FIXED INCOME

HYBRID FORMS

FIXED INCOME

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1. Bonds with warrants

Warrants are securities issued by a company that give the holder the right to acquire the company’s common stock at a fixed price, called the exercise price, within a specified period. Warrants are often issued in conjunction with a bond issue to make the issuance more attractive to investors. Moreover, the warrants related to a bond issue can be detached and traded separately. Offering warrants is also popular at corporate restructuring processes and corporate mergers. Although warrants usually give the right to purchase a company’s stock, in the past they have been used to give the buyer the right to purchase a bond of the issuer. From here on, we consider exclusively warrants on equity rather than debt.

1.1 Investment characteristics

Warrants are similar to call options, however there are some key differences. A company itself issues warrants on its own common stock as the underlying security. When warrants are exercised the company’s balance sheet is affected due to the issuing of common shares and the receipt of cash. Options, on the other hand, are created and traded by individuals and institutions that need to have no connection with the company; and the exercising of options does not affect the company's balance sheet. In addition, warrants often have long maturity periods (during which they can be exercised) while options usually have to be exercised within a few months.

Some key terms associated with warrants are discussed below:

1.1.1 Conversion Ratio

The number of warrants needed to purchase one share of common stock is called the conversion ratio of the warrant. The conversion ratio need not be one warrant per share.

> **Example:**
> Call warrants issued by Nestlé[1] in September 2014 had a conversion ratio of 15:1. Thus, 15 of these warrants were needed to purchase one registered share of Nestlé.

1.1.2 Strike Price or Exercise Price

The price at which the warrant holder can purchase one share of common stock is termed the strike price or the exercise price of the warrant. The strike price is protected against the dilution of common stock due to stock dividends or stock splits, just as in the case of convertibles.

[1] Original document: http://www.cu3.ch/zkb/mail//files/derivate/de_CH0245304925_20140922_15-52-03-232.PDF

1.1.3 Maturity Date

The maturity date is the date on which the warrant expires. In case of “American” style warrants the holder may exercise the warrant before maturity. There are some options trading in the U.S. market which do not have any maturity dates. Such instruments are referred to as perpetual warrants.

1.1.4 Option Type

The warrant prospectus will also specify the type of options inherent in the warrant. The “American” style option will allow the holder to exercise the warrant any time during the life of the warrant. The “European” style option can only be exercised at the maturity date of the warrant.

1.1.5 Dilution Effect

The exercise of a regular call option on a company’s share has no impact on the number of outstanding shares of the company. Warrants are different from regular call options, the exercising of a warrant leads to the company issuing more shares and then selling them to the holder of the warrant for the strike price. If the strike price is lower than the market price, it has a negative effect on the share of the existing stockholders, this effect is called dilution.

1.1.6 Traditional and Covered Warrants

The most common type of warrant is the traditional warrant (also referred to as a company warrant). Traditional warrants are options issued by a corporation on its own shares. Covered (or synthetic) warrants are an innovative form of warrants on publicly traded shares. As the underlying shares are publicly quoted, the financial institution can sell warrants on these shares to investors and then purchase the underlying shares directly from the stock exchange. In this case there is no dilution effect; the exercise of warrants has no impact on the balance sheet of the company issuing the underlying shares.

1.2 Valuation of warrants

The valuation concept of warrants is similar to the valuation of call options, but it is somewhat more complicated due to the dilution effect. Warrants can be priced approximately by using the Black-Scholes Option Pricing Model if the new shares created by the exercise of warrants will not have a significant effect on the company’s balance sheet. However, if the exercise of warrants is likely to impact the company’s balance sheet significantly, special pricing models have to be used to value warrants.

Warrant pricing models have the following form:

$$W = f\,(X, T, S, D, r, s)$$

where:

W	Value of the warrant
X	Exercise price of the warrant
T	Time to maturity of the warrant
S	Current market price of the underlying common stock
D	Potential dilution of common stock on exercise of warrants
r	'risk-free' interest rate
s	implied volatility

The above model will apply in the case of stocks not paying dividends. The model will get more complicated for dividend paying stocks.

Using the assumptions of the Black-Scholes Model, the value of the warrant can be calculated using the value of a regular call option on the company's stock:[2]

$$W = C \times \frac{N}{N+M}$$

where:

W	Value of the warrant
C	Value of the regular call option according to the Black-Scholes Model
N	The number of the company's shares before the new issuance
M	The number of the company's new shares

Example:
Suppose that Syngenta AG has 76 million shares worth CHF 295 each is considering issuing 1 million warrants each giving the holder the right to buy one share with a strike price of CHF 320 in 3 years. The interest rate is 1% per annum, and the volatility is 30% per annum. Suppose that the company pays no dividends. The value according to the Black-Scholes Model of a regular 3-year European call option on the stock with the same parameters is 54.59. How much does the warrant worth?

In this case the value of N is 76,000,000 and the value of M is 1,000,000, the calculation is the following:

$$W = 54.59 \times \frac{76{,}000{,}000}{76{,}000{,}000+1{,}000{,}000} = 53.88$$

The value of each warrant is CHF 53.88.

2 For a mathematical derivation, see: HULL, J. C., 2012, "Options, Futures and Other Derivatives", Prentice Hall

1.3 Empirical Studies and Market

Academic studies investigated the relationship of the covered warrants and options market and found that covered warrants are more expensive than options with the same investment characteristics. This overpricing can be explained by the liquidity differences between covered warrants and options.[3] Counterparty risk which is virtually eliminated through the clearing house in the case of standard options can be another reason for the price difference. Note that in contrast to standard call options covered warrants have a counterparty risk towards the issuing company.

Warrants came into use during the bull market of the 1920s, but it became important financial products in the 1960s. Today, the global warrants market is broadly diversified in terms of underlying security: equities, commodities, currencies, indices can also be underlying securities of warrants.

1.4 Exotic Types of Warrants

There are exotic types and features of warrants due to the investors demand, some of these exotic types are briefly described below:

Callable Warrants: Some warrants may be issued with a call feature, which gives the issuing company the right to call the warrants back at a specified price. Such warrants will involve a call risk to the investors.

Basket Warrants: The payoff of a basket warrant is related to the value of the underlying assets in a specified (hypothetical) portfolio. Basket warrants are always covered warrants and cash settled. They can be used to hedge the risk of specific sectors of the stock market.

Index Warrants: Index warrants are specific types of basket warrants, where the underlying portfolio is a particular index, for example the DAX index. The exercise level (rather than exercise price) is expressed in index points, and the exercise price = exercise level · price of the index point.

[3] LI, G. – ZHANG, C., 2011, “Why Are Derivative Warrants More Expensive Than Options? An Empirical Study ”, Journal of Financial and Quantitative Analysis

2. Convertible bonds

A typical convertible bond is a debt instrument that provides investors the option of converting the bond into the issuing company's shares. In general, this option is attractive to investors as they benefit from rising share price at reduced downside risk compared to a direct equity investment. If the company's stock price falls to low levels, investors may simply wait until the convertible's maturity when the principal amount is repaid.

Convertible bonds are hybrid instruments as their risk profile may vary from bond- to equity-like. If the underlying company's share price is sufficiently low, a convertible bond behaves like a straight corporate bond as future conversion at low levels is unlikely. In contrast, if the underlying share price is high, convertible bonds move together with the company's stock and show significantly higher volatility.

Since an investor has the choice of holding a convertible bond to keep receiving coupon payments from the company or exchanging the bond for a specific number of common shares of the company, the convertible bond can also be thought of a portfolio of two securities:

1) a straight bond paying regular coupon and the principal amount at redemption

2) a number of call options (equal to the conversion ratio) on the company’s common stock.

Since the investor has the conversion privilege, he is willing to accept a lower coupon rate for a convertible bond compared to an otherwise identical non-convertible bond (i.e., a non-convertible bond of the same company with same characteristics such as maturity and seniority). From the company’s perspective it may appear that a convertible bond has a lower cost of capital compared to a non-convertible bond with similar attributes. This comes however at the price of giving away company shares in the case of conversion.

First convertible bonds were issued by US railroad companies in the 19th century. While the market share of US companies on the global convertible bond market is still highest with over 50%, Europe increased to 30% and Asia to 20% over time (with Asia ex-Japan growing strongly since 2000). The global convertible bond market is also broadly diversified in terms of sectors: from blue-chip IT companies to more exotic issues e.g. the energy sector; a large variety of convertibles can be found on the market.

The variety of convertible bonds is also reflected in their specifications defined in the convertible bond prospectus. The prospectus which is often a document of by far over 100 pages, defines all characteristics of a convertible bond. These include among others maturity, coupon, conversion features, potential dilution protection and the rank in the company's capital structure.

There are also significant differences between warrants and convertible bonds. The exercise of warrants incurs cash outflow for investors, while convertible bonds are only exchanged for common stock without any cash outlay. Conversion of convertibles only brings about structural changes in the balance sheet of issuing companies. Exercising of warrants, however, involves cash inflows and increase in the number of common shares outstanding.

2.1 Investment characteristics

Due to the unique nature of convertible bonds, investors must be conversant with the special terminology associated with a convertible issue. This section will define the terms relevant for the understanding of convertible bonds. Apart from the conversion features, convertible bonds have the same specifications as corporate bonds.

Maturity Date: Date on which the face value of the bond will be paid back.

Face Value: The principal amount of the bond which will be paid back at the Maturity Date.

Coupon Payment: The annual interest payment of the bond expressed as a percentage.

Other typical corporate bond characteristics that are defined in the convertible bond's prospectus include coupons frequency and seniority in the capital structure. According to the conversion characteristics of convertible bonds the relationship between the bond-part and the equity-part has to be specified. This is done by the conversion ratio.

2.1.1 Conversion Ratio

Conversion Ratio defines the number of company shares which will be received by the investor at the time of conversion. This ratio is determined at the time of the issue of the convertible security and will usually remain constant over the life of the security. The investor is often protected against losses caused by stock splits or large stock dividends. If the original conversion ratio is 10 to 1 and the issuing firm announces a 2 for 1 stock split, the conversion ratio will automatically become 20 to 1.

2.1.2 Bond Floor (Investment Value)

This is the present value of the cash flows of the bond while neglecting any possible conversion. It excludes any income coming from the equity-part of the convertible bond. In other words the bond floor of a convertible bond is the price of a non-convertible bond with characteristics identical to those of the convertible bond under consideration.

> **Example:**
> Assume that a company has two bond issues outstanding in the market: a convertible bond issue and a non-convertible bond issue. Also assume that both issues have the same coupon rate, term to maturity and credit rating. The convertible bond is selling at 109 while the non-convertible bond is valued at 87.[4] What is the value of the bond floor of the convertible bond?
>
> The bond floor of the convertible bond is 87, which is the value of the non-convertible bond with the investment characteristics identical to those of the convertible bond under consideration.

As we can see in this example, the investors are willing to pay a premium of 22 for the privilege of being able to convert the bond into common shares; this value is called investment premium (see the next definition).

[4] Most convertible bonds are quoted as a percentage of the face value, 109 means 109% of the face value, for example if the face value is CHF 1,000, the value of the CB is 109% × CHF 1,000 = CHF 1,090. (Exception to this rule are typically French convertible bonds traded in units.)

2.1.3 Investment Premium

The investment premium shows how much an investor is willing to pay for the option embedded in the convertible bond compared to a non-convertible bond with the same investment characteristics. It can be calculated as the following:

$$\text{Investment Premium} = \text{Price of the Convertible Bond} - \text{Bond Floor}$$

It also can be calculated as a percentage amount compared to the bond floor too:

$$\text{Investment Premium (\%)} = \frac{\text{Price of the Convertible Bond} - \text{Bond Floor}}{\text{Bond Floor}}$$

Example:
Assume that the bond floor of a convertible bond is 100 and the convertible bond is valued at 120 on the market. What is the value of the investment premium and the investment premium expressed as a percentage?

The value of the investment premium is 120-100 = 20. The value of the investment premium as a percentage is 20/100 = 20%.

2.1.4 Parity (Conversion value)

Parity is defined as the amount that an investor can receive by immediately exchanging the convertible bond for common shares and selling the shares at the prevailing market price of the common stock.

Thus, parity is calculated as:

$$\text{Parity} = \text{Conversion Ratio} \times \text{Market Price of the Underlying Share}$$

Example:
Swiss Life Holding AG issued CHF 500 million senior unsecured convertible bonds due 2020 on 25/11/2013 with face value of CHF 5,000 and with a conversion ratio of 20.4943:1.[5] If Swiss Life common stock is currently selling for CHF 218.30, what is the parity of the Swiss Life's convertible bond?

Using the formula for parity, the answer is:

$$\text{Parity} = 20.4943 \times \text{CHF } 218.30 \cong \text{CHF } 4{,}473.91$$

2.1.5 Conversion Premium

The conversion premium shows how much a convertible bond investor is willing to pay to own the convertible bond as opposed to the underlying shares. It can be calculated as the following:

$$\text{Conversion Premium} = \text{Market Price of the Convertible Bond} - \text{Parity}$$

5 Source: http://www.swisslife.com/en/home/media/mediareleases/news_feed/2013/20131113_1600.html

It also can be calculated as a percentage amount compared to the parity:

$$\text{Conversion Premium (\%)} = \frac{\text{Market Price of the Convertible Bond} - \text{Parity}}{\text{Parity}}$$

Example:
What is the value of the conversion premium of the Swiss Life's convertible bond described in the previous example if the market price of the convertible bond is 116?

The convertible bond is quoted as a percentage of the face value, so the price of the CB in CHF is 116% × CHF 5,000 = CHF 5,800. The value of the conversion premium is CHF 5,800 – CHF 4,473.91 = CHF 1,326.09.

2.1.6 Conversion Price

Conversion price represents the price that an investor notionally pays when the convertible security is exchanged for shares of common stock. In some cases, the prospectus of a convertible bond will specify the conversion price instead of the conversion ratio. In any case, the conversion price and the conversion ratio are related to each other as shown in the following formula:

$$\text{Conversion Price} = \frac{\text{Face Value of the Convertible Bond}}{\text{Conversion Ratio}}$$

It says what price the common stock must reach before it is worthwhile to convert.

Example:
What is the conversion price of the Swiss Life's convertible bond described in the previous example?

The conversion price is calculated using the above equation, as shown below:

$$\text{Conversion Price} = \text{CHF } 5{,}000 / 20.4943 = \text{CHF } 243.97$$

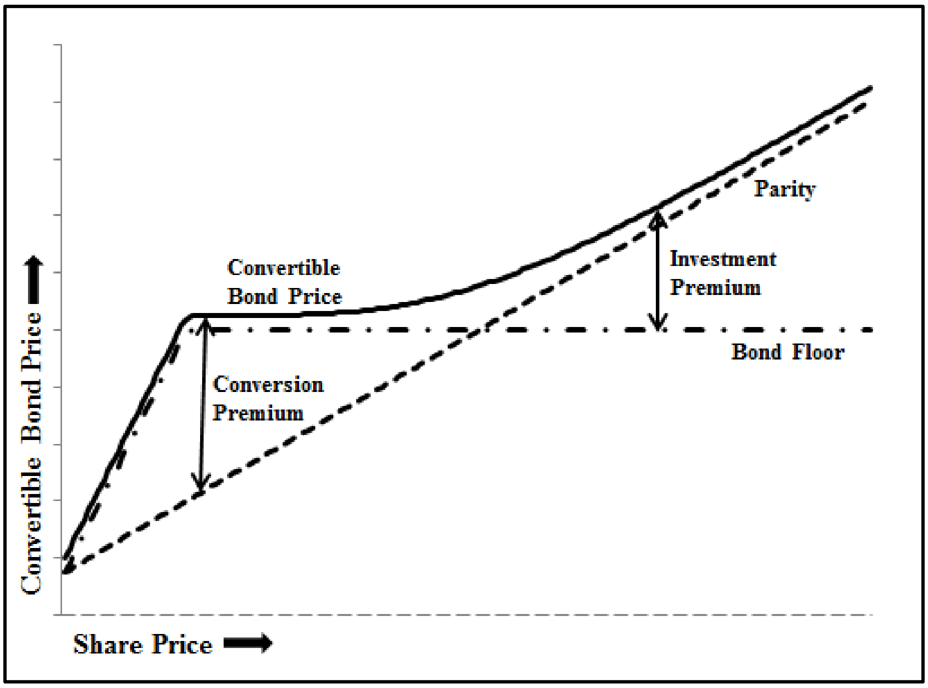

Figure 2-1: Convertible Bond Price, Parity and Bond Floor

Figure 2-1 shows that the price of the CB can be decomposed into the bond floor plus the investment premium or the parity plus the conversion premium.

2.1.7 Distressed, OTM, ATM and ITM Convertible Bonds

Figure 2-1 has shown the value decomposition of the convertible bonds into the equity-part and bond-part using the bond floor and the parity as key concepts. Figure 2-2 shows the second dimension of convertible bonds, the behaviour of convertible bonds according to the value of the underlying share price.

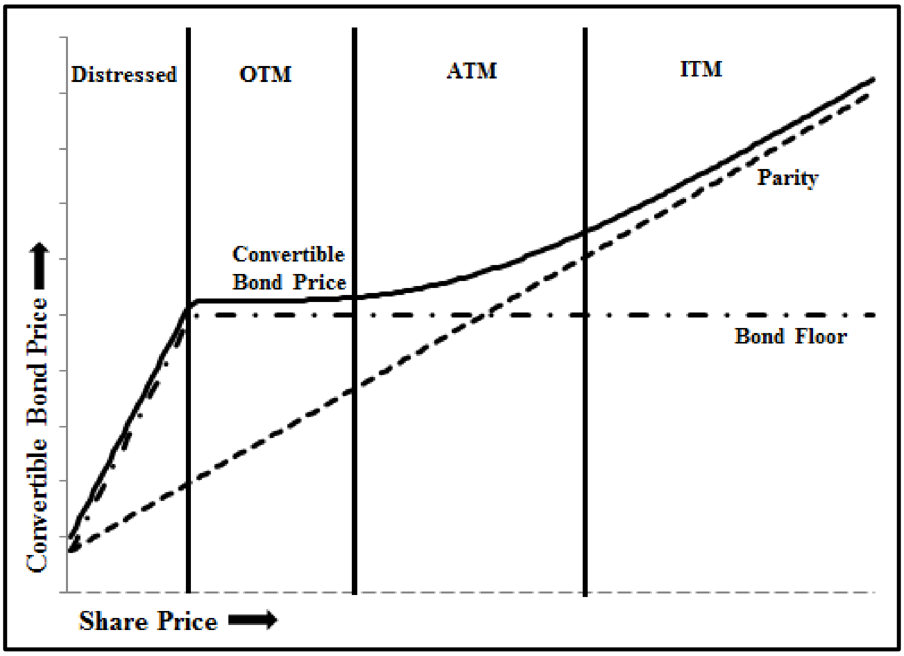

Figure 2-2: Four Regions of a Convertible Bond's Behaviour

Based on Figure 2-2, the behaviour of convertible bonds can be split into the following four regions:

1.) Equity-like (In The Money (ITM)): The share price trades above the conversion price, the probability of conversion is high and the conversion premium is low. The price of the convertible bond is near to the parity and it is more sensitive to the changes in the price of the underlying equity than interest rate and credit moves.

2.) Balanced (At The Money (ATM)): The share price trades close to the conversion price and the convertible bond is sensitive to the changes in the price of the equity as well as in interest rate and credit moves.

3.) Bond-like (Out of The Money (OTM)): The share price trades below the conversion price, the conversion is unlikely and the investment premium is low. The price of the convertible bond is near to the bond floor and the sensitivity to the changes in the price of the underlying equity is negligible.

4.) Distressed: In this case the share price is low and the default risk of the convertible bond is high. The convertible bond's bond floor falls together with the company's stock and does not provide protection to the CB's value any longer: The value of the convertible bond converges to parity. In this region the sensitivity of the underlying equity is higher than in the OTM region and the convertible bond is equity-like.

2.2 Convertible Bond Features

The above section has described the typical characteristics of convertible bonds. However, the market is more complex and a variety of other features may be specified for convertible bonds. Understanding them is important to investors to adequately assess their risk.

2.2.1 Callability

During call periods (when callability is active) the issuer can enforce the early redemption amount at a predefined call price to the investor. If parity is above call price, investors will exercise the conversion option (or sell the convertible bond) and the convertible bond's price approaches parity. The call price is usually expressed as a percentage of the face value. Callability can be active in discrete set of dates (discrete type) or during a specific time interval (continuous type). In most cases callability is a continuous type feature and is deferred, i.e., the bonds become callable after a certain period of time following the issue date. Thus, a callable convertible bond issued in 2015 may become callable after 2020.

> **Example**
> Steinhoff International Holdings Ltd., the South African household goods manufacturer and distributor announced to call its 390m EUR outstanding convertible bond on March 3, 2015 effective on April 6, 2015. As the convertible bond's price was above 180% in March 2015 and thus, clearly above the call price of 105.8%, investors converted (or sold) the convertible bond at parity before April 6.

Technically, if the issuer calls the bond back, the investor receives a call notice from the issuer about the call action and during the so-called call notice period (typically a couple of months) the investor can decide whether to convert the convertible bond into shares, this is called forced conversion.

2.2.2 Soft Callability

In the periods when soft callability applies the issuer is restricted from calling the bond unless the underlying equity trades above a certain threshold. The soft callability threshold is usually expressed as a percentage of the conversion price. Soft callability is related to the callability feature; therefore it is usually connected to the time interval of the callability feature. If there is no soft callability threshold given for a convertible bond and the convertible bond can be called by the issuer unconditionally, it is called hard callability.

2.2.3 Putability

In the put periods (when putability is active) the bond holder can sell the convertible bond back to the issuer at a predetermined amount called the put price. The put price is usually expressed as a percentage of the face value. Putability is normally active at a discrete set of dates, but it can be continuous feature as well. Putability is advantageous to investors as it gives them the right for a redemption before maturity. It can be observed that deep out of the money convertibles approach their put price as put date comes closer: since conversion is unlikely at maturity, investors prefer to exercise their put prematurely, that is to say, re-sell their convertible bond to the issuer. This mechanism provides downside protection to convertibles.

2.2.4 Contingent Conversion

In the contingent conversion periods (when contingent conversion is active) there is a restriction on the conversion, the investor cannot convert the bond into shares unless the price of the underlying equity is higher than a contingent conversion threshold. The contingent conversion threshold is expressed as a percentage of the conversion price. This is a restriction on the rights of the investor and lowers the price of the convertible bond.

2.2.5 Makewhole Clause

Under the makewhole clause investors get an extra payout when the convertible bond is called by the issuer. This payout can happen as an extra cash payment to the investor upon call or as an increased conversion ratio upon call, hence in the case of a forced conversion the investor will get an extra amount of shares. The makewhole clause makes the convertible bond more attractive to investors and compensates them for the missed coupons.

2.3 Valuation of convertible bonds

Valuation of convertible bonds is complex due to the wide range of exotic features shown previously. The Black-Scholes option pricing model could be used to approximate the value of the conversion option; but this would imply a number of unrealistic assumptions regarding call policy (no call), put policy (no put) and stock's volatility (constant). After the valuation of the conversion option we can value the convertible bond as the value of the non-convertible bond plus the conversion option. A typical rule of thumb is that ITM CBs can be valued as the value of the parity and OTM CBs can be valued as the value of the bond floor.

Other option pricing models, such as the binomial model are more realistic and could include exotic features. This section introduces a simple example of the binomial pricing model.

The binomial model includes following steps:

1) The first step is to build a stock price tree according to the Cox-Ross-Rubinstein model until the maturity of the convertible bond. In this model the current stock price (S) moves up, by a proportion u, or down, by a proportion d in each period. The risk-neutral probability (p) is calculated in the model using the risk-free interest rate (r) and the time step (Δt) as the following:

$$p = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

2) The second step is to compute the value (V) of the convertible bond at maturity using the following formula:

$$V = \max(C_r \cdot S + c, N + c)$$

where C_r is the conversion ratio, N is the face value and c is the coupon of the convertible bond.

3) The third step is to move backwards in the tree by one time step and calculate the continuation value (H) of the nodes. The continuation value (or holding value) is calculated as the following:

$$H = e^{-(r+CS) \cdot \Delta t} \cdot (p \cdot V_u + (1-p) \cdot V_d) + c$$

where CS is the credit spread added to the risk-free rate reflecting the default risk of the convertible bond, V_u is the up-value and V_d is the down-value of the convertible bond in the consecutive level in the tree.

4) The value at each node before the maturity including callability is the following:

$$V = \max(C_r \cdot S + c, \min(H, K + c))$$

where K is the early redemption amount and we assume that the investor gets the coupons paid at time t in case of a call event.

This form of the binomial model assumes no dividend yield of the underlying equity and represents the default risk with a credit spread above the risk-free interest rate. The advantage of this model compared to the Black-Scholes option pricing model is that it can include American-style features such as callability in the pricing model. For simplicity, this model does not include other exotic features, but it can easily be modified to include other path-dependent features. The following example shows a simple two-period calculation according to this model:

Example:
Consider a convertible bond with the following parameters:

Maturity	2 years	Volatility	20%
Face value	100	Stock price	100
Conversion ratio	0.8	Interest rate	3%
Coupon	3%	Credit spread	100 bps
Callability	Only at year 1	CRR (up movement)	1.2214
Call price	100	CRR (down movement)	0.8187

Firstly, the stock price tree is constructed using the up and down movement given in the example according to the Cox-Ross-Rubinstein model assuming that Δt=1 year, see on the left side of Figure 2-3. For example the highest price at year 2 is calculated as 100*1.2214*1.2214=149.18. The risk-neutral probability, p is equal to the following:

$$p = \frac{e^{0.03 \cdot 1} - 0.8187}{1.2214 - 0.8187} = 0.5258$$

Stock Prices			**Convertible Bond Prices**		
0	**1**	**2**	**0**	**1**	**2**
		149.18			**122.34**
	122.14			**103.00**	
100.00		**100.00**	**101.49**		**103.00**
	81.87			**101.96**	
		67.03			**103.00**

Figure 2-3: Stock Prices and Convertible Bond Prices

Secondly, we calculate the values at the last level of the convertible bond tree (at time 2) according to the formula shown previously at step 2. For example the value at the highest node is equal to 122.34 = max(149.18*0.8+100*3%, 100+100*3%).

The third step is to calculate the previous levels of the tree step by step. First, we calculate the continuation values and then the value of the convertible bond at time 1.
The continuation values of the up-node and down-node respectively at time 1 are the followings:

$$H(up) = e^{-(3\%+1\%)\cdot 1} \cdot (0.5258 \cdot 122.34 + (1 - 0.5258) \cdot 103.00) + 3 = 111.73$$

$$H(down) = e^{-(3\%+1\%)\cdot 1} \cdot (0.5258 \cdot 103.00 + (1 - 0.5258) \cdot 103.00) + 3 = 101.96$$

The values of the convertible bond at the up-node and down-node at time 1 including callability with K=100 (Call price) are the followings (see also at level 1 of the convertible bond tree):

$$V(up) = \max(0.8 \cdot 122.14 + 3, \min(111.73, 100 + 3)) = 103.00$$

$$V(down) = \max(0.8 \cdot 81.87 + 3, \min(101.96, 100 + 3)) = 101.96$$

Finally, we calculate the continuation value and the value of the convertible bond at time 0, without callability, because the bond is only callable at time 1 according to the example.

$$H(today) = e^{-(3\%+1\%)\cdot 1} \cdot (0.5258 \cdot 103.00 + (1 - 0.5258) \cdot 101.96) + 3 = 101.49$$

$$V(today) = \max(0.8 \cdot 100 + 3, 101.49) = 101.49$$

The value of the convertible bond is 101.48 today according to the binomial model.

2.4 Investment Strategies

Investors of convertible bonds are usually divided into two groups: long-only investors and convertible bond arbitrageurs.

Long-only investors usually buy the convertible bonds because of their positive view on the underlying share or the undervaluation of the convertible bond and may hedge the currency risk (they do not hedge other risks related to the convertible bond). The strategy of long-only investors can be divided further by regions (e.g. US, Europe etc.), the position of the convertible bond on the payoff curve (ITM, ATM, OTM etc.) or credit risk categories (investment-grade etc.). For example, the so-called balanced strategy concentrates on the ATM convertible bonds, where the payoff of the convertible bond is convex. Most investment products following long-only strategies are actively managed mutual funds. Only few passive vehicles (e.g. ETFs) exist on convertibles.

Convertible bond arbitrageurs usually buy convertible bonds and simultaneously short-sell the underlying share.[6] The proportion of the long and short part is determined by the delta of the convertible bond and the strategy is obtained by dynamic delta hedge. The potential profit is caused by the convexity of the convertible bond, but the strategy is not riskless due to CBs' time-decay and liquidity difference between equities and CBs. Other more complex convertible bond arbitrage strategies are used by derivatives desks, for example hedging the default risk, interest rate risk or volatility risk of the convertible bonds.

2.5 Risk Management of Convertible Bonds

This section provides an introduction into the risk management of convertible bonds. Our focus is on market risk in this section, but several other types of risks could be important in the risk management process such as liquidity-, model- and operational risk. Market risk of convertible bonds depends on several parameters: share prices, interest rates, dividend yield, credit spreads etc. For understanding the market risk of convertible bonds we need measures from the "equity world" (Greeks) and the "fixed income world" (e.g. duration).

2.5.1 Greeks

Greeks quantify the sensitivities of the convertible prices with respect to a small change in an underlying variable.

Delta: measures the change in the price of the convertible bond with respect to changes in the underlying equity's price.

Gamma: measures the change in delta for a small change in the level of the underlying share. Gamma is a measure for a CB's convexity and describes the ratio of upside return potential vs. downside risk. It is important to determine the frequency of the delta hedge of the convertible bond.

Rho: measures the price sensitivity of the convertible bond for a small change in interest rates.

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6 The word „arbitrage" is misleading here, as the strategy is not riskless – however, this terminology is commonly used by market participants (like in the case of statistical arbitrage).

Vega: measures the change in the price of the convertible bond with respect to changes in the volatility.

Theta: measures the sensitivity of the value of the convertible bond to the passage of time. Time is a deterministic parameter in the pricing models, it is not a real measure in the sense of risk, but it is linked to the gamma and traders usually keep an eye on this.

Omicron: measures the sensitivity of the value of the convertible bond to changes in the credit spread. It is also called as CreditDV01.

2.5.2 Fixed Income Measures

Duration: This measure is a weighted average of the payment time of different cash flows of a fixed income security. Duration is applied only on the bond floor of the convertible bond ignoring call, put and conversion etc. events.

Yield to maturity: This measure is the internal rate of return (IRR) of the convertible bond, calculated from the actual price of the convertible bond ignoring call, put and conversion etc. events. Note that a convertible bond's yield to maturity can be strongly negative especially if it is deep in-the-money. As this measure assumes that a convertible bond is repaid at the redemption price and ignores the possibility of conversion, it can be seen as the yield that is realised if the stock price falls below conversion price at maturity. Consequently, a yield-to-maturity of e.g. -20% simply means that the convertible bond is deep in-the-money.

Yield to call and yield to put: These yield figures are relevant for callable and putable bonds assuming that a convertible bond is called or put, respectively.

Coupon yield: As the often negative yield to maturity figures of convertible bonds can be misleading, often a convertible bond's coupon rate (coupon divided by current price) is published on e.g. funds' factsheets. This number is per definition non-negative but also does not account properly for convertible bonds' risk as it ignores its conversion features.

2.6 Empirical Studies

Pricing models usually assume that the rational issuer will call the bond back if the price of the CB exceeds the call price. However, empirical studies show that the call of the CB can cause significant reaction in the underlying equity's price and issuers may not call the bond back immediately above the call price. This reaction is caused by the signalling effect and the price pressure effect.

The signalling effect is based on the asymmetric information between the issuer and the bond holder. In case of in the money CBs a call notice has a negative signal effect, because the issuer is forcing an early conversion instead of redeeming the bonds with cash later.

The price pressure effect is the result of trades in the underlying equity after the call announcement. In case of out of the money convertible bonds, after the call notice, convertible bond arbitrageurs have to close their short positions in the underlying stock as the long position in the CB closed. This buying of the underlying stock has a positive effect on the underlying share's price.[7]

Another field of studies is related to the motivation of issuing CBs. The sequential-financing hypothesis suggests that CBs are the most effective way for firms with growth potential to finance their projects since CBs reduce issue cost of the future projects.

2.7 Contingent Convertibles

Contingent Convertibles (CoCos) are debt instruments which are automatically converted into common stock or are written off if a predefined trigger event occurs. At first CoCos seem like a type of convertible bond, but in reality they have different risk profiles. In case of a convertible bond the investor has limited downside risk, if the share price declines, conversion is unlikely to occur and the convertible bond is priced as a non-convertible bond.[8] CoCos are different, as they have significantly higher downside risk and limited upside potential. To compensate for this, they typically offer higher coupons than convertible or even traditional corporate bonds. Another difference to convertible bonds is their ranking in a company's capital structure. CoCo's are subordinated debt while the majority of convertible bonds is senior. Most importantly, it has to be noted that in the case of a standard convertible it is usually the investor who decides about the conversion (or the issuer in case of an anticipated trigger). In the case of CoCo's, it is the issuing prospectus and the financial situation of the issuer which triggers the conversion.

The key question in the pricing of CoCos is to determine the trigger event. The most commonly used groups of trigger events are accounting trigger, market trigger and regulatory trigger. Accounting triggers (e.g. the Tier I capital) are reported with a time lag (e.g. quarterly), and do not show the actual economic situation of the issuer. Market triggers (e.g. share price) show the current market situation of the issuer, but it can be manipulated by large trades. Regulatory triggers depend on the decision of the regulators therefore it leads to uncertainty and reduces the marketability of CoCos.

CoCos have first been created as a reaction to the 2008 financial crisis, the automatic conversion into shares made them attractive from a regulatory point of view. The issuer does not need to reach out to new investors to increase capital.

The first issuers of CoCos were Lloyds Banking Group (2009), Rabobank (2010), Bank of Cyprus (2011) and Credit Suisse (2011). For example Lloyds issued GBP 7 billion Enhanced Capital Notes (ECNs) in November 2009. These ECNs will automatically convert into equity if the bank's Tier I capital falls below 5%.[9]

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7 BECHMANN, K. L. – LUNDE, A. – ZEBEDEE, A. A., 2014, "In- and out-of-the-money convertible bond calls: Signaling or price pressure?", Journal of Corporate Finance

8 Except the case of distressed convertible bonds, where the low equity price has effect on the credit quality of the bond.

9 Original document: http://www.lloydsbankinggroup.com/globalassets/documents/investors/2009/2009nov3_lbg_exchange_offer_publication_of_exchange_offer_memo.pdf

3. Callable bonds

3.1 Investment characteristics

A callable bond can be thought of as a combination of a non-callable (bullet) bond and a short position in a call option. The call option gives the issuer the right to call the bond away at a specified price (called the call price) at a certain time (called call date). The call price is generally the par value plus a premium (called the call premium). The earliest call date and corresponding call price are specified when the bonds are issued.

Price of the callable bond = Price of the non-callable (bullet) bond – Value of the call option

The presence of the short position in a call option results in two drawbacks for the bondholder:

- Reinvestment risk: as the market yield decreases under the issue's coupon rate, the issuer will call the bond, as it is possible for him to borrow at a lower yield. The investor will have to reinvest at a lower interest rate.
- Price compression: as interest rates fall, the price of straight bonds increases. In the case of callable bonds, the price appreciation is limited by the call price.

Therefore, there must be some compensation for these disadvantages. Either the bond will be sold under par value or its coupon rate will be higher than the coupon of a straight bond so that the bond will sell at par value.

For a putable bond, the same concept is valid, but the put option price must be added from the non-putable bond price.

$$\text{Price of the putable bond} = \text{Price of the non putable bond} + \text{Value of the put option}$$

3.1.1 Price-yield relationship for a callable bond and negative convexity

The following figure plots the price/yield relationship for callable and putable bond prices.

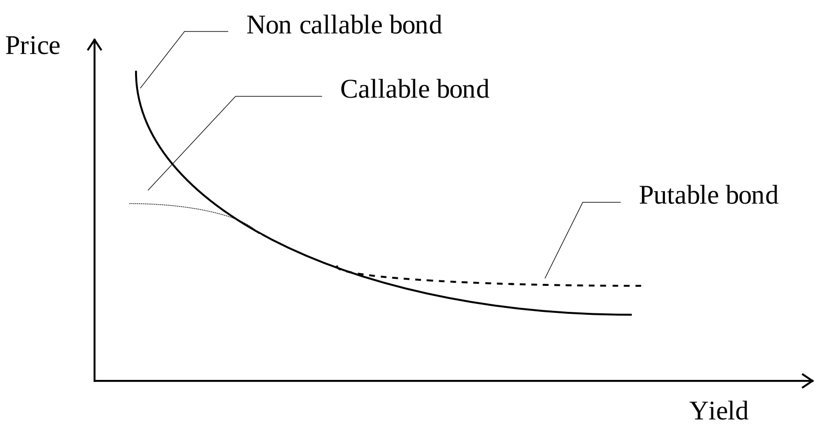


Figure 3-1: Callable and putable bond's price/yield relationship

At low yield levels, the price/yield relationship for a callable bond will depart significantly from the price/yield relationship for the non-callable (bullet) equivalent bond. It is due to the price compression (there is a limited price appreciation as the yield declines). Hence, we can observe a **negative convexity**.

3.2 Valuation and duration

3.2.1 Determining the call option value

If the bond is **callable and may be retired early**, we cannot use our traditional valuation model. The effective maturity of a callable bond can be anywhere between the first call date and its maturity date due to the presence of the call feature.

The most natural way to analyse a callable bond is to treat it as a combination of a long position in a call-free bond and a short position in a call option on the same bond. This sort of investment strategy is referred to as a “covered call writing” in the terminology of the options market, as the investors holds the bond and sells a call on this bond.

$$\text{Callable bond price} = \text{Call} - \text{free equivalent bond price} - \text{Call option price}$$

The value of the call-free equivalent bond price can be calculated with the traditional model (sum of the present value of the non-callable (bullet) bond cash flows discounted at different spot rates).

The call option has many complexities and it is not easy to evaluate: it combines the features of American and European options by providing a call period, it can have a non fixed strike price, it can have a non fixed volatility, etc.

Besides the level of interest rates volatility plays an important role for the value of a callable bond. For an investor in callable bonds falling or stable interest rate volatility is a good environment. Falling volatility leads to a lower option value – the price of the callable bond rises. Stable volatility means the investor can cash in the higher current yield of the callable bond (compared to non-callable bonds).

3.2.2 Call-adjusted yield

For a callable bond, the **call-adjusted yield** is simply the yield to maturity at the grossed-up, non-callable equivalent bond price.

Example:
Assume XYZ Inc. can issue at par (= 100.00) a 8% coupon, 10-year bond, with an 8% yield to maturity. The bond is callable in 5 years at 108.00.

At the same time, a similar long term government bond yields 7%. At first, one may think that the investor receives 1% (= 100 basis points) additional yield for the credit risk of XYZ.

But if (for example) the call option is valued at 4.00 points, we have:

$$\text{Price of the non callable bond} = \text{Price of the callable bond} + \text{Value of the call option}$$

or

$$\text{Price of the non callable bond} = 100.00 + 4.00 = 104.00$$

The yield to maturity of our bond at par is 8% (quoted yield). But the yield to maturity of the same bond if the price was 104.00 is 7.41 (call-adjusted yield). Thus, our investor receives only 0.41% (41 basis points) for the credit risk of XYZ.

You will note that in order to calculate the call-adjusted yield we need to have some idea what the call option is worth (in this case 4%).

3.2.3 Option-adjusted spread

The formula explained in the paragraph 3.2.1 enables the investors to compute the value of a bond with an embedded option as the difference between the value of an option-free bond and the value of the option linked to the bond to be evaluated. However, investors usually prefer to think in terms of yield spread measures rather than in terms of price differentials.

A yield spread is the difference between the yield to maturity on a particular bond and that on a Government bond of comparable maturity.

The **option-adjusted spread** (OAS) is the constant spread that, added to the spot rates used to discount the bond cash-flows, makes the bond theoretical price equal to the observed market price. It is called option-adjusted because the cash-flows of the bond are adjusted to reflect the embedded option.

Hence, the OAS measures the yield spread of a fixed income instrument that is not attributable to embedded options. It holds

$$\text{OAS} = \text{spread} - \text{option value (in interest rate basis points)}$$

Example:
A callable bond with a 10-year maturity and a coupon rate of 6% is selling for 116.22. The yield to maturity of this bond is 4%. If 10-year government bonds have a yield to maturity of 3.15%, the spread is 85 basis points.
Suppose that the estimated value for the call option is 3.64. Then the implied value of the non-callable (bullet) bond is 119.86 and the yield of this bond is now 3.6% (the call-adjusted yield). One would say that the option-adjusted spread is 45 basis points (opposed to the raw spread of 85 basis points).

The OAS can then be compared to the available spread on a non-callable (bullet) security of similar credit quality with the same maturity. If the callable security's adjusted spread is less than the spread on a non-callable (bullet) security, the non-callable (bullet) security may be a better investment choice.

Example:
A callable bond might be trading at a spread to Treasuries of 180 basis points. Of that, 60 basis points might be attributable to the bond's call feature, with the remaining 120 basis points attributable to such factors as the bond's credit risk, liquidity, etc. For that bond, the option-adjusted spread would be 120 basis points.

The calculation of the option-adjusted spread is quite complex and is beyond the scope of this chapter. In summary, it requires the determination of the interest rate volatility, which is essential in order to determine the value of the embedded option. The interest rate volatility is then used to calculate a binomial interest rate tree in which each node represents a time period. Then, the value of the bond at each node has to be calculated as the present value of the expected cash flows (using the one-year forward rate at the node as the discount rate).

Option price models for bonds are more complex than for equities. The reason is the pull to par effect of bond prices, which leads to a non-constant volatility of interest rates over longer timeframes. Additionally there is an interaction between bond prices and interest rates used in the models.

3.2.4 Effective duration and convexity

The Macaulay duration is the present value weighted average maturity of a bond’s total cash flow stream, using the internal rate of return (yield to maturity) as the discount rate for all of the bond’s cash flow. **Its calculation assumes that when the yield changes, the cash flows of the bond will not change.** This is an unrealistic assumption for callable (and putable) bonds: as the interest rates decrease, the probability for a callable bond to be called back increases.

Example[10]:
The following table shows the duration of an American 10.5% 10 years, semi-annual coupon paying bond (5.25% per semi-annum) that is callable in 5 years, in comparison with a 5 years and a 10 years 10.5% semi-annual coupon paying non-callable (bullet) bond.

	Durations (years)		
Yield (%)	**5 years non callable**	**Callable**	**10 years non callable**
8	4.07	4.49	6.73
9	4.05	4.74	6.62
10	4.03	5.03	6.51
10.5	4.02	5.17	6.45
11	4.01	5.31	6.40
12	3.99	5.55	6.29
13	3.97	5.71	6.18

Graphically, we can see that the duration of a callable bond may increase with the yield, as its expected maturity varies between the call date and the final maturity date. At a yield of 10.5% the duration of the callable bond is approx. in the middle of both non-callable bonds. At 10.5% yield the chances of rates rising or falling over time is approx. 50% each (assuming no “reversion to the mean” effects). So the duration of the callable bond has to be in between both possible outcomes (call after 5 years or redemption after 10 years). As soon as yields move away from 10.5% one outcome becomes more probable and due to that the duration move in that direction.

10 DOUGLAS L, 1988, “Yield Curve Analysis: The Fundamentals of Risk and Return”, New York Institute of Finance, New York.

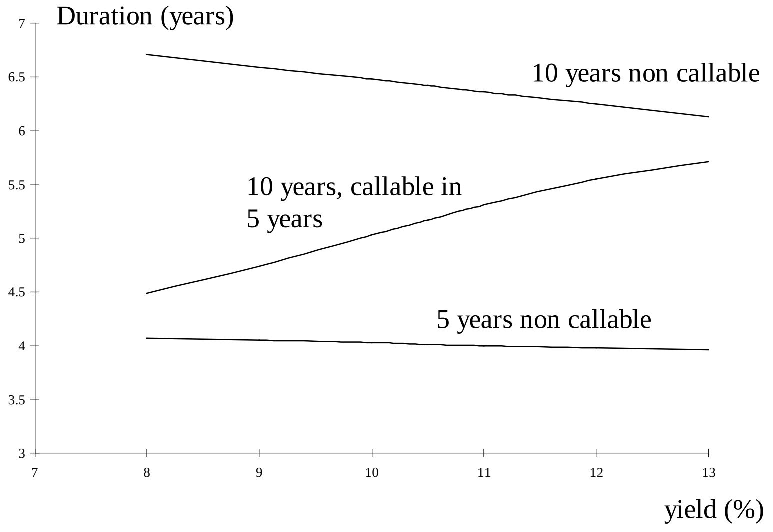


Figure 3-2: Duration of a callable bond

Duration and convexity formulae can be adjusted to take into account the embedded call option. For example, the call adjusted duration and convexity are defined as[11]:

$$\begin{matrix}\text{Call adjusted} \\ \text{duration}\end{matrix} = \frac{\text{Price}_{\text{callfree}}}{\text{Price}_{\text{callable}}} \cdot \begin{pmatrix}\text{Duration of} \\ \text{call - free bond}\end{pmatrix} \cdot (1-\delta)$$

$$\begin{matrix}\text{Call adjusted} \\ \text{Convexity}\end{matrix} = \frac{\text{Price}_{\text{callfree}}}{\text{Price}_{\text{callable}}} \cdot \left[\begin{pmatrix}\text{Convexity of} \\ \text{call - free bond}\end{pmatrix} \cdot (1-\delta) - \begin{pmatrix}\text{Price of} \\ \text{call - free} \\ \text{bond}\end{pmatrix} \cdot \gamma \cdot \begin{pmatrix}\text{Duration of} \\ \text{call - free} \\ \text{bond}\end{pmatrix}^2 \right]$$

where δ and γ are the delta and gamma of the call option embedded in the bond.

[11] For a mathematical derivation, see: FABOZZI Frank J. Edward, 1988, “Fixed Income Mathematics”, Chicago, Probus Publishing , Appendix A.

4. Floating rate notes

4.1 Investment characteristics and types

Floating rate notes or FRNs are long term securities wherein coupon rates are adjusted periodically (at the **reset period**) according to changes in a base or benchmark rate (the **reference rate**) such as a Libor rate observed at a given date or during a given period on the **reference market** (for example, the London interbank market for the Libor). Typical reset periods are every six or three months, depending on the frequency of payment of coupons in order to keep maturities of both base rate and coupon payment matched. Usually the coupon of the floating rate note is reset on the basis of a financial index (typically a market interest rate), but in some cases the benchmark is given by a non financial index, like the price of a commodity (gold, oil) or a price index (like CPI). The benchmark employed for the coupon rate adjustment has to be publicly available or, at least, publicly announced by the trustee.

> **Examples:**
> In February 2003 Australian and New Zealand Banking Group has issued a floating rate note maturing on February 4th, 2013 with a total face value of 550 Mio USD. The bond is callable as of February 2008 at par, and pays interest quarterly equal to USD 3-month LIBOR plus 55 basis points up to February 2008, and from then on equal to USD 3-month LIBOR plus 105 basis points.
>
> The US Government began issuing Floating Rate Notes (FRNs) in January 2014. Issued for a term of two years, FRNs pay varying amounts of interest quarterly until maturity. Interest payments rise and fall based on discount rates in auctions of 13-week Treasury bills.

The rate adjustment mechanism can be straightforward or very complicated; in the simplest cases, the floating coupon is set at a fixed spread called quoted margin (for example 50 *basis point*s) over (or, in some cases, under) the base rate (for example the Libor). The size of the spread is given by the characteristics of the issue (credit risk, marketability...). The quoted margin is typically fixed up to the maturity of the FRN (exceptions to this are callable FRNs that have step-up clauses – the margin increases if the issuer does not call the bond (see first example above).

Besides these “plain vanilla” structures there are also complicated coupon formulas. It is important to note that each complication leads to potentially very different price and risk behaviour. The variable coupon can have a cap, a floor or a collar. In the first case, the variable rate cannot exceed a given level so that the bond has a maximum cost for the issuer; in the second case, the variable coupon cannot fall below a given level and in the last case the coupon may only vary between a maximum (cap rate) and a minimum (floor rate). In a special case, the floor rate, once touched, remains fixed for the whole remaining life of the bond; issues carrying such provision are called ***drop-lock bonds***.

Cap’s, floor’s and collar’s are options on interest rates. A cap of 5% for 10 years with quarterly coupon payments requires 40 interest rate options on the relevant base rate (typically Libor). These 40 options are traded as a package – called cap (or floor). A collar is the combination of a cap and a floor.

The floating rate can be also inversely related to the coupon of the bond: it is the case of **inverse floaters** where the coupon is given by a fixed level (say 15%) minus the floating rate (say Libor) and so the coupon rate changes in the opposite direction of the base rate (and thus of market interest rates). The inverse floater has always a floor (minimum interest rate), which cannot be fixed below zero (the investor never has to pay interests to the issuer).

Floating rate notes often carry provisions entitling the holder (putable bond) or the issuer (callable bond) to terminate early the life of the bond. These options have a much smaller impact on the valuation of “plain vanilla” FRNs compared to fixed coupon bonds. This is due to the much smaller interest rate sensitivity of FRNs. Calls still have an influence on the pricing because they influence how long the quoted margin as a compensation of the credit component is paid to the investor. As soon as the paid margin deviates from the market spread for a specific issuer, the interests of investor and issuer start to diverge. Lower market spreads will lead the issuer to call existing FRNs and issue new bonds with lower margins. Higher credit spreads will in the tendency lead the FRN not to be called and the investor facing a too low quoted margin up to the final maturity of the bond.

The majority of all floating rate issuers are financial institutions. Only in the last years more non-financial corporates and governements have started to play a larger role in this market segment. Also on the investor side financial institutions with outstanding floating rate liabilities hold an important market share. If interest rates are expected to rise, the note-holder can expect increase in coupon rates of such bonds at the next reset date.

4.2 Yield mesures for floating rate notes

The main feature of **floating** rate notes is the impossibility to know in advance the cash flows that the investor will get from the security (please notice that this is not the case with **variable** rate notes whose interest rate changes over time but with a predefined schedule).

One basic and straightforward yield calculation is the **current yield**[12], determined as the ratio of the current coupon to the price of the bond. Since some investors take floating rate notes as a close substitute for money market instruments (given their indexation to money market rates) the aforementioned measure can be appropriate for short term investment horizon and assuming that, in the short term, the price of the bond is not going to change.

The floating coupon feature tends to ensure that floating rate notes pay returns in line with current interest rates and thus their market value is likely to be close to the face value as far as interest rate risk is concerned.

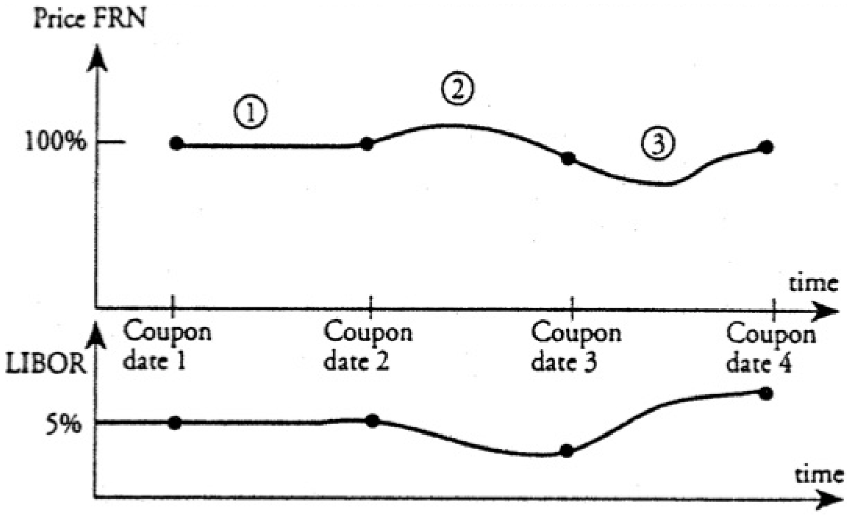
As long as there is no change in the relevant benchmark rate the price will stay constant at par (assumption: quoted margin exactly matches needed remuneration for credit risk) (1 of chart below).

Only between coupon dates the price will deviate from par value (2 and 3 of chart below). The quoted price will tend toward par as we get closer to the next coupon payment date. Generally, the coupon rate for the current period is set at the previous coupon payment date, and equals the market rate at that time. Thus, just after the coupon payment, the bond should be quoted at par.

On the trading side in normal markets FRNs have relatively low bid-ask spreads because they exhibit lower price volatilities than fixed coupon bonds. The lower volatility is due the regular adjustment of the coupon to market levels and the corresponding pull to par at coupon reset dates.

[12] See “General Principles”.

Of course, significant deviations from parity may occur due to changes in the creditworthiness of the issuer (or of the guarantor).



Between two coupon dates, the floating rate bond can be considered as a short-term zero-coupon bond that will pay interest and principal at the next coupon date (as it is possible to sell the bond at par just after the coupon payment date).

Thus, the price of a floating rate bond should theoretically be given by

$$P_{cum} = P_{ex} + f \cdot C_1 = \frac{C_1}{(1+R_{0,1})^{1-f}} + \frac{100}{(1+R_{0,1})^{1-f}}$$

where P_{cum} is the price of the bond with accrued interest (or **gross price**), P_{ex} is the price of the bond without coupon, f is the fraction of that is time elapsed since the last coupon payment date, and C_1 is the next coupon.

Example:
Company XYZ has issued a EUR floating rate bond, 2016-2026. Coupons are paid semi-annually on the 31st of March and the 30th of September. The coupon rate for the following period is set at the same dates equal to the 6-month EUR LIBOR rate.

On the 6th of September 2016, the next coupon to be paid was set to 1.75% (annual rate). The six-month risk-free interest rate is 0.75% (annual rate). What is the price of such a bond?

Since the last coupon payment, the time elapsed is f = 156 / 180 = 0.866 semester. The accrued interest is equal to 0.866 · 0.875 ≅ 0.76. The six-month rate is $(1.0075^{0.5}-1) = 0.374\%$.

Thus, the bond price is

$$P_{cum} = P_{ex} + f \cdot C = \frac{0.875+100.0}{1.00374^{1-0.866}} = 100.82$$

that is, a quoted price of:

$$P_{ex} = 100.82 - 0.76 = 100.06$$

But in practice, things are more complicated as the coupon rate is not always equal to the market spot rate for the next period, and default risk exists and the price of credit risk changes over time.

Because the level of the benchmark rate in the future is not known the cash flows of a floating rate note are also not known in advance and thus a conventional measure of yield to maturity cannot be derived.

A simple measure employed with floating rate instruments is the **effective margin** that measures the average spread over the benchmark rate that the bondholder earns during the whole life of the security. The idea behind the calculation of the effective margin is to assume that the base rate does not change over the residual life of the security and to discount the cash flows thus determined using the reference rate plus a margin. If the calculated present value is equal to the market price of the security, then the effective margin is equal to the margin employed in the calculation of the present value; otherwise the procedure has to be repeated with a different margin until the present value is equal to the market price. Of course, the results change if the reference rate changes during the life of the bond. But the impact of changes in the reference rates on the level of the effective margin is relatively small.

Example[13]:
The following table shows how to calculate the effective margin of a Floating Rate Note having a maturity of 5 years, a coupon rate calculated as reference rate plus 50 *basis point*s (reference rate = 8%), semi annual coupon payments; let's assume that the current reference rate is equal to 8%. According to the third column of the table, the cash flows of the first 9 periods are equal to one half the current reference rate (4%) plus one half of the spread (0.25), multiplied by 100 (nominal value of the security). The last three columns give the calculation of the present value of the bond at different effective margins (spreads). Therefore, if the price of the bond is 100.40, then the effective margin is 40 *basis point* (on annual basis; 20 *basis point*s semi annually); when the price is 99.60 the effective margin is 60 *basis points*.

			PV of cash flows with spread at		
Period	**Refer. Rate**	**Cash Flow**	**50**	**40**	**60**
1	8%	4.25	4.08	4.08	4.07
2	8%	4.25	3.91	3.91	3.91
3	8%	4.25	3.75	3.76	3.75
4	8%	4.25	3.60	3.61	3.59
5	8%	4.25	3.45	3.46	3.44
6	8%	4.25	3.31	3.32	3.30
7	8%	4.25	3.18	3.19	3.17
8	8%	4.25	3.05	3.06	3.03
9	8%	4.25	2.92	2.93	2.91
10	8%	104.25	68.76	69.09	68.43
		Present value	100.00	100.40	99.60

A somewhat more precise and very commonly used measure of yield for floating rate notes is the so called **discount margin**. The idea behind the discount margin is very similar to the adjusted margin concept. Again the assumption is made that the actual benchmark rate (Libor) does not change over the residual life of the security; assumed index = actual Libor.

In a first step the future value of the floating rate note at the next coupon reset is calculated – by deducting the actual coupon and adding the actual refinancing rate to the maturity of the next coupon (Libor to next coupon date). Then similar to the calculation of the adjusted margin the discount margin is calculated that equals the value of the future cash flows to the price of the FRN at the next coupon date.

13 From Fabozzi Frank, J., *Bond Markets, Analysis and Strategies*, Prentice-Hall, 2003, 5th edition.

$$\text{price} + \text{ accrued interest } + \left(\text{Libor to next coupon} + \text{discount margin}\right) \cdot \frac{\text{d}}{360} - \text{coupon}$$

$$= \sum_{i=1}^{n} \frac{\text{assumed index } + \text{ quoted margin}}{\left(1 + \text{assumed index} + \text{discount margin}\right)^{t}}$$

d = days to the next coupon

Again by assuming a constant benchmark rate we make a quite crude assumption about the future behavior of the benchmark rate. A more accurate approach would be to use the market implied forward rates for the assumed index. But this leads to differing results depending on the assumptions and data used. Using actual benchmark levels has the advantage that every market participant gets the same number for the discount margin – this makes communication between market participants easier – at the price of losing accuracy at the margin.

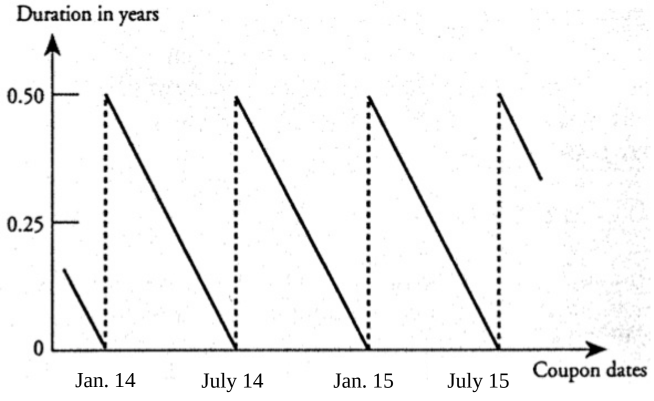
The discount margin is also the first step in order to compare yields of FRNs with the yield of similar fixed coupon bonds. The link between both instruments is the swap market. An interest rate swap can move us from a floating rate to a fixed rate. By adding the discount margin of an FRN to the swap rate for the same maturity we get to a fixed rate equivalent yield. This can then be compared to the yield to maturity of fixed coupon bonds (a not totally accurate comparison due to the coupon effects within the yield to maturity concept but still a good approximation).

Example:
A 10-year General Electric floating rate yields a discount margin of 50 bps. At the same time a 10-year General Electric bond with a 2.5% coupon yields 2.6%. Which investment is more attractively priced? 10-year swap rates trade at 2.05-2.00%.

FRN swapped to fixed coupon = discount margin FRN + swap rate = 0.5% + 2% = 2.5%. The coupon bond is more attractive – it yields 10 bps more (2.6%).

4.3 Risk measures – interest rate versus credit duration

With regards to interest rate risk floating rate notes behave approximately like a zero bond with maturity at the next coupon reset date. This is due to the tendency of the FRN price to revert to par value (100%) at each coupon reset. A zero bond too reaches and returns par value (100%) at maturity. The chart below shows the duration pattern for a 6-month reset FRN. Duration is always fluctuating between 0 and 6 months.



At this point it is crucial to differentiate between **interest rate duration** of a floater – which is always short – and the **credit duration**. As long as we want to quantify the impact of a change in the level of interest rates the concept of interest rate duration works well. If we are faced with a yield movement due to a change in the level of creditworthiness of the issuer or a change in the pricing of credit risk things are different.

Since the compensation for credit risk in the form of the quoted margin of an FRN is fixed investors are not compensated for credit risk changes – this is exactly the same reality faced by investors in normal fixed coupon bond with the same remaining maturity. A 10-year FRN will suffer the same price impact due to a weakening credit quality as a 10-year fixed coupon bond. If the credit spread increases by 1% both bonds are short a 1% cash flow stream for the next 10 years. So both instruments have the same credit duration – and it is long for both bonds. In order to exactly calculate the credit duration for an FRN we have to assume a Libor rate for all coupon dates – typically again a constant Libor is assumed. Then we apply the standard duration formulas.

As soon as we start investing in FRNs we have to consider this duality with regards to duration – interest rate duration versus credit duration. In the financial crisis 2008 many money market funds that invested partially in longer dated floating rate notes had to face significant price reductions due to a marked rise in credit spreads.

This split between interest and credit duration also occurs as soon as swaps or futures are used to manage interest rate risk. If a 10-year coupon bond (duration = 8 years) is fully hedged to floating rates with the use of an interest swap we have the same constellation as owning an FRN. Interest rate duration is fully hedged – depending on the details of the transaction interest rate duration falls to between 0 and 6 months. But the credit duration stays long (8 years) – we aren't compensated for changes in credit spreads due to the fixed coupon of the underling bond.

4.4 Complex FRN's

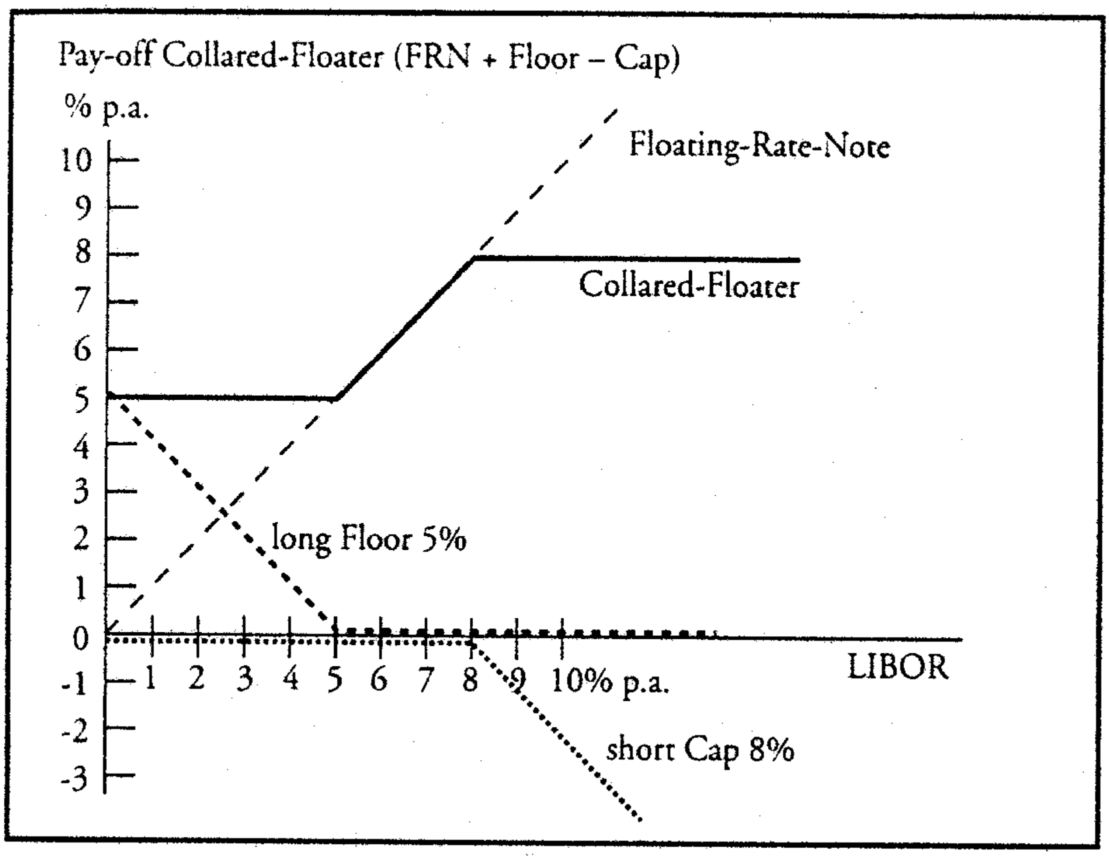
4.4.1 Collared floaters

Coupons paid by collared floaters are restricted to a specific range by a cap and a floor (for example 5%-8%). Within the range they are set according to the Libor rate (plus potentially a margin).

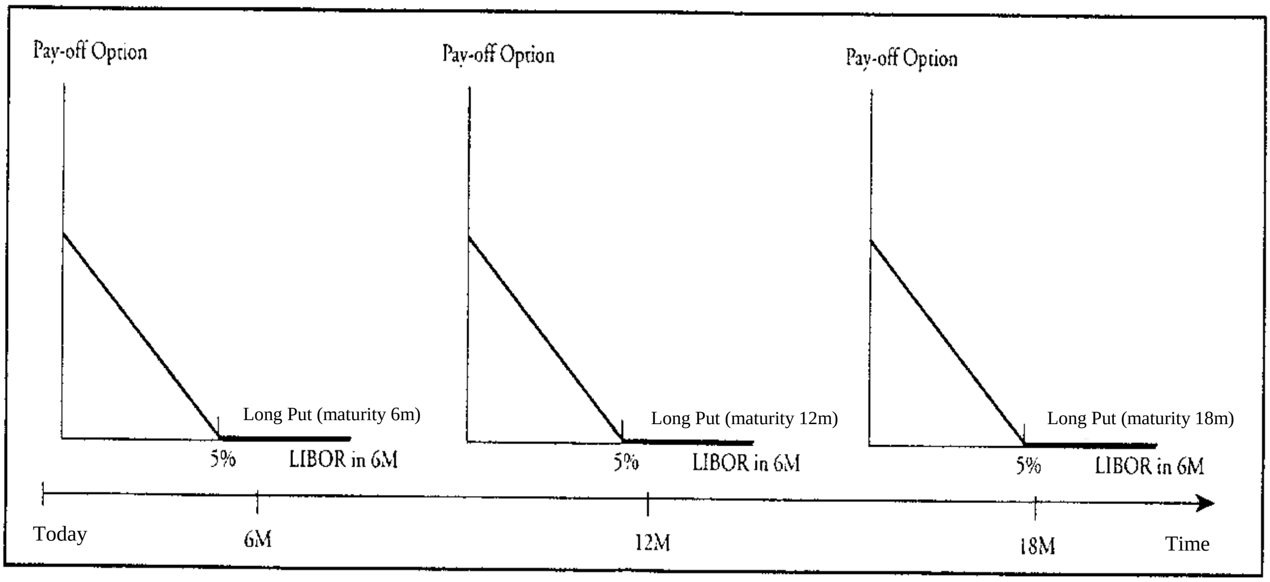
This structure builds on two elements: a "plain vanilla" FRN + an interest rate collar.

A collar is the sum of two further elements: a long floor and a short cap. Floor's and cap's are options on Libor rates. The most important pricing factors for these options are:
- cap rate
- floor rate
- maturity
- interest rate volatility (for each coupon maturity)
- Libor rates (actual rate and all implied forward rates for the maturity of the transaction)

Pay-off profile of a collared-floater:



As shown in this graph the long floor ensures that the investor always gets a coupon of at least 5%. The short cap limits the coupon paid to a maximal rate of 8%.



In order to get this payoff structure at each coupon date we need a put and call option for each coupon date. So a 10-year collared floater on a 3-month Libor rate consists of 40 put options (long floor) and 40 call options (short cap). Due to that the pricing of such a FRN is dependent on the full Libor forward curve up to 10 years. A 10-year collard floater has typically a duration of around 4-5 years (depending on floor and cap rates). Again at first sight an unsophisticated investor could see this instrument as something very similar to a "plain vanilla" FRN – but it carries a clearly totally different price risk and reacts to movements of the whole yield curve (in this case up to 10 years).

4.4.2 Inverse floaters

As far as the **valuation of an inverse floater** is concerned, the starting point is to consider that an inverse floater can be created on the basis of a fixed rate bond, which is called **collateral**. The collateral generates two bonds: a floating rate note and an inverse floater. The two securities are designed so that in any period the total coupon paid on the two bonds is lower than or equal to the collateral's coupon and that the total value of the two bonds is lower than or equal to the collateral's value.

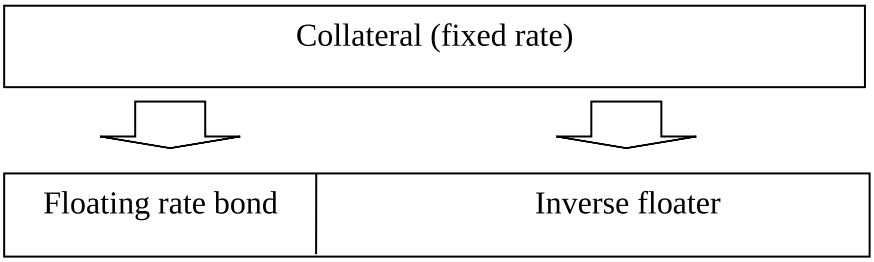


Figure 4-1: Generation of an Inverse Floater

Thus the price of an inverse floater can be derived on the basis of the price of the collateral and that of the floater:

inverse floater's price = collateral's price – floater's price

This implies a very long duration for an inverse floater. The above equates to a long position in two coupon bonds and a short position in a FRN (all with the same maturity).

Let us assume the following: we start with a USD 100m collateral that pays a 6% coupon. Now we generate a USD 50m FRN and a USD 50m inverse floater out of that. So we can pay 12% coupon – Libor on the inverse floater. 12% comes from earning 6% on USD 100m that we can pay to only USD 50m of nominal value.

Thinking in bond equivalents the inverse floater equates to owning two long positions of USD 50m in a 6% coupon bond and a short position of USD 50m in an FRN paying Libor. This package ensures the payment of the coupon of 12% (2 x 6%) – Libor. And also the nominal at maturity matches – we receive two nominals (two coupon bonds) and we pay one nominal to the FRN.

So a 10-year inverse floater has approximately the double duration of a 10-year fixed coupon bond (2 x duration of a 10 year coupon bond – duration of a 10 year FRN). This is clearly an unexpected outcome for something called floating rate.

One potential shortcoming of the aforementioned valuation method is that it does not take into account that usually the floater has a floor rate equal to zero. This is a sort of put option in the hands of the inverse bondholder that may have a value and has to be properly evaluated. The valuation of the floor is made with the appropriate option valuation formula and is beyond the scope of this section.

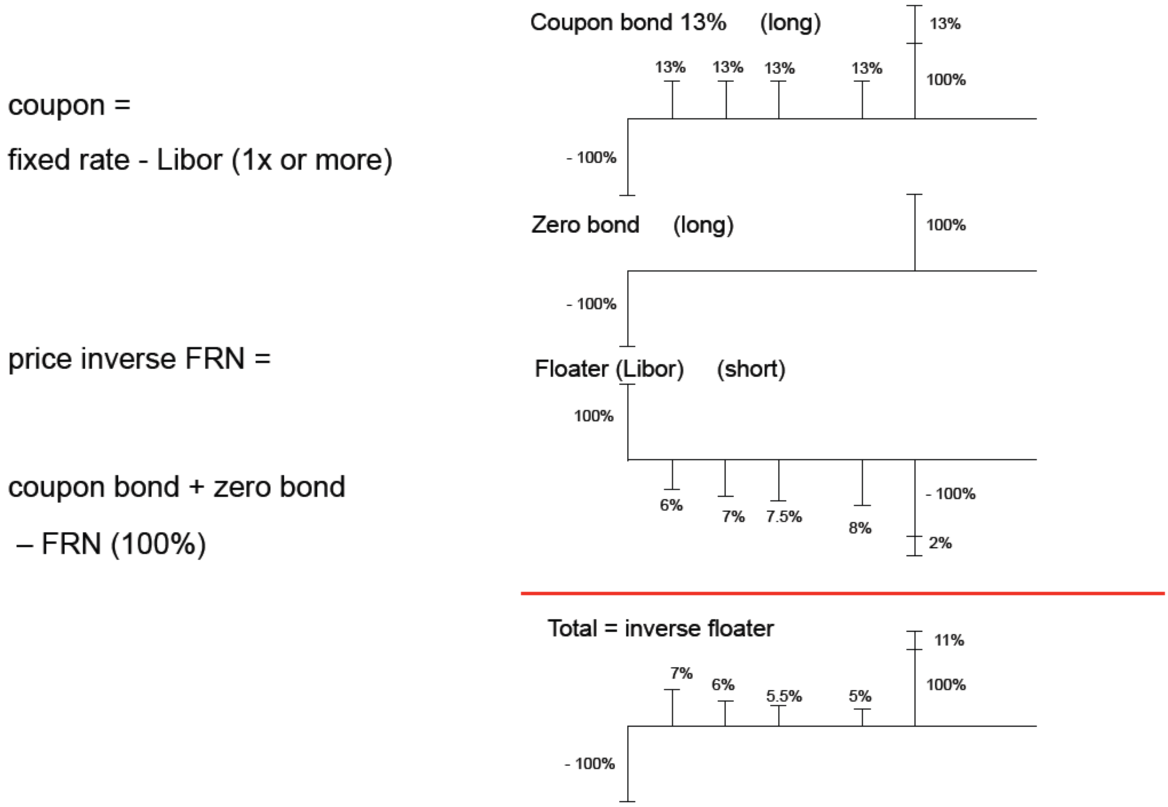


Figure 4-2: Price and duration of an inverse floater

4.4.3 Auction rate securities (ARS) and variable rate demand bonds (VRDB)

Auction rate securities (ARS) are FRNs with a variable interest rate the value of which is calculated periodically by an auction process. These auctions are usually organised every 7, 28, or 35 days. The issuer pays the coupon amount on the same dates.

In some cases, the contractual provisions of the loan provide for daily auctions with a coupon payment at the beginning of each month.

As well as determining the next coupon to be paid, the auction process allows bondholders to sell their bonds to investors who wish to buy them. The transaction price of the bond is always at par[14].

[14] This means that the buyer must pay the seller the nominal value of the bond. The notion of par will be described later.

Bondholders and investors do not have direct access to the auctions. They must send in their bids to their financial intermediary (broker/dealer) who in turn sends them on to the agent responsible for organising the auction. This agent is appointed by the issuer and is usually an investment bank.

People who wish to take part in the auction specify in their bid the number of bonds and the minimum coupon rate at which they wish to buy this quantity. Specifically, a bondholder may send in the following types of bid:

Hold: the bondholder does not wish to sell his position whatever interest rate results from the auction process. By sending in this bid, the bondholder tells his financial intermediary that he wishes to maintain his position whatever the coupon rate paid on the bond. His position will therefore not be available for auction.

Hold at Rate: the bondholder wishes to maintain his position only if the bond's coupon rate is higher than the minimum rate stated in his bid.

Example: A bondholder has 100 XYZ ARS bonds. Each bond has a nominal value of USD 25,000. The investor wishes to maintain his position only if the coupon rate resulting from the auction is at least 2 percent.

This investor should therefore send his financial intermediary the following bid:

Hold at Rate, quantity = 100, minimum rate = 2%.

This means that if the coupon rate resulting from the auction is less than 2 percent, 100 bonds should be sold at par. The number of bonds specified in his bid is therefore available for sale at a minimum rate of 2 percent.

Sell: the bondholder wants to sell his position whatever the coupon rate resulting from the auction.

Example: A bondholder has 50 XYZ ARS bonds. Each bond has a nominal value of USD 25,000. As he needs liquidity, this investor wants to sell half his position, or 25 bonds, whatever the coupon rate resulting from the auction.

Consequently, this investor should send in the following bid to his financial intermediary:

Sell, quantity = 25.

The number of bonds specified in his bid will be put up for sale during the auction with no conditions as to the value of the coupon rate. This number will therefore be sold if there are enough buyers!

For their part, investors who want to buy ARS bonds send in their **buy** bids to their financial intermediary specifying the number and the minimum coupon rate required.

Example: An investor wants to buy 20 XYZ ARS bonds as long as the coupon rate on the bonds is at least 2.1 percent. Each bond has a nominal value of USD 25,000.

This investor should send in the following bid to his financial intermediary:

Buy, minimum rate = 2.1%.

If the coupon rate resulting from the auction is less than 2.1 percent, the bid will not be executed.

Note that investors may also send in buy bids without specifying a minimum coupon rate. The number of bonds specified in their bid will therefore be bought during the auction whatever the coupon rate as long, of course, as there are enough sellers!

The table below shows how the ARS coupon rate is determined during the auction process.

The auction concerns the XYZ ARS bond. The total amount of the XYZ bond issue is USD 50,000,000. The XYZ loan consists of 2,000 bonds. Each bond has a nominal value of USD 25,000.

All the bids, whether to buy or sell, have been recorded in the system and sorted from the lowest to the highest coupon rate specified in participants' bids[15].

Participant	Bid type	Quantity	Minimum interest rate	Cumulative quantity
1	Buy	100	-	100
2	Hold at Rate	200	0.95%	300
3	Hold at Rate	100	0.97%	400
4	Hold at Rate	200	0.98%	600
5	Buy	300	0.99%	900
6	Buy	200	1.1%	1100
7	Hold at Rate	100	1.15%	1200
8	Buy	400	1.2%	1600
9	Hold at Rate	100	1.22%	1700
10	Sell	100	-	1800

The quantity available for sale, and therefore for auction, is determined by the total number of **sell** and **hold at rate** bids, in this example 800 bonds.

Purchase bids have been made for 1000 bonds.

The coupon rate, known as the clearing bid, will be the lowest rate that meets the quantity available for auction. In our example, this rate is 0.99 percent. The issuer will therefore pay a coupon rate of 0.99 percent on its bond issue until the next auction date.

The table below shows the transactions that will be made at the end of the auction.

15 Note that a buy bid with no rate limit is a bid that must be executed whatever the coupon rate. If it comes to it, this bid must be executed even if the coupon rate is 0, which is why it is at the top of the list. A sell bid with no rate limit is a bid that must be executed whatever the coupon rate. If it comes to it, this bid must be executed even if the coupon rate is infinite, which is why it is at the bottom of the list.

Participant	Bid type	Quantity	Minimum interest rate	Cumulative quantity	Bids executed
1	Buy	100	-	100	Yes
2	Hold at Rate	200	0.95%	300	Yes
3	Hold at Rate	100	0.97%	400	Yes
4	Hold at Rate	200	0.98%	600	Yes
5	Buy	300	0.99%	900	Yes, in part (200)
6	Buy	200	1.1%	1100	No
7	Hold at Rate	100	1.15%	1200	Yes
8	Buy	400	1.2%	1600	No
9	Hold at Rate	100	1.22%	1700	Yes
10	Sell	100	-	1800	Yes

Note that although 800 bonds are available for sale during the auction, only 300 will effectively be **exchanged**. Specifically, participants 7, 9 and 10 will sell 300 bonds **at par** to participants 1 and 5.

Of course, it will clearly not be possible to determine a coupon rate by this process if the quantity available to auction is zero. This situation will occur if all the bondholders have sent in hold bids to their agent.

Should this arise, the coupon rate that will be paid on the bond issue until the next auction date will be the **all hold** rate. The value of this rate is contractual. It is fixed on the loan issue date in accordance with the reference interest rate calculated by the Bond Market Association (BMA) or in accordance with the prevailing rate for short-term commercial paper.

Auctions will be deemed to have **failed** if the quantity requested for purchase is less than the quantity offered for sale. In our previous example, the auction would be deemed to have failed if purchase requests are received for fewer than 800 bonds.

As the coupon rate cannot be set by auction, the issuer must pay the maximum coupon rate on its loan until the date of the next auction. The value of this rate is contractual and is fixed at the time of issue.

Although they are not legally obliged to do so, the brokers/dealers who took part in the bond issue process[16] will participate in the auctions offering liquidity. In other words, they will ensure that the auction process is not deemed to have failed by offering to buy the number of bonds that bondholders wish to sell in their bids.

Unfortunately, the so-called sub-prime crisis, which began in 2007, revealed the limits of the auction process for investors but also for issuers of ARSs. During the crisis period, investors were unable to sell their positions on the auction dates and issuers paid the maximum coupon rate provided for contractually when the auction status was “failed”.

Indeed, at the end of 2007 and the beginning of 2008, the banks stopped buying excess sell bonds on their own account as they were having serious difficulties in borrowing the necessary liquidity on the interbank market to finance this activity.

As well as these liquidity problems, the banks no longer wished to participate in the auction process as they too wanted to limit their exposure to interest rate risk in order to reduce the amount of regulatory capital.

[16] The issue process is described in detail in another chapter.

As it was customary for investment banks to distribute these bonds to their clients as forming part of short-term instruments[17], some investors found themselves in turn facing serious liquidity problems as it was impossible to sell the bonds which they regarded as an alternative to cash.

For their part, the issuers, mainly US American municipalities, were also seriously disadvantaged by the number of auctions with failed status. They were therefore obliged to pay the maximum coupon rates provided for in the bond issue contract thus very significantly increasing their financing costs.

Investors and issuers, who felt very seriously disadvantaged and misled by the arguments they had been given by the investment banks, began legal proceedings.

UBS, for example, was forced by law to repay nearly USD 20 billion of ARSs to its customers and to pay compensation of USD 150 million. Other banks, such as Citigroup, Bank of America, RBC Capital Market, Merrill Lynch and Wachovia were forced by the SEC to reach agreements with their clients to redeem their positions[18].

In effect, the SEC took the view that these banks had not properly informed their clients of the liquidity risks of these bonds. Quite the reverse, the banks were distributing the bonds to their clients arguing that they were a very good alternative to cash.

Note that the auction process is not used simply to set the coupon rate on bonds. It is also used to set the remuneration of certain US American mutual funds[19] and the dividend rate for some preferred stocks[20].

Variable rate demand bonds (VRDB) are debt instruments that usually have long maturities but may be redeemed early at the request of the investor or of the issuer at any time during the bond's life, usually giving seven days' notice. These are therefore bonds containing put and call provisions of the American type.

Note that the very short period of notice required by the issuer leads investors to regard these debt instruments as having the same characteristics as a money market instrument.

As the period is usually too short for the issuer to obtain the funds needed to redeem the bonds (exercise of the put option), these issues have a special provision by which a bank guarantees to advance cash to the issuer when redemption requests are made. In most cases, a letter of credit provided by the guarantor bank gives concrete form to this provision.

VRDBs are usually unsecured bonds. They nevertheless benefit from insurance purchased by the issuer from an insurance company against the issuer defaulting on payment of the interest and principal. This type of insurance is often taken out when the bond issuer is a United States municipality.

Unfortunately, the sub-prime crisis demonstrated the limits of liquidity guarantees provided by banks and the limited effectiveness of payment default insurance.

During the crisis, banks faced serious liquidity problems causing them to fail to respect the liquidity guarantees they had provided in the past. Because of this, many investors were unable to redeem their bonds in the period defined contractually. As these bonds were regarded as an alternative to cash, affected investors in turn found themselves facing serious liquidity problems. Fortunately, massive intervention from the central banks limited the damage to the real economy!

17 Their argument and hypothesis was that their own intervention prevented an auction having failed status.

18 All these decisions can be consulted on the SEC's web site at the following address: http://www.sec.gov/news/press/2008/2008-290.htm

19 That of certain Nuveen and Blackrock mutual funds, for example.

20 Preferred stocks will be presented in another chapter.

Similarly, payment default insurance can only be effective if the insurance company itself is solvent. However, the crisis showed that these companies did not have enough capital to meet their obligations if there were failures in the chain of insured institutions.

In view of this scenario, the rating agencies drastically reduced the ratings of bond issues insured by these companies. Institutional investors, who no longer respected their investment constraints in terms of ratings, were forced to request the immediate repayment of the VRDBs they were holding in their portfolios, thus accelerating the liquidity problems of the banks that had guaranteed liquidity on those bonds.

5. Inflation-linked bonds

Inflation is an economic term that describes the general rise in prices of goods and services. As prices rise, a unit of money can buy less goods and services. Hence, inflation is an important risk factor for investors, because it can erode the purchasing power of their investments. For example, suppose an investor acquires a one-year nominal bond at par value with an annual nominal rate of 5%. If annual inflation turns out to be 2%, then the investor's purchasing power increases by roughly 3%, but if actual inflation is instead 7%, then the investor's purchasing power would indeed decrease by 2%.

Inflation-linked bonds are securities that are designed to protect investors from this inflation risk. Like other bonds, they are issued with a fixed coupon rate and a fixed maturity date, but their payoffs are linked to inflation. So, if prices go up, payments of these bonds increase too, maintaining the purchasing power of its holders.

5.1 Real and break-even rates

Conventional bonds promise fixed nominal coupons and the redemption payment of the principal, but the real value, that is how many products and services can be bought with these payments when they are received, is unknown because it depends on the future inflation rate. On the contrary, the payments of inflation-linked bonds are indexed to inflation and therefore guarantee a fixed real return irrespective of inflation.

The yield of a conventional bond consists mainly of three parts, a real yield which is a compensation for delayed consumption and an expected inflation over the life of the bond, which is a compensation for the expected loss in purchasing power due to inflation. So, conventional bonds also compensate for inflation, but it is ex ante. If realized inflation turns out to be higher than expected, then the investor suffers a loss in purchasing power. Due to this risk, the third part of the yield of a conventional bond is an inflation risk premium to compensate the investor. So,

$$Nominal\ \ Yield = Real\ \ Yield + Expected\ \ Inflation\ \ Rate + Inflation\ \ Risk\ \ Premium$$ [21]

On the other hand, the yield of an inflation-linked bond consists only of two parts, a real yield and the realized inflation over the life of the bond, but as this inflation compensation is applied after issuance, ex post, the rate when the bond is issued consists only of the real yield.

$$Inflation\text{-}linked\ \ Yield = Real\ \ Yield + Actual\ \ Inflation\ \ Rate$$

[21] This formula is an approximation, the actual formula is $(1 + Nominal\ \ Yield) = (1 + Real\ \ Yield) \cdot (1 + Expected\ \ Inflation\ \ Rate) \cdot (1 + Inflation\ \ Risk\ \ Premium)$ This approximation is appropriate when yields and inflation are low and therefore is a general industry practice.

From the nominal yields of a conventional bond and of a comparable inflation-linked bond of the same maturity, an estimation of the inflation expected by the market over the term of the bonds can be obtained. This is called the breakeven inflation, and is calculated, ignoring the inflation risk premium[22], as:

$$Breakeven\ Inflation = Nominal\ \ Yield - Inflation\text{-}linked\ Yield$$ [23]

If actual inflation exceeds breakeven inflation over the lives of the bonds, then inflation-linked bonds would provide superior returns to a similar conventional bond. Conversely, if actual inflation is less than breakeven inflation, conventional bonds would provide superior returns to a similar inflation-linked bond. If actual inflation is equal to breakeven inflation, then an investor would be indifferent between both of them.

5.2 Investment characteristics

Some of the characteristics of inflation-linked bonds are equivalent to that of conventional bonds, such as time to maturity, coupon payment frequency, etc. This section considers the key choices and constraints that face the designers of inflation-linked bonds and that are specific for this asset class.

5.2.1 Reference price index

The first task is to determine how realized inflation is measured. Cash flows are linked throughout the life of the bond to changes in a specified price index. Inflation-linked bonds are usually linked to the national consumer price index. The protection an investor attains depends on the correlation between the basket used to form the index and the investor's own consumption.[24] The benefits of consumer price indices are that they are known to the general public, that they are published monthly and that their calculations are reliable.

5.2.2 Indexation

Inflation-linked bonds are designed to protect investors against the erosion of the bonds' cash flows due to inflation, but this protection can take a variety of forms.

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[22] Inflation-linked bonds are priced on the market, and this market is less liquid than the market for conventional bonds, so a liquidity risk premium is demanded for these bonds. Thus, both inflation-linked and conventional bonds exhibit a risk premium. Assuming both premiums are of similar value; i.e., that they compensate each other, then they can be ignored for the calculation of the breakeven inflation.

[23] This formula is also an approximation, the actual formula is $Breakeven\ Inflation = \frac{(1+Nominal\ \ Yield)}{(1+Inflation\text{-}linked\ Yield)} - 1$

[24] Countries from the European Union use the European Harmonised Index of Consumer Prices, formed with household costs across different European countries, instead of their own consumer price index, to increase liquidity. Bonds that are linked to the same reference price index can be traded more easily between those countries, and the more tradeable they are, the lower the liquidity premium gets, and therefore the less the country needs to pay to issue them.

The majority of inflation-linked bonds are structured similar to nominal bonds except that the principal and the coupon payments are not fixed, they change following movements of the reference inflation index used, this structure is known as **capital indexed**. In particular, the principal is continually indexed to realized inflation and the coupons are set as a fixed percentage of this value.

The principal indexed in each period t is calculated as:

$$N_t = N_{t-1} + (N_{t-1} \cdot \pi_t) = N_{t-1} \cdot (1 + \pi_t)$$

where:

π_t inflation accrued at time t

N_t principal inflation indexed at time t

Thus, cash flows at each coupon payment t are:

$$CF_t = CR \cdot \mathrm{N}_t$$

where:

CR real coupon rate of the bond

And at maturity T we simply add the indexed principal repayment:

$$CF_T = CR \cdot N_T + N_T$$

Example:
An inflation-linked bond was issued by the German government at January 2009, with a real coupon rate of 3%, face value of EUR 1000, maturing in five years. The inflation during these five years turned out to be 0.8, 1.3, 2, 2 and 1.4, respectively. Which were the cash flows of the bond using the capital indexation structure?

t	Inflation	Principal	Coupon	Cash Flow
1	0.8	1008	30.24	30.24
2	1.3	1021.10	30.63	30.63
3	2.0	1041.53	31.25	31.25
4	2.0	1062.36	31.87	31.87
5	1.4	1077.23	32.32	1109.55

Another popular indexation structure is to only index the coupons, while the principal redemption value remains constant. This structure is known as **coupon indexed**. The coupons are a variable percentage of the constant principal. Thus, the indexed coupon interest rates (ICR) are calculated simply by adding the inflation rate of the period to the coupon rate of the bond. So, at each period t are:

$$ICR_t = CR + \pi_t$$

Thus, cash flows at each coupon payment t are:

$$CF_t = ICR_t \cdot \mathrm{N}$$

Example:
An inflation-linked bond was issued by the German government the first day of 2009, with a real coupon rate of 3%, face value of EUR 1000, maturing in five years. The inflation during these five years turned out to be 0.8, 1.3, 2, 2 and 1.4, respectively. Which were the cash flows of the bond using the coupon indexed structure?

t	Inflation	Principal	Coupon	Cash Flow
1	0.8	1000	38	38
2	1.3	1000	43	43
3	2.0	1000	50	50
4	2.0	1000	50	50
5	1.4	1000	44	1044

5.2.3 Indexation lag

Ideally, bond cash flows would have to be in line with realized inflation. However, this is not possible in practice because the length of time it takes to compile data and perform calculations means that price indexes are published with a delay of a few months. So bond payments are linked to a lagged value of the price index, this is called **indexation lag**. For most countries this indexation lag is currently three months,[25] so a coupon payment of an inflation-linked bond in April is based on the inflation accrued until January.

The indexation lag causes that there is a period at the end of a bond's life when there is no inflation protection at all, counterbalanced by a period of equal length before it is issued for which inflation compensation is paid. Unless both periods have the same inflation rate, the real return of the inflation-linked bond would not be fully invariant to inflation. This problem is more relevant the longer the indexation lag and the shorter the bond's time to maturity.

Example:
As in the previous examples, assume an inflation-linked bond that was issued in January 2009, maturing in 5 years. If the indexation lag was three months, the annual cash flows would be based on the inflation accrued from October to September, both included. So, for the three months from October to December 2013, there is no inflation protection, compensated by the protection for the period from October to December 2008.

5.2.4 Deflation floor

Most inflation-linked bonds include a deflation floor, a guarantee that an inflation-linked bond's principal repayment is never less than the original par amount. So, if at maturity, the indexed principal is below par the investor will receive the original principal amount. Thus,

$$Principal\ \ repayment\ \ at\ \ maturity = Maximum\ \ (Par\ \ Value,\ \ N_T)$$

Generally, only the principal repayment at maturity is protected against deflation. That is, if during the life of the bond the price index falls below its value at the bond issuance, coupons will be paid off on sub-par principal. Australia is the only important market where coupons are also protected against deflation.

[25] In the U.K., prior to 2005, the indexation lag was eight months.

Example:
A one-year capital indexed inflation-linked bond with an annual real coupon rate of 3% is issued with a face value of EUR 1000. What would be its payoff at maturity if inflation turns out to be -1%?

Principal indexed at maturity:

$$N_T = 1000 \cdot (1 - 0.01) = \text{EUR } 990$$

So, the payoff would be:

$$CF_T = 3\% \cdot 990 + \max(1000,\ 990) = \text{EUR } 1029.7$$

A deflation floor can be thought of as a put option embedded in the bond, which costs investors a certain amount. At issuance this put is at par, and the more inflation accrued during the life of the bond, the further out-of-money it is. As long periods of deflation are rare, being Japan the exception in recent times, the longer the maturity of an inflation-linked bond, the less likely the principal would be sub-par at maturity, therefore the lower the embedded price of this option.

Currently, most of the countries include a deflation floor on their inflation-linked bonds, exceptions being for example the U.K. and Canada. Japan includes a deflation floor in its issuances since 2013.

5.2.5 Dual duration[26]

Duration is a measure of the average time for which capital is tied up in a bond and also reflects how sensitive its value is to changes in interest rates. The duration for inflation-linked bonds can be calculated in the same way as for nominal bonds, but needs to consider the indexation of the coupons and the principal. Capital indexed inflation-linked bonds tend to have higher durations than comparable bonds as principal repayment at maturity is usually larger due to its indexation and as the early coupons of inflation-linked bonds are smaller than those of traditional bonds.

Example:
A bond with a 10-year maturity pays a real annual coupon rate of 6%, and has a face value of EUR 100. The annual expected inflation equals 2%. Its yield to maturity is k=10%. What is its Macaulay duration using the coupon indexed structure and the capital indexed structure?

[26] For a mathematical approach of this concept, see Siegel and Waring (2004).

First, consider the coupon indexed bond:

T (Years) [1]	Principal	Coupon	Cash flow CF [2]	PV (CF) $[3]=[2]/(1+k)^t$	CF weight [4] = [3] / Price	Time weighted by CF weight [5] = [1] · [4]
1	100	8	8	7.27	0.0829	0.083
2	100	8	8	6.61	0.0754	0.151
3	100	8	8	6.01	0.0685	0.206
4	100	8	8	5.46	0.0623	0.249
5	100	8	8	4.97	0.0566	0.283
6	100	8	8	4.52	0.0515	0.309
7	100	8	8	4.11	0.0468	0.328
8	100	8	8	3.73	0.0425	0.340
9	100	8	8	3.39	0.0387	0.348
10	100	8	108	41.64	0.4747	4.747
			Price:	**87.71**	**Duration:**	**7.04**

The duration of the bond is 7.04 years. It is easy to see, that this structure is equivalent to a nominal bond that pays an 8% annual coupon.

Second, consider the capital indexed bond:

T (Years) [1]	Principal	Coupon	Cash flow CF [2]	PV (CF) $[3]=[2]/(1+k)^t$	CF weight [4] = [3] / Price	Time weighted by CF weight [5] = [1] · [4]
1	102	6.12	6.12	5.56	0.0636	0.064
2	104.04	6.24	6.24	5.16	0.0589	0.118
3	106.12	6.37	6.37	4.78	0.0546	0.164
4	108.24	6.49	6.49	4.44	0.0507	0.203
5	110.41	6.62	6.62	4.11	0.0470	0.235
6	112.62	6.76	6.76	3.81	0.0436	0.261
7	114.87	6.89	6.89	3.54	0.0404	0.283
8	117.17	7.03	7.03	3.28	0.0375	0.300
9	119.51	7.17	7.17	3.04	0.0347	0.313
10	121.9	7.31	129.21	49.82	0.5691	5.691
			Price:	**87.54**	**Duration:**	**7.63**

The duration of the bond is 7.63 years, which is higher than for the coupon-indexed bond and a comparable nominal bond.

However, this does not mean that the risk of price changes is higher for capital indexed inflation-linked bonds than for nominal bonds, because the duration of nominal bonds is calculated with nominal coupons, while duration of inflation-linked bonds is calculated with real coupons. Therefore, it is not possible to compare both durations. To do this comparison, it has to be taken into account that there are two types of durations, a concept known as dual duration. Real rate duration is the sensitivity to changes in the real rates, while inflation duration is the sensitivity to changes in inflation.

Nominal bonds are sensitive to both changes, therefore for nominal bonds:

$$\textit{nominal bond duration} = \textit{real duration} = \textit{inflation duration}$$

But, inflation-linked bonds are protected against price movements, so a change in inflation will not affect the market value of the bond, therefore:

$$\textit{inflation-linked bond duration} = \textit{real duration}$$
$$\textit{inflation duration} = 0$$

The best way to compare the duration of conventional and inflation linked bonds is to make a clear separation of the two types of duration calculations. The best tool for these calculations is to use key rate durations.[27]

5.3 Market situation[28]

Although the first ever inflation-link bond was issued back in 1780,[29] the market has only been growing strongly over the last ten years. In 1981, the United Kingdom was the first industrialized country to supplement its government bond issue with inflation-linked bonds. The United States with its first issuance in 1998 or Germany in 2006 are further examples of how this asset class has expanded in recent years. As of 2014, all of the G7 countries are issuing inflation-linked bonds and the number of countries issuing these securities is currently expanding with India in 2013 and Spain in 2014 being the most recent countries to join. The total market value of inflation-linked bonds issued worldwide currently amounts to around USD 2.7 trillion (see Figure 5-1), being the U.S., the U.K. and Brazil the leading countries by market value.

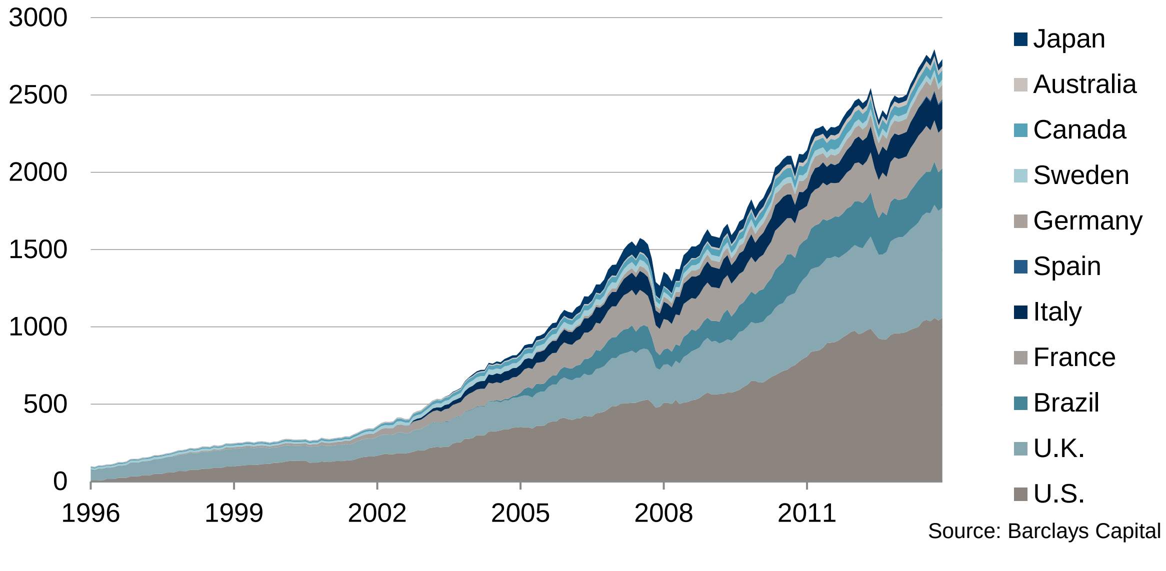

Figure 5-1: Evolution of the market of inflation-linked bonds

[27] See the module about “Interest rates - term structure and applications”.

[28] Most inflation-linked bonds are issued by countries, only a very small number is issued by companies. Thus, we concentrate on government bonds in this section.

[29] By the state of Massachusetts due to inflation caused during the American War of Independence.

Prior to 1981, inflation-linked bonds were only issued by countries that were experiencing episodes of high inflation, such as Brazil and Argentina in the 1950s and 1960s, because it was practically the only way to finance their debt over the long term. The reason was that investors wished to protect their purchasing power against the erosion of the currency's value. What are the reasons for the recent growth of this market, especially for countries where inflation seems to be under control?[30]

- The most cited explanation is to reduce the borrowing costs. This can be achieved in two ways. First, by issuing inflation-linked bonds, countries can avoid paying the inflation risk premium found on conventional bonds. Second, if the government believes that future inflation will be lower than that implied by the market, then it will expect to reduce its debt refinancing costs through inflation-linked bonds.[31]
- To reach new investor groups who are attracted to inflation-linked bonds, because, for example, their liabilities are linked to the cost of living; i.e., such as pension funds.
- To help bolt confidence in the government's fight against inflation, because the state cannot reduce the real cost of borrowing through inflation. This reason is particularly important for emerging countries.

[30] For a more complete analysis see Deacon, Derry and Mirfenderesky (2004), chapter 4.

[31] This latter reason was an important one for the issuance of the U.K. in 1981.