FIXED INCOME

GENERAL PRINCIPLES

FIXED INCOME

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1. Introduction: the debt instrument concept

1.1 Introduction

From a **legal point of view**, a bond is a **debt security**. The security is issued by a borrower as an acknowledgement of a debt. By way of the security, the borrower promises to repay the holder the amount he has lent him (represented by **the nominal or face value**) and pay him interest on this amount (**coupons**) on future dates defined at the time of issue.

The date on which the borrower agrees to repay the amount lent is known as the **maturity date** of the debt instrument.

The legal structure of a bond includes the fact that the nominal value of the bond (or the principal to be repaid at maturity) is distinct from its coupons (which are considered to be payments of interest rather than principal). As we explain below, this structure, which allows the possibility of a distinction in treatment between payments of interest and principal, can lead to arbitrage, and has in the past led to the development of some key instruments[1].

A bond is also commonly called a **note** by market participants. “Note” and “bond” are used interchangeably by bond market players to describe the same type of debt. There is no difference in either form or content between a bond and a note.

The existence of this dual terminology to describe the same type of debt is historical. Market participants customarily used the term “bond” when referring to debt securities with a maturity date of over seven years, and the term “note” when referring to debt securities with a maturity of less than seven years.

These days, a debt security is often called a “note” if it is issued under a program (MTN or EMTN) and/or if the coupon amount paid by the issuer varies over the life of the loan.

From a **financial point of view**, a bond or a note is simply a series of expected cash flows.

1.2 Economic role of bond issues

Issuing bonds is an important alternative source of finance for corporate entities institutions that want to borrow for a period of more than one year.

Instead of borrowing from banks, bond issuers turn to institutional and private investors to finance their activities over the medium or long term.

[1] Note that Swiss law recognises a debt security as a bond only if the loan meets the following criteria: “The bond issue is a loan for a large amount, portions of which are offered to a large number of creditors on identical terms. Only loans put out to public subscription or listed on the stock exchange on the strength of a prospectus are bond issues within the meaning of Art. 727, paragraph 1, number 1, letter b, Swiss Code of Obligations -CO (Art. 1156, paragraph 1 CO). The issue prospectus is public if it is sent to an unspecified circle of recipients or to a minimum of twenty recipients. Only loans for a total amount of over two million francs, divided into portions and put out to public subscription are deemed bond issues”.

When issuing bonds or notes, their issuers obtain financing for a period of more than one year what distinguishes these instruments from euro commercial paper (ECP) and/or T-Bills, for example.

With a view to accounting, the amount borrowed by the bond issuer appears under liabilities in its balance sheet.

Holding bonds gives investors (bondholders) a series of cash flows over a predefined period. The timing of these cash flows is determined at the time of issue in the loan contract (**indenture**).

Investors may be households (retail investors) or institutional investors such as pension funds, banks, commercial enterprises, insurance companies or investment funds.

The decision to invest in the issuer's bonds rather than its equities (shares) is strategic and takes account of several factors. For example, bondholders will be better protected than the holders of equities (shareholders) should the issuer go into liquidation or become insolvent. In fact, as bondholders are creditors of the company, they will have priority over the shareholders for repayment of their claim.

With a view to accounting, an investor's bonds position is reported under assets in its balance sheet.

It is very important for an economy to have an efficient **domestic bond market** as it helps reduce issuers' dependence on banks and foreign countries to finance themselves while offering private and institutional investors a long-term investment vehicle.

1.3 Bond issuers

Bond issuers are institutions under public or private law who wish to obtain financing in the medium or long term.

For investors, the issuer's creditworthiness is a very important characteristic of a bond as it helps them assess the probability of receiving the cash flow amounts the issuer has committed to pay.

The following table shows the structure of the global bond market by regions and main issuer segments:

Country	Total	%	Government	Financial Institutions	Corporates
United States	35113	43.0%	15210	14857	5046
Euro Area	19621	24.0%	9247	8967	1407
Japan	12063	14.8%	8976	2383	703
Other Mature Markets	9405	11.5%	4766	3270	1371
Emerging Markets	5508	6.7%	3209	3731	1480
Total	81710		41408	33208	10007
%			50.7%	40.6%	12.2%
(in billions of US dollars)					
Source: BIS, Quartarly Review March 2015					

The largest issuers are sovereign states. They issue bonds to finance their expenditure. Sovereigns are large issuers because the general government debt as a percentage of gross domestic product (GDP) even for developed nations in many cases is above 60% and as high as approx. 200% in the case of Japan. In financial terms, such bonds are known as **government bonds**. These issuers' creditworthiness depends on their ability to subsequently raise taxes and/or borrow again in the future. The US and Japan are the two largest issuers – emerging markets still are relatively small issuers, at least in their own currencies (as shown in the table above). Emerging market issuers still finance a large part of their needs in hard currencies (mainly USD and to some extent EUR).

Provincial or State bonds are issued by sub-national government entities, such as Canadian Provinces, Australian States, Swiss Cantons. Provinces and States are assumed also to have tax-raising powers to support the bond repayments. A special case are municipal bonds in the US since they are tax-exempt for certain investors. They form a sub-unit of the US bond market known as the ***municipal bond market***.

Government agency bonds are issued by government-related or government-sponsored enterprises (GSE), such as Fannie Mae or Freddie Mac in the US. They do not usually benefit from a direct government guarantee.

A government agency is a dedicated autonomous structure for implementing a clearly defined policy.

Supranational agency bonds are issued by a small number of entities, usually banks, which are themselves owned by a number of sovereign states. Examples are the International Bank for Reconstruction and Development (otherwise known as the World Bank), the European Investment Bank, the Asian Development Bank, the African Development Bank, and the Inter-American Development Bank. They issue in accordance with their statutes, and in the course of normal banking business, but their governmental shareholding structure is generally perceived as providing extra security.

It may be that the government guarantees the payments of bonds issued by other institutions. In financial jargon, such bonds are known as **Government guaranteed bonds**. It is assumed that the entity has the ability to generate its own repayments, but the state guarantee is an additional safeguard.

Corporate bonds are issued by corporations (public or private).

Corporate bonds can be classified by the nature of the issuer - financial and non-financial corporates are the two main segments. Within these the traditional classification is: utilities; transportation; industrials; banks and finance companies, where industrials include not only manufacturing companies but also service and commercial companies. There are numerous sub-classifications such as oil and gas, telecommunications, construction, insurance and so on. Various sector classifications systems are maintained by ratings agencies and index providers. For non-financial corporates the US has historically been the most important market and even today still is dominant in terms of size (see table above).

Though it is generally assumed that the repayments of a corporate bond will be supported by the business activities of the issuer, the credit quality of corporate bonds can be very variable. Useful information about the credit quality of a bond at the moment of issuance can be derived from the "offer document" in which the issuer provides details about its creditworthiness.

As time passes that information becomes outdated and the investor has to rely on ratings assigned by rating agencies that assign ratings to bonds when they are issued and constantly monitor credit quality during the bond's life. Rating is a synthetic grade given to a bond's issue and based on an analysis of the issuer's financial condition and management, economic and debt characteristics as well as the revenue sources securing the bond.

Bonds are said to be **secured** if they are guaranteed by specific assets of the issuer. If the issuer becomes insolvent, the secured bondholders have access to the assets pledged by the issuer (collateral) as a means of repaying of their claims. The amount that secured bondholders will recover depends on the wind-up value of the collateral. It therefore does not depend on the ranking of their claim in the creditor compensation process should the issuer fail.

Covered bonds[2] are examples of secured bonds. Financial institutions (such as banks) issue covered bonds to finance property or financial assets.

By investing in covered bonds, investors benefit from the issuer's direct guarantee, but also from the pool of assets (covered pool) pledged, whether in terms of the value of the assets or the cash flows received from them.

The cover pool is usually dynamic and made up of assets which are separated from the other assets on the issuer's balance sheet should it become insolvent. The assets making up the cover pool are usually residential or commercial mortgages, but also government bonds.

One of the special features of covered bonds is that the issuer's insolvency does not automatically affect the covered bondholders. If the issuer is insolvent, the cover pool becomes static and is put up for sale. If the sum recovered is enough to repay the principal and interest, the covered bondholders cannot take recourse to the acceleration clause which provides that the amount of interest and principal becomes payable immediately should the issuer become insolvent.

Unsecured bonds, also known as **debentures**, are not guaranteed by the issuer's tangible or financial assets. If the issuer fails, unsecured bondholders only have a right to the wind-up value of any of the issuer's assets that have not been pledged to secure its other debts.

Issuers borrow by means of unsecured bonds because they do not usually have enough assets to secure their entire debt. As unsecured bondholders take on more risk, the coupon rate paid on unsecured issues is higher than on secured issues.

Note that in case of insolvency, the company will be liquidated under the insolvency laws in force in the issuer's country of residence.

2 This type of bond is used extensively by issuers in Germany and in Switzerland under the name "Pfandbrief" (which is a special type of a covered mortgage bond). In Switzerland, covered mortgage bonds are issued by the Swiss cantonal banks' central mortgage bond institutions (centrales des lettres de gage, centrali delle banche per le obbligazioni fondiarie, Pfandbriefzentralen) and the Swiss mortgage institutions' mortgage bank (banque des lettres de gage, banca di obbligazioni fondiarie, Pfandbriefbank). The objective of these issues is to obtain liquidity from investors to refinance banks that grant mortgages. Note that covered bonds have particularly extensive guarantees and are governed by law.

In most cases, an official receiver is appointed to sell the company's assets making up the insolvency estate. However, it is very unusual for the income from the sale to be enough to cover all the company's debts. In fact, apart from the risk linked to the assets' value, the administrative costs of insolvency proceedings (administration, lawyers, etc.) are usually very high. A typical recovery rate (what is left for investors after insolvency) is approx. 40%.

As an example, the Enron insolvency cost USD 159 million in liquidation expenses. As to the Lehman insolvency, experts estimate that over USD 200 million in winding-up costs could be incurred, which would make it the most expensive insolvency in history.

As it is very likely that the income from the sale will not be enough to repay all creditors after liquidation expenses have been deducted, national regulations usually give priority ranking to the repayment of certain types of debt such as employees' salaries, social security contributions or taxes.

These are known as **senior** claims in relation to the company's other debts. It is therefore only after creditors with legal priority have been repaid in full that unsecured bondholders can be paid[3].

To protect unsecured bondholders from any degradation in their creditor status that could result from subsequent secured issues, restrictive provisions known as negative pledge provisions are usually inserted in unsecured issue contracts. We shall come back to theses restrictive provisions later in this document.

Corporate bonds from the same issuer can therefore range from **senior** (ranking uppermost in the list of creditors in the event of default), and **subordinated** (for example, to senior notes). There are numerous other levels of ranking that can be interposed among those stated, such as **junior subordinated**.

For financial issuers the landscape with regards to debt ranking is in total change after the financial crisis. Various new legislations that allow for "bail in" changes the rules of debt ranking in stress situations. Even senior bonds can in certain cases end up being highly subordinate.

Contingent convertible bonds (CoCos) are the newest concept of subordination. These debt instruments are automatically converted into common stock or written off if a predefined trigger event (too low capital ratios or regulatory concerns on the viability of the issuer) occurs. In a certain sense they are even subordinate to equity instruments – a contradiction to the standard concept of bond instruments. (See also the chapter "Hybrid Forms")

In the context of structured debt instruments market players often use the term **mezzanine** to refer to a subordinated debt with the characteristics of both debt and equities. The creditor of such a debt will find itself between the senior creditors and the shareholders should the company fail[4].

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3 It should be noted that banks give companies loans on condition that they have priority in case of insolvency (senior). Consequently, it is only after bank loans have been repaid that the unsecured bondholders can recover their debts, depending on the subordination ranking of their bond.

4 Note that in some cases, banks have the right to consider their mezzanine debt as forming part of their statutory capital.

Finally, bonds may be issued by **Special Purpose Vehicles**[5] (SPV). Unlike commercial enterprises, these entities have no commercial activity. They are usually set up in offshore jurisdictions that have a low (or even no) rate of corporate income tax and where setting up a company is very quick and cheap. Such companies may also be established on-shore depending on the rules set out in the country's regulations for this type of enterprise.

An SPV's objective is to acquire assets (usually financial) from an **originator** (which may or may not be a bank). These assets may be residential or commercial mortgages, bonds issued by commercial enterprises, consumer loans, or any other asset on which there will be a future payment flow. By issuing bonds, the SPV obtains cash for its acquisitions.

Because an SPV has no commercial activity, the payment of coupons and redemption of the bonds it has issued depend solely on cash flows and the value of the assets acquired by the SPV plus the debt's degree of seniority.

In financial terminology, these bonds are known as **asset backed securities** (ABS). If the SPV's assets are mortgages, we call them **mortgage backed securities**. A distinction is customarily made between commercial and residential mortgages. The term **RMBS** is used if the assets backing the bond are residential mortgages, or **CMBS** if the mortgages are commercial. We shall address ABSs in detail in a separate chapter.

Note that a bond issue may have a payment guarantee provision granted by a third party institution. An issue with this type of provision is known as a **guaranteed bond**. This situation often arises if the parent company of a group of companies guarantees notes issued by its subsidiaries.

The issue will also be **guaranteed** if the issuer has bought insurance against its own payment default from an insurance company.

1.4 Bond characteristics

To respond simultaneously to investors' requirements and their own financing needs, bond issuers (with the assistance of their investment bank) have been and continue to be very creative. It is therefore extremely difficult, or even impossible, to produce an exhaustive inventory of all the types of bond on the market.

The objective of this section is to familiarise the reader with the different characteristics of bonds, in other words the possibilities that may exist to define the cash flows that will be paid on a bond. Accordingly, we have adopted the criteria used by the International Monetary Fund (IMF) when it publishes its statistics.

Using this approach, we can define a bond as a set of contractual provisions that will legally determine the structure and amount of the cash flows the issuer of the loan will pay to the bondholders.

These provisions appear in the **indenture**, which is the formal contract of the bond issue, and appear in full in the issue **prospectus** produced for the attention of investors.

[5] These companies are known as Special Purpose Entities (SPE).

Within the prospectus an important feature are the bond covenants. They specify a promise that certain activities will or will not be carried out. Lower rated bonds carry a higher default risk. Due to that they often have more covenants in order to better protect investors. The strictness of covenants is cyclical – in strong markets issuers have to offer less covenant protection to investors than in more difficult market environments. Covenants are the result of a market driven bargaining process between the issuer and the investors.

Typical covenants for investment grade bonds are:

Pari passu: the term is used to refer to claims that have the same subordination or seniority ranking, in other words that will be treated in the same way should the issuer. Having this clause in the prospectus the investor makes sure that no other bonds are treated in a senior manner in the case of a default.

Negative pledge: an indenture stating that the corporation will not pledge any of its assets if doing so gives the lenders less security. By including a negative pledge clause, the bondholders of the current bond issue prevent the issuance of any debt in the future that would jeopardize their current priority claim on the company's assets.

Cross default: makes sure that an issuer can't selectively default on one bond alone. If one bond defaults all bonds of the same issuer are set into default and protects investors from unequal treatment.

Change of control: protects investors in the case of a change of control. It allows investors to sell their bonds back to the issuer at par value, if the company concerned is acquired (poison put).

Lower rated bonds have additional covenants that restrict the flexibility of the issuing company in order to protect the bond holders. Examples are: limitation on additional indebtedness, limitation on restricted payments (restricts share buy backs or cash moves to other subsidiaries), limitations on liens (restricts pledge of assets), limitation on sales of assets (describes permitted use of proceeds from asset sales).

1.4.1 Bond maturity

A bond's *maturity* refers to the contractual date set for repayment of the bond's nominal value.

The maturity date may be firm or may depend on economic, legal or fiscal conditions set at the time of issue.

In financial jargon, a bond with a fixed maturity is known as a **bullet bond**. The period of a *bullet* bond may be between one year and over one hundred years.

For example, in 1993 Coca-Cola issued a *bullet* bond for USD 150 million with a fixed maturity date of 2093, in other words a period of one hundred years.

On 1 March 2009, the municipality of New York repaid USD 1,000 to the holder of a bullet bond issued 135 years earlier. That municipal bond issue had financed a road to the Bronx. Throughout those years, the municipality of New York paid interest at 7 percent on the issue. Thirty-eight other holders of the same bond issue should be repaid by the municipality of New York between 2009 and 2147!

Note that in most cases, the period of a *bullet* loan is more likely to be between one and thirty years.

In practice, it is usual to regard a bond with a maturity of between one and five years as “short term”, a bond with a maturity of between five and twelve years as “medium term” and a bond with a maturity of more than twelve years as “long term”.

Bullet bonds will therefore be repaid **as a lump sum**, in other words a single payment is made on the maturity date to repay the loan.

The number of years, months or days remaining before the issuer repays the bond is known as the **residual maturity**.

For example, a bond issued on 13 July 2009 with a maturity date of 13 July 2019 will have a residual maturity of nine years on 13 July 2010.

In most cases, the bond’s maturity clause specifies the exact due date. However this clause may also indicate only the maturity month rather than the exact date.

In the latter case, it is important for investors to find out how many days notice the issuer must give bondholders regarding the repayment of the loan in the maturity month.

Bonds may also be issued without a maturity date. These are known as **perpetual bonds**. This applies to UK *consols,* for example.

Consols are bonds issued by the UK government in the 18th century. Although they have an early redemption provision, they will certainly never be repaid given the low interest rate (2.5 percent) that the UK government has to pay on its borrowing.

Nowadays, banks issue most perpetual bonds with early redemption provisions in favour of the issuer. The aim was to have legally speaking long term financing for regulatory capital but on the other hand to have economically cheaper short to medium term financing. After long years regulators have restricted the option of the issuing banks, but still almost all perpetual bank bonds have call provisions in favour of the issues even today. Therefore, these are at least in normal times not real perpetual bonds. For example Allianz in Swiss franc 3.25% perpetual CHF 500m – first call 4.7.2019.

As we shall see later, the bank’s main reason for issuing this type of loan is to have long-term financing, as it is the case with the issue of equities (shares), and include the amount borrowed as part of the *regulatory capital* while enjoying tax deductions on the coupons paid[6].

The issuer may also include **sinking-fund provisions** in the loan conditions. These provisions require the issuer to retire a portion of the outstanding debt, designated as the **sinker percentage**, each year. The sinking fund provisions can be satisfied in many different ways: the retired bonds can be purchased on the open market, or the portion to be redeemed can be selected by a lottery, etc.

[6] The Basel II regulations require banks to have enough capital to support the risks of their activities.

Example 1: Sinking-fund provision

An investor can buy a 5 percent coupon, EUR 1000 nominal value, ten-year bond with a quoted price of 102.00. The bond has a 90 percent sinker[7] at par value. What are the annual cash flows paid by the bond?

The sinking funds payments should commence at the end of the first year, repaying 10 percent of the bonds annually through the nine years. The final 10 percent repayment will be at the maturity date. All repayments are made at par value.

The annual cash flows are as follows:

Time	Total amount due	Interest payment (5%)	Sinking payment	Principal repayment	Total annual payment
		[1]	[2]	[3]	[1] + [2] + [3]
1	1000 EUR	50 EUR	100 EUR		150 EUR
2	900 EUR	45 EUR	100 EUR		145 EUR
3	800 EUR	40 EUR	100 EUR		140 EUR
4	700 EUR	35 EUR	100 EUR		135 EUR
5	600 EUR	30 EUR	100 EUR		130 EUR
6	500 EUR	25 EUR	100 EUR		125 EUR
7	400 EUR	20 EUR	100 EUR		120 EUR
8	300 EUR	15 EUR	100 EUR		115 EUR
9	200 EUR	10 EUR	100 EUR		110 EUR
10	100 EUR	5 EUR		100 EUR	105 EUR

Some of the issue contract provisions may provide for early or deferred redemption of the issue at the option of the issuer and/or bondholder.

A **callable bond** gives the issuer the right to repurchase the bond at a pre-determined price (called the call price), at a certain time (called the call date). The call price is often the par value plus a premium (called the call premium). The earliest call date and corresponding call price are specified when the bonds are issued.

Example 2: Callable bonds

Bond 1:
Ineos Group Holdings Plc has issued in 2006 a EUR 1.75 billion bond with coupon 7.875 percent, maturity 15 February 2016 callable on the dates shown (European call) at the following prices:

On 15 February 2011	103.938
On 15 February 2012	102.625
On 15 February 2013	101.313
On 15 February 2014	100

Bond 2:
UBS AG/Jersey Branch has issued in 2006 a GBP 300 million bond with coupon 5.25 percent, maturity 21 June 2021 callable on and anytime after the dates shown at the following prices:

on 6/21/16 at 100
on 6/21/17 at 100
on 6/21/18 at 100
on 6/21/19 at 100
on 6/21/20 at 100

The first bond is only callable on set dates, and at no time in between. Such a call is known as a European call, as noted in the text of the example. The second bond carries a call feature allowing the issuer to call the bond at par at any time starting from a certain date. Such a "continuous" call is known as an American call.

If the interest rates fall substantially, issuer of callable bonds can exercise the call option and get refinancing at lower coupon rates. Hence, it protects the issuer from being compelled to continue paying high coupons if interest rates drop.

A call provision reduces the value of the bond to an investor because it does not give the investor the advantage of enjoying higher returns when interest rates drop. For example, if a company has issued a bond with a 10 percent coupon several years ago and can now borrow money at 5 percent, the call provision gives the company an opportunity to refinance the bonds and save on interest payments. (See also the chapter "Hybrid Forms")

7 The sinker percentage is the percentage of bonds retired before maturity. In this case, the annual retirement is 10 percent of the nominal value (= 90% / (10 – 1)).

Putable bonds are just the reverse of callable bonds. In putable bonds, the bondholder has right to sell the bond to the issuer at one or several set dates at pre-determined prices. Just so, a put provision increases the value of the bond to an investor, since it provides the investor with protection against the bond trading below the put price.

Variable rate demand bonds (VRDB) are debt instruments that usually have long maturities but may be redeemed early at the request of the investor or of the issuer at any time during the bond's life, usually giving seven days' notice. These are therefore bonds containing put and call provisions of the American type. (See also the chapter "Hybrid Forms")

Retractable bonds are bonds carrying the option (for both the issuer and the investor) for early redemption at one or several fixed dates. They differ from a VRDB in that redemption requests cannot be made at any time during the life of the bond but only on the dates fixed at the time of issue.

Some bond issues contain provisions giving the issuer or bondholders the right to extend the loan by changing the bond's maturity date after it has been issued. These bonds are known as **extendable bonds**. If this right is exercised, the bond continues to offer the same payment terms. An issuer or the bondholder will exercise this right if interest rate conditions at maturity are unfavourable[8] compared with the current terms of the bond.

1.4.2 Interest rate

An important provision of the bond relates to the way the bondholder is paid by the issuer.

A bond's income is known as the **coupon**. The coupon amount is determined by the bond's coupon rate (interest rate). It may be paid annually, semi-annually, quarterly or at any other interval defined at the time of issue.

The periodic coupon amount is calculated as follows:

$$\frac{\text{Nominal value x interest rate (as a percentage)}}{\text{Annual divisor}} \text{ x number of days}$$

Example 3: Coupon calculation

Nominal value: USD 5,000
Coupon rate (annual): 2%
Semi-annual coupon, number of days: 180
Annual divisor: 360

A bond's coupon rate may be **fixed** or **variable** throughout the life of the loan, it may be fixed for a certain period and then variable, it may be variable and determined according to the value of an index or, generally, it may have any property that meets the needs of investors and/or issuers.

8 As we shall see, this applies if interest rates on the market are higher and/or the issuer creditworthiness has declined.

Note that structured products are usually issued in bond form (under the name *notes* or *certificates*) by a bank, a specially formed commercial company or an SPV.

These companies have highly creative payment structures for bond issues, and it is very difficult, indeed impossible, to draw up an exhaustive inventory of all the provisions that exist in practice to determine the coupon amount. Consequently, the object of this section can only be to familiarise the reader with the diversity of provisions for determining the coupon.

A **fixed-rate bond** is a bond with an interest rate that is set in advance for the entire period of the loan. The rate is determined at the time of issue.

The value of this rate may be fixed over the full loan period, for example 3 percent for the entire life of the loan, or it may be changed depending on the period under consideration. For example, the value of the coupon rate of a ten-year bond may be 2 percent for the first two years, then 4 percent for the next five years, and 3 percent for the three remaining years. All these values are set out in a contract provision at the time of issue.

A bond that does not have an early redemption provision and pays a coupon rate set at the time of issue that does not change throughout the life of the loan, is known as a **straight bond**.

Medium-term notes[9] are examples of straight bonds. They are issued by banks and usually have a maturity of between two and eight years. These bonds pay the same coupon rate throughout the life of the loan. The rate is determined at the time of issue.

Step-up bonds are bonds with an interest rate fixed on issue, but which increases over time. For example, a step-up bond with a ten-year maturity may pay 3 percent for the first five years, 3.2 percent for the next three years and 3.4 percent for the last two years. The cash flow of these bonds is therefore known in advance and increases over time.

Step-down bonds are bonds with an interest rate fixed on issue, but which decreases over time. For example a step-down bond with a ten-year maturity may pay 3 percent for the first five years, 2.8 percent for the next four years and 2.5 percent for the last year. The cash flow of these bonds is therefore known in advance and decreases over time.

One of the major disadvantages for issuers when the interest rate is fixed at the time of issue is that they may not benefit immediately from lower costs if interest rates fall on the market or if their creditworthiness improves over time[10].

In practice, issuers protect themselves by including a **call** provision allowing them to repay the loan early if market conditions are favourable. However, to exercise the call, issuers must have enough liquidity. If the funds raised from the loan have enabled them to finance a long-term investment, it is highly unlikely that the issuers will have enough liquidity to exercise their option.

9 We shall see that Swiss Federal law makes a clear distinction between a bond and a medium-term note.

10 The remuneration required by investors depends on the risk they take by investing in an instrument. All other things being equal, an improvement in the issuer's creditworthiness will reduce the remuneration required by investors. In other words, the issuer may offer a lower coupon rate if its creditworthiness improves.

In practice, issuers usually float a new loan under new terms and use the funds raised to repay the initial loan. However, as we shall see in detail, the costs associated with a loan issue are usually very high, thus reducing the benefits of early redemption of the initial bond.

An alternative to bonds with rates fixed in advance is to issue floating rate bonds.

Floating rate notes (FRN) or **floaters** are bonds with a coupon rate that varies over the life of the loan and is adjusted periodically based on a reference interest rate or index observed on the adjustment date.

The timing of the coupon rate adjustment and the formula for calculating the new rate are set at the time of issue and described in a contract provision.

For example, the bond provision may provide for the coupon rate to be adjusted every three months with a calculation formula based on the three-month USD Libor rate plus 50 bp.

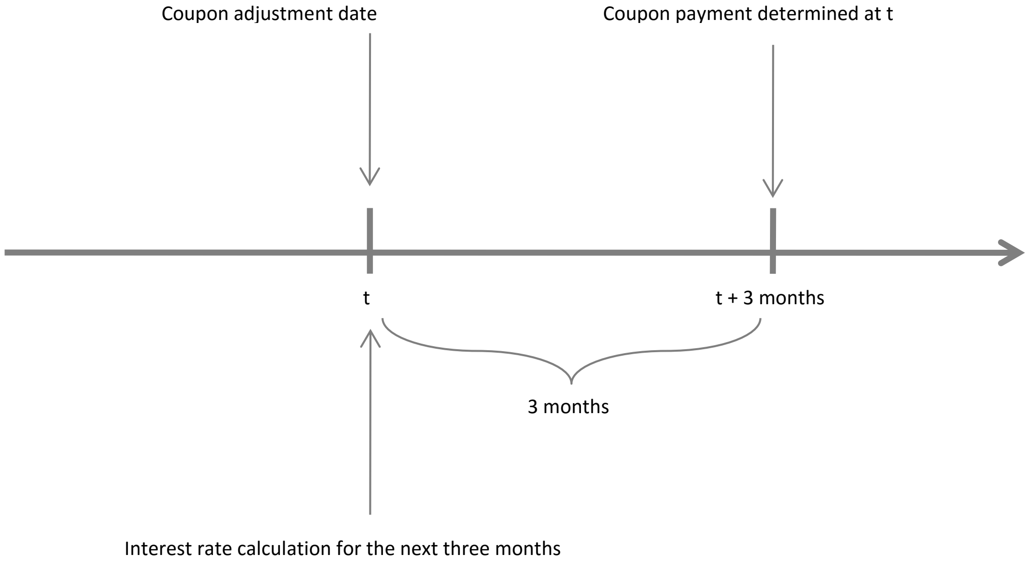

Figure 1-1: FRN coupon review and payment

In practice, the exact coupon rate adjustment dates are described in a contract provision, for example every 28th of January, April, July and October. Coupon rates are therefore adjusted in relation to the reference interest rate observed on those dates.

However, it is possible that one of these dates may fall on a bank holiday during the life of the loan and therefore the reference interest rate cannot be calculated on that date. It is therefore important for the contract to make provision for alternative measures, known as business day conventions.

A business day is a day on which the commercial banks and foreign exchange market are open for transactions and to settle payments.

Some participants may define a business day in their contracts as one when a clearing system or a particular stock exchange is open. For example, the TARGET day (Trans-European Automated Real-Time Gross settlement Express Transfer system), the NYSE day or the FED day.

The business day conventions used and mentioned specifically in the loan contract are:

Following business day convention: the adjustment date will be the first following business day.

Modified following business day convention (or simply “modified”): the adjustment date will be the first following business day unless that day falls on the first day of the following month. If it does, the adjustment date will be the preceding business day.

For example, if the coupon rate adjustment date is 28 February, and this date falls on a Sunday, the coupon adjustment date will be Friday 26 February as the following business day is the first day of the following month, or 1 March.

Preceding business day convention: the coupon rate adjustment date is the first preceding business day.

The rates usually used by issuers as a reference are the interest rates prevailing on the interbank market. These rates are usually based on the London Interbank Offered Rate (LIBOR), the London Interbank Bid Rate (LIBID), the London Interbank Mean Rate (LIMEAN, which is an average of the LIBOR and LIBID rates) or the Euribor for issues in euros.

One of the main disadvantages of variable-rate bonds for issuers is that they risk an increase in their financing costs if the reference rate increases over time. To reduce this risk, issuers may lay down contractual provisions which limit any increase in the coupon amount that must be paid.

For investors, one of the major disadvantages of variable rate bonds is that they risk a fall in the amounts received over the life of the bond if the reference rate falls over time. To reduce this risk, issuers may include contractual provisions that limit any reduction in the coupon amount that will be paid to investors. By so doing, they ensure that they meet investors’ needs and obtain the financing they seek.

The **drop-lock bond** (DL bond) combines the features of both floating- and fixed-rate securities. The DL bond is issued with a floating-rate interest that is reset semi-annually at a specified margin above a base rate, such as six months LIBOR. This continues until the base rate is at or below a specified trigger rate on an interest fixing date or, in some cases, on two consecutive interest-fixing dates. At that time the interest rate becomes fixed at a specified rate for the remaining life of the bond[11]. This provision therefore limits the risk that the coupon may fall below a certain threshold. It is therefore favourable (unfavourable) to the investor (issuer).

Capped FRNs are FRNs with a variable coupon rate but a provision stating the maximum value of this rate for the life of the bond. This provision therefore helps limit the risk that the coupon may rise above a certain threshold. It is therefore favourable (unfavourable) to the issuer (investor).

[11] Source: Definition taken entirely from the International Monetary Fund (IMF).

Collared (mini-max) FRNs are FRNs with a variable coupon rate but a provision stating the minimum and maximum values of the rate for the life of the bond. This type of provision therefore limits the variation of the coupon rate over a given interval and allows issuer and investors to reduce their interest rate risk.

A **ratchet bond** is an FRN with a coupon rate that may vary but only downwards. The contract provisions for this type of bond generally stipulate paying a coupon rate fixed at issue for an initial period, for example 3 percent for the first two years. Once this period has elapsed, the coupon rate is periodically adjusted based on a reference interest rate plus a margin fixed on issue.

If the rate calculated is lower than the rate fixed initially, for example 1 percent, the bond's coupon rate is revised to equal the rate calculated, 1 percent in our example. Conversely, if the rate calculated is higher than the rate fixed initially, for example 4 percent, the coupon rate is not revised. The issuer continues to pay the rate fixed initially, 3 percent in our example, until the next review date.

Note that this provision is clearly to the advantage of the issuer. It enables it to know in advance the coupon amount that will have to be paid on the loan while enjoying better conditions on the market. It is therefore an excellent alternative to a fixed-rate bond issue with an early redemption call provision. In fact, the issuer may benefit from a fall in the rate without having to support the administrative costs of a new issue which is usually required to finance repayment of the initial loan.

Because this ratchet provision is not advantageous to investors, the bond issue contract usually contains a put provision which gives investors the right to demand repayment of their bonds on specific dates but only if the coupon rate has been revised.

Convertible floating rate notes are FRNs with provisions giving the issuer or investors the right to convert a fixed-rate bond to a variable-rate bond or a variable-rate bond into a fixed-rate bond.

Auction rate securities (ARS) are FRNs with a variable interest rate the value of which is calculated periodically by an auction process. These auctions are usually organised every 7, 28, or 35 days. The issuer pays the coupon amount on the same dates. (See also the chapter "Hybrid Forms")

Generally, the interest payment period on an FRN coincides with the coupon rate review period. For example, and as illustrated in Figure 1-1: above, an FRN indexed to the 3-month Libor will find its rate revised at "t" for a coupon payment in t+3 months. The coupon amount that will be paid in t+3months will therefore be based on the 3-month Libor noted at "t" for a period of three months[12].

Nevertheless, some FRNs have provisions in which the coupon rate review dates are more frequent than the coupon payment dates. These FRNs are known as **mismatch FRNs**.

For example, an FRN with a coupon payment every three months and a coupon rate that is revised every month based on the 1-month Libor is considered a *mismatch*. This situation is illustrated in diagram 2 below.

[12] The term "in arrears" is often used to refer to a coupon payment for a period based on a rate determined at the beginning of that period.

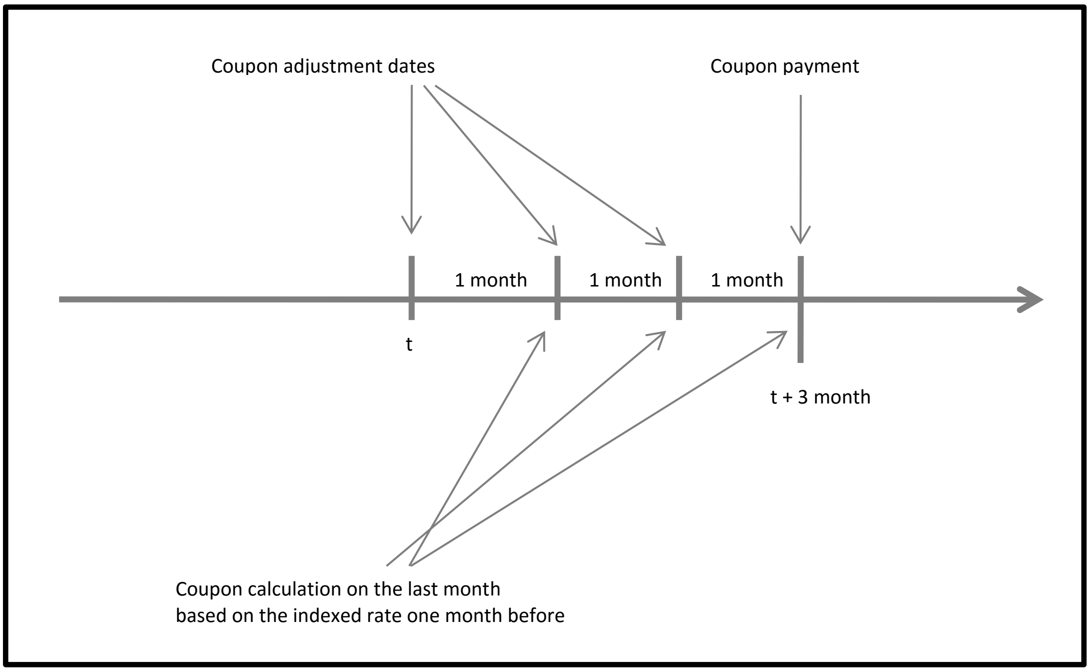

Figure 1-2: mismatch FRN coupon review and payment

The advantage for investors is that the coupon paid quarterly on a *mismatch* FRN may be higher than that paid on a *normal* FRN if the 1-month Libor rates rise. The advantage is the same for the issuer if the 1-month Libor rates fall.

Another example of a *mismatch* FRN is an ARS with daily auctions but coupon payment at the beginning of each month.

A **flip-flop note** is a debt instrument that contains a provision allowing its holder to switch between a fixed-rate instrument and a variable-rate instrument.

If the coupon rate on the variable-rate instrument is higher than the coupon rate on the fixed-rate instrument, the holder will choose the variable-rate instrument.

If the coupon rate on the variable-rate instrument is lower than the coupon rate on the fixed-rate instrument, the holder will choose the fixed-rate instrument.

This type of provision is therefore equivalent to setting a minimum coupon rate provision on a FRN.

Reverse FRNs, also known as **reverse floaters or inverse floaters**, are bonds with a coupon rate whose value increases (reduces) when the value of the reference rate or index reduces (increases).

In practice, a fixed rate is determined at the time of issue and serves as a basis for calculating the inverse floater's coupon rate. This rate will be equal to the fixed rate less the floating reference rate – typically a Libor rate.

<u>Example</u>: A reverse FRN revised quarterly on the basis of the 3-month Libor. The fixed rate that was determined at the time of issue is 6 percent. Supposing the 3-month Libor rate on the issue date is 3 percent, the coupon rate that will be paid for the first three months is 3 percent (6 to 3 percent).

On the next review date, in other words three months after the issue, we find that the 3-month Libor rate is 4 percent. The coupon rate that will be paid for the next three months will be 2 percent (6 to 4 percent).

There is no formal rule for the formula for calculating the bond's coupon rate. The generic formula for a Reverse FRN is as follows:

bond interest rate = [fixed rate – multiple x (floating rate)].

The values for the fixed rate and the multiple applied to the reference rate are usually negotiated between the issuer and the investors at the time of issue. The higher the value of the multiple, the greater the impact of a change in the reference rates on the bond's coupon rate, all other things being equal.

Income bonds are bonds that pay interest only if profits of the issuing entity are adequate enough to pay interest; therefore, a missed coupon payment in the case of income bonds is not viewed as a default. Some income bonds are cumulative, which means that if a coupon payment is missed, it must be made up before dividends are paid to shareholders. This structure is extremely closely related to that of preferred or preference shares.

Zero-coupon bonds do not pay any interest during their life. **Deep-discount bonds** generally mature at nominal value, but are issued at a large discount to the nominal value. Hence, the return on such a bond is derived from the difference between the issue price and the nominal value. **Capital Growth Bonds** are issued at par (100 percent) with redemption at a multiple of that amount.

These zero-coupon bonds may be issued *directly*, in other words to finance the issuer's activities or *indirectly*, in other words based on an existing fixed-rate bond.

For example, on 18 February 2009, the European Bank for Reconstruction and Development issued a zero-coupon bond worth USD 10 million with a maturity date of 18 February 2039. This bond was issued at par and has an annual call redemption provision. The capital that will be repaid by the issuer is calculated using an accrued interest rate of 5.75 percent per year. Thus, the lump sum that will be repaid to the creditors will be USD 53,507,083.72[13] on 18 February 2039.

This zero-coupon bond issue was therefore issued *directly* by the European Bank for Reconstruction and Development to finance its activities.

Zero-coupon bonds may also be issued *indirectly* by an investment bank by **stripping** an existing bond which pays fixed coupons throughout the life of the loan. These are known as **stripped bonds** and are usually issued to meet the investment needs of financial players.

Bonds are stripped as follows. The investment bank buys a large number of top quality bonds on its own account, usually issued by the government and with no early redemption (call) provision. It then uses the cash flows received on this bond to issue zero-coupon bonds, with identical maturity dates to the payment dates of the bond that pays the fixed coupon.

For example, a government bond with a ten-year maturity, a nominal value of USD 1000 and an interest rate of 5 percent paid semi-annually may be broken down into twenty-one zero-coupon bond issues.

[13] The detailed calculation is as follows: $10{,}000{,}000 \times (1 + 0.0575)^{30} = 53{,}507{,}083.72$

The first twenty zero-coupon issues will have a nominal value of USD 50 (corresponding to the coupon amount paid on the government bond) and a maturity date identical to one of the coupon payment dates on the government bond. The first issue will thus have a six-month maturity, the second a one-year maturity, the third an 18-month maturity, and so on.

The twentiest zero-coupon bond will have a nominal value of USD 1000 and a maturity date identical to the maturity date of the government bond.

To begin with, these zero-coupon bonds created by the investment banks and called **treasury investment growth receipts** (TIGRs)[14], **certificates of accrual on treasury securities** (CATS), **certificate of government receipts** (COUGRs), **Lehman investment opportunity note** (LIONs) were very successful with investors.

Since 1985, the US American government has had its own **strips programme**. Under this programme, a financial institution may present a standard bond issued by the government to the US American Treasury for stripping into **treasury strips**. The US American Treasury will therefore strip the bond by separating the individual cash flows to create zero-coupon bonds as explained earlier. Each treasury strip thus has its own security number and the issuer remains the government.

Stripped bonds also exist on the French government bonds market and on the UK gilt-edged securities (Gilts) market, to quote just these examples. As well as being a useful instrument in practice, the concept of a zero-coupon bond is central to the understanding of financial analysis, and key to financial engineering: this should become apparent when one realises that all bonds are composed of discrete cash flows, each one of which can be imagined as being a single zero-coupon bond. This leads to a number of further concepts, such as the spot curve, which will be explored in greater detail in the chapter “Interest rates - term structure and applications”

Inflation-linked bonds pay a coupon rate that is adjusted for inflation according to a specified inflation index. The main advantage of this type of indexation is that it protects investors from a loss of buying power on the coupons received due to a general rise in prices. In this sense inflation-linked bonds are the safest and lowest risk bond investments (See also the chapter “Hybrid Forms”)

1.4.3 Currency

Another important provision concerns the **loan repayment currency** and the **coupon payment currency**.

In most cases, the repayment currency and the coupon payment currency are identical. For example, if the issuer borrowed CHF, it will repay its loan in CHF and pay the coupons in CHF.

Bonds where the loan repayment currency and the coupon payment currency are different are known as **dual currency bonds**.

[14] The first bank to offer this type of bond was Merrill Lynch in 1982.

As an example, **foreign interest payment security** (FIPS)[15] are fixed rate bonds issued on the Swiss capital market and repaid in CHF but the coupon is paid in a currency other than CHF.

Adjustable long-term putable securities (ALPS) are variable-rate dual currency bonds which have early redemption put provisions.

Example:
In 1999 RESEAU FERRE DE FRANCE issued a EUR 500 million bond with maturity in 2029, with annual coupon of 5.05 percent until July 2015 in EUR, then 5.35 percent in GBP until maturity. The bond is putable on 12 July 2015 at 100, if not the put bond is redenominated into GBP at GBP 0.652 / EUR1.

Multiple currency clause bonds are bonds which have a provision allowing the holder to choose the loan repayment currency and/or the coupon payment currency.

Special drawing right (SDR) bonds are bonds issued by the International Monetary Fund (IMF) in SDR.

The SDR is a monetary unit and not a traded currency. Its value is determined by a basket of currencies of IMF member countries. The percentage of each currency in the basket depends on the importance of the currency internationally.

Exchange rates for an SDR against the EUR, JPY, USD and GBP are calculated by the IMF every Monday.

The interest rate on the SDR is also set each Monday by the IMF based on a weighted average of the short-term interest rates of the currencies making up the basket.

Note that only member countries and certain central banks are eligible to buy bonds issued by the IMF.

1.4.4 Conversion or exchange provisions

Bond issues may also have conversion or exchange provisions in their contracts.

The holder of a **convertible bond** can exchange the security for a fixed number of shares of the common stock of the issuing company in accordance with terms set forth in the bond indenture. The option to convert is solely at the discretion of the holder (although some corporations may have the ability to force conversion). The conversion cannot be reversed.

A **reverse convertible bond** is a bond with a provision allowing the issuer to repay the loan in the existing shares of an underlying company, determined at the time of issue, which has no economic relationship with the issuer or the bond guarantor.

This type of provision is usually used by banks when issuing structured products.

Exchangeable bonds are bonds that can be exchanged for shares or bonds issued by a third party at prices determined at time intervals defined in advance.

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[15] Be careful not to confuse this with the FIPS (Fixed Income Pricing System) offered by NASDAQ to value high yield bonds.

1.4.5 Price quotes

A price of a bond is the market value at which the bond is currently traded.

The price of a bond is generally quoted as **percentage of the face value**. To convert the price quote to a cash figure, multiply the quoted price by the face value and divide the result by 100.

> **Example:**
> A bond with a par value of 5'000 CHF is traded at 86.70. What is its price in CHF?
> The bond price is:
>
> $$(86.7 \cdot 5'000 \text{ CHF}) \ / \ 100 \ = \ 0.867 \cdot 5'000 \text{ CHF} \ = \ 4'335 \text{ CHF}.$$

Some US Treasury bonds, however, are quoted in percentage of the face value and in 32nds **of a percent**.

> **Example:**
> An American T-Bond with a par value of 1'000 USD is traded at 89-16. What is its value in USD?
> Its value is 895 USD, as
>
> $$1'000 \cdot (89 \ + \ 16 \ / \ 32) \ / \ 100 \ = \ 1'000 \cdot (89.50) \ / \ 100 \ = \ 895 \text{ USD}$$

This pricing method arises from the trading convention of 1/32nd point (0.03125 decimal) as the minimum price change for this type of bonds.

Prices for US Treasury bonds are often expressed as in the example above using a hyphen ("-") instead of a decimal point: thus 103-14 means 103 14/32 (=103.4375).

1.4.6 Accrued interest

Since the dates of interest payments vary from bond to bond, the comparison of the prices of two bonds is difficult. Therefore, bond prices are generally quoted **net of interest**, i.e., as if the coupon had just been paid and the full period (one year for an annual coupon, six month for a semi-annual coupon, etc.) is left until the next coupon.

In practice, when the bond is purchased between two coupon payment dates, the buyer pays an **accrued interest** amount on pro-rata basis to the seller in addition to the quoted price. The buyer in turn will get this amount back when he receives the next coupon from the issuing company, as he retains the entire coupon amount.

$$\text{Full Price of bonds} \ = \ \text{Quoted price} \ + \ \text{Accrued interest}$$

This implies that when one buys a bond between two coupon payment dates, the full purchase price (also called **gross price** or **dirty price**) is higher than the quoted price (also called **flat price** or **clean price**).

Example:
On the 20th August 2016, a 1999-2019 bond with a 2.5% coupon, 200'000 JPY face value, is quoted at 104.55. The annual coupon is paid on the 25th of June. What is the price paid by the buyer of such a bond?

Accrued interest for 55 days (from 25th June 2016 to 20th Aug 2016) has to be paid to the bond seller. The interest for the remaining period until the next coupon payment (i.e., from 20th August 2016 to 25th June 2017) is payable to the buyer. Though the bond buyer will pay the interest accrued till the date of purchase to the bond seller, at the next coupon payment she will receive the total coupon.

On the 20th August 2016, the buyer will pay the bond price:

$$1.0455 \cdot 200'000 \text{ JPY} = 209'100 \text{ JPY}$$

plus the accrued interest on 55 days:

$$\frac{55}{360} \cdot 2.5\% \cdot 200'000 = 763.89 \text{ JPY}$$

which makes a total payment to the seller of 209'863.89 JPY. At the next coupon payment (25th June 2017), the buyer will receive the total coupon, i.e.

$$2.5\% \cdot 200'000 \text{ JPY} = 5'000 \text{ JPY}$$

But she has paid 763.89 JPY of accrued interest; so, in fact, its net income will be

$$5'000 \text{ JPY} - 763.89 \text{ JPY} = 4'236.11 \text{ JPY}$$

which corresponds to 305 days of interest (from 21st August 2016 to 25th June 2017).

$$\frac{305}{360} \cdot 2.5\% \cdot 200'000 \text{ JPY} = 4'236.11 \text{ JPY}$$

One should also note that the computation rules vary from one market to another and from one type of bond to another. For example:

- for **Swiss bonds**, **German domestic bonds and Eurobonds** one year is assumed to have 360 days, ie.12 months of 30 days each. The formula to calculate the accrued interest is:

$$\text{Accrued interest} = \frac{30 \cdot m + d}{360} \cdot C$$

  where C is the coupon rate, m is the number of months and d the number of days since the last coupon payment.

- for **US Treasury Bonds**, the exact number of days (actual/actual) are used in the accrued interest formula. Note that for US treasury bonds the coupon is paid semi-annually, hence the coupon rate is divided by 2. So, the formula to calculate accrued interest becomes:

$$\text{Accrued interest} = \frac{\text{Exact number of days since last coupon}}{\text{Exact number of days between coupons}} \cdot \frac{C}{2}$$

where C is the annual coupon rate.

- for **US domestic bonds** (corporate, yankees, federal agency issues excluding T-Bonds, ...), a 360 days year (30/360) is used. Since, the coupon is paid twice a year the accrued interest can be calculated using:

$$\text{Accrued interest} = \frac{30 \cdot m + d}{180} \cdot \frac{C}{2}$$

- for **Japanese** or **English government bonds**, accrued interest is calculated using actual number of days since the last coupon payment and a 365 days year:

$$\text{Accrued interest} = \frac{\text{Exact number of days since last coupon}}{365} \cdot C$$

- for **French bonds**, accrued interest is calculated using actual number of days in the denominator as well as the numerator:

$$\text{Accrued interest} = \frac{\text{Exact number of days since last coupon}}{\text{Exact number of days between coupons}} \cdot C$$

where C is the annual coupon rate.

These different methods of computing accrued interest may lead to different interest figures.

Example:
On the 15th July 2015, what is the accrued interest on a US T-bond and a US corporate bond each with a 5% annual coupon rate, if both have paid a semi-annual coupon, on the 15th May 2015?

For the T-Bond, we have an actual /actual basis. In the 15th May to 15th July period, we have 61 days (16 + 30 + 15); in the 15th May to 15th November period, we have 184 days (16 + 30 + 31 + 31 + 30 + 31 + 15). The accrued interest is

$$\frac{61}{184} \cdot 5\% \cdot \frac{1}{2} = 0.828804\%$$

For the corporate bond, we have a 30/360 basis. In the May 15 to July 15 period, we have 60 days (15 + 30 + 15); in the May 15 to November 15 period, we have 180 days. The accrued interest is

$$\frac{60}{180} \cdot 5\% \cdot \frac{1}{2} = 0.833333\%$$

1.5 Preferred stocks

To end this chapter, we would like to devote a section to **preferred stocks (shares)**[16].

A **preferred stock** is a share, not a bond. Like the holder of an ordinary share, the holder of a preferred stock is a shareholder of the company. He has the right to a dividend but usually no voting rights (in some cases there are very restricted voting rights in special situations – such as issuance of new shares).

[16] The word "stock" is used in the US whereas the word "share" is used in the UK.

We decided to include a section on these instruments because the economic characteristics of preferred stocks are very similar to those of bonds.

The first similarity is that the dividend amount paid on preferred stocks is determined as a **percentage of the share's nominal value** and not as an amount fixed periodically by the shareholders at the annual general meeting.

The second similarity is that preferred stocks may have a determined maturity date. Preferred stocks with no repayment date provision are known as **perpetual preferred stocks**.

The third similarity is that the vast majority of preferred stocks issued on the market have **sinking-fund** provisions, in other words provisions for repayment of the amount issued by amortisation.

The fourth similarity is that some issues may **provide for conversion** into the issuer's ordinary shares. This conversion may even be contractually obligatory when the preferred stocks mature.

As the shareholders do not set the dividend of preferred stocks, it is important for the issuer to plan how it will determine the dividend rate to be paid to preferred stockholders. There are three ways of setting this rate.

The first way is simply to determine a fixed value for the rate at the time of issue (**fixed-rate preferred stock**) in the same way as the coupon rate of fixed-rate bonds is determined.

The second way is to periodically reassess the dividend rate based on the value of an interest rate noted on the adjustment date. These preferred stocks are known as **adjustable-rate preferred stocks** (ARPS). Note that a supplementary provision sets out the minimum and maximum rates that may be paid on the instrument.

The third way of setting the dividend rate is to use an auction process. These securities are known as **remarketed preferred stocks** (RP). The auction process is similar to the one used for Auction Rate Securities and is usually held every 7 or 49 days.

Preferred stock issues are usually favoured and used if the government decides to give assistance by providing liquidity to a company in difficulty to avoid system risk.

This is what happened, for example, when the American government intervened in 2008 to help the AIG insurance company. Instead of lending AIG funds, the government became a shareholder by acquiring preferred stocks specially issued for the purpose by the company.

Unlike a bond, the issuer is not regarded as being in payment default if it does not pay holders of preferred stocks a dividend. However, contractual provisions govern the treatment of unpaid dividends.

If the unpaid dividend provision is ***cumulative***, the unpaid amount is accumulated (accrued) until the dividend has been paid in full.

If the unpaid dividend provision is ***non-cumulative***, the unpaid amount will never be paid. To protect preferred stockholders from this, other provisions listed in the Certificate of Designation may provide for a dividend payment priority compared with ordinary shareholders making preferred stockholders senior in relation to ordinary shareholders. These provisions may also give preferred stockholders voting rights if the dividend is not paid.

It should be noted that as preferred stocks are not view as debt for the issuer, the dividends paid are not tax-deductible by the issuer.

1.6 Conclusion

In this chapter, we have described the main provisions that may characterise a debt instrument. As we shall see in the chapter dealing with operational processes, all these provisions must be correctly recorded and described in the financial institutions' securities databases to avoid risks arising that may have a considerable financial, legal or reputational impact for the institution.

2. Time value of money

The **time value of money principle** states that any sum today is worth more than that same sum tomorrow. This is based on any number of reasons, but which can all be summarized in that the present is certain, and the future is not. For example, in most periods the phenomenon of inflation can be noted, and this means that something cost EUR 100 today may well cost more than EUR 100 in a year's time. In addition there is the "opportunity cost" of deferring a purchase: the desired article may not just be more expensive later, it may well not be for sale. More philosophically present consumption is often considered to be better than deferring gratification to the future, and for this reason, an incentive is required to motivate us to defer gratification. This incentive is represented by the interest rate. An economist might phrase the same reasoning by saying that one of the functions of interest is to compensate for the loss of utility due to the existence of risk and the preference for the present. Interest rates are usually expressed on an annual basis, and express the remuneration that has to be paid from the borrower to the lender for the service of lending money. There are several types of interest rates.

2.1 Simple versus compound interest

Simple interest assumes that interest does not itself earn interest, and is calculated by the following formula:

$$\text{Simple interest} = (\text{initial value}) \cdot (\text{interest rate}) \cdot (\text{number of years})$$

The initial value is the principal amount on which interest is paid over a given period.

Example:
What is the simple interest earned on EUR 1,000 invested at 7% p.a. (per annum) after 10 years?

The answer is

$$\text{EUR } 1{,}000 \cdot 0.07 \cdot 10 = \text{EUR } 700$$

However in the real world interest payments received are often reinvested to earn more interest in subsequent periods.

Compound interest assumes that interest is reinvested; so compound interest is simple interest plus interest earned on interest paid earlier. The formula to calculate compound interest on a given initial value over a given period is:

$$\text{Compound interest} = (\text{initial amount}) \cdot \left[(1 + \text{interest rate})^{\text{number of years}} - 1 \right]$$

Example:
What is the compounded interest earned on EUR 1,000 invested at R=7% p.a. (per annum) after 10 years?

The answer is

$$1{,}000 \text{ EUR} \cdot \left[(1+0.07)^{10} - 1 \right] = 967.15 \text{ EUR}$$

Proof:

Year	Capital at begin of period [1]	Interest [2]=[1]·R	Capital at end of period [1]+[2]
1	1,000	70	1,070
2	1,070	74.9	1,144.9
3	1,144.9	80.14	1,225.04
…	…	…	…
10	1,838.46	128.69	1,967.15

So, we see that, after ten years, the compound interest is 967.15 EUR, as opposed to the simple interest of 700 EUR.

2.2 Present and future value

The process of determining the **present value** of a future payment (or receipt) or series of future payments (or receipts) is called **discounting**. The compound interest rate used for discounting cash flows is also called the **discount rate**. So the **present value** (or **actual value**) of a future income is given by:

$$\text{Present value} = \frac{\text{Future value}}{(1+\text{Interest rate})^{\text{number of years}}}$$

The present value of a promised future cash flow is inversely related to both the length of the investment period and the level of interest rates.

Example:
A financial firm offers to pay you EUR 100,000 in 10 years, if you give the firm EUR 60,000 today. Using a 7% interest rate, the present value of EUR 100,000 in 10 years is:

$$\text{Present value} = \frac{100'000}{(1.07)^{10}} = \text{EUR } 50'835$$

Accepting this offer would imply paying 60,000 EUR for something that is worth EUR 50,835. You should definitely refuse!

The process of finding the **future value** of the payment (or receipt) or series of payments (or receipts) using the concept of compound interest is known as **compounding**. The general formula for compounding is:

$$\text{Future value} = (\text{Present value}) \cdot (1+\text{Interest rate})^{\text{number of years}}$$

Example:
EUR 100 are deposited in a bank account with a 5% annual interest rate. What is the balance of the account at the end of the first and the second year respectively?

At the end of the first year, we have

$$100 \cdot (1 + 0.05) = \text{EUR } 105$$

At the end of the second year, we have

$$105 \cdot (1 + 0.05) = \text{EUR } 110.25$$

Why not just EUR 110? Because we also have earned a 5% interest on the 5 EUR paid at the end of the first year (5 · 5% = 0.25). We could also have directly stated:

$$100 \cdot (1 + 0.05)^2 = \text{EUR } 110.25$$

A high interest rate environment and long investment period lead to greater accumulation of compound interest.

Example:
Suppose that 1,000 EUR were deposited in a saving account on 1st of January 1934. What is the balance on 31st of December 2000, if interest was paid at a rate of 10.5%?

There have been 67 years of compounding. The final balance is:

$$1{,}000 \cdot (1.105)^{67} = 804{,}030.69 \text{ EUR}$$

In the case of simple interest, we would have a balance of only

$$1{,}000 + (1{,}000 \cdot 0.105 \cdot 67) = 8{,}035 \text{ EUR}$$

2.3 Annuities

In the special case of an annuity, a fixed amount of money is paid each year for a specified number of years. The present value of this series of cash flows is given by the following formula:

$$\text{Present value} = \sum_{t=1}^{n} \frac{CF}{(1+R)^t} = \frac{CF}{R} \cdot \left(1 - \frac{1}{(1+R)^n}\right)$$

When using this formula we suppose that the first payment is received in one year from now.

Example:
The same financial firm as above offers to pay you 10,000 EUR at the end of each year during 10 years, if you lend the firm 70,000 EUR today. Using a 7% interest rate, the present value of this 10-year annuity is:

$$\text{Present value} = \frac{10'000}{0.07} \cdot \left(1 - \frac{1}{(1+0.07)^{10}}\right) = 70'236$$

Accepting this offer would imply paying 70,000 EUR for something that is worth 70,236 EUR. You should definitely accept it!

The future value of the same series of cash-flows assumes that all individual cash-flows are reinvested at the same interest rate R, and can be calculated with the following formula:

$$\text{Future value} = \sum_{t=0}^{n-1} CF \cdot (1+R)^t = CF \cdot \left(\frac{(1+R)^n - 1}{R} \right)$$

Like for the present value, the formula above supposes that the first cash flow is received in one year from now.

Let us illustrate this with a simple example.

Example:
What is the future value of the series of coupons of a 7%, 10-year bond purchased at par (1'000 EUR), if we assume that all payments are reinvested at a 7% rate?

The answer is:

$$70 \cdot \left(\frac{(1+0.07)^{10} - 1}{0.07} \right) = 967.15$$

The simple interest in this case would have been:

$$1,000 \cdot 0.07 \cdot 10 \;=\; 700 \text{ EUR}$$

267.15 EUR (= 967.15 EUR – 700 EUR) is the additional amount earned by reinvesting the coupon payments and earning interest on the interest.

2.4 Continuous discounting and compounding

Compounding can take place not only with annual frequency, but also with higher frequency. It can be shown that if there are m compounds per year (i.e. interest is paid m times per year) then an initial amount N_0 invested at an annual interest rate R during n years becomes $N_0 \cdot \left(1 + \frac{R}{m}\right)^{m \cdot n}$. When m tends to infinity (i.e. interest is paid at every instant), it can be shown that this formula becomes $N_0 \cdot e^{R \cdot n}$, where e ≅ 2.718 is the Euler number. So we can use the following formula:

$$\text{Future value} \;=\; (\text{Actual value}) \cdot e^{\text{Time} \cdot \text{Instantaneous interest rate}}$$

Continuous compounding will lead to a higher future value. As interest is paid continuously, there is more interest on interest.

Example:
100 EUR are deposited in a bank account with a 5% continuous annual interest. What is the account balance at the end of the first year and second year respectively, using continuous compounding?

At the end of the first year, the balance is

$$100 \cdot e^{0.05 \cdot 1} \;=\; 105.13 \text{ EUR}$$

At the end of the second year, the balance is

$$100 \cdot e^{0.05 \cdot 2} \;=\; 110.52 \text{ EUR}$$

2.5 Bond valuation

Bond valuation is based on the discounted cash flow method and on the equilibrium concept.

2.5.1 Valuation of a zero-coupon bond

The simplest bond to consider is a zero-coupon bond which pays a single cash flow CF_t at the end of period t. The price of such a bond, denoted by $B_{0,t}$, is equal to the present value of its final (and only) cash flow:

$$B_{0,t} = \frac{CF_t}{(1+k)^t}$$

where CF_t is the cash flow received at the end of period t, and k is the appropriate discount rate.

Example:
What is the price today of a zero-coupon bond that will pay EUR 1,000 in exactly 5 years, assuming a discount rate of 7%? How about a 7-year bond, still assuming a discount rate of 7%?

$$B_{0,5} = \frac{1,000}{(1.07)^5} = \text{EUR } 712.99$$

And

$$B_{0,7} = \frac{1,000}{(1.07)^7} = \text{EUR } 622.75$$

In the previous example, the discount rate was assumed to be the same regardless of the maturity of the bond. But generally the discount rate varies from maturity to maturity. If we denote the annual rate of return demanded by a lender to lend money from time 0 to time t (also called the **spot rate**) by $R_{0,t}$ and if only one final payment CF is made towards both interest and principal, the price of such a zero-coupon bond will be calculated as:

$$B_{0,t} = \frac{CF_t}{(1+R_{0,t})^t}$$

The above formula allows us to use different discount rates for different maturities.

Example:
What is the price today of a zero-coupon bond that will pay 1,000 EUR in exactly 5 years, assuming a 5-year spot rate $R_{0,5}$ = 5%? How about a 7-year bond, assuming a 7-year spot rate $R_{0,7}$ = 6%?

$$B_{0,5} = \frac{1,000}{(1.05)^5} = \text{EUR } 783.53$$

And

$$B_{0,7} = \frac{1,000}{(1.06)^7} = \text{EUR } 665.06$$

Given the spot rates, we are now able to price zero-coupon bonds of different maturities. This principle can be extended to calculate the value of coupon-bearing bonds, as follows.

2.5.2 *Static arbitrage and valuation of coupon bonds*

A coupon-bearing bond can be visualised as a stream of future cash flows. Such a stream can be replicated by **a portfolio of zero-coupon bonds**.

Example:
Investor A holds one straight bond with a 4-year maturity, EUR 1,000 face value, 6% coupon rate. Investor B wants to replicate the same cash flows as investor A's bond, but he can only use zero-coupon bonds. What should he buy?

Investor B should buy:

- a zero-coupon bond that matures in exactly one year and pays EUR 60.
- a zero-coupon bond that matures in exactly two years and pays EUR 60.
- a zero-coupon bond that matures in exactly three years and pays EUR 60.
- a zero-coupon bond that matures in exactly four years and pays EUR 1,060.

With such a portfolio, he will receive exactly the same cash flows as investor A.

What should the price of such a portfolio be? By definition, it should be exactly the same as the price of the replicated bond, as both have exactly the same cash flows. Otherwise, there would be scope for arbitrage (buy the cheaper of the two bonds, sell short the other bond, match the cash flows, and keep the price difference as risk-free profit). So, we can calculate the price of the bond using the following relationship:

Bond price = Price of replicating zero-coupon bond portfolio

And as the portfolio price is the sum of all the zero-coupon bond prices, the price of any coupon-bearing bond is the sum of the present value of all its individual cash payments:

$$P = \sum_{t=1}^{T} \frac{CF_t}{\left(1+R_{0,t}\right)^t} = \frac{CF_1}{\left(1+R_{0,1}\right)^1} + \frac{CF_2}{\left(1+R_{0,2}\right)^2} + \ldots + \frac{CF_T}{\left(1+R_{0,T}\right)^T}$$

where CF_t is the cash flow received at the end of period t (coupons or repayment), and T is the number of years remaining until maturity (time to maturity), and, as usual, $R_{a,b}$ is the spot rate for a single payment invested at time a and payable at time b.

Example:
What is the price of a straight bond with a 4-year maturity, EUR 1,000 face value, 6% coupon rate? The spot rates are $R_{0,1}$ = 7%, $R_{0,2}$ = 8% $R_{0,3}$ = 8.5% and $R_{0,4}$ = 9%.

The following table represents the stream of cash flows:

time	0	1	2	3	4	4
Cash Flows in EUR		60	60	60	60	1,000
Discount rate	0	0.07	0.08	0.085	0.09	0.09

Therefore the bond price is:

$$\text{P} = \frac{\text{USD}\,60}{\left(1+0.07\right)} + \frac{\text{USD}\,60}{\left(1+0.08\right)^2} + \frac{\text{USD}\,60}{\left(1+0.085\right)^3} + \frac{\left(\text{USD}\,1000+\text{USD}\,60\right)}{\left(1+0.09\right)^4} = \text{USD}\,905.42$$

2.5.2.1 Special cases

If the bond pays a **semi-annual coupon**, we can still use the same formula

$$P = \sum_{t=1}^{T} \frac{CF_t}{\left(1+R_{0,t}\right)^t} = \frac{CF_1}{\left(1+R_{0,1}\right)^1} + \frac{CF_2}{\left(1+R_{0,2}\right)^2} + \ldots + \frac{CF_T}{\left(1+R_{0,T}\right)^T}$$

where CF_t is the cash flow received at the end of the semi-annual period t (coupons or repayment), T is the number of semi-annual periods remaining until final maturity, and $R_{0,t}$ the required rate of return to lend money from time 0 to the end of the **semi-annual** period t i.e. the spot rate from time 0 to time t.

2.5.2.2 Influences on the bond price

The impact of the coupon rate

The bond price depends on the promised payments (or the expected cash flows), which appear in the numerators of the summation; hence, it is directly related to the bond's coupon rate.

$$P_0 = \sum_{t=1}^{T} \frac{CF_t}{\left(1+R_{0,t}\right)^t} = \frac{CF_1}{\left(1+R_{0,1}\right)^1} + \frac{CF_2}{\left(1+R_{0,2}\right)^2} + \ldots + \frac{CF_T}{\left(1+R_{0,T}\right)^T}$$

A higher coupon bond will be worth more than a lower coupon issue with the same maturity, since the expected cash flows are higher.

Example:
Consider two bonds with 4-year maturities, EUR 1,000 face value and a 6% and 7% coupon rate. What are their prices, if the spot rates are $R_{0,1}$ = 7%, $R_{0,2}$ = 8%, $R_{0,3}$ = 8.5% and $R_{0,4}$ = 9%?

The following table represents the cash flows for both bonds:

Time	0	1	2	3	4	4
Cash flows for bond 1 in EUR		60	60	60	60	1,000
Cash flows for bond 2 in EUR		70	70	70	70	1,000
Discount rate	0	0.07	0.08	0.085	0.09	0.09

Hence, the prices of the bonds are:

$$P_{bond\ 1} = \frac{60}{(1+0.07)} + \frac{60}{(1+0.08)^2} + \frac{60}{(1+0.085)^3} + \frac{(1000+60)}{(1+0.09)^4} = 905.42 \text{ CHF}$$

$$P_{bond\ 2} = \frac{70}{(1+0.07)} + \frac{70}{(1+0.08)^2} + \frac{70}{(1+0.085)^3} + \frac{(1000+70)}{(1+0.09)^4} = 938.25 \text{ CHF}$$

As expected, the price of the second bond is higher.

The impact of the discount rate of interest

The bond price also depends on the discount rates, which appear in the denominators in the summation; hence, the bond price is inversely related to the discount rates of interest. (The bond's value rises if any of the discount rates is reduced, and falls if any of the discount rates is increased).

Example:
Consider a bond with a 4-year maturity, EUR 1,000 face value, 6% nominal interest rate. What is its price, if the spot rates are $R_{0,1}$ = 7%, $R_{0,2}$ = 8%, $R_{0,3}$ = 8.5% and $R_{0,4}$ = 9%? What happens if the 2-year spot rate increases by 0.5%?

The following table represents the stream of cash flows of the bond:

Time	**0**	**1**	**2**	**3**	**4**	**4**
Cash flows on EUR		60	60	60	60	1,000
Discount rate	0	0.07	0.08	0.085	0.09	0.09

Price of the bond is:

$$P = \frac{\text{EUR } 60}{(1+0.07)} + \frac{\text{EUR } 60}{(1+0.08)^2} + \frac{\text{EUR } 60}{(1+0.085)^3} + \frac{(\text{EUR } 1000 + \text{EUR } 60)}{(1+0.09)^4} = \text{EUR } 905.42$$

If there is an increase in the 2-year spot rate, we have $R_{0,2}$ = 8.5%, and the new price for our bond is:

$$P = \frac{\text{EUR } 60}{(1+0.07)} + \frac{\text{EUR } 60}{(1+0.085)^2} + \frac{\text{EUR } 60}{(1+0.085)^3} + \frac{(\text{EUR } 1000 + \text{EUR } 60)}{(1+0.09)^4} = \text{EUR } 904.95$$

As expected, the new price is lower.

2.6 Price / yield relationship

The total return realized from holding a bond during a given time-period can be broken down into three components

$$\text{Total return} = \text{Price return} + \text{Coupon return} + \text{Reinvestment return}$$

The price component is the variation in the quoted price, the coupon component is the periodic payment from the bond, the reinvestment return is the income generated by reinvesting all previously received cash flows ("interest on interest").

The price return itself can be decomposed in two components:

$$\text{Price return} = \text{Price return due to yield change} + \text{Amortisation of premium / discount}$$

The price return due to the yield change comes from the change of at least one of the spot rates (used in the discounting process). The amortisation of the premium or discount reflects the fact that the bond price will tend to converge toward the final payment value.

As we have seen already, the bond's current yield varies inversely with the bond price, and that the price is inversely proportional to the current yield (except, of course, for zero-coupon bonds); but the current yield is not a good estimate of the total return, as it focuses solely on the coupon component of the return.

There is also an inverse relationship between the price and the yield to maturity, but it is not linear.

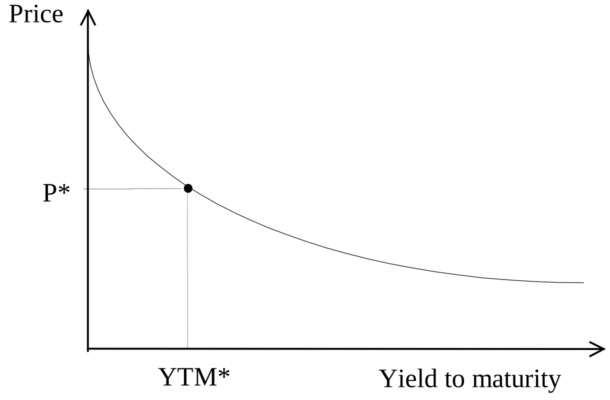


Figure 2-1: Yield to maturity and price relationship

We have seen that the yield to maturity is the internal rate of return of the bond. Despite this, **it can only serve as a proxy for the total return under a series of very restrictive assumptions**:

- **All coupon reinvestments are made at the yield to maturity rate**. In reality, each coupon is reinvested at the market rate prevailing on the date of cash receipt. If this rate differs from the yield to maturity, the total return will differ from the yield to maturity. Higher (lower) reinvestment rate lead to higher (lower) total return compared to the yield to maturity.
- The **reinvestment rate risk** is particularly important with long term bonds, and high coupons rates, for which the realised return may differ significantly from the one inferred by the yield to maturity.
- **For bonds paying semi-annual coupon, an annual basis-point add-on is required to determine the equivalent annual return**. This conversion is necessary due to the reporting of total returns on an annualised basis.

In most cases, using the yield to maturity as an expected return proxy implies that whatever the maturity, there is only one prevailing rate to lend and borrow money, and that this rate will never change in the future.

The important thing to remember is that yield to maturity and total return are two totally different concepts.

Example:
Assume that a bond with a 4-year maturity, 6% annual coupon has been bought at par in year 0; hence with a yield to maturity of 6%. Assume further that in the following years interest rates follow a downtrend, so that the coupon received in year 1 is invested for three years at 4.5%; the coupon received in year 2 is reinvested for two years at 3%, and the coupon received in year 3 is reinvested for 1 year at 2%. What's the total realized return in year 4?

The final capital in year 4 (for a 100 initial investment) is: $6 \cdot 1.045^3 + 6 \cdot 1.03^2 + 6 \cdot 1.02 + 106 = 125.33$.

This means that the realized return is $\sqrt[4]{\frac{125.33}{100}} - 1$ =5.81% p.a., lower than 6%!

3. Bond yield measures

3.1 Current yield

The **current yield** of a bond is simply the annual coupon payment divided by the market price of the bond (excluding accrued interest), otherwise known as the **Clean Price** (also net price or flat price).

$$\text{Current yield} = \frac{\text{Annual coupon}}{\text{Net Price}}$$

When the coupon rate is fixed (which is generally the case), the bond's current yield varies inversely with the bond's price. As the bond's price rises (declines), its current yield falls (increases) since its coupon return is now a lesser (larger) amount of the bond value. So, all other factors held constant, a bond with a higher current yield sells at a lower price.

Example:
The XYZ 2002-2012 3.25% bond is quoted at 98.1 on the 13.06. Its current yield is 3.25/98.1 = 3.31%.

The current yield only takes into account the annual coupon income of the bond, and therefore, it is not an adequate tool to compare two bonds. For example,

- the current yield of a zero-coupon bond is zero as it pays no coupon.
- All other things being equal, the current yield of a bond priced under par will decrease as the bond approaches maturity (the coupon is fixed, but the price rises toward par as the bond approaches maturity, since par is the redemption paid at maturity). In the case of a bond trading above par the price will fall towards par as the bond approaches maturity.

3.2 Yield to maturity

The **yield to maturity** (YTM) is the discount rate that equates the present value of the bond's future cash flows till maturity with the current market price of the bond. Just after the coupon payment, we have :

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+YTM)^t} = \frac{CF_1}{(1+YTM)^1} + \frac{CF_2}{(1+YTM)^2} + \ldots + \frac{CF_T}{(1+YTM)^T}$$

where:

P	price of the bond (market price)
CF_t	cash flow received at the end of period t (coupons or repayment)
T	remaining life of the bond (time to maturity).

The yield to maturity is also known as the **internal rate of return** (IRR) of the investment in the bond.

Example:
An investor can buy a bond for 116.00 with a 10% coupon, 1'000 EUR face value and 4 years until maturity. The coupon has just been paid. What is the bond's yield to maturity?

The yield to maturity YTM solves the following equation:

$$\frac{100}{(1+\text{YTM})}+\frac{100}{(1+\text{YTM})^2}+\frac{100}{(1+\text{YTM})^3}+\frac{1'100}{(1+\text{YTM})^4}=1'160$$

By iteration with a computer, we find the correct answer, which is YTM = 5.44%.

The yield to maturity assumes that the bond is held to maturity, and that all cash flows are received as scheduled through final maturity, and are immediately reinvested at the same yield to maturity. As we have seen (in paragraph 2.6), the yield to maturity should not be confused with the total return on the bond investment.

The case of semi-annual coupons is easy to handle. First, calculate a semi-annual yield to maturity YTM_S:

$$P=\sum_{t=1}^{T}\frac{CF_t}{(1+YTM_S)^t}=\frac{CF_1}{(1+YTM_S)^1}+\frac{CF_2}{(1+YTM_S)^2}+\ldots+\frac{CF_T}{(1+YTM_S)^T}$$

where CF_t is the cash flow received at the end of period semester t (coupons or repayment), and T is the number of semesters in the remaining life of the bond (time to maturity).

Then, convert this semi-annual yield in an annual yield YTM_A using one of the following formulae:

- on a Euromarket:

$$YTM_A=(1+YTM_S)^2-1$$

- on the US or English market:

$$YTM_{US_A}=2\cdot YTM_S$$

by convention, the annual yields are not compounded on these markets (which gives a lower yield than on the Euromarkets).

The following example will illustrate this concept.

Example:
A 10-year eurobond pays semi-annually an annual coupons of 6% and is quoted at 110.00. The coupon has just been paid. What is its yield to maturity? What would be its yield to maturity if the coupon was paid annually?

The yield to maturity solves:

$$110 = \frac{3}{(1+YTM_S)} + \frac{3}{(1+YTM_S)^2} + \ldots + \frac{103}{(1+YTM_S)^{20}}$$

The solution is YTM_S = 2.37%. The annual corresponding yield is

$$YTM_A = (1+YTM_S)^2 - 1 = 4.79\%$$

If the coupon was paid annually, we would have a yield of 4.72%, which is lower (this clearly is only true if the price stays unchanged at 110.00). This is predictable, as a semi-annual interest payment allows interest compounding.

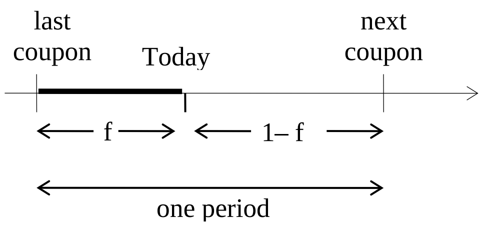
Note that for a US bond, we would express the yield of 4.79% as 2 · 2.37% = 4.74%!

The same methodology can be used in the case of quarterly coupons.

Using different yield conventions lead to differing results for yields. The prices of the bonds do not change irrespective of the yield methodology used. But still it is important to use consistent yield methodologies as soon as one wants to compare bonds on the basis of their yield levels.

3.2.1 Yield to maturity between two coupon payment dates

Between two coupon payment dates, the buyer of a bond must pay the accrued interest to the seller. The accrued interest is due for the fraction f of the total period between two coupon dates



Hence, the total price to be paid for the bond is (in the case of an annual coupon payment):

$$\text{Total price} = \text{Quoted price (or Net Price)} + f \cdot \text{Coupon}$$

The formula to calculate the yield to maturity must be modified to include accrued interest in the total price:

$$P + f \cdot C = \sum_{t=1}^{T} \frac{CF_t}{(1+YTM)^{t-f}} = \frac{CF_1}{(1+YTM)^{1-f}} + \frac{CF_2}{(1+YTM)^{2-f}} + \ldots + \frac{CF_T}{(1+YTM)^{T-f}}$$

$$= (1+YTM)^f \cdot \left[\frac{CF_1}{(1+YTM)^1} + \frac{CF_2}{(1+YTM)^2} + \ldots + \frac{CF_T}{(1+YTM)^T} \right]$$

Let us illustrate this.

Example:
A bond with an annual coupon of 6% is quoted at 108.00 on the market, 9 years and 3 months before its maturity. What is its yield?

We have:

$$f = 9 \text{ months} = 9/12 \text{ years} = 0.75 \text{ years}$$

The effective price paid is

$$108 + 0.75 \cdot 6 = 108 + 4.5 = 112.5$$

and the yield to maturity solves:

$$112.5 = (1+\text{YTM})^{0.75} \cdot \left[\frac{6}{(1+\text{YTM})} + \frac{6}{(1+\text{YTM})^2} + \ldots + \frac{106}{(1+\text{YTM})^{10}} \right]$$

The solution is YTM = 4.90% (by iteration).

While the methodology is general, one should always keep in mind that the way to calculate the number of days for the accrued interest may vary from one country to another.

3.2.2 Influences on the yield to maturity: the coupon effect

The yield to maturity of two bonds having the same maturity but different cash flows is not necessarily the same, even if the interest they pay is reinvested at the same rates. This is called the **coupon effect** or **coupon bias**[17].

In the following example we calculate the present value of the two bonds using the rates obtainable for investments making single repayments at the end of one year and at the end of two years. Thus here the two year rate is the rate for a single payment at the end of two years, with no interest payment in the intervening period. These rates are called the one-year spot rate and the two-year spot rate respectively. In the example $R_{0,1}$ refers to the rate between now (0) and one year from now (1), and thus $R_{0,2}$ refers to the two year spot rate, and in further examples $R_{a,b}$ refers to the spot rate for the period from a to b. We will examine spot rates in more detail later.

Example:
Bond A is a two-year 10% coupon, while bond B is a two-year 5% coupon. The returns on money obtained for one year and two years are $R_{0,1}$ = 6%, and $R_{0,2}$ = 7% respectively.

The bonds prices are

$$P_A = \frac{10}{1.06} + \frac{110}{1.07^2} = 105.512$$

$$P_B = \frac{5}{1.06} + \frac{105}{1.07^2} = 96.428$$

But the yields to maturity of the two bonds are different:

17 Note that there is no coupon effect with zero-coupon bonds.

$$P_A = \frac{10}{(1+k_A)} + \frac{110}{(1+k_A)^2} = 105.512 \Rightarrow YTM_A = 6.953\%$$

$$P_B = \frac{5}{(1+k_B)} + \frac{105}{(1+k_B)^2} = 96.428 \Rightarrow YTM_B = 6.975\%$$

The differences in the bond yields arise because the yield to maturity is a complex average of the spot rates applied to one and two years cash investments. In our example, bond B has a greater fraction of its value tied to the higher two years interest rate.

If one compares one formulation of the price of a coupon bearing bond

$$P = \sum_{t=1}^{T} \frac{CF_t}{\left(1+R_{0,t}\right)^t} = \frac{CF_1}{\left(1+R_{0,1}\right)^1} + \frac{CF_2}{\left(1+R_{0,2}\right)^2} + \ldots + \frac{CF_T}{\left(1+R_{0,T}\right)^T}$$

with the formulation used in the definition of the yield to maturity

$$P = \sum_{t=1}^{T} \frac{CF_t}{\left(1+YTM\right)^t} = \frac{CF_1}{\left(1+YTM\right)^1} + \frac{CF_2}{\left(1+YTM\right)^2} + \ldots + \frac{CF_T}{\left(1+YTM\right)^T}$$

it is clear that **the yield to maturity is a complex average of the spot rates**. Hence, the reader should carefully distinguish between yield to maturity and the spot rate $R_{0,T}$.

In fact, if the series of rates $R_{0,t}$ are increasing as t increases (i.e. if the spot curve is positive, as discussed in the section on yield curves below), one can show that the yield to maturity (YTM) will underestimate the corresponding spot rate $R_{0,T}$.

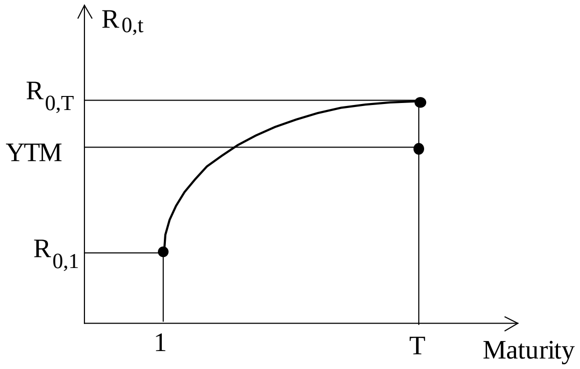


Figure 3-1: Yield to maturity versus spot rate

We can illustrate this by the following example[18]:

Example:
Let us consider the following increasing spot rates: $R_{0,1}$ = 2%, $R_{0,2}$ = 4%, $R_{0,3}$ = 5%, $R_{0,4}$ = 5.5%, $R_{0,5}$ = 6%. If we select three bonds A, B, and C differing only by their coupon rates, we can compute their prices and their returns to maturity.

	A	B	C
Maturity	5 years	5 years	5 years
Annual coupon rate	0%	3%	10%
Repayment	100%	100%	100%
Price	**74.73**	**87.69**	**117.96**
Yield to maturity	6%	5.91%	5.76%

Therefore, it is clear that using coupon-paying bonds, we will underestimate the effective spot rate for the considered maturity (6% for 5 years in our example). The bias will increase for larger coupon rates.

If the series of rates $R_{0,t}$ are decreasing, one can show that the yield to maturity (YTM) will overestimate the corresponding spot rate ($R_{0,T}$).

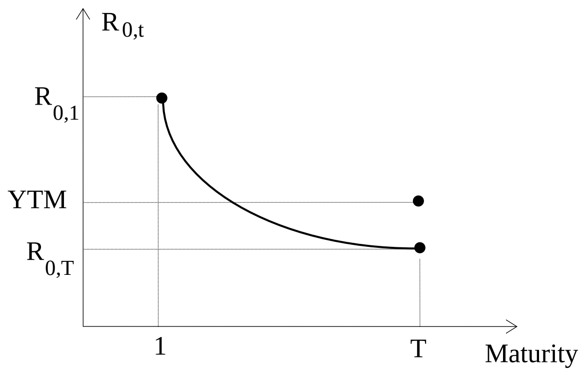


Figure 3-2: Yield to maturity versus spot rate

We can illustrate this by the following example:

Example:
Let us consider the following decreasing spot rates: $R_{0,1}$ = 6%, $R_{0,2}$ = 4%, $R_{0,3}$ = 3%, $R_{0,4}$ = 2.5%, $R_{0,5}$ = 2%. If we select three bonds A, B, and C differing only by their coupon rates, we can compute their prices and their yields to maturity.

	A	B	C
Maturity	5 years	5 years	5 years
Annual coupon rate	0%	3%	10%
Repayment	100%	100%	100%
Price	**90.57**	**104.35**	**136.52**
Yield to maturity	2%	2.08%	2.21%

Therefore, it is clear that using coupon-paying bonds, we will overestimate the effective spot rate for the considered maturity (2% for 5 years in our example). The bias will increase for larger coupon rates.

[18] Source : DUMONT Pierre André, 1995, "Les obligations ordinaires: typologie, procédures d'émission et aspects boursiers", HEC-University of Geneva, Geneva.

In the case of a coupon-bearing bond, the spot rate ($R_{0,T}$) and the yield to maturity (YTM) are rarely the same. But this issue will be dealt with later.

3.3 Yield to call

For callable bonds, the **yield to call** is the discount rate YTM_C that equates the present value of the bond's future cash flows received through the call date to the bond's current market price:

$$P = \sum_{t=1}^{TC} \frac{CF_t}{(1+YTM_C)^t} = \frac{CF_1}{(1+YTM_C)^1} + \frac{CF_2}{(1+YTM_C)^2} + \ldots + \frac{CF_{TC}}{(1+YTM_C)^{TC}}$$

where CF_t is the cash flow received at the end of period t (coupons or repayment), and TC is the remaining time until the call date. **It assumes that the bond will be called**, and that all cash flows are received as scheduled through the call date.

Example:
An investor can buy the same bond as in the previous example (10% coupon, 1,000 EUR face value, 4-year bond, quoted price of 116.00). The bond is callable in three years at 103. What is the bond's yield to call?

The yield to call YTM_c solves the following equation:

$$\frac{100}{(1+YTM_C)} + \frac{100}{(1+YTM_C)^2} + \frac{1030+100}{(1+YTM_C)^3} = 1160$$

By successive approximation, we find the correct answer, which is YTM = 5.07%.

The yield to call differs from the yield to maturity as the discounting period is shorter (since the call date precedes the maturity date), and the final cash flow is often higher (since the call price is often above par-value).

3.4 Other yields

Japanese Yield

One should note that some Japanese bond dealers still use a modified version of the current yield, also called Japanese yield, defined as:

$$\text{Japanese current yield} = \frac{\text{Annual coupon} - \dfrac{\text{Price (\%)} - 100}{\text{Remaining life}}}{\text{Price}}$$

The Japanese yield considers not only the coupon payment, but also the capital gains/losses made by the investor over the life of the bond. Thus, a bond purchased at premium will have a lower current yield due to capital losses suffered over its life-time, while bonds purchased at discount will exhibit higher current yield due to the capital gains component.

For more on the Japanese yield see http://www.mof.go.jp/english/bonds/guide.htm

Yield to average life

The **yield to average life** YTM_{AL} is only used to compare bonds with a series of principal repayment (like sinking fund bonds, mortgage backed securities, ...) and bullet bonds that repay principal at maturity. For simplicity, the full principal repayment is supposed to occur on the **average life date**. The average life of a bond is the weighted average maturity of the principal repayment (note that the coupon rate plays no role in the average life, as it only considers principal repayments):

$$\text{Average life in years} = AL = \sum_{t=0}^{T} \frac{\text{Principal paid at time t}}{\text{Total principal to be repaid}} \cdot t$$

The yield to average life is simply the internal rate of return (IRR) to the average life date (as if the average life date was the final maturity date of the bond):

$$P = \sum_{t=1}^{AL} \frac{CF_t}{(1+YTM_{AL})^t} = \frac{C_1}{(1+YTM_{AL})^1} + \frac{C_2}{(1+YTM_{AL})^2} + \dots + \frac{C_{AL}}{(1+YTM_{AL})^{AL}} + \frac{\text{Principal}}{(1+YTM_{AL})^{AL}}$$

where C_t is the coupon received at the end of period t (without repayment), and AL is the average life of the bond. All principal repayment are assumed to be made on the average date.

Example:
An investor can buy a 5% coupon, 1,000 EUR face value, 10-year bond, quoted at a price of 102. The bond has a 90% sinker[19], with sinking funds payments starting at the end of the first year, and repaying 10% of the bonds annually through the ninth year. All repayments are made at par value.

The average life is:

$$AL = \frac{100}{1000} \cdot 1 + \frac{100}{1000} \cdot 2 + \dots + \frac{100}{1000} \cdot 10 = 5.50 \text{ years}$$

The yield to average life solves the following equation:

$$\frac{50}{(1+YTM_{AL})} + \frac{50}{(1+YTM_{AL})^2} + \dots + \frac{50}{(1+YTM_{AL})^5} + \frac{1025}{(1+YTM_{AL})^{5.5}} = 1020$$

Note that the last payment is 1,025 EUR because we only earn interest (of 25 EUR) for half a year, as the average life is five years and a half.

By iteration, we find the correct answer, which is YTM_{AL} = 4.58%. **In essence, our bond is roughly comparable with a bullet bond giving a 4.58% yield to maturity, and maturing in 5.5 years.**

3.5 Other basic concepts

Three dates are essential while determining any rate of interest:

- the commitment date, which is the date at which the borrower and lender set the fixed rate on the loan.
- the lending date, at which the money is to be loaned.
- the repayment date, at which the money is to be repaid.

[19] The **sinker percentage** is the percentage of bonds retired before maturity.

In the following discussion, we will use the concept of spot rates (which we came across above) and forward rates which are defined below.

3.5.1 Spot rates

The **spot rate**, denoted by $R_{0,t}$, is defined as the annual interest rate received on a pure discount security[20] (zero-coupon bond) maturing at time t. It is, at time 0, the required rate of return to lend money from time 0 to time t, if only one final payment is made for both interest and principal.

One should remember that:

- the commitment date and the lending date are the same.
- spot rates are interest rates on loans or bonds that pay only **one** cash flow to the investor.

Generally, spot rates are quoted as annual rates.

Example:
A bond involves an investment of 797.19 EUR and returns a principal of 1,000 EUR in exactly two years. What is the two years spot rate? What about the one year spot rate?

As the bond is a pure zero-coupon bond, its return will be the two-year spot rate.

$$797.19 = \frac{1'000}{(1+R_{0,2})^2} \quad \Rightarrow \quad R_{0,2} = 12\% \text{ p.a.}$$

The rate is expressed on an annualised basis. Nothing can be said about the one-year spot rate.

3.5.2 Forward rates

The **forward rate**, denoted by $F_{t,h}$, is the rate of interest on a bond where the commitment date (0) and the date the money is lent (t) are different. If a commitment is made today on a two-years loan (h=2) to begin in one year (t), the annualised interest rate from year t to year t+h (from year t to year t+ h) is a forward rate.

One should remember that:

- the commitment date is today (t=0), but the lending date differs (t).
- forward rates are interest rates on loans or bonds that pay only one cash flow to the investor.

Forward rates are also generally quoted as annual rates.

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20 Note that the exact definition of a pure zero-coupon bond is a bond that pays 1 CHF at maturity.

Example:
A commitment involves a loan of 841.68 EUR in one year and a principal and interest repayment of 1,000 EUR in three years. What is the two-year forward rate to begin in year one?

As the bond is a pure zero-coupon bond, its return will be the two-year forward rate.

$$841.68 = \frac{1'000}{(1+F_{1,3})^2} \quad \Rightarrow \quad F_{1,3} = 9\% \text{ p.a.}$$

The rate is expressed on an annualised basis.

3.5.3 Relation between spot rate and implicit forward rate

Generally, the implicit forward rate can be calculated as the ratio of the end-of-period wealth to the beginning-of-period wealth, or as the ratio of the corresponding spot rates:

$$F_{t1,t2} = \sqrt[t2-t1]{\frac{\left(1+R_{0,t2}\right)^{t2}}{\left(1+R_{0,t1}\right)^{t1}}} - 1$$

Example:
The spot rates are $R_{0,1}$ = 6%, $R_{0,2}$ = 7% and $R_{0,3}$ = 7.5%. What is the implicit one-year forward rate at the end of the first year? At the end of the second year?

The one-year forward rate at the end of the first year can be derived by using two consecutive spot rates. If the investor invests 100 EUR for two years at the current spot rate $R_{0,2}$, he will receive at the end of the second year:

$$100 \cdot (1 + 0.07) \cdot (1 + 0.07) = 114.49 \text{ EUR}$$

Another solution would be to invest 100 EUR for one year at the current spot rate $R_{0,1}$, and to reinvest the proceeds at the forward rate $F_{1,2}$. Note that all positions in this trading strategy are determined at time 0. At the end of the first year, the investor will receive

$$100 \cdot (1.06) = 106.00 \text{ EUR}$$

He will reinvest this amount at the forward rate, and he should end with the same amount as the two-year strategy. Hence:

$$106.00 \cdot (1 + F_{1,2}) = 114.49 \text{ EUR}$$

and we have:

$$F_{1,2} = \frac{\text{Wealth position (end of year 2)}}{\text{Wealth position (end of year 1)}} - 1 = \frac{114.49}{106.00} - 1 = 0.0801 = 8.01\%$$

Similarly, we can calculate the implicit one year forward rate at the end of the second year:

$$F_{2,3} = \frac{\text{Wealth position (end of year 3)}}{\text{Wealth position (end of year 2)}} - 1 = \frac{100 \cdot (1.075)^3}{114.49} - 1 = 0.0851 = 8.51\%$$

By construction, the **spot rate** may also be seen as the **geometric average** of implicit consecutive forward rates:

$$\left(1+R_{0,t}\right) = \left[\left(1+R_{0,1}\right)\cdot\left(1+F_{1,2}\right)\cdot\left(1+F_{2,3}\right)\ldots\left(1+F_{t-1,t}\right)\right]^{\frac{1}{t}}$$

3.6 Yield Curves

While the concept of a yield curve might seem intuitively obvious, the expression itself is essentially a market term of relatively recent origin: it is rare to find references to it prior to the early 1970s[21]. Prior to that there had been some academic work on the term structure of interest rates, but one of the earliest, seminal, articles to have included both expressions was only published in 1971[22]. Formally, the term structure deals with the relationship between zero-coupon yields (or spot rates) and time to maturity, whereas the yield curve deals with yield to maturity and time to maturity. Market practice is to use yield curves rather than term structures when dealing in the market, though much bond and economic analysis will involve term structures rather than yield curves.

3.6.1 Market Curves (Observed)

The easiest form of yield curve, and that most commonly used in the market, simply involves a graph whose two axes are time or maturity (x axis) and yield (y axis) plotting points representing the observed market yields of bonds of different maturities from the same issuer (or very similar issuers). The most commonly used yield curve is that of the relevant sovereign issuer: thus in the US market, the most commonly used yield curve is that portraying US Treasury issues, and that in Japan shows Japanese Government Bond issues. The situation is somewhat more complicated in the Eurozone: prior to the adoption of the single currency, in each of the countries that adopted the EUR it was possible simply to draw a line joining a relatively small number of bonds across the maturity spectrum to produce a standard "market" government benchmark curve. The situation in the Eurozone market is not so clear today, and no established standard procedure currently exists for establishing the default-free bond benchmark. Given that Germany has the best mix of relevant market size, lowest yield and good liquidity, German government bonds are often used as reference. Partly as a result of these complications to find the right benchmark bonds there is an increased reliance on the swap curve (see below), since there is only one swap curve for the whole Eurozone.

Market yield curves are not limited to depicting government bond markets, and market curves need not even map yield versus maturity. Some of these curves are examined in greater detail below.

3.6.1.1 Bond Yield

As mentioned, the most commonly currently used curve in markets today is the government yield curve. However in recent years, its dominance has been challenged by the swap curve, which we describe further below. In addition, while it is true that bond valuation is best achieved using a term structure model in order to discount the individual cash flows of a bond, market practice is to indicate a yield by references to some benchmark yield curve

[21] The earliest article we have found on the subject is "Yield curves and representative yields on British government securities" in the Bank of England Quarterly Bulletin, March 1967.

[22] J. Huston McCulloch – "Measuring the Term Structure of Interest Rates" - Journal of Business, Vol. 44, No. 1 (Jan., 1971), pp. 19-31.

The government yield curve is generally created using only the most recent or benchmark issues[23]. Since bonds with maturities of less than one year are usually considered money market instruments rather than bonds[24], the shortest maturity in a bond market yield curves is usually one year. An ideal yield curve might have any number of points, but in practice the aim is to include one bond per year of maturity. The longest maturity mapped depends of course on the market, but is usually the maturity of the longest recently issued bond. The observed yields are plotted against their maturities[25], and the points are then generally joined by a straight line. Although practitioners are aware that such a method is not precise, it is considered to be adequate. There are of course numerous numerical methods for drawing curves through or close to a set of points, but since no model is clearly better than all the others, and since there is otherwise no market standard, a straight line fit is considered the clearest way to communicate. The yield imputed for a maturity with no observation attached is simply the yield corresponding to the point on the segment of the yield curve consisting of the straight line joining the points with maturities immediately shorter and longer than the maturity sought. This is known as straight-line interpolation.

As described above, market practice in drawing any yield curve based on observed yields has been simply to draw some form of line “through” the points observed. Smoothing techniques, such as fitting a cubic spline[26] to the yields can mean that the drawn yield curve does not necessarily cross through all points, but that line will still be based on some form of minimisation of the distance between the line drawn and the observed points. An essential aspect of such a yield curve is that it is no more than a market artefact, used because it is useful (particularly to communicate) but without any claim to being, for example, a true “par” curve. That this should be the case is evident from the fact that few if any of the observed bonds will be trading at par. Indeed it will often be the case that the observed bonds will be trading at a significant premium or discount from their par value, particularly if the observed bond is designed to be re-opened and tapped over time. This is the single strongest argument against using such yield curves for accurate valuations, such as may be necessary for risk management, structured product creation, and client or regulatory reporting.

We will examine some of the methods used to model yields and the yield curve below when we look at term structure. But an alternative is to fit a yield curve to the data without imposing an “explanatory” model to it. Here we shall study the basic problem that any numerical solution must address.

We can state the problem as follows: we wish to build a spot or zero-coupon curve based on observed market prices/yields that would most accurately predict those observed prices/yields. This is an easy task only if those prices are all at par, and that the bonds are all an exact number of years from maturity. In such an unlikely case, we can calculate the corresponding spot curve with accuracy using a bootstrapping method. In general however we are not in this position and the general problem to be solved can be restated as follows:

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[23] In markets, such as the JGB market, where there is only one recognised benchmark, the most recently issued bond in a number of maturities will be used as well as the only benchmark.

[24] Most bond indices exclude bonds with maturities below one year: bonds with less than one year to maturity are eliminated whenever bond indices are rebalanced (usually once a month).

[25] Bonds are often mapped as having a maturity of the closest single year unit: thus a bond with 9 years and 10 months to maturity may well be mapped as a ten-year bond, particularly if it was issued two months ago and no ten-year bond has since been issued by the same issuer.

[26] A spline is an interpolation where the interpolant is piecewise ploynomial, in our case cubical.

We have a number of bonds through which we want to fit a yield curve, or more accurately, from which we want to derive a spot curve. These bonds can be imagined as a portfolio, **C**. The portfolio **C** can be further described as a matrix **C** of cash flows (coupon payments and principal), where each row consists of the cash flows from a single one of the bonds, arranged in columns according to time. Thus $c_{i,j}$ represents the cash flow paid by bond *i* at time *j*. Since $c_{i,j}$ is a zero-coupon bond (with maturity *j* and yield z_j), the price of that cash flow is the discount factor $d_j = \frac{1}{\left(1+z_j\right)^j}$. **D** is the column vector of all the discount factors *d*, and **P** is the vector of the market prices (including accrued interest) of the bonds in portfolio/matrix **C**. We therefore know **P** and we also know **C**, so we just need to solve for **D**. Since the prices observed must each be the sum of the cash flows of the relevant bond multiplied by the corresponding discount factor, **P** = **CD** (a matrix multiplication). In the real world, it is unlikely that this relationship will hold precisely, so that we can modify our matrix equation to **P** = **CD** + ε, where ε is the vector of errors which must be minimised. Unfortunately for various reasons, this equation is unlikely to be solved straightforwardly[27].

There is a large number of numerical methods which can be used to fit smoothed curves to a set of points, and the literature in the case of fitted yields curves is extensive. Although there are a number of computer programs which can handle different methodologies for fitting a curve, there is certainly still no market standard, hence why a relatively crude methodology is still employed.

3.6.1.2 Swaps

A swap is an agreement between two counterparties to exchange (or "swap") future cash flows. Although there are numerous types of swaps, by far the most common is the Interest Rate Swap (IRS), and of these the basic model is the fixed-for-floating IRS. When no other qualification is attached to the term, it is to this kind of swap that we refer. In such a swap, one counterparty will agree to pay the other counterparty a fixed rate at fixed intervals for a fixed period, and simultaneously to receive a floating rate payment on the same dates over the same period. By market convention, the party paying the fixed rate (or "fixed leg") is called the buyer or payer and is said to be "long the swap", and the party paying the floating rate is called the seller or receiver and is "short the swap".

Although the size of a swap is measured in terms of the notional principal on which the fixed and floating rate are paid, that notional is never exchanged[28]. The only cash flows that are exchanged are the interest payments, and even these are "netted"; in other words, only the difference between the two interest payments ever changes hands. The floating rate will be specified, but is almost certain to be 3- or 6-months LIBOR.

Thus if LIBOR is 4% a swap of EUR 10,000,000 where the fixed rate is 5% and the floating rate is LIBOR + 100 will not on this occasion involve any money changing hands (since the difference between the fixed rate of 5% and the LIBOR of 4% + 100 basis points is 0). Note that the "tenor" (or maturity) of the swap is irrelevant in this instance.

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27 For example there are likely to be more cash flow periods than there are bond prices, which makes a direct solution impossible.

28 And if it were the amounts exchanged would obviously be exactly the same anyway: the netting process makes any exchange redundant. Hence the notional amount of the swap is never at risk.

If LIBOR were to fall to 3% for the next exchange of payments, the total exchanged would then be:

$$\text{EUR } 10,000,000 \cdot (5\% - (3\% + 1\%) = \text{EUR } 10,000,000 \cdot 1\% = \text{EUR } 100,000.$$

EUR 100,000 would be paid BY the party "long the swap".

If instead LIBOR were to rise to 4.5% for the next exchange of payments, the total exchanged would be:

$$\text{EUR } 10,000,000 \cdot (5\% - (4.5\% + 1\%) = \text{EUR } 10,000,000 \cdot 0.5\% = \text{EUR } 50,000.$$

EUR 50,000 would be paid TO the party "long the swap".

In passing it should be mentioned that this brief analysis of netting highlights just how much the idea of measuring the swap market in terms of the notional involved gives an over exaggerated impression of the cash flows that might be implied: in fact, the flows in these cases ranges from EUR -50,000 to EUR 100,000, or in other words, at worst, 1% of the notional involved, even though the fixed leg was set at 5%.

The swap market is a large and liquid market in most significant currencies, and generic swaps can be priced for almost any maturity up to that of the longest government bond (and sometimes beyond). Recently, higher capital charges are imposed in the Basel III risk framework on transactions with longer maturities and this has led to a decrease in liquidity for long maturity transactions. Swaps do not usually "exist" before they are agreed in a trade, and the benchmark maturities on any given day are always an exact number of years from that day. This is in contrast to bonds, which have a fixed maturity date: so that the ten-year benchmark has a maturity date that does not change each day, but rather only changes when a new benchmark is issued. As a result, the yield curve for swaps is relatively easy to draw, even using the rather crude methodology of simply joining points with a straight line: there are generally more points, and they are evenly spaced. The swap curve depicts the fixed-rate leg of a plain vanilla swap against the floating leg of a six-month LIBOR. It assumes that the counterparties are highly rated (so almost default-risk-free).

In the last years the convenience of the swap curve and the ease with which it can be constructed have led to its increasing use as the benchmark for an everyday pricing tool in the corporate bond market. In particular it is increasingly used in the Eurozone because there is currently no accepted sovereign yield curve there.

In addition, there seems to be less basis risk[29] taken when a corporate bond is hedged using swaps rather than the traditional government bonds or futures. This is particularly true in volatile markets. The usefulness of swaps as a hedge became particularly apparent in 1998, during the Russian default and the Long Term Capital Management debacle, when spreads between risky bonds and safe Treasury securities increased dramatically. Firms that had hedged their bond positions by going short government bonds or futures found that these hedges did not offset loss quite as well as hoped: the value of their (risky) bonds fell, and the value of the (risk-free) governments increased, so that the value of the short position in governments fell as well. Rather than mitigating them, these hedge positions in governments amplified losses, which is just the reverse of what a hedge is supposed do (as highlighted in the previous footnote). Swaps resemble risky bonds more than risk-free government bonds do, so a short position in swaps (i.e. paying the floating rate) may be a better hedge than the traditional government bonds or government-based futures. After 1998 corporate bond trading desks at banks have switched from government bonds and futures to swaps for the hedging of their trading books.

Finally, the total notional amount of swaps outstanding is not rigidly fixed unlike the stock of government bonds (although this constraint does not apply to government bond-based futures contracts): they are created by the mere agreement of the two counterparties. This means that there is never any difficulty in “finding” the right side of a swap with which to hedge.

When a new standard swap transaction is traded, the level of the fixed rate is set in order to assure that the transaction has a starting market value of zero. Given that a swap consists of two payment streams or legs – fixed and floating rates - this implies that the present value of all fixed rate payments have to equal the present value of all floating rate payments. This also implies that the fixed rate of a swap transaction can be read as a coupon of an fixed coupon bond that trades at par. The fixed coupon of the bond equals the fixed rate of the swap. Since the price trades at par the yield of this implied bond equals the coupon rate.

So a swap curve that we observe in the market with yields for various maturities can be interpreted as coupons and yields of par bonds for the respective maturity of the swap transaction. Swap curves are therefore often said to represent par curves.

It should come as no surprise that there is a derivative curve based on the difference between the government yield curve and the swap curve, which is simply the difference between the two, and which is known as the Swap Spread Curve.

[29] Basis risk is a specific form of spread risk applied to hedging. When a hedge is put on, it can only be perfect if the instrument used to hedge is the same as the instrument being hedged. Since obviously the instrument used to hedge is not exactly the same instrument, it is not guaranteed to completely hedge all price movements of the instrument being hedged. In particular, the spread (or in this case therefore, the basis) may change as well as the prices of the two instruments: i.e. the hedge may not be perfect. The risk of this imperfection is called the basis risk, and may work both ways: a move in the basis may just as easily benefit the hedger as be to his or her detriment. It should be remembered that a hedge should be neutral: its purpose is neither to lose nor to gain money.

3.6.1.3 Credit and Spread Curves

Credit is the term adopted to refer to bonds which have any risk of default. Typically these will be issued by corporations: hence the term corporate bonds. But in addition the term may encompass bonds issued by sovereign issuers in currencies other than their own (and occasionally even to government bonds issued in the government's own currency). There are numerous forms of credit curves, from the general ratings-based ones to term structures for individual issuers. Sophisticated analysis of individual issuer curves allows for the same kinds of yield curve strategies described further below, but the freedom of action is not quite as large as for developed government markets: liquidity and a relative paucity of issues are just two of the causes. Nevertheless, the creation of credit yield curves is of particular interest in markets where the corporate bond sector is growing fast. As their name implies, spread curves are a measure of the spread of the yields of bonds with different maturities, but of the same issuer, to the relevant maturity on the government curve. They are essentially a derivative of the credit curve and the government yield curve.

3.6.2 Theoretical Curves (Imputed)

3.6.2.1 Term Structures

Term structure is the name given to a model of the zero-coupon or spot curve. In addition to the information contained in a standard market yield curve, a term structure model should have some explanatory value: where a market yield curve simply joins points together because they are there, a term structure attempts to explain why the points are where they are, or at least to create a functional form for the curve.

Term structure analysis usually involves creating or using mathematical models, and financial analysts and economists employ the results for a wide variety of applications. Some of the most common include:

- appreciating the dynamics of financial markets
- forecasting future interest rates
- constructing and managing portfolios
- building and managing hedging strategies
- implementing models to predict economic growth formulating monetary policy
- estimating the overall cost of capital over longer periods
- estimating future inflation

There are a number of term structure models in use and the key theories are expectations hypothesis, liquidity preference, and market segmentation or preferred habitat theories. They will be explored in greater detail but in brief: the expectations hypothesis holds that the shape and level of the yield curve reflects the market expectations of future rates; the liquidity preference theory holds that investors prefer liquid securities, and that since longer-term bonds are more risky, they will demand a liquidity, risk, or term, premium, leading to higher yields for longer bonds; finally market segmentation or preferred habitat theory holds that different investors have different maturity preferences, and as a result there is limited substitutability between bonds of different maturities.

3.6.2.2 Parametric modelling

There are a number of models of the yield curve which depend on fitting a functional form of curve to the observed form. For example, the idea of the Nelson-Siegel, and of the Nelson-Siegel-Svensson approach is to fit the empirical form of the yield curve with a pre-specified functional form of the spot rates which is a function of the time to maturity of the bonds. Though the models are parsimonious (use few variables), the fit is not closed-form and is achieved through optimisation techniques.

In addition there are factor models, originating with Vasicek (1977) but also Cox-Ingersoll-Ross (1985), Heath-Jarrow-Morton (1992) and Duffie-Kan (1996). And there are also binomial term structure models, such as Black-Derman-Toy (1990) and Black-Karasinski (1991). This is only a partial list of the models currently in use, and it is beyond the scope of this course to explain any or all of these in significant detail. However it should be noted that some of these models constrain the shape of possible yield curves, sometimes excluding existing (observed) ones. Furthermore, it is claimed that a version of the Duffie-Kan model does in fact allow for all the yield curves observed in relatively mature markets subject to certain conditions[30].

3.7 Yield spread analysis

Fixed income instruments differ in a variety of ways (marketability, tax status, credit risk). It is thus possible to examine the impact of the various differences in the characteristics of bonds on their yield to maturity. It is often helpful to consider one difference at a time: for example, when the only difference is given by the maturity, attention is paid to the term structure of yields; when the bonds are equal other than in their credit risk, the attention is focused on the risk structure of yields and so on. In any case, the differential in the yields of two or more bonds is called **yield spread** and the analysis of the causes and consequences of those spreads is called **yield spread analysis**. The yield spread of a given instrument is usually measured against the yield of a government security having comparable maturity since Government securities are the instruments of the highest quality in the fixed income market in terms of marketability, credit risk and often of tax status. The yield spread between a given security and the corresponding Government security has the nature of risk premium because it reflects the risks (lower liquidity, higher credit risk ...) that an investor has to face when he invests in non-Government securities.

Normally yield spreads are measured in terms of basis points, where a basis point is equal to 0.01%: so, for example, a given bond A has a yield to maturity equal to 7% and another to 7.50%, the yield spread is 0.50%, that is 50 basis points.

Moreover, one can also measure yield spread in terms of the yield level and calculate the **relative yield spread** defined as

$$\text{relative yield spread} = \frac{\text{yield bond B - yield bond A}}{\text{yield bond A}}$$

30 See Brousseau (2002)

or the **yield ratio**

$$\text{yield ratio} = \frac{\text{yield bond B}}{\text{yield bond A}}$$

In our example, the relative yield spread is 0.0714 (0.50/7) and the yield ratio 1.0714.

3.7.1 Types of spreads

As we have seen above historically market participants mainly compared bond yields with the respective government bond with the same maturity – the **government yield spread** of a bond.

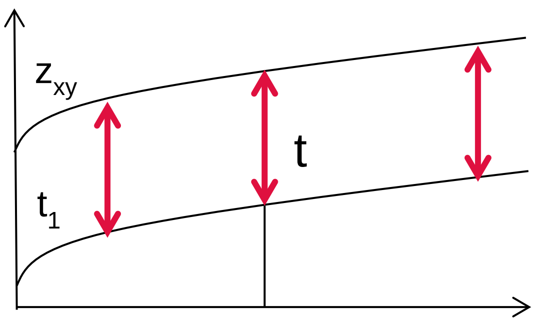
Given today's importance of the swap markets, often yield spreads are measured against the correspondent swap yield with the same maturity – the **swap spread** of a bond.

Both spreads are only an approximation of the effective yield difference. As soon as we have differences in coupon levels between the bonds or swaps (for swaps yields equal the implied coupons) we face the coupon effect (as described in section 3.2.2) that distorts yield to maturity.

In order to overcome this shortcoming a measure called z-spread has been developed. Thanks to progress on the side of market information systems (in this case mainly driven by Bloomberg), this concept has gained acceptance in the market place mainly amongst fixed income specialist.

The z-spread tries to eliminate the coupon effect when comparing yields of bonds to swap rates. In order to compute the z-spread we first have to calculate a spot rate curve for swap yields. A traditional swap curve is based on par-bond yields. Using bootstrapping (concept is explained in the appendix of the chapter "Interest rates - term structure and applications") zero or spot yields can be derived for swaps.

Having spot rates for all relevant maturities (swap curve z_0) we can discount each cash flow of a given bond with the relevant spot rate. This results in a present value that is higher than the market price of a bond that carries credit risk. In order to equal the present value to the market price we add a spread z_{xy} (the z-spread – constant for all maturities) to each spot rate. By iteration we find the level of the z-spread that equates the present value of the discounted bond cash flows to the observed market price (see graph).



The advantage of the z-spread concept is that it is a very accurate measure of credit risk and isn't distorted by coupon effects. But on the other side it needs a lot of data, especially for a historical spread comparison over time. Due to this the z-spread has become market standard only a few years ago.

3.7.2 Spread curves

The bond market can be subdivided into sectors by type of issuer (Government, corporate, financial institutions), credit quality (summarized by the rating), maturity (short, medium and long term) and level of coupon. Each sector of the market is characterized by a yield spread versus other sectors of the bond market.

3.7.3 Determinants of yield spreads

Many factors may affect the yield spread; in principle, any difference in any characteristics between two bonds should be reflected in a yield differential. For simplicity, the determinants of the yield spread are usually classified as:

- maturity of the instrument;
- creditworthiness of the issuer;
- embedded options;
- tax status of the instrument;
- liquidity of the security.

As far as the first item is concerned the reader is referred to the chapter "Interest rates - term structure and applications".

The creditworthiness (probability of default) of the issuer clearly affects the yield: as long as there is a possibility of default, the **expected yield** (which takes into account the likelihood of default and the loss in the event of a default) is lower than the **promised yield** (which is calculated on the promised cash flows taken at their face value). The greater the likelihood of default (default probability) and the greater the amount lost in case of default (the loss in the event of default), the wider the spread between promised and expected yield (yield spread).

An example of the relationship between credit risk and expected yield is depicted in Figure 3-3 (**risk structure of interest rates**): higher risk bonds command higher yields to maturity.

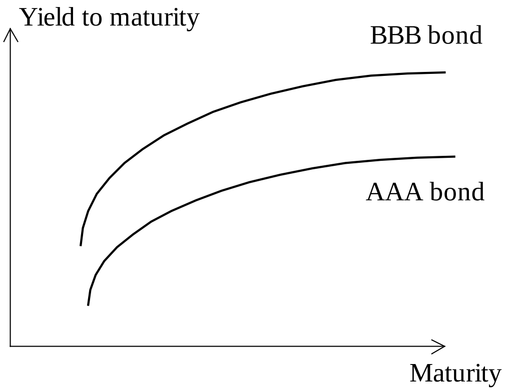


Figure 3-3: Term structure of different rating grade bonds

Yield spreads due to creditworthiness tend to widen when the economy is likely to face a recession and narrow in boom phases. This reflects the fact that investors see a higher probability of default and a greater severity of losses when the economy is not in good health. Consequently they assign an increasing probability of default to lower quality borrowers and require a higher risk premium.

When the credit risk is taken into account, the yield earned by the investor must be high enough to account for default probability and for the losses incurred in the event of default. The following relationship holds, where r_f is the risk free rate, r_p is the risky rate and (1-p) is the default probability (so that p is the probability of exact fulfilment of the payment obligations). Let's assume at first that, in the event of default, the bondholders do not receive any payment (i.e. that they receive a payment of 0). In that case:

$$p \cdot (1 + r_p) + (1 - p) \cdot 0 = 1 + r_f$$

hence

$$r_p = \frac{1 + r_f}{p} - 1$$

and the premium for the credit risk is

$$\text{premium} = r_p - r_f = \frac{(1 + r_f)}{p} - (1 + r_f)$$

If the bondholder, as it is generally true, can get a fraction γ of the amount owed by the issuer, then

$$p \cdot (1 + r_p) + \gamma \cdot (1 - p) \cdot (1 + r_p) = 1 + r_f$$

and the credit risk premium is

$$r_p - r_f = \frac{(1 + r_f)}{(\gamma + p - \gamma \cdot p)} - (1 + r_f)$$

As far as **embedded options** are concerned, many bond issues include provisions granting either the issuer or the bondholder or both, one or more options to take some actions such as repaying the bond before the maturity (for the issuer), or being redeemed before maturity (for the borrower). The first of these is called a **call** option, and the second a **put** option. Clearly, the option benefits the party who can choose to exercise it (the holder of the option) at the cost of the party who has written it. The advantage of holding an option has to be paid for by a **yield differential**. For example, the call provision allows the issuer to shorten the maturity of the bond; that option will be exercised only when it is advantageous for the issuer who may, for example, be able to replace a higher yield instrument with a lower yield one. Since each option has to be paid for, a bond with an embedded call provision should command a higher yield (the issuer has to pay for the option) than a similar bond without the call provision itself. Conversely, a putable bond (a bond that the issuer must redeem at a given price if the investor wishes) should carry a lower yield (the bondholder has to pay for the option he holds) than a bond similar but for the put provision.

The effect of the **tax status** is quite clear: since investors will base their bond investment decisions on the net yield a taxable bond (like a corporate bond, for example) has to pay a higher gross yield in order to compete with an exempt bond (like a government or municipal bond that is tax exempt for investors, for example).

The after tax (net) yield on a taxable fixed income security is defined as:

$$\text{after}-\text{tax } (\text{net}) \text{ yield } = \text{ pretax } (\text{gross}) \text{ yield} \cdot (1- \text{ tax rate})$$

Example:
If the yield on a given taxable bond is 10% and the relevant tax rate is 30%, the after tax yield is

$$10\% \cdot (1-30\%) = 7\%$$

Likewise, it is possible to calculate the equivalent taxable yield of a tax exempt security with the following formula:

$$\text{equivalent taxable yield} = \frac{\text{tax exempt yield}}{1-\text{tax rate}}$$

As far as the **liquidity** is concerned, the greater the expected liquidity, the lower the yield required by the investors. The reason is straightforward: when an issue is illiquid, the investor might experience some problems in selling the bond before the maturity. The liquidity of bonds has three dimensions:

- marketability: that is the existence of a broad and deep market on a given instrument;
- time to maturity: because at maturity (unlike stocks for example) the bond will be paid back and so return to cash;
- financiability: to the extent that a given issue can be utilized as a collateral in order to borrow funds.

Even within the same segment of bonds there can be small differences in liquidity. So for example so called “on the run” US treasury bonds sometimes trade at slightly lower yields than “off the run” US treasury bonds. “One the run” bonds are the last issued benchmark maturity (5, 10 or 30 years) bonds. As soon as a new new issue is placed in the market the older bonds become “off the run” bonds. The reason for this is that an important part of all trading activity concentrates on the “on the run” treasuries.

