# KCSE MATHEMATICS 2016
## PAPER 1
### SECTION I (50 marks)

*Answer **all** the questions from this section in the spaces provided.*

1. Without using a calculator evaluate, $\dfrac{-2(5+3)-9 \div 3 + 5}{-3 \times -5 + -2 \times 4}$ (3 marks)

2. Simplify $\dfrac{p^2 + 2pq + q^2}{p^3 - pq^2 + p^2q - q^3}$ (4 marks)

3. The external length, width and height of an open rectangular container are 41 cm, 21 cm and 15.5 cm respectively. The thickness of the materials making the container is 5 mm. If the container has 8 litres of water, calculate the internal height above the water level. (4 marks)

4. The figure below shows a net of a solid (measurements are in centimetres).
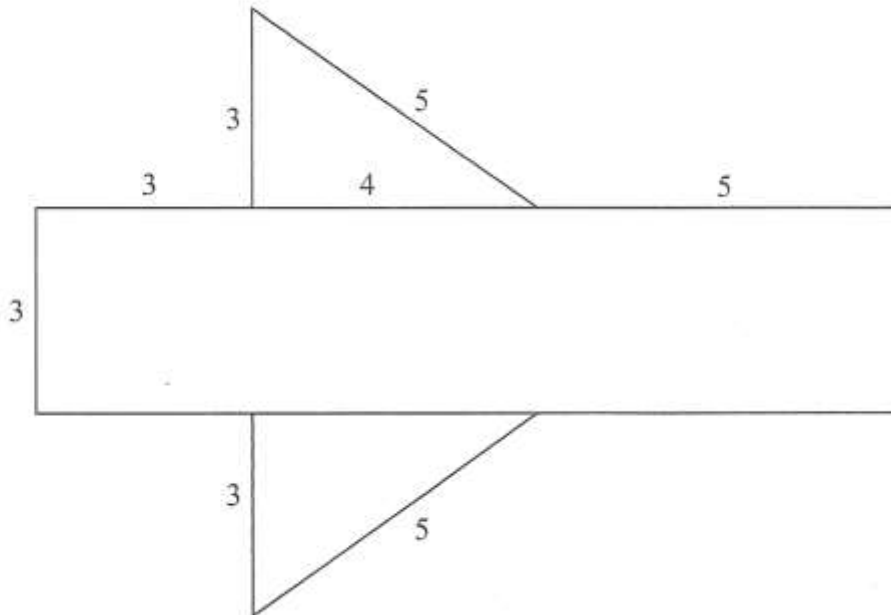


Below is a part of the sketch of the solid whose net is shown above. Complete the sketch of the solid, showing the hidden edges with broken lines. (3 marks)
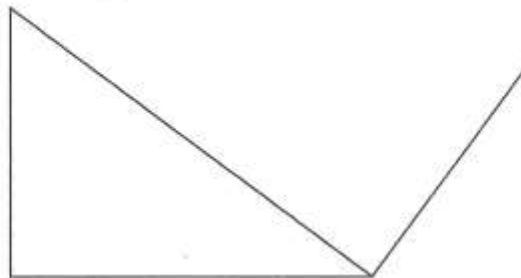
5. Given that OA = 2i + 3j and OB = 3i − 2j, find the magnitude of AB to one decimal place. (3 marks)

6. A bus travelling at an average speed of 63 km/h left a station at 8:15 a.m. A car later left the same station at 9:00 a.m. and caught up with the bus at 10:45 a.m. Find the average speed of the car. (3 marks)

7. Given that $x$ is an acute angle and $\cos x° = \frac{2}{5}\sqrt{5}$ find, without using mathematical tables or a calculator, $\tan(90 − x)°$. (2 marks)

8. Without using mathematical tables or a calculator, evaluate $27^{\frac{2}{3}} \times \left(\frac{81}{16}\right)^{-\frac{1}{4}}$ (3 marks)

9. A minor arc of a circle subtends an angle of 105° at the centre of the circle. If the radius of the circle is 8.4 cm, find the length of the major arc. (Take $\pi = \frac{22}{7}$). (3 marks)

10. The gradient of the tangent to the curve $y = ax^3 + bx$ at the point $(1,1)$ is −5. Calculate the values of $a$ and $b$. (4 marks)

11. A line with gradient of −3 passes through the points $(3, k)$ and $(k, 8)$. Find the value of $k$ and hence express the equation of the line in the form $ax + by = c$, where $a$, $b$ and $c$ are constants. (3 marks)

12. Points L and M are equidistant from another point K. The bearing of L from K is 330°. The bearing of M from K is 220°.

    Calculate the bearing of M from L. (3 marks)

13. In this question, mathematical tables should not be used.
    A Kenyan bank buys and sells foreign currencies as shown below:

|  | Buying (In Kenya Shillings) | Selling (In Kenya Shillings) |
|---|---|---|
| 1 Hong Kong Dollar | 9.74 | 9.77 |
| 1 South African Rand | 12.03 | 12.11 |

A tourist arrived in Kenya with 105 000 Hong Kong Dollars and changed the whole amount to Kenya Shillings. While in Kenya, she spent Sh 403 879 and changed the balance to South African Rands before leaving for South Africa. Calculate the amount in South African Rand, that she received. (3 marks)

14. A small cone of height 8 cm is cut off from a bigger cone to leave a frustum of height 16 cm. If the volume of the smaller cone is 160 cm³, find the volume of the frustum. (3 marks)

15. The production of milk, in litres, of 14 cows on a certain day was recorded as follows:

    22, 26, 15, 19, 20, 16, 27, 15, 19, 22, 21, 20, 22 and 28.

    Determine:

    (a) the mode; (1 mark)

    (b) the median. (2 marks)

16. Given that Log 4 = 0.6021 and Log 6 = 0.7782, without using mathematical tables or a calculator, evaluate log 0.096. (3 marks)

## SECTION II (50 marks)

*Answer any five questions from this section in the spaces provided.*

17. (a) Solve the equation $\dfrac{x+3}{24} = \dfrac{1}{x-2}$ (4 marks)

(b) The length of a floor of a rectangular hall is 9 m more than its width. The area of the floor is 136 m².

(i) Calculate the perimeter of the floor. (4 marks)

(ii) A rectangular carpet is placed on the floor of the wall leaving an area of 64 m². If the length of the carpet is twice its width, determine the width of the carpet. (2 marks)

18. Three business partners: Asha, Nangila and Cherop contributed Ksh 6 000, Ksh 85 000 and Ksh 105 000 respectively. They agreed to put 25% of the profit back into business each year. They also agreed to put aside 40% of the remaining profit to cater for taxes and insurance. The rest of the profit would then be shared among the partners in the ratio of their contributions. At the end of the first year, the business realised a gross profit of Ksh 225 000.

(a) Calculate the amount of money Cherop received more than Asha at the end of the first year. (5 marks)

(b) Nangila further invested Ksh 25 000 into the business at the beginning of the second year. Given that the gross profit at the end of the second year increased in the ratio 10:9, calculate Nangila's share of the profit at the end of the second year. (5 marks)

19. The frequency table below shows the daily wages paid to casual workers by a certain company.

| Wages in Shillings | 100–150 | 150–200 | 200–300 | 300–400 | 400–600 |
|---|---|---|---|---|---|
| No. of workers | 160 | 120 | 380 | 240 | 100 |

(a) In the grid provided, draw a histogram to represent the above information. (5 marks)

(b) (i) State the class in which the median wage lies. (1 mark)

(ii) Draw a vertical line, in the histogram, showing where the median wage lies. (1 mark)

(c) Using the histogram, determine the number of workers who earn Sh 450 or less per day. (3 marks)

**20.** In the diagram below, the coordinates of points A and B are (1, 6) and (15, 6) respectively. Point N is on OB and that 30N = 20B. Line OA is produced to L such that OL = 30A
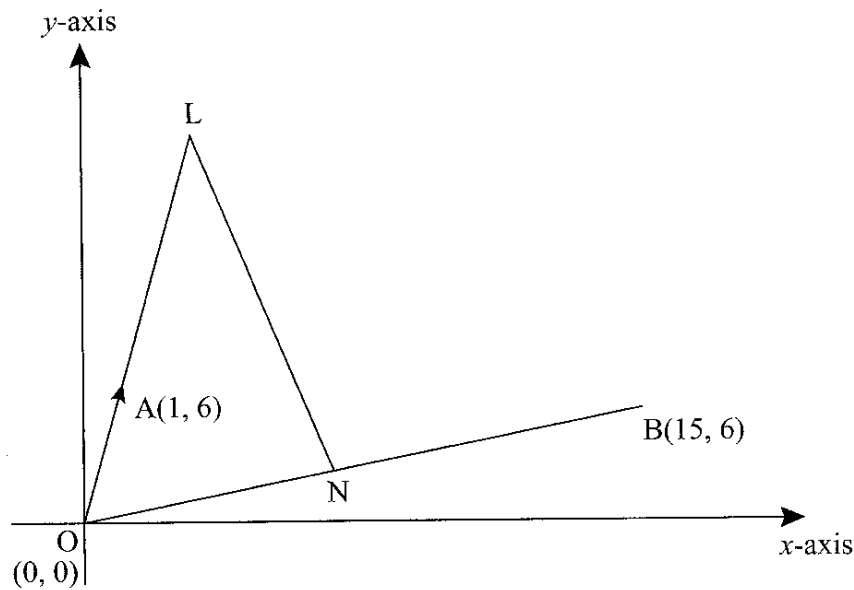


(a) Find vector LN. (3 marks)

(b) Given that a point M is on LN such that LM:MN = 3:4, find the coordinate of M. (2 marks)

(c) If line OM is produced to T such that OM:MT = 6:1

(i) Find the position vector of T. (1 mark)

(ii) Show that points L, T and B are collinear. (4 marks)

**21.** (a) The ratio of Juma's and Akinyi's earnings was 5:3. Juma's earnings rose to Ksh 8 400 after an increase of 12%.

Calculate the percentage increase in Akinyi's earnings given that the sum of their new earnings was Ksh 14 100. (6 marks)

(b) Juma and Akinyi contributed all the new earnings to buy maize at Ksh 1175 per bag. The maize was then sold at Ksh 1762.50 per bag. The two shared all the money from the sales of the maize in the ratio of their contributions.

Calculate the amount that Akinyi got. (4 marks)

**22.** Using a pair of compasses and ruler only, construct:

(a) (i) triangle ABC in which AB = 5 cm, $\angle$BAC = 30° and $\angle$ABC = 105°. (3 marks)

(ii) a circle that passes through the vertices of the triangle ABC. Measure the radius. (3 marks)

(iii) the height of triangle ABC with AB as the base. Measure the height. (2 marks)

(b) Determine the area of circle that lies outside the triangle correct to 2 decimal places. (2 marks)

**23.** The equation of a curve is $y = 2x^3 + 3x^2$.

    (a)    Find:

        (i)    the $x$-intercept of the curve:        (2 marks)

        (ii)    the $y$-intercept of the curve.        (1 mark)

    (b)    (i)    Determine the stationary points of the curve.    (3 marks)

        (ii)    For each point in (b) (i) above, determine whether it is a maximum or a minimum.    (2 marks)

    (c)    Sketch the curve.    (2 marks)

**24.**    (a)    On the grid provided, draw a graph of the function $y = \frac{1}{2} x^2 - x + 3$ for $0 \leq x \leq 6$.    (3 marks)

    (b)    Calculate the mid-ordinates for five strips between $x = 1$ and $x = 6$, and hence use the mid-ordinate rule to approximate the area under the curve between $x = 1$, $x = 6$ and the $x$ – axis.    (3 marks)

    (c)    Assuming that the area determined by integration to be the actual area, calculate the percentage error in using the mid-ordinate rule.    (4 marks)

# KCSE MATHEMATICS 2016
## PAPER 2

### SECTION I (50 marks)

*Answer **all** the questions from this section in the spaces provided.*

1.  Simplify $\dfrac{4}{\sqrt{5}+\sqrt{2}} - \dfrac{3}{\sqrt{5}-\sqrt{2}}$ (3 marks)

2.  By correcting each number to one significant figure, approximate the value of $788 \times 0.006$. Hence calculate the percentage error arising from this approximation. (3 marks)

3.  The area of triangle FGH is $21\ cm^3$. The triangle FGH is transformed using the matrix

    $$\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$$

    Calculate the area of the image of triangle FGH (2 marks)

4.  Make $s$ the subject of the formula.

    $$w = 3\sqrt{\dfrac{s+t}{s}}$$  (3 marks)

5.  Solve the equation
    $2 \log x - \log (x-2) = 2 \log 3$. (3 marks)

6.  Kago deposited Ksh 30 000 in a financial institution that paid simple interest at the rate of 12% per annum. Nekesa deposited the same amount of money as Kago in another financial institution that paid compound interest. After 5 years, they had equal amounts of money in the financial institutions.

    Determine the compound interest rate, to 1 decimal place for Nekesa's deposit. (4 marks)

7.  The masses in kilograms of 20 bags of maize were:
    90, 94, 96, 98, 99, 102, 105, 91, 102, 99, 105, 94, 99, 90, 94, 99, 98, 96, 102 and 105.

    Using an assumed mean of 96 kg, calculate the mean mass, per bag of the maize. (3 marks)

8.  The first term of an arithmetic sequence is $-7$ and the common difference is 3.

    (a)  List the first six terms of the sequence; (1 mark)

    (b)  Determine the sum of the first 50 terms of the sequence (2 marks)

**9.** A bag contains 2 white balls and 3 black balls. A second bag contains 3 white balls and 2 black balls. The balls are identical except for the colours. Two balls are drawn at random, one after the other from the first bag and placed in the second bag.

Calculate the probability that the 2 balls are both white. (2 marks)

**10.** An arc 11 cm long, subtends an angle of 70° at the centre of a circle. Calculate the length, correct to one decimal place, of a chord that subtends an angle of 90° at the centre of the same circle. (4 marks)

**11.** Given that $qi + \frac{1}{3}j + \frac{2}{3}k$ is a unit vector, find q. (2 marks)

**12.** (a) Expand the expression $(1 + \frac{1}{2}x)^5$ in ascending powers of $x$, leaving the coefficients as fractions in their simplest form. (2 marks)

(b) Use the first three terms of the expansion in (a) above to estimate the value of $(1\frac{1}{20})^5$. (2 marks)

**13.** A circle whose equation is $(x - 1)^2 + (y - k)^2 = 10$ passes through the point (2,5). Find the value of $k$. (3 marks)

**14.** Water and milk are mixed such that the volume of water to that of milk is 4:1. Taking the density of water as 1 g/cm³ and that of milk as 1.2 g/cm³, find the mass in grams of 2.5 litres of the mixture. (3 marks)

**15.** A school decided to buy at least 32 bags of maize and beans. The number of bags of beans were to be at least 6. A bag of maize costs Ksh 2 500 and a bag of beans costs Ksh 3 500. The school had Ksh 100 000 to purchase the maize and beans.

Write down all the inequalities that satisfy the above information. (4 marks)

**16.** Find in radians, the values of $x$ in the interval $0^c \leq x \leq 2\pi^c$ for which $2\cos^2 x - \sin x = 1$. (Leave the answer in terms of $\pi$) (4 marks)

## SECTION II (50 marks)

*Answer any five questions from this section in the spaces provided*

**17.** A garden measures 10 m long and 8 m wide. A path of uniform width is made all round the garden. The total area of the garden and the path is 168 m².

(a) Find the width of the path. (4 marks)

(b) The path is to be covered with square concrete slabs. Each corner of the path is covered with a slab whose side is equal to the width of the path. The rest of the path is covered with slabs of side 50 cm. The cost of making each corner slab is Sh 600 while the cost of making each smaller slab is Sh 50.

Calculate:

(i) the number of the smaller slabs used. (3 marks)

(ii) the total cost of the slabs used to cover the whole path. (3 marks)

**18.** In the figure below, P, Q, R and S are points on the circle with centre O. PRT and USTV are straight lines. Line USTV is a tangent to the circle at S. ∠ RST = 50° and ∠ RTV = 150°.
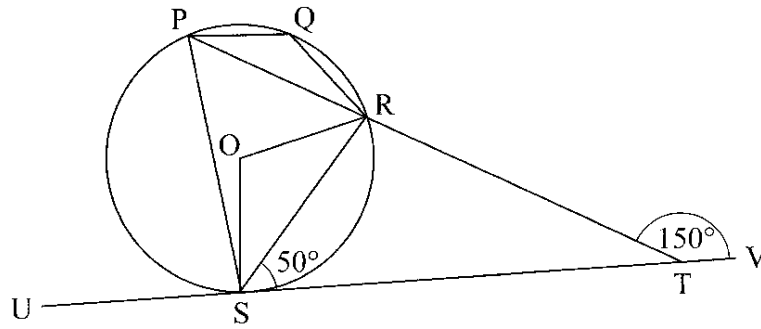


(a) Calculate the size of:

    (i) ∠ QRS; (2 marks)

    (ii) ∠ USP; (1 mark)

    (iii) ∠ PQR. (2 marks)

(b) Given that RT = 7 cm and ST = 9 cm, calculate to 3 significant figures;

    (i) the length of line PR; (2 marks)

    (ii) the radius of the circle. (3 marks)

**19.** The figure ABCDEF below represents a roof of a house.
AB = DC = 12 m, BC = AD = 6 m, AE = BF = CF = DE = 5 m and EF = 8 m.
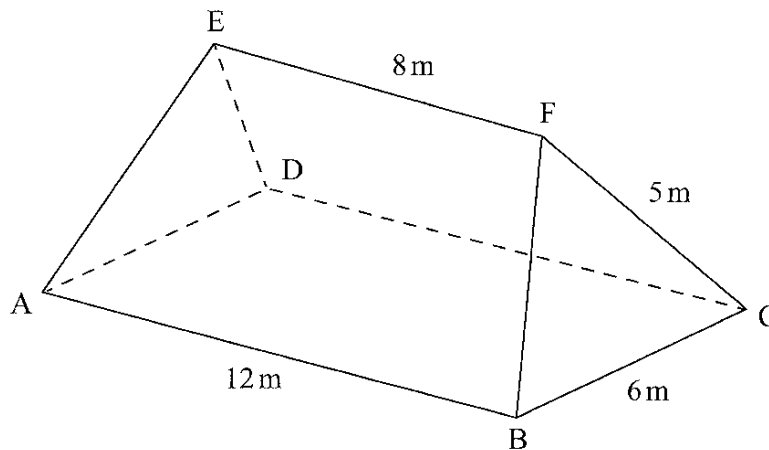


(a) Calculate, correct to 2 decimal places, the perpendicular distance of EF from the plane ABCD. (4 marks)

(b) Calculate the angle between:

    (i) the planes ADE and ABCD; (2 marks)

    (ii) the line AE and the plane ABCD, correct to 1 decimal place; (2 marks)

    (iii) the planes ABFE and DCFE, correct to 1 decimal place. (2 marks)

20. A water vendor has a tank of capacity 18 900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000 cm³/minute through pipe A and 120 000 cm³/minute through pipe B.

    (a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank. (3 marks)

    (b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

        (i) Calculate the time it took to fill the tank on that day. (4 marks)

        (ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6 300 litres, calculate, in litres, the amount of water that was wasted. (3 marks)

21. A tourist took 1 hour 20 minutes to travel by an aircraft from town T(3°S, 35°E) to town U(9°N, 35°E). (Take the radius of the earth to be 6370 km and $\pi = \frac{22}{7}$),

    (a) Find the average speed of the aircraft. (3 marks)

    (b) After staying at town U for 30 minutes, the tourist took a second aircraft to town V(9°N, 5°E). The average speed of the second aircraft was 90% that of the first aircraft. Determine the time, to the nearest minute, the aircraft took to travel from U to V. (3 marks)

    (c) When the journey started at town T, the local time was 0700h. Find the local time at V when the tourist arrived. (4 marks)

22. The gradient function of a curve is given by the expression $2x + 1$. If the curve passes through the point $(-4, 6)$;

    (a) Find:

        (i) The equation of the curve, (3 marks)

        (ii) The values of $x$ at which the curve cuts the $x$-axis, (3 marks)

        (iii) Determine the area enclosed by the curve and the $x$-axis. (4 marks)

23. A quadrilateral with vertices at K(1,1), L(4,1), M(2,3) and M(1,3) is transformed by a matrix.

    $T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ to a quadrilateral K'L'M'N'.

    (a) Determine the coordinates of the image. (3 marks)

    (b) On the grid provided draw the object and the image. (2 marks)

    (c) (i) Describe fully the transformation which maps KLMN onto K'L'M'N'. (2 marks)

        (ii) Determine the area of the image. (1 mark)

    (d) Find a matrix which maps K'L'M'N' onto KLMN.

**24.** The first, fifth and seventh terms of an arithmetic progression (AP) correspond to the first three consecutive terms of a decreasing Geometric Progression (G.P.). The first term of each progression, is 64, the common difference of the AP is d and the common ratio of the G.P. is r.

(a) (i) Write two equations involving d and r. (2 marks)

(ii) Find the values of d and r. (4 marks)

(b) Find the sum of the first 10 terms of:

(i) The arithmetic progression (A.P.); (2 marks)

(ii) The Geometric Progression (G.P.). (2 marks)