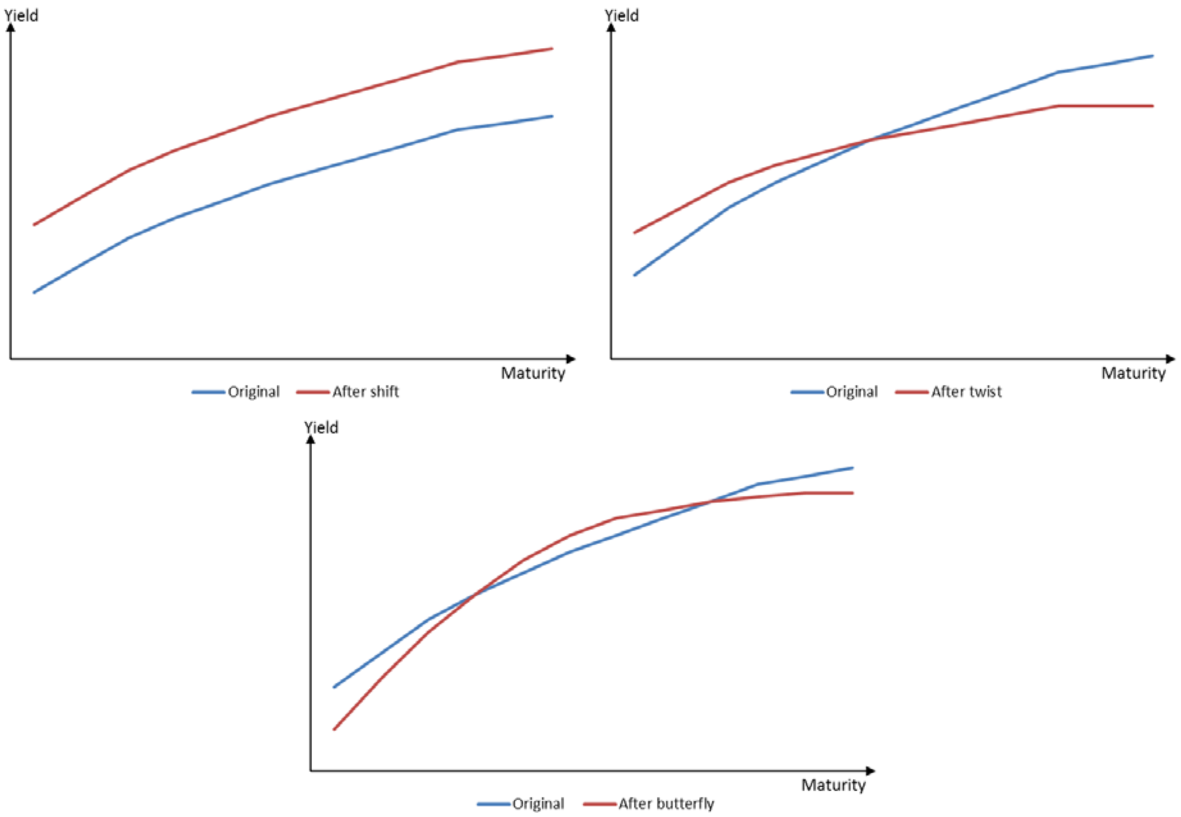

Figure 3-1: Shift, twist and butterfly movements

Therefore, with these three factors we can represent the return of a coupon bond for a short period as:

$$R_i^{t,t+1} = zs_i^t \cdot RS^{t,t+1} + zt_i^t \cdot RT^{t,t+1} + zb_i^t \cdot RB^{t,t+1}$$

where:

$RS^{t,t+1}$	is the return of the shift factor between t and t+1,
$RT^{t,t+1}$	is the return of the twist factor between t and t+1,
$RB^{t,t+1}$	is the return of the butterfly factor between t and t+1.
zs_i^t	is the exposure of bond i to the shift factor,
zt_i^t	is the exposure of bond i to the twist factor,
zb_i^t	is the exposure of bond i to the butterfly factor.

Each of the factor exposures is the sum of the exposures of the discount bonds. We therefore have reduced the multifactor model of the previous section to a three factor model which allows the implementation of strategies based on complicated movements of the interest rate term structure through three basic movements, i.e. a parallel shift, a steepening or flattening, and a move toward more or less convexity of the discount yield curve.

As with the previous model, as soon as a forecast of the covariances is provided we have a predictive bond risk model calculating the variance of the bond i.[31]

[31] For the mathematical calculation of the variance, see the Appendix of this module.

3.1.4 Summary and needed enhancements*

We have shown over the last three sections several risk models for bonds. As already mentioned, we have focused only on explaining bond returns by changes in the discount rate term structure. The following table summarises the level of explanation for variances in bond returns by focusing on the non-callable US Treasury market:[32]

Model	Number of Factors	Percent of Explained Variance[33]
Duration	1	75.8
Duration & Convexity	2	81.1
Shift Factor[34]	1	82.4
Shift & Twist Factor[35]	2	87.0
Full MFM	10	88.0

We can see that the simple duration model can be improved to a large extent. Between it and the full MFM there is 12% of additional explanation of the variance of bond returns. The second point worth mentioning is that the restriction of the full MFM to a three-factor model[36] only very marginally reduces the explanatory power.

Up till now, we have dealt only with factors related to movements of the term structure. There are many other risk related issues that we will mention briefly.

Among the issues not discussed, we must first mention the existence of call, put options, or sinking fund provisions embedded into bonds. One way to take into account these important features is to adjust the cash flows of the option free bond using a valuation model for the embedded options.[37]

Also till now, we have not taken into account other characteristics of the bonds which help explain their yield spread without involving default risk. Among the risk factors that are usually included along these lines, we can mention:

- benchmark issue factor to capture the liquidity premium of heavily traded bonds,
- coupon related factor to assess the market view of the tax advantage between low and high coupon bonds,
- perpetual factor to value the advantages or disadvantages of perpetual bonds,
- etc.

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32 This market is the closest in terms of specification with respect to the models we described as there are no call options and no default risk involved.

33 This table comes from Kahn (1995).

34 Using the first component.

35 Using both the first and second components.

36 The table provides a model with only the shift and twist factors.

37 For more details see Kahn (1995).

Finally, a comprehensive model for bond risk should also include factors to deal with the default risk. Each sector of the bond market may have its own yield spread for an identical level of bond rating, e.g. AAA. Furthermore, each quality rating can have its own spread to assess the changing market views on their respective default risks. All these different spreads can be modelled through factors.

Notice that most of the factors that have been described are related to the technical characteristics of the bonds, but not to the economic situation of the debtor behind the bond. This means that the role of the specific part in the explanation of the return of a bond is relatively small compared to what we experience at the equity level. To some extent this does explain why bonds are more interchangeable and why less emphasis on diversification is expressed in the bond literature: in the broadest possible sense, bond prices tend to all move in the same direction more often than equity prices do, and hence the correlation of returns between bonds is much more likely to be high than those of equities in general.

3.2 Interest rate anticipation strategies*

Active bond management relies necessarily on an economic scenario for the forecast of the movements of the yield curve.[38]

The skill of the portfolio manager is to build a portfolio with risk exposures consistent with his prediction of the term structure movements:

- If he forecasts a downward parallel shift of the term structure, he must then make sure that the managed portfolio is more exposed to the shift factor than the benchmark ($zs_P > zs_B$).
- If he forecasts that the slope of the term structure will become steeper or flatter, then the exposure of the managed portfolio to the twist factor must be higher than that of the benchmark ($zt_P > zt_B$).
- If he forecasts the curvature of the term structure change its level of concavity, then the portfolio manager will ensure that the managed portfolio is more exposed to the butterfly factor than the benchmark ($zb_P > zb_B$).
- If he forecasts that the spread of a specific sector will decrease relative to any other sector, then he must overweight this sector relative to the other sector in question.
- If he forecasts that the spread of lesser quality bonds will decline, then he must overweight these types of bonds, and vice versa
- And so on….

[38] A more bond specific approach is possible for portfolio managers who specialize in forecasts on the default risk side of bonds.

Thus, there is a wide variety of ways to build the bond portfolio along the lines described for equities:

- The portfolio manager just makes sure that basic statistics (i.e. duration, convexity etc.) of the managed portfolio are consistent with his scenario.
- He can use a risk model to ensure that he is taking the desired risk exposures.
- He can use an optimiser to minimise the tracking error of his portfolio, subject to the constraints on risk factor exposures, which are necessary to implement his active policy.
- Having forecasts for the factor returns, he can apply a full-fledged optimisation technique to maximise the forecasted risk adjusted active return of the bond portfolio.

With this increase in sophistication in building a portfolio, we move from a largely judgmental approach to a more structured and quantitative form of bond management.

4. Computing the hedge ratio: the modified duration method*

Consider the situation where a position in an interest rate dependent asset (such as a bond portfolio or a money market security) is being hedged using an interest rate futures contract.

Define

- $F_{0,T}$ quoted price for the interest rate futures contract. To simplify things, we will generally assume that it is expressed in decimal form.[39]
- S_0 value of the asset being hedged (bond portfolio or money market security) expressed in decimal form.
- MD_F modified duration of the asset underlying the futures contract, that is, modified duration of the cheapest to deliver bond
- MD_S modified duration of the asset being hedged

We assume that the change in the yield, Δy, is the same for all maturities (which means that only parallel shifts in the yield curve can occur). From the modified duration definition,

$$\Delta S = -S_0 \cdot MD_S \cdot \Delta y$$

To a reasonable approximation, it is also true that

$$\Delta F = -F_{0,T} \cdot MD_F \cdot \Delta y$$

Since the Δy's are assumed to be the same, we have $\rho_{\Delta S,\Delta F}=1$. Combining the two previous equations, we can write:

$$\Delta S = \frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F} \cdot \Delta F$$

It follows that[40]

$$\frac{\sigma_{\Delta s}}{\sigma_{\Delta F}} = \frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F}$$

[39] In decimal form means per one unit (CHF, USD, etc.) of face value. If the quoted futures price is 90.30, its decimal form expression is 0.9030.

[40] Using the fact that

$$\Delta S = \frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F} \cdot \Delta F$$

we have

$$\sigma^2_{\Delta S} = \left(\frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F}\right)^2 \cdot \sigma^2_{\Delta F} \Rightarrow \sigma_{\Delta S} = \left(\frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F}\right) \cdot \sigma_{\Delta F}$$

and the optimal hedge ratio (HR) to use for hedging is

$$HR = \rho_{\Delta S,\Delta F} \cdot \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = \frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F}$$

Using it results in equating the duration of the combined position equal to zero.

However, the hedge obtained is by no means perfect:

- it assumes that Δy is the same for all yields whereas in practice short-term yields are usually more volatile and have low correlation with long-term yields; as a result, the performance of the hedge can be disappointing, particularly if there is a big difference between MD_S and MD_F.
- it omits convexity: if the convexity of the asset underlying the futures contract is markedly different from the convexity of the asset being hedged and there is a large change in interest rates, the hedge performance may be worse than expected.
- in order to calculate MD_F, an assumption on what will be the cheapest-to-deliver bond is necessary. If the cheapest-to-deliver bond changes, MD_F changes and the optimal number of contracts also changes.

Knowing the hedge ratio, what is the number of futures contracts to be purchased? As we have seen already, the number of futures contracts required is given by

$$N_F = -HR \cdot \frac{N_S}{k} = -\frac{S_0 \cdot MD_S}{F_{0,T} \cdot MD_F} \cdot \frac{N_S}{k} = -\frac{N_S \cdot S_0 \cdot MD_S}{k \cdot F_{0,T} \cdot MD_F}$$

where NS is the number of units of the spot asset to be hedged, and k is the contract size. In fact, we have just found another way of estimating the hedge ratio using regression. This can be interpreted as:

$$\begin{array}{c}\text{Number of}\\ \text{futures}\\ \text{contracts}\end{array} = -\frac{\begin{pmatrix}\text{Market value}\\ \text{of portfolio to hedge}\end{pmatrix}}{\begin{pmatrix}\text{Market value of}\\ \text{one futures contract}\end{pmatrix}} \cdot \frac{\begin{pmatrix}\text{Modified duration}\\ \text{of portfolio}\end{pmatrix}}{\begin{pmatrix}\text{Modified duration}\\ \text{of CTD}\end{pmatrix}}$$

As we know[41] that $F_{0,T} \cdot CF_{CTD,0} = S_{CTD,0}$, where $CF_{CTD,0}$ is the conversion factor of the cheapest to deliver bond at time 0 and $S_{CTD,0}$ its price, we can replace $F_{0,T}$ in the above formula. Thus we get:

$$N_F = -\frac{N_S \cdot S_0 \cdot MD_S}{k \cdot F_{0,T} \cdot MD_F} = -\frac{N_S \cdot S_0 \cdot MD_S}{k \cdot S_{CTD,0} \cdot MD_F} \cdot CF_{CTD,0}$$

which can be interpreted as:

$$\begin{matrix}\text{Number of}\\ \text{futures}\\ \text{contracts}\end{matrix} = -\frac{\begin{pmatrix}\text{Market value}\\ \text{of portfolio to hedge}\end{pmatrix}}{\begin{pmatrix}\text{Futures}\\ \text{size}\end{pmatrix} \cdot \begin{pmatrix}\text{CTD spot}\\ \text{price}\end{pmatrix}} \cdot \frac{\begin{pmatrix}\text{Modified duration}\\ \text{of portfolio}\end{pmatrix}}{\begin{pmatrix}\text{Modified duration}\\ \text{of CTD}\end{pmatrix}} \cdot \begin{pmatrix}\text{Conversion}\\ \text{factor}\\ \text{of CTD}\end{pmatrix}$$

4.1 Examples of hedging strategies using longer bond futures*

By holding a long position in interest rate futures, an investor can "freeze" the higher interest rate today for a future fixed-interest investment. This is called a **long-hedge**. There are numerous different bond futures trading in different exchanges: in this instance we will use the example of the Eurex CONF futures contracts.

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41 Recall that the invoice price (price for the bonds delivered on an interest rate futures contract) is calculated as the decimal futures settlement price (i.e. the price per one currency unit of face value) times the conversion factor of the delivered bond times the contract size plus the accrued interest on the delivered bond. If we short deliver the cheapest to deliver bond,

$$\text{Invoice price} = F_{t,T}^{(\text{decimal})} \cdot k \cdot CF_{CTD,t} + AI_{CTD,t}$$

In theory, this invoice price should be equal to the purchase price of the delivered bond. Assuming again that the shorts deliver the cheapest to deliver bond, we have

$$\text{Purchase bond price} = S_{CTD,t}^{(\text{decimal})} \cdot k + AI_{CTD,t}$$

The equality of the purchase price and the invoice price gives:

$$F_{t,T}^{(\text{decimal})} \cdot CF_{CTD,t} = S_{CTD,t}^{(\text{decimal})}$$

which is also valid in a non decimal form.

Example:[42]
A portfolio manager expects redemption of a fixed-interest investment of 10 million CHF in mid-December.He would like to invest the money on the Swiss bond market with a modified duration MD_S=6.00 years. Since he expects falling interest rates, he would like to hedge the present lower cost price for the planned investment in bonds as bond prices will rise when interest rates will fall.

By purchasing the December CONF futures and later selling the futures position at a higher price, a profit is realised on the futures position that compensates the loss from the price increase for the planned bond purchase.

Let us assume that the futures price is $F_{0,T}$=112.90 and that the cheapest to deliver bond has a modified duration of MD_F=6.54 years and a conversion factor of 1.031156. The contract size is 100'000 CHF.

The number of contracts to hold (computed by the duration method) is:

$$\begin{matrix}\text{Number of}\\ \text{futures}\\ \text{contracts}\end{matrix} = -\frac{\begin{pmatrix}\text{Market value}\\ \text{of portfolio to hedge}\end{pmatrix}}{\begin{pmatrix}\text{Market value of}\\ \text{one futures contract}\end{pmatrix}} \cdot \frac{\begin{pmatrix}\text{Modified duration}\\ \text{of portfolio}\end{pmatrix}}{\begin{pmatrix}\text{Modified duration}\\ \text{of CTD}\end{pmatrix}}$$

$$= -\frac{-10'000'000}{112'900} \cdot \frac{6.00}{6.54} = 81.26 \text{ contracts}$$

Thus the manager decides to purchase 81 December CONF futures at 112.90.
In December, the yield in the long term sector has fallen by 0.30 percentage points, the futures is quoted at 115.13, and the price increase of the planned portfolio amounts to 182’000 CHF. The manager closes his futures position.

The result of the strategy is

- a “loss” (higher bond purchase price) of 182'000 CHF, as the value of the portfolio in December is 10'182'000 CHF.
- a profit on the long CONF futures position of 180'630 CHF (that is, 2.23·81·1'000, where 2.23 is the difference between the purchase price of 112.90 and the selling price of 115.13).

This gives a total loss of 1'370 CHF. Without any hedge, the extra-cost for the portfolio would have been 182'000 CHF.

An investor can “freeze” the lower interest rate today for a future selling of a fixed-interest portfolio by holding a short position in interest futures. This is called a **short-hedge**.

Example:[43]
In October, an institutional investor manages a bond portfolio with a market value of 40'000'000 CHF and a modified duration of 8.20 years. The investor expects rising interest rates in the near future and he would like to hedge his portfolio against expected price losses. The December futures is quoted at 112.90, the cheapest to deliver bond has a modified duration of 6.54 years and a conversion factor of 1.031156.

By selling the December CONF futures at 112.90, and later, buying the futures position back at a lower price, a profit is realised on the futures position that compensates the price decline of the bond portfolio.

[42] Adapted from [SOFFEX].

[43] See in the Eurex website at www.eurexchange.com the publication: “Interest Rate Derivatives - Fixed Income Trading Strategies”.

The number of contracts to hold (computed by the duration method) is:

$$\begin{matrix}\text{Number of}\\ \text{futures}\\ \text{contracts}\end{matrix} = -\frac{\begin{pmatrix}\text{Market value}\\ \text{of portfolio to hedge}\end{pmatrix}}{\begin{pmatrix}\text{Market value of}\\ \text{one futures contract}\end{pmatrix}} \cdot \frac{\begin{pmatrix}\text{Modified duration}\\ \text{of portfolio}\end{pmatrix}}{\begin{pmatrix}\text{Modified duration}\\ \text{of CTD}\end{pmatrix}}$$

$$= -\frac{40'000'000}{112'900} \cdot \frac{8.20}{6.54} = -444.22 \text{ contracts}$$

Thus the investor decides to sell 444 December CONF futures at 112.90.

In December, the yields in the long term sector have risen by 0.30 percentage points, the futures is quoted at 110.74, and the loss on the portfolio is 970’000 CHF. The investor closes his futures position by buying back December CONF futures at 110.74.

The result of the strategy is

- a “loss” (lower bond price) of 970'000, as the value of the portfolio in December is now 39'030'000 CHF.
- a profit on the short CONF futures position of 959'040 CHF (that is, 2.16·444·1'000, where 2.16 is the difference between the selling price of 112.90 and the buy-back price of 110.74).

which gives a total loss of 10'960 CHF. Without any hedge, the loss of the portfolio would have been 970'000 CHF.

Thus by freezing the actual price level, an investor has the option to hedge existing or planned commitments on the interest rate market against upward or downward price movements.

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