Advanced Management Accounting

Advanced Management Accounting Notes

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ACKNOWLEDGMENT

We gratefully acknowledge permission to quote from the past examination papers of the following bodies: **Kenya Accountants and Secretaries National Examination Board (KASNEB); Chartered Institute of Management Accountants (CIMA); Chartered Association of Certified Accountants (ACCA).**

We would also like to extend our sincere gratitude and deep appreciation to **Mr. Cyrus Iraya** for giving his time, expertise and valuable contribution, which were an integral part in the initial development of this Management Accounting Notes. He holds the following academic honours, **MBA, B.COM (Accounting), CPA**, currently pursuing his **Phd in Finance at the University of Nairobi**.

INSTRUCTIONS FOR STUDENTS

This study guide is intended to assist distance-learning students in their independent studies. In addition, it is only for the personal use of the purchaser see copyright clause. The course has been broken down into nine lessons each of which should be considered as approximately one week of study for a full time student. Solve the reinforcement problems verifying your answer with the suggested solution contained at the back of the distance learning pack. When the lesson is completed, repeat the same procedure for each of the following lessons.

At the end of lessons 2, 4, 6 and 8 there is a comprehensive assignment that you should complete and submit for marking to the distance learning administrator.

SUBMISSION PROCEDURE

1. After you have completed a comprehensive assignment clearly identify each question and number your pages.
2. If you do not understand a portion of the course content or an assignment question indicate this in your answer so that your marker can respond to your problem areas. Be as specific as possible.
3. Arrange the order of your pages by question number and fix them securely to the data sheet provided. Adequate postage must be affixed to the envelope.
4. While waiting for your assignment to be marked and returned to you, continue to work through the next two lessons and the corresponding reinforcement problems and comprehensive assignment.

On the completion of the last comprehensive assignment a two week period of revision should be carried out of the whole course using the material in the revision section of the Management Accounting Notes. At the completion of this period the final Mock Examination paper should be completed under examination conditions. This should be sent to the distance learning administrator to arrive in Nairobi at least five weeks before the date of your sitting the KASNEB Examinations. This paper will be marked and posted back to you within two weeks of receipt by the Distance Learning Administrator.

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Management Accounting Course Description

CONTENTS

The course covers the range of topics needed to successfully sit the appropriate examination in the CPA syllabus.

The range of topics is extremely wide *(see the syllabus copied in the Management Accounting Notes)* and serious students must plan to work through the many exercises provided.

This requires the devotion of the students prime time to his studies for without this commitment, the benefit of this material will not be realised.

Students require a working knowledge of four previous syllabi, Costing in Section 2, Business Finance and STAMIS in section 3 and Quantitative Techniques in Section 4. These must be revised in order to appreciate this course and its particular orientations.

Students mainly find difficulty because of time given for preparation, and lack of material, this course overcomes the second cause. It is the onus of each individual student to overcome the first.

RECOMMENDED TEXT

Management and Cost Accounting by Colin Drury

LESSON ONE

INTRODUCTION TO MANAGEMENT ACCOUNTING

OBJECTIVES

Define and introduce Management Accounting

Explain the decision making process

INSTRUCTIONS

1. Read the Study Text and Chapters 1, 2 and 12 of Management and Cost Accounting by Colin Drury

2. Attempt the reinforcing questions at the end of the lesson under examination conditions

3. Compare your answers with those given in Lesson 10

CONTENTS

Key Definitions

Definition of accounting

Accounting is the process of identifying measuring and communicating economic information to permit informed judgements and decisions by users of information.
It is therefore concerned with providing information that will help decision makers make good decisions.

To understand accounting one must understand:

- The attributes of good information
- Process of measuring and communicating information
- The decision making process
- Users of information
- The above points are briefly discussed below:

Users of information

The users of information can be divided into two:

- Internal users who are parties within the organization e.g. the management or the employees.
- External users who on the other hand, are parties outside the organisation e.g. the shareholder, creditors, government, customers, etc

From the users point of view accounting can be divided into two:

Management Accounting

- Which is concerned with provision of information to people within the organisation to help them make better decisions. Management accounting is concerned with the provision and interpretation of the information required by management at all levels for the following purposes:
- Formulating the policies of the organisation
- Planning the activities of the organisation in the long-term, medium-term and short-term (Strategic to operational planning)
- Controlling the activities of the organisation
- Decision-making
- Performance appraisal
- Management accounting can also be said to be concerned with data gathering (both from internal and external sources), analysing, processing, interpreting and communicating the resulting information for use within the organisation, so that management can more effectively plan, make decisions and control operations.

Financial Accounting

Which is concerned with the provision of information to external parties outside the organization. It's the process of measuring, classifying, summarising and reporting financial information used in making economic decisions. It's concerned with the preparation of financial statements to be used by the firm's external stakeholders.

The key differences between Management Accounting (MA) and Financial Accounting (FA) can be summarised as follows:

	MA	FA
Users	Internal	External
Nature	Future	Historical
Details	More detailed	Summarised
Legality	Not legal	Legal
Format	Not standard	standard

It is important to define cost accounting at this point.

Cost accounting

It's the process of cost ascertainment and cost control. It is a formal system of accounting by means of which cost of product and services are ascertained and controlled.

1.2 Attributes of good information

- Information is anything that is communicated and is sometimes said to be processed data. It is data processed in such a way as to be of meaning to the person receiving it. Good information should have the following attribute:
- It should be relevant to a user's needs. For the communicator this requires the following:
- Identifying the user: Information must be suited and sent to the right person i.e. the person who requires it to do his job.
- Getting the purpose right: It is effective only when it helps the user to make decisions.
- Getting the volume right: Information must be complete for its purpose and should not omit any necessary item. It should be no greater in volume than the users would find helpful or be able to take in.
- It should be accurate within the user's needs i.e. should be correct and error free.
- It should inspire the user's confidence i.e. information should not give the user any reason to mistrust it, disbelieve it or ignore it by making sure the information is neutral.
- It should be timely: The information must be readily available within the time period which makes it useful. It must be at the right place at the right time.
- It must be appropriately communicated: Information will lose its value if it is not clearly communicated to the users in a suitable format and through a suitable medium.
- It should be cost effective: Good information should not cost more than its worth. Gathering, storing, retrieving and communicating an item of information may require expenses in form of time, energy and resources. If the expense is greater than the potential value of the item, then it should not be communicated.

1.3 Role of the Management Accountant in the Management Process

The management process involves planning, organising, controlling, directing, communicating and motivating. We will explain each of these functions:

Planning

Planning is the basic function of the management by means of which the managers decide:

- What goals are to be accomplished
- How they will be accomplished
- Planning gives the manager a warning of possible future crisis and therefore they avoid the need to make unplanned decisions.
- The management accountant helps to formulate future plans by providing information to assist in deciding what product to sell, in what market and at what prices and in evaluating proposals for capital expenditure.
- In the budgeting process the management accountant provides data on past performance, establishes budget procedures and budget time tables.

Control

- Control involves a comparison of actual performance with the plan so that deviation from the plan can be identified and corrective action taken.
- It can be defined as the process of compelling events to conform to a plan. The management accountant aids the control process by providing performance reports that compare the actual performance with the planned outcome for each responsibility centre.
- A responsibility centre may be defined as a segment (e.g. a division) of an organisation where an individual manager holds delegated authority and is responsible for the segments performance.
- The management accountant also draws a manager's attention to those specific activities that do not conform to a plan. This aids the process of *Management By Exception*

Organising

- It is the establishment of the framework within which the required activities are to be performed and the designation of who should perform these activities. It involves the establishment of decision units such as departments, sections, branches, etc.
- The management accountant will provide information on the performance of each of these segments.

Motivation

Motivation involves influencing human behaviour so that the participants identify with the objectives of the organisation and makes decisions that are in harmony with these objectives. Budgets and performance reports produced by management accountants motivate the firm's employees. To be motivating however, targets should be challenging but achievable.

Communication

To communicate means to make known, impart or transmit the information. The management accountant aids the communication process by installing and maintaining an effective communication system such as the Management Accounting Information System (MAIS). An example of a MAIS is the budgetary system.

1.4 Decision Making Process

Decision making is the process of choosing among alternatives. There are 7 steps that should be followed as shown in figure 1 below:

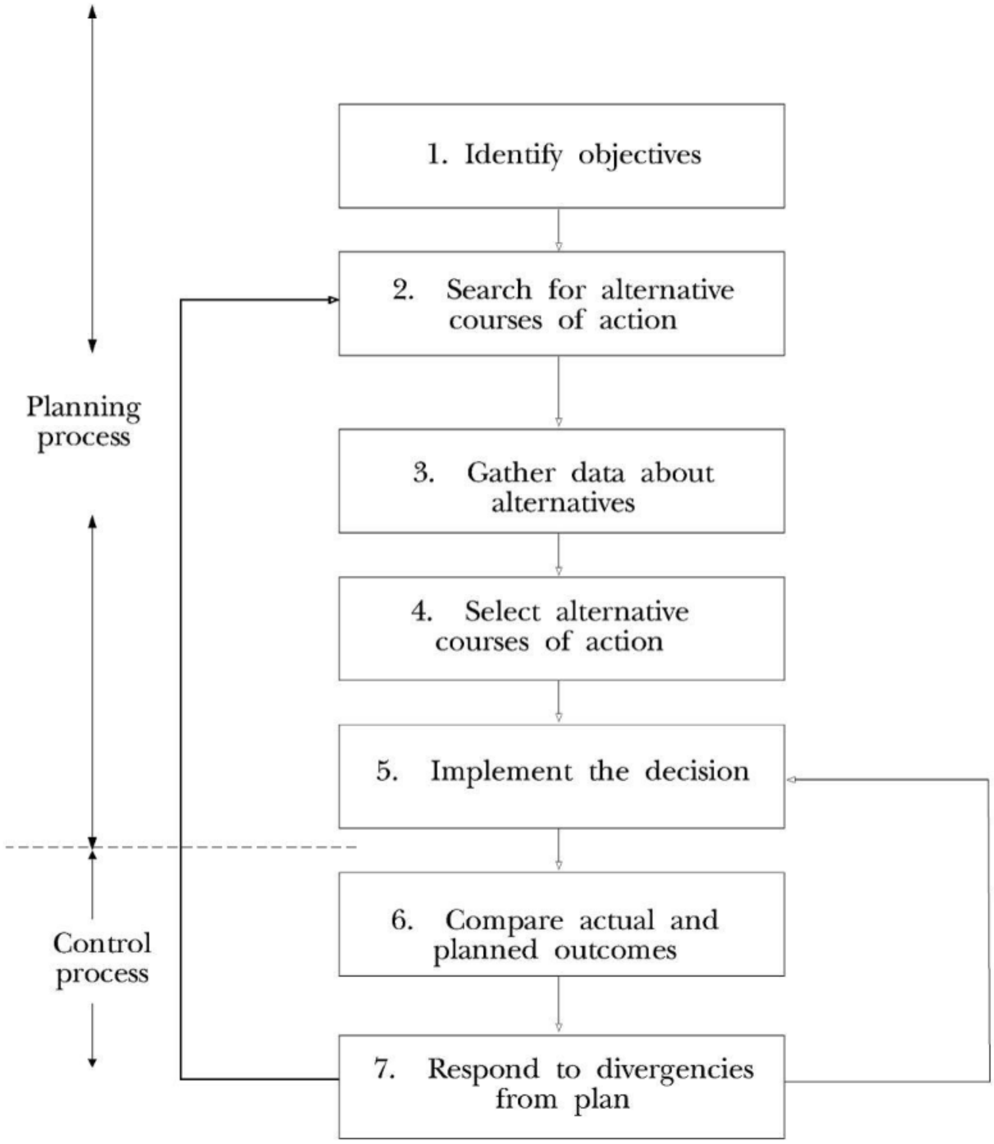


Figure 1 The decision-making, planning and control process

Figure 1 above represents a diagram of a decision-making model. The first five stages represent the decision-making of the planning process. Planning involves making choices between alternatives and is primarily a decision-making activity. The final two stages represent the control process, which is the process of measuring and correcting actual performance to ensure that the alternatives that are chosen and the plans for implementing them are carried out. Let us now consider each of the elements of the decision-making and control process.

Identifying Objectives

Before good decisions can be made there must be some guiding aim or direction that will enable the decision-makers to assess the desirability of favouring one course of action over another. Hence the first stage in the decision-making process should be to specify the objectives of the organisation.

The Search for Alternative Courses of Action

The second stage of the decision-making model is a search for a range of possible courses of action (or strategies) that might enable the objectives to be achieved. If the management of a company concentrates entirely on its present product range and markets, and market shares and cash flows are allowed to decline, there is a danger that the company will be unable to generate sufficient cash flows to survive in the future. To maximise future cash flows, it is essential that management identifies potential opportunities and threats in its current environment and takes any developments which may occur in the future. In particular, the company should consider one or more of the following courses of action:

1. Developing new products for sale in existing markets;
2. Developing new products for new markets;
3. Developing new markets for existing products.

The search for alternative courses of action involves the acquisition of information concerning future opportunities and environments. It is the most difficult and important stage of the decision-making process. Ideally, firms should consider all alternative courses of action, but, firms will in practice consider only a few alternatives, with the search process being localised initially. If this type of routine search activity fails to produce satisfactory solutions, the search will become more widespread.

Gather Data about Alternatives

When potential areas of activity are identified, management should assess the potential growth rate of the activities, the ability of the company to establish adequate market shares, and the cash flows for each alternative activity for various states of nature. Because decision problems exist in an uncertain environment, it is necessary to consider certain factors that are outside the decision-maker's control, which may occur for each alternative course of action. These uncontrollable factors are called states of nature. Some examples of possible states of nature are economic boom, high inflation, recession, the strength of competition, and so on.

The course of action selected by a firm using the information presented above will commit its resources for a lengthy period of time, and the overall place of the firm will be affected within its environment—that is, the products it makes, the markets it operates in and its ability to meet future changes. Such decisions dictate the firm's long-run possibilities and hence the type of decisions it can make in the future. These decisions are normally referred to as long-run possibilities and hence the type of decisions it can make in the future. These decisions are normally referred to as long-run or strategic decisions. Strategic decisions have a profound effect on the firm's future position, and it is therefore essential that adequate data is gathered about the firm's capabilities and the environment in which it operates. Because of their importance, strategic decisions should be the concern of top management.

Besides strategic or long-term decisions, management must also make decisions that do not commit the firm's resources for a lengthy period of time. Such decisions are known as short-term or operating decisions and are normally the concern of lower-level managers. Short-term decisions are based on the environment of today, the physical, human and financial resources presently available to the firm.

These are, to a considerable extent, determined by the quality of the firm's long-term decisions. Examples of short-term decisions include the following:

1. What selling prices should be set for the firm's products?
2. How many units should be produced of each product?

3. What media shall we use for advertising the firm's product?
4. What level of service shall we offer customers in terms of the number of days required to deliver an order and the after-sales service?

Data must also be gathered for short- term decisions; for example, data on the selling prices of competitor's products, estimated demand at alternative selling prices, and predicted costs for different activity levels must be assembled for pricing and output decisions. When the data has been gathered, management must decide which course of action to take.

Select Appropriate Alternative Courses of Action

In practice, decision-making involves choosing between competing alternative courses of action and selecting the alternative that best satisfies the objectives of an organisation. Assuming that our objective is to maximise future net cash inflows, the alternative selected should be based on a comparison of the differences between the cash flows. Consequently, an incremental analysis of the net cash benefits for each alternative should be applied. The alternatives are ranked in terms of net cash benefits, and those showing the greatest benefits are chosen subject to taking into account any qualitative factors. We shall discuss how incremental cash flows are measured for short-term and long-term decisions and the impact of qualitative factors.

Implementation of the Decisions

Once alternative courses of action have been selected, they should be implemented as part of the budgeting process. The budget is a financial plan for implementing the various decisions that management has made. The budgets for all the various decisions are expressed in terms of cash inflows and outflows, and sales revenues and expenses. The budgets are merged together into a single unifying statement of the organisation's expectations for future periods. The statement is known as a master budget. The master budget consists of a budgeted profit and loss account, cash flow statement and balance sheet. The budgeting process communicates to everyone in the organisation the part they are expected to play in implementing management's decisions.

Comparing Actual And Planned Outcomes And Responding To Divergences From Plan

The final stages in the process outlined in Figure 1 of comparing actual and planned outcomes and responding to divergences from plan represent the firm's control process. The managerial function of control consists of the measurement, reporting and subsequent correction of performance in an attempt to ensure that the firm's objectives and plans are achieved. In other words, the objective of the control process is to ensure that the work is done so as to fulfil the original intentions.

To monitor performance, the accountant produces performance reports and presents them to the appropriate managers who are responsible for implementing the various decisions. Performance reports consisting of a comparison of actual outcomes (actual costs and revenues) and planned outcomes (budgeted costs and revenues) should be issued at regular intervals. Performance reports provide feedback information by comparing planned and actual outcomes. Such reports should highlight those activities that do not conform to plans, so that managers can devote their scarce time to focusing on these items. This process represents the application of management by exception. Effective control requires that corrective action is taken so that actual outcomes conform to planned outcomes.

1.5 Decision Making Environment

There a four main environment within which decisions can be made. These are:

- Certainty
- Risk
- Fundamental uncertainty
- Competition

- Certainty environment

In this environment complete information is available as to which states of nature will occur. The decision making process just involves picking the best alternative.

Risk

Risk involves situations or events which may or may not occur but whose probability of occurrence can be predicted from past records. In this environment, the states of nature are not certain but probability distribution can be assigned.

Fundamental uncertainty

Uncertain events are those whose outcome cannot be predicted with statistical confidence. In this environment the states of nature are not known nor are their probability distribution. The decision making process depends on the risk attitude of the decision maker.

Competition

In this environment the decisions made by the firm are affected by decisions made by other firms with opposing interests.

Decision Making Under Risk and Uncertainty

Before looking at the various methods of making decisions under risk, we shall look at the three main risk attitudes that distinguish different decision makers. These are:

Risk seeking

A risk seeker is a decision maker who is interested in the best possible outcome no matter how small the chance that they may occur i.e. he takes high risks in anticipation of high profitability. For such a decision maker, the marginal utility for wealth is positive and increasing.

2. Risk neutral

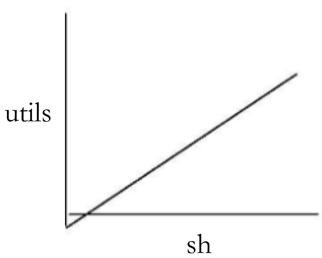
A decision maker is risk neutral if he is concerned with what will be the most likely outcome i.e. he is indifferent to risk. For such a decision maker the marginal utility of wealth is positive and constant.

3. Risk Averse

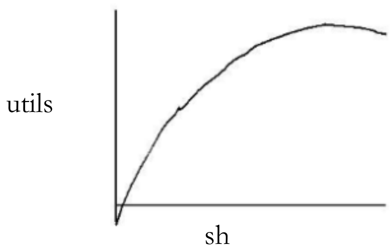
A decision maker is risk averse, if he acts on the assumption that the worst possible outcome will occur, and chooses the decision with the least risk possible.
For such a decision maker, the marginal utility of wealth is positive but decreasing.

These risk attitudes can be illustrated by the diagrams below:

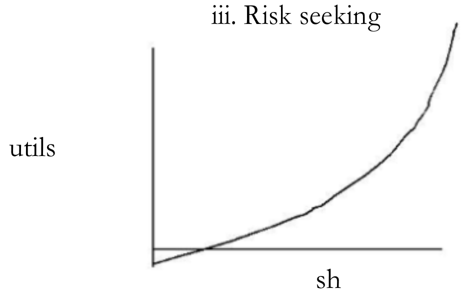
i. Risk neutral



ii. Risk averse



iii. Risk seeking



Measure of Risk

Std deviation (δ)

$$\delta = \sqrt{\sum_{t=1}^{n}(MV_t - EMV)P_t}$$

Where
MV_t is the monetary value under condition t.
EMV is the expected monetary value
Pt is the probability of condition t occurring n
is the number of different conditions.

Coefficient of variation.

It is a relative measure of risk and it is used to compare alternatives of different magnitudes based on their risk return consideration.

$$C\,V = \frac{\delta}{EMV}$$

EMV = ε MV_t P_t

Methods Of Decision Making Under Uncertainty

To discuss these methods we shall use illustration 1.1

Illustration 1.1

Assume that ABC Ltd is trying to set the selling price for one of its products and three prices are under consideration. These are Sh.4, Sh.4.30 & Sh.4.40
The following information is also provided

Alternatives

Conditions	Sh.4.00	Sh.4.30	Sh.4.40
Best possible	16,000	14,000	12,500
Most likely	14,000	12,500	12,000
Worst possible	10,000	8,000	6,000

Fixed costs = Sh. 20,000
variable cost per unit = Sh. 2

Required:

Advice the company on the best price to set.

Solution

The first step is to prepare as payoff table a shown below:

Payoff matrix (profits in Shs.)

Conditions	Sh.4	Sh.4.30	Sh.4.40
Best possible	12010	12200	10000
Most likely	8000	8750	8800
Worst possible	0	(1600)	(5600)

Decision Making Criteria

Maximax decision rule

This decision rule looks at the best possible result and it chooses the maximum payoff for each alternative and then the maximum of this maximum.

Conditions	Sh.4	Sh.4.30	Sh.4.40
Best possible	12010	12200	10000
Most likely	8000	8750	8800
Worst possible	0	(1600)	(5600)
Maximum	12010	12200	10000

The decision is to set a price of Sh.4.30 since it maximises the maximum pay off.
This criterion appeals to risk takers or optimists who are ready to undertake huge losses if they occurred. Small and new companies should not use this method.

Maximin decision rule

Under this criterion, the decision maker looks at the worst possible outcome of each decision alternative and then chooses the alternative that offers the least unattractive (worst) outcome i.e. He chooses the alternative that maximises the minimum profit.

Conditions	Sh.4	Sh.4.30	Sh.4.40
Best possible	12010	12200	10000
Most likely	8000	8750	8800
Worst possible	0	(1600)	(5600)
Minimum	0	(1600)	(5600)

The decision is to set a price of Sh.4.00 since it maximises the minimum payoff. This criterion appeals to risk averse decision makers since it is a criterion of extreme caution. It can be applied by those firms which cannot be able to absorb huge losses if they occurred.

Laplace Criterion of Rationality

This criterion holds that if decision makers do not know the probabilities of the various states of nature and have no reason to think otherwise, then the states of nature should be considered to be equally likely. On the basis of this assumption, the expected monetary value for each alternative is calculated and the alternative with the highest expected monetary value is chosen.

Conditions	Probability	Sh.4.00	Sh.4.30	Sh.4.40
Best possible	1/3	12010	12200	10000
Most likely	1/3	8000	8750	8800
Worst possible	1/3	0	(1660)	(5600)
EMV		6667	6450	4400

Workings.

EMV $_{Sh.4}$ = 1/3 (12010) + 1/3 (8000) + 1/3 (0) = 6667

Others are computed in the same way.

Decision

Set a price of Sh.4.00 since it maximises the expected monetary value.

Minimax Regret Criterion

This method seeks to minimise the maximum regret that would occur from choosing a particular strategy or alternative. The regret is the opportunity loss that occurs from taking one decision given that a certain contingency occurs.

For each state of nature
Opportunity loss = Max pay off – Payoff under each alternative

Conditions	Sh.4.00	Sh.4.30	Sh.4.40
Best possible	200	0	2200
Most likely	800	50	0
Worst possible	0	1600	5600
Maximum regret	800	1600	5600

Decision
Set a price of Sh.4.00 since it minimises the maximum regret

Methods of Decision Making Under Risk

In this environment, it is possible to attach probabilities to the various states of nature. The decision criteria would either be:

The expected monetary value
The expected opportunity loss

The two criteria are similar since the choice that maximises the expected monetary value also minimises the expected opportunity loss (EOL)

Illustration 1.2

Assume in the ABC pricing decision (illustration 1.1) that the probability of the best possible outcome is 0.2, most likely outcome is 0.6 and the worst possible outcome is 0.2.
Required: Advice the Company on the best price to set.

Solution

Conditions	Probability	Sh.4.00	Sh.4.30	Sh.4.40
Best possible	0.2	12010	12200	10000
Most likely	0.6	8000	8750	8800
Worst possible	0.2	0	(1660)	(5600)
EMV		7200	7370	61600

Decision

Set a price of Sh.4.30 since it maximises the expected monetary value.

Conditions	Probability	Sh.4.00	Sh.4.30	Sh.4.40
Best possible	0.2	200	0	2200
Most likely	0.6	800	50	0
Worst possible	0.2	0	1600	5600
EOL		520	350	1560

Decision

Set a price of Sh.4.30 since it minimises the EOL

Once the EMV has been calculated, the standard deviation and the coefficient of variation can also be computed as shown below:

$$\delta \text{Sh.4} = \sqrt{\left[(12000 - 7200)^2\ 0.2 + (8000 - 7200)^2\ 0.6 + (0 - 7200)^2\ 0.2\right]}$$

$= 3919$

The others are:

$\delta_{\text{sh 4.3}} = 4679$
$\delta_{\text{sh 4.4}} = 5898$

Decision

Set a price of Sh.4 because it minimises the standard deviation. The assumption here is that the decision maker is risk averse.

C.V $\dfrac{\delta}{\text{EMV}}$

C.V sh4.00 = 0.54
C.V sh4.30 = 0.63
C.V sh4.40 = 0.96

Decision
Set a price of Sh.4 because it minimises the C.V (coefficient of variation)

1.6 Multi-stage decision making under
risk (The use of decision trees)

Sequencing is concerned with the selection of an appropriate sequence or order of performing a series of jobs to be done on a finite number of machines or service facilities in some well defined technological order so as to optimize some measure of performance of the system, such as minimising overall cost, total elapsed time. Decision trees are used in solving sequential problems where there is an element of uncertainty. We use expected values to find the best alternative.

A decision tree is a graphical representation of decision process indicating decision alternatives, states of nature, associated probabilities and conditional pay-offs for each combination of decision alternatives and states of nature. It shows all the possible choices that can be made as branches on the tree. And all the possible outcomes for each choice as subsidiary branches on the tree.

A decision tree is beneficial for several reasons including:
It provides a pictorial representation of a sequential decision process.
It makes the expected value calculations easier because these calculations can be performed directly on the tree diagram.
The actions of more than one decision maker can be considered.

Steps followed in making the tree are:

- Define the problem or identify the objectives
- Identify the possible causes of action, (decision alternative)
- (Identify the possible states of nature/conditions
- Estimate the probabilities for each state of nature
- Estimate the conditional pay-off for each alternative and states of nature
- Draw the decision tree

Calculate the expected monetary value at each state of nature node using the roll back method.

Illustration 1.3

A company making roof tiles has been considering the likely demand for the roof tiles over the next six years and think that demand pattern will be as follows:

Situation	Prob.
High demand for 6 years	0.5
Low demand for 6 years	0.3
High demand for 3 years followed by low demand for 3 years	0.2

There is no possibility of low demand followed by high demand. Enlargement of capacity is required and the following are the available options.

Option A Install fully automatic facilities immediately at a cost of Sh.5.4 million.

Option B Install semi-automatic facilities immediately at a cost of Sh.4 million.

Option C Install the semi-automatic facilities immediately as in B and upgrade to fully automatic at an additional cost of Sh.2 million in 3 years time provided demand has been high for 3 years.

The returns expected for the various demand and capacity options are estimated to be:

	If high demand	If low demand
Option A	Sh.1.6m p.a.	Sh.0.6 m p.a.
B	Sh.0.9 p.a for 3 years then Sh.0.8 m p.a.	0.5 p.a. for 3 years
C	Sh.0.9 p.a. for 3 years, then Sh.1.1 for 3 years	Sh.0.8m p.a. for 3 years then Sh.0.3m p.a. for 3 years

What decisions should the firm take assuming that the objective is to maximise expected value?

Solution

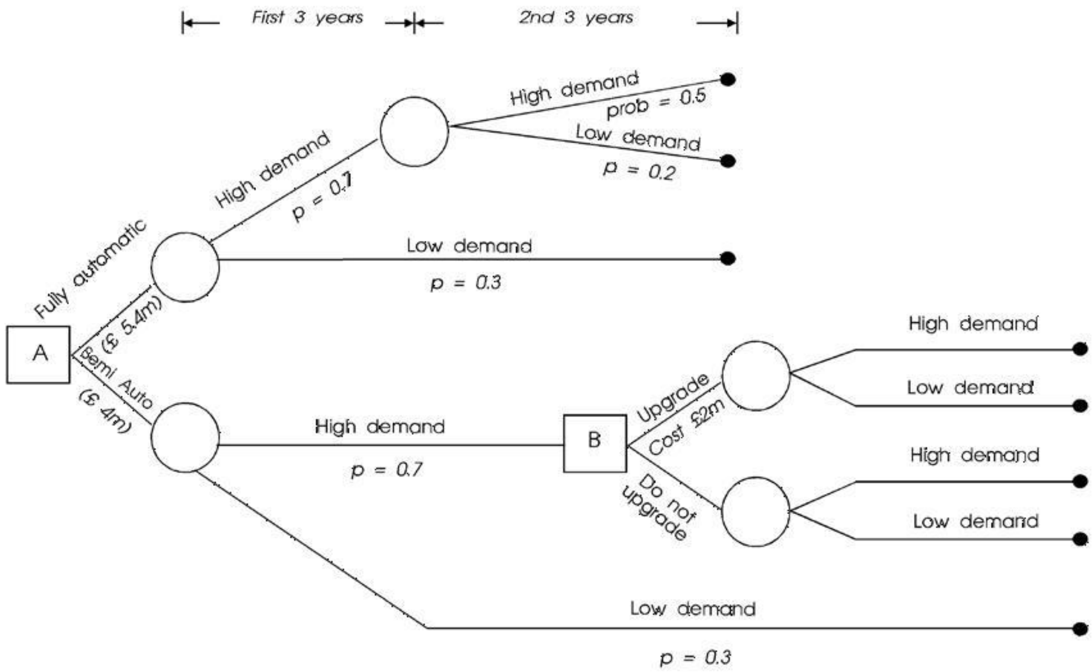
We develop the decision tree in two stages:

(a) Forward Pass (starting from the left and moving towards the right)

(b) Backward Pass (we start from the end and move backward as illustrated in the solution)

Forward Pass

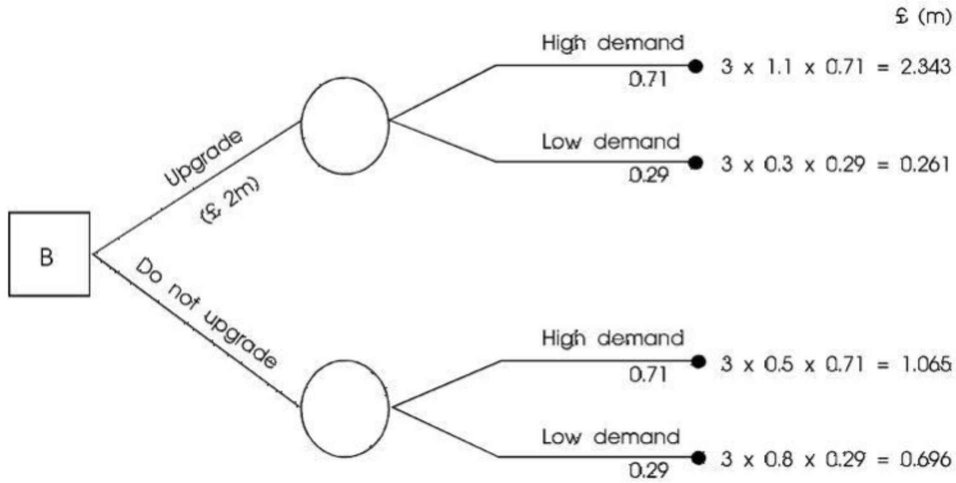
We draw the decision tree according to the information given the example.



Backward Pass

Note that there are two decision points A and B, A at the start and B at the end of the first three years.

To evaluate expected values at A, it is necessary to evaluate expected values at B because the decision at B will affect the decision at A. This is why this stage is known as the backward pass.



Therefore the expected value of upgrading is 2.34 + 0.261 = 0.604

i.e. Sh. 0.604m

The expected value of not upgrading is (Sh.1.065 + 0.696) m

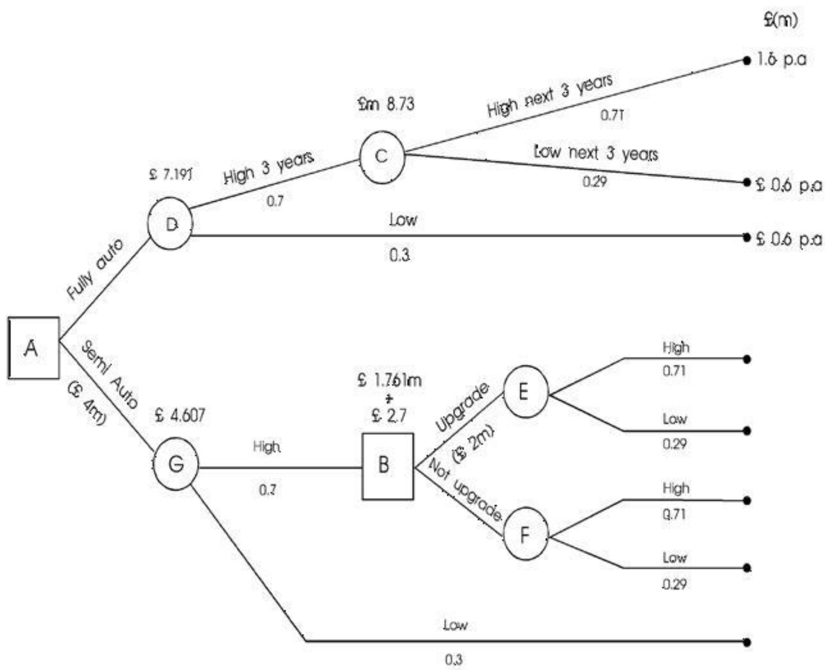
= Sh.1.761m

The probability of high during the 2nd 3 years using conditional probability $= \dfrac{0.5}{0.5 + 0.2}$

≈ 0.71

Decision at B — Do not upgrade

The final diagram with exported values



Outcome at C

Last 3 years	1.6 x 3 x 0.71	=	3.408
	+ 0.6 x 3 x 0.29	=	<u>0.522</u>
			3.930

At C, outcome from 1st three years = $\frac{3}{}$ x 1.6
= 4.8

Total outcome at C = 3.93 + 4.8
= 8.73

Outcome at D

8.73 x 0.7 + 0.6 x 6 x 0.3 = 6.111 + 1.08
= Sh. 7.191 m

Outcome at B

For last 3 years	=	Sh.1.761
plus 0.9 x 3	=	<u>2.7</u>
Total	=	Sh.4.461

Outcome at G

4.461 x 0.7 + 0.8 x 0.3 x 6 = 3.167 + 1.44

= Sh.4.607 m

Outcome at A

From Fully automatic = (7.191 - 5.4)
= Sh.1.791 m

From Semi-automatic = (4.607 - 4)
= Sh.0.607 m

Final Decision

Put in the fully automatic machinery at the outset.
(Please note that these answers have been worked out using approximate values for probabilities)

$\frac{5}{7}$ = 0.71 and $\frac{2}{7}$ = 0.29

Perfect and Imperfect Information

The uncertainty about the future outcome from taking a decision can be reduced by obtaining more information fast about what is likely to occur.

That information can be obtained from various sources e.g.
Market surveys
Conducting a pilot test
Building a prototype model
Hire consultants

Information can be categorised depending upon how reliable it is likely to be for predicting what would happen in the future and for helping managers to make better decisions.

Perfect information (PI) is information that can be guaranteed to predict the future with 100% accuracy, which, although it might be quite good, it could be wrong in it's prediction of the future.

Both perfect and imperfect information is costly and its value must be determined.

Value of perfect information (Pi) = EMV with PI – EMV without PI.

Illustration 1.4

Consider the ABC pricing decision (Illustration 1.1) and assume that it is possible to obtain ideal information at a cost of Sh.500

Required

Advice the company on whether to acquire the perfect information

EMV with PI = 0.2(12200) + 0.6 (8800) + 0.2(0)
= 7720

Value of perfect information = 7720 –7370 = Sh.350
The decision is not to acquire perfect information since it costs more than its worth.

Imperfect Information (IPI)

Market research finding or information from pilot are likely to be reasonably accurate but can still be wrong in prediction. They provide imperfect information.

The value of IPI = EMV with IPI = EMV without IPI.

Illustration 1.5

The financial director of Spinney Electrics is considering the national launch of a new washing machine. The potential sales of the product during its lifetime are classified as being either high, medium or low and the net present value of the machine sales under each of these three conditions is estimated to be Sh.50 million, Sh.10 million, and Sh.20 million, respectively. The marketing director of Spinney Electrics estimates that there is a 0.4 probability that sales will be high, a 0.25 probability that they will be medium and a 0.35 probability that they will be low.

Required:

(a) Assuming the company's objective is to maximise expected net present value, determine whether or not the new product should be launched. (4 marks)

(b) Explain the meaning of `expected value of perfect information'. Find the expected value of perfect information for this situation. (5 marks)

(c) The financial director also has an alternative option. Instead of proceeding directly with a full national launch the company could test the market for the washing machine in their Midlands sales region. This would delay the national launch, and this delay, together with other outlays associated with testing the market, would lead to costs having a net present value of Sh.0.25 million. The test marketing in the Midlands sales region would yield information indicating whether the national launch is likely to be successful or unsuccessful. The following table shows the reliability of each of the possible indications.

		Actual national sales			
		Probability	High	Medium	Low
Test marketing indication	Successful national launch	0.6	0.6	0.15	0.25
	Unsuccessful national launch	0.4	0.1	0.4	0.5

For example,

If the test market indicates a successful national launch then the probability of low sales would be 0.25. Also prior to the test market it is thought that the test market has a probability of 0.6 of indicating a successful national launch and 0.4 for an unsuccessful launch.

i. Represent this information in a decision tree and calculate the value of this imperfect information. (7 marks)

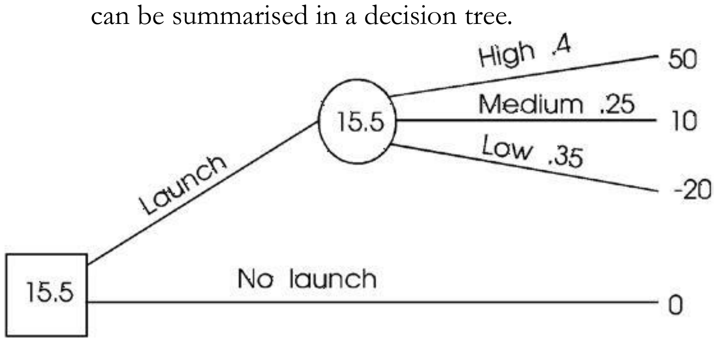
ii. Give advice to the financial director as to whether or not the company should test the market in their Midlands region. In your advice explain why this method of analysis should not be relied upon entirely when making appropriate decisions. (4 marks)

(20 marks)

Solution

This question concerns the use of decision trees (or decision tables) to structure and resolve a decision problem.

(a) The initial problem is to determine whether or not to launch the new product. The information can be summarised in a decision tree.



The values given in the nodes of the decision tree are the expected values of the remainder of the tree from that point. If the launch takes place the expected profit is Sh.15.5 million, compared with zero if no launch takes place. Clearly, it is advisable to launch the product, given the information available.

(b) The expected value of perfect information gives the increase in profit that would be achieved if the decision maker knew what the outcome to the decision would be. The value of perfect information is calculated by finding the expected profit that would be achieved with perfect information and subtracting from it the expected profit from the strategy which is best in the absence of perfect information. Although perfect information is rarely available, it can be used as a benchmark against which the cost of information can be compared.

In this example, if the decision maker knew beforehand that sales would be either medium or high he would still launch the product, but if he knew it would be low he would recommend no such launch.

Hence the expected payoff from perfect information is

EPPI = (50 x 0.4) + (10 x 0.25) + (0 x 0.35) = 22.5 (Sh.m)

Hence the expected value of perfect information is

22.5 — 15.5 = Sh.7 million

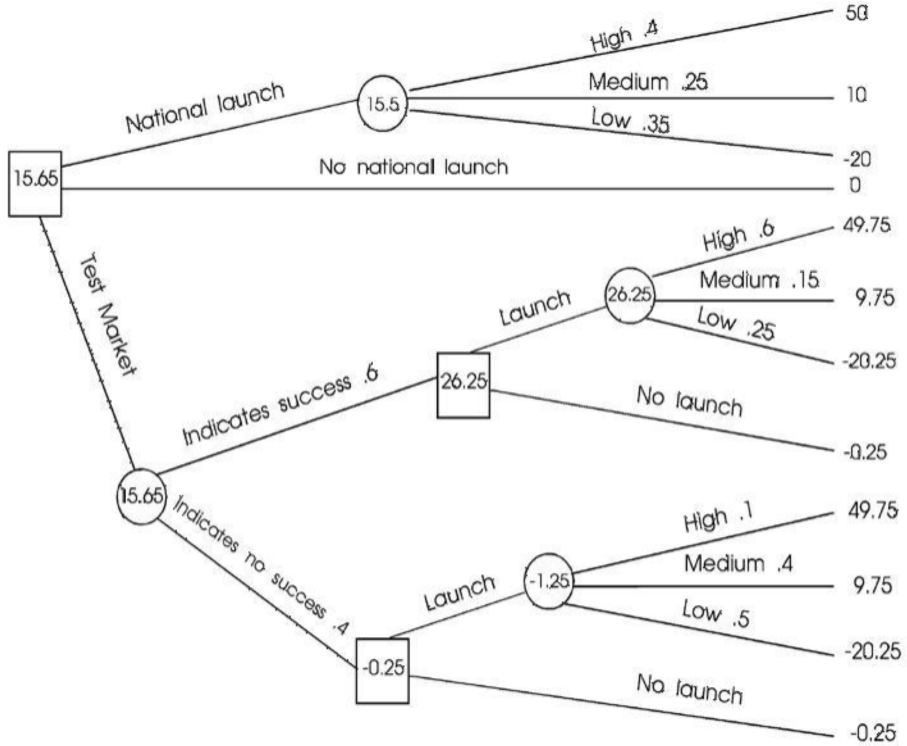
(c) i.The table can be converted to probabilities in the following way:

S = indication of successful national launch
U = indication of unsuccessful national launch
H = high sales in national launch
M = medium sales in national launch
L = low sales in national launch

Hence we have

P(S)	= 0.6	P(U)	= 0.4		
P(HIS)	= 0.6	P(MIS)	= 0.15	P(LIS)	= 0.25
P(HIU)	= 0.1	P(MIU)	= 0.4	P(LIU)	= 0.5

A further branch is now added to the decision tree described in part (a).



Using rollback analysis for decision trees, the expected profit if the market is tested in the Midlands sales region is Sh.15.65 million. Hence the value of this imperfect information is

Sh.15.65m — Sh.15.5m = Sh.150,000

ii. From a financial point of view, it is clear that the market should be tested prior to the national launch.

However the whole analysis is fraught with difficulties and approximations. For example, the potential sales of the product would cover a large number of possible values, and there is considerable approximation in classifying into only three categories. In addition, it is likely that the assigned probabilities are only approximations. Also the sales take place in the future when different market conditions may occur. We should also be aware that the financial amounts might not be the most appropriate unit on which to base the decision.

This type of analysis can give an indication of a likely outcome and is useful in ensuring that a manager does not make inconsistent decisions.

Illustration 1.6

Assume that a small oil company is trying to decide whether or not to drill on a particular site. The chief engineer has made the following estimate from past experience.
P(Oil) = 0.2
P(no oil) =0.8

It is possible for the company to hire a firm of international consultants to carry out a survey of the site. The company has estimated.

If there is oil then there is a 95% chance that the report will be favourable
If there is no oil there is a 10% chance that the report will be favourable

The cost of drilling	= Sh.10 million
Value of benefit if oil is found	= Sh.70 million
Cost of information	= Sh.3 million

Required:

Advice the company on whether to hire the consultant and compute the value of sample (imperfect) information.

Solution

Let F be Favourable report showing there is oil
F^1 be adverse report showing there is no oil
O be there is oil
N be there is no oil

The prior probabilities are:

$P(F/0) = 0.95$ $\quad$ $P(F^1/0) = 0.05$

$P(F/N) = 0.10$ $\quad$ $P(F^1/N) = 0.90$

The first step is to revise the probabilities using Bayes theorem which is given as follows:

$$P(B/A) = \frac{P(B)*P(A/B)}{P(A)} = \frac{P(B \text{ and } A)}{P(A)}$$

Therefore:

$P(O/F) = \frac{P(0) \text{ X } P(F/0)}{P(F)}$

$P(F) = P(0) \text{ X } P(F/0) + P(N) \text{ X } P(F/N)$
$= 0.2 \text{X} 0.95 + 0.8 \text{ X } 0.1 = 0.27$

$P(F') = 1 – 0.27$
$= 0.73$

$P(0/F) = \frac{P(0) \text{ X } P(F/0)}{P(F)} = \frac{0.2 \text{ X } 0.95}{0.27} = 0.7037$

$P(N/F) = 1 – 0.704$
$= 0.296$

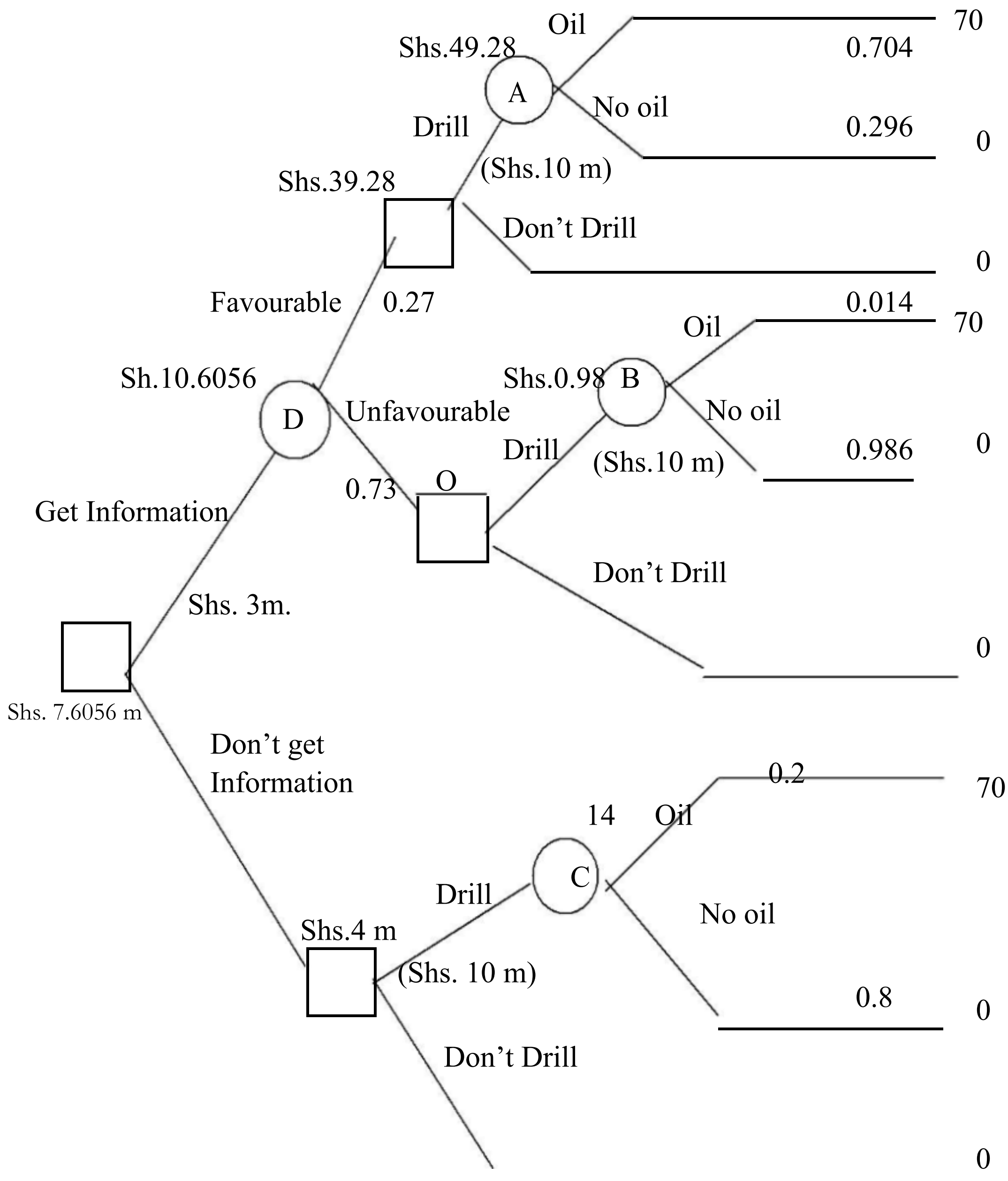
$P(N/F') = \frac{P(N) \text{ X } P(F'/N)}{P(F')}$

$= \frac{0.8 \text{ X } 0.9}{0.73}$

$= 0.9863$

$P(0/F') = 1 – 0.986$
$= 0.014$

Using the above probabilities a decision tree can be constructed as follows:



Evaluation using EMV

Emv @ A = 70 (0.704) + 0 (0.296) = 49.28

Emv @ B = 70 (0.014) + 0 (0.986) = 0.98

Emv @ C = 70 (0.2) + 0 (0.8) = 14

Emv @ D = 39.28 (0.27) + 0 (0.73) = 10.6056

Decision

Hire the consultant and if the report is favourable drill and if the report is adverse don't drill. The expected monetary value would be Sh.7.6056 million made up of a profit of Sh.57 million with a probably 0.27 x 0.704 = 0.19
A loss of Sh.13 million, with prob. 0.296 x 0.27 =
0.08 Loss of Sh.3 million pounds with prob. 0.733

EMV = (57 X 0.19) * (13 X 0.08) – (3 X 0.73)
=7.6 million

Value of imperfect information

= EMV with IPI – EMV without IPI
= Sh.10.61m - Sh.4m =Sh.6.61 million

REINFORCING QUESTIONS

QUESTION ONE

The Oil Kenya Company currently sells three grades of petrol, regular, premium and `regular extra' which is a mixture of regular and premium. Regular Extra is advertised as being "at least 50 percent premium". Although any mixture containing 50 per cent or more premium fuel could be sold as `regular extra' it is less costly to use exactly 50 per cent. The percentage of premium fuel in the mixture is determined by one small valve in the blending machine. If the valve is properly adjusted, the machine provides a mixture which is 50 percent premium and 50 percent regular. Assume that if the valve is out of adjustment the machine provides a mixture which is 60 percent premium and 40 percent regular. Once the machine is started it must continue until 100,000 litres of `regular extra' have been mixed.

The following data is available:

	Shs
Cost per litre—premium	3.20
Cost per litre—regular	3.00
Cost of checking valve	800.00
Cost of adjusting the valve	400.00

Subjective estimates of the probabilities of the valve's condition are estimated to be:

Event	Probability
Valve in adjustment	0.7
Valve out of adjustment	0.3

Required:

(a) The expected cost of checking the valve and adjusting it if necessary. (5 marks)

(b) The conditional cost of not checking the valve when it is out of adjustment (5 marks)

(c) Using the criterion of minimum expected cost, calculate the probability at which there will be need to check if the valve is out of adjustment. Comment on the results. (5 marks)

(d) Comment on the results obtained in (a) and (b) above. (5 marks)

(Total: 20 marks)

QUESTION TWO

Chakula Engineering Company Limited (CECL) recently sent their chief designer to the USA and UK to review developments in the American and British markets. He has now returned with details of a new type of food mixer that is being developed over there. CECL are considering the design and manufacture of a liquidizer gadget attachment to be used as an extra gadget for the new mixer when it is sold in Kenya. The chief designer's notes show that 10% of the experts he questioned in both the UK and USA believed the new mixer would reach the Kenyan market in a year's time, whereas 30% thought it would be launched in four years time, and the remainder suggested a five-year delay before it reached Kenya. The present value (PV) of net cash flows from making and selling liquidizer are estimated by the company to be Shs 8 million, if the market develops four years from now and 3.2 million if it develops five years from now.

CECL have not developed a liquidizer before, and whilst its immediate development would cost Shs 2 million, they feel they have only a 50% chance of a successful development at present. A number of alternative courses of action present themselves. The company could abandon the whole project, or wait for one year to see if the mixer has penetrated the Kenyan market. They could then abandon or develop the liquidizer at a PV cost of Shs 1.8 million, with a 70% chance of success, but they would be late into the market and the PV of their receipts they estimate at Shs 4.8 million. A further alternative is that the company could delay a decision for a second year, and then abandon or develop the project. Development costs at that stage would have a PV of Shs 1.4 million, including the expenditure of Shs 400,000 on acquiring extra product data during the second year of delay, and the chance of a successful development would be 90%. At this point, however, the mixer could only come on the market at the four or five year point from now.

Required:

Using a decision tree approach, advise the company on the course of action to adopt. **(20 marks)**

QUESTION THREE

Siku Kuu Ltd. Manufactures and distributes a line of Christmas gifts. The company had neglected to keep its gifts line current. As a result, sales have decreased to approximately 25,000 units per year fro a previous high of 125,000 units. The gifts have been redesigned recently and are considered by company officials to be comparable to its competitors' models. The company plans to redesign the gifts each year in order to compete effectively. Kama Kawaida, the Sales Manager, is not sure how many units can be sold next year, but she is willing to place probabilities on her estimates. Kama Kawaida's estimates of the number of units that can be sold during the next year and the related probabilities are as follows:

Estimated **Sales in units**	**probabilities**
50,000	0.10
75,000	0.40
100,000	0.30
125,000	0.20

The units would be sold for sh.500 each. The inability to estimate the sales more precisely is a problem for Siku Kuu Ltd. the number of units of this product is small enough to schedule the entire year's sales in one production run.

If the demand is greater than the number of units manufactured, then sales will be lost. If the demand is below supply, the extra units cannot be carried over to the next season and would be given away to various charitable organizations.

The production and distributions cost estimates are as follows:

Units manufactured		50,000	75,000	100,000	125,000
Variable costs	(Sh)	9,900,000	14,850,000	19,800,000	24,750,000
Fixed costs	(Sh)	7,700,000	7,700,000	8,800,000	8,800,8000
Total costs	(Sh)	17,600,000	22,550,000	28,600,000	33,550,000

The company intends to analyze the data to facilitate making a decision as to the proper size of the production run.

Required:

a) Prepare a payoff table for the different sizes of production runs required to meet the four sales estimates prepared by Kama Kawaida for Siku Kuu Ltd.
If Siku Kuu Ltd. relied solely on the expected monetary value approach to make decisions, what size of production run would be selected? (6 marks)

b) Identify the seven basic steps that are taken in any decision process. Explain each step by reference to the situation presented by Siku Kuu Ltd. and your answer to requirement (a) (14 marks)

(Total: 20 marks)

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

LESSON TWO

COST ESTIMATION AND FORECASTING

OBJECTIVES

Examine the methods of estimating and forecasting costs.

INSTRUCTIONS

1. Read the Study Text and Chapters 24 of Management and Cost Accounting by Colin Drury.

2. Attempt the reinforcing questions at the end of the lesson under examination conditions

3. Compare your answers with those given in Lesson 10

CONTENTS

STUDY TEXT

In this chapter, you will learn to use cost estimating relationships to estimate and forecast costs. As the name implies, a cost estimating relationship (CER) is a technique used to estimate a particular cost or price by using an established relationship with an independent variable. If you can identify an independent variable (cost driver) that demonstrates a measurable relationship with contract cost or price, you can develop a CER. Note that CER may be mathematically simple in nature (e.g., a simple ratio) or it may involve a complex equation (e.g. Multiple regression analysis).

2.1 STEPS OF DEVELOPING A COST ESTIMATING RELATIONSHIP.

Strictly speaking, a CER is not a quantitative technique. It is a framework for using appropriate quantitative techniques to quantify a relationship between an independent variable and cost or price. Development of a CER is a 6-step process shown below:

Step 1. Define (or select) the dependent variable (Y)

Will the CER be used to estimate price, cost, labor hours, material cost, or some other measure of cost? Will the CER be used to estimate total product cost or estimate the cost of one or more components? The better the definition of the dependent variable, the easier it will be to gather comparable data for CER development.

The dependent variable is the cost to be predicted and it is choice depends on the purpose of the cost function. It may also be referred to as response variable.

Step 2. Select the cost driver(s)

This may also be referred to as independent, explanatory or predictor variable. A cost driver can be defined as any factor whose change causes a change in the total cost of an activity.

Examples of potential cost driver

- Direct labour hours
- Machine hours
- Number of units
- Number of production runs
- Number of orders, etc.

The potential cost driver should be plausible (i.e. make economic sense) and should be accurately measured.

In selecting potential cost drivers for CER development Consider the following factors:

Variables should be quantitatively measurable. Parameters such as maintainability are difficult to use in estimating because they are difficult to measure quantitatively.

Data availability is also important. If you cannot obtain historical data, it will be impossible to analyse and use the variable as a predictive tool. For example, an independent variable such as physical dimensions or parts count would be of little value during the conceptual phase of system development when the values of the independent variables are not known. Be especially wary of any CER based on 2 or 3 data observations.

If there is a choice between developing a CER based on performance or physical characteristics, performance characteristics is generally the better choice, because performance characteristics are usually known before design characteristics.

Step 3. Collect data concerning the relationship between the dependent and independent variables.

Collecting data is usually the most difficult and time-consuming element of CER development. It is essential that all data be checked and double checked to ensure that all observations are relevant, comparable, relatively free of unusual costs.

A sufficient number of past observations must be obtained to derive an acceptable cost function. This should be adjusted to reflect any change of circumstances e.g. changes in price levels caused by inflation, changes in types of equipment used, etc. The time period used to measure the dependent variable and the cost driver should be the same.

Step 4 Plot the data on a graph

The graph (usually referred to as a scatter diagram) will indicate the general relationship between the dependent variable and the cost driver and will give a visual indication as to whether a lineal cost function can approximate the cost behaviour. It will also highlight extreme observations (outliers).

Step 5. Select the relationship that best predicts the dependent variable.

After exploring a variety of relationships, you must select the one that can best be used in predicting the dependent variable. Normally, this will be the relationship that best predicts the values of the dependent variable. A high correlation (relationship) between a potential independent variable and the dependent variable often indicates that the independent variable will be a good predictive tool. However, you must assure that the value of the independent variable is available in order for you to make timely estimates. If it is not, you may need to consider other alternatives.
There are various methods that can be used to estimate the cost function. Examples include: Engineering method
Account analysis
Regression analysis
High low method
Time series analysis
Simulation analysis

Step 6: Test the reliability of the cost function

There are three main tests that should always be done. These include:
Logical relationship tests
Goodness of fit test
Specification tests (Tests of the assumptions of the model)

2.2 COST ESTIMATION METHOD

Engineering method

These methods are based on the use of engineering analysis of technological relationship between inputs and outputs e.g. method studies and time and motion studies.

The procedure in such a study is to make an analysis based on direct observation of the underlying physical quantities required for an activity and then to convert the final result into cost estimate.

This method is useful for estimating costs of repetitive processes where input and output relationship is clearly defined e.g. the cost associated with direct materials, direct labour and machine time.

Account analysis (Inspection of accounts) method

This method requires that departmental managers and the accountant inspect each item of expenditure within the accounts for some output level and then classify each of these items as wholly fixed, wholly variable or mixed.

A single average unit cost figure is selected for the items categorised as variable whereas a single total cost for the period is used for the items categorised as fixed.
Mixed costs are decomposed into their variable and fixed components.

High low method (Two point method)

Under this method, records of costs in the previous period are reviewed and the costs of 2 periods are selected. These are the period with the highest level of outputs and the period with the lowest output. A line passing through these two points is then established and used in estimating costs.

Illustration 2.1

The production manager of XYZ Company, is concerned abut the apparent fluctuation in efficiency and wants to determine how labour costs (in Sh.) are related to volume. The following data presents results of the 12 most recent weeks.

Week No.	Units Produced(X)	Labour Costs(Y)
1	34	340
2	44	346
3	24	287
4	36	262
5	30	220
6	49	416
7	39	337
8	21	180
9	41	376
10	47	295
11	34	215
12	24	275

Required:

Estimate the cost function using:
The high low method
Regression analysis

Assume that the Company intends to produce
45 units
34 units next period
Estimate the labour cost to be incurred.

Solution

We will first use the high-low method to establish the cost function.

High low method

	X	Y
Highest point		
416		
Lowest point	21	180
Difference	28	236

Gradient/ slope $= \frac{236}{28} = 8.43$

The function will be:

$$Y = a + bx$$

We can Substitute the lowest points (21,180)

180 = a + 8.43(21)

a = 2.97. This can be approximated to 3

The predicting equation is therefore Y = 3 + 8.43 x

i. if X=45 units

Y = 3 + 8.43*45
= Sh.382.35

ii. 34 Y = 3 + 8.43(34)
= Sh.289.62

Note:

The main problems of the high low method are:
Reliability is low
It Ignores all the other points except the highest and lowest which in most cases are outliners.

2.3 REGRESSION ANALYSIS

A regression equation identifies an estimated relationship between a dependent variable (the cost) and one or more independent variables (the cost driver). When the equation includes only one independent variable then it is referred to as simple regression and its form is:

$\tilde{Y} = a + bx$

Where, $\tilde{Y}$ is the predicted value of Y
a and b are Constant
x is the cost driver

When the equation includes 2 or more independent variables, it is referred to as multiple regression and is of the form:

$Y = a + b_1 x_1 + b_2 x_2 + \ldots\ldots b_n x_n$ for n independent variables.

Simple Regression

Regression analysis determines mathematically the regression line of best fit. It is based on the principle that the sums of squares of the vertical deviation from the line established is the least possible

I.e. $\sum(Y - \hat{Y})^2$ is minimised
where Y is the observed value of the dependent variable
$\hat{Y}$is the predicted value of Y

The equation can be solved by the use of normal equations and these are:

1. $\Sigma y = na + b(\Sigma x)$

$\Sigma xy = a(\Sigma x) + b(\Sigma x^2)$

From these normal equations:

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$a = \frac{\Sigma Y}{n} - \frac{b\Sigma x}{n}$$

Looking at illustration 2.1, then we first compute the sum of X, Y, XY, X^2 and Y^2

The table below shows these summations.

Week No.	Units X)	L.Costs(Y)	XY	X^2	Y^2
1	34	340	11560	1156	115600
2	44	346	15224	1936	119716
3	24	287	8897	961	82369
4	36	262	9432	1296	68644
5	30	220	6600	900	48400
6	49	416	20384	2401	173056
7	39	337	13143	1521	113569
8	21	180	3780	441	32400
9	41	376	15416	1681	141376
10	47	295	13865	2209	87026
11	34	215	7310	1156	46225
12	24	275	6600	576	75625
	430	3549	132,211	16234	1104005

Value of b can be calculated as follows:

$$b = \frac{12(132211) - 430(3549)}{12(16234) - (430)^2} = 6.10$$

$$a = \frac{3549}{12} - 6.10\frac{(430)}{12} = 77.08$$

Therefore the predicting function is $\hat{Y} = \underline{77.08 + 6.1X}$

b. i. If X = 45 units, then

$\hat{Y} = 77.08 + (6.1 \times 45)$
$= \text{Sh.}351.58$

ii. If X = 34 units, then

$\hat{Y} = 77.08 + (6.1 \times 34)$
$= \text{Sh.}284.48$

Illustration 2.2

Assume that the company (in illustration 2.1) intends to spend Sh.400 on labour cost next period. Compute the number of units that the company may produce.

Solution

Note:

$\hat{Y} = a + bx$ is a regression of Y on X i.e. $Y = f(x)$

We require a regression of X on Y. i.e. $X = g(Y)$ to answer the above question. The general format of the equation is:

$$X = a^1 + b^1 Y$$

$$b = \frac{n \sum xy - \sum x \sum y}{n\sum Y^2 - (\sum Y)^2}$$

$$a = \frac{\sum X}{n} - \frac{b\sum Y}{n}$$

$$b^1 = \frac{12(132{,}211) - (430 \times 3549)}{12(1{,}104{,}005) - (3549)^2}$$

$$= 0.0926$$

$$a^1 = \frac{430}{12} - \frac{0.0926(3549)}{12}$$

$$a^1 = 8.3286$$

Therefore the predicting equation is $\hat{X} = 8.33 + 0.093Y$

Thus if the Company intends to spend Sh.400 on labour, the number of units to be produced will be:

$$\hat{X} = 8.33 + 0.093(400)$$
$$= 45.56 \text{ units}$$

Approximately 46 units

2.4 EVALUATION OF THE REGRESSION MODEL

The regression equation calculated above was based on the assumption that cost varied with the units produced. However, a number of different activity measures exist such as direct labour hours, direct labour cost, number of production runs, etc.

It is important therefore to determine the reliability of the estimated cost function. Various tests of reliability can be applied. These tests can be grouped into 3:

Logical relationship tests
Goodness of fit tests
Specification tests

Logical relationship tests

These tests, also referred to as economic plausibility test, are used to determine whether there is an expected logical relationship between the independent and the dependent variable.

To carry out this test, it is important to understand the input-output relationship in the company.

For the illustration there is an expected logical relationship between the number of units and the labour cost mainly because the higher the number of units, the higher the number of labour hours and therefore the higher the labour cost.

Goodness of fit tests

These tests can be divided into two:

Testing the whole model
Testing the slope

Testing the whole model

Tests of the whole model are used to determine the reliability of all the independent variables taken together. The measures used are:

Coefficient of determination (r^2)
Std error of the estimate
F-test

I. Coefficient of Determination (r^2)

If the regression line calculated by the least square method were to fit the actual observations perfectly, then all observed points would lie on the regression line. The coefficient of determination, r^2, explains the amount of variation in Y which is explained by the introduction of X in the model. A perfect linear relationship between X and Y would result in r^2 being equal to 1.

$$r^2 = \frac{\text{explained variation}}{\text{Total variation}}$$

$$= \frac{\Sigma\,(\hat{Y} - \acute{y})^2}{\Sigma\,(Y - \acute{y})^2}$$

Where ý is the mean value of Y

For computation purposes r^2 can be given by

$$r^2 = \frac{(n\,\Sigma\,xy - \Sigma\,x\,\Sigma x)^2}{[n\,\Sigma x^2\ -(\Sigma x)^2]\,[n\,\Sigma y^2 - (\Sigma y)^2]}$$

From the illustration 2.1

$$= \frac{[12(132211) - (430)\ (3549)]^2}{[12\ (16234) - (430)^2]\ \{12\ (1104005 - (3549)^2\}}$$

$$= 0.565$$

About 56.5% of the variations in labour cost can be explained by variations in units produced while about 43.5% of the variation in labour cost is explained by other independent variables and the error term.

Note

The higher the r^2, the better the function is. As a rule of thumb, r^2 must be at least equal to 0.8.

II. Standard error of estimate (S_e)

The coefficient of determination r^2 gives us an indication of the reliability of the estimate of total cost based on the regression equation but it does not give us an indication of the absolute size of the probable deviations from the line established. This information can be obtained by calculating the standard error of estimate given by the following formula.

$$S_e = \sqrt{\sum \frac{(Y - \hat{Y})^2}{n-2}}$$

For computation purpose,

$$S_e = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-k-1}}$$

Where k is the no. of independent variables

For illustration 1.2

$$S_e = \sqrt{\frac{1104005 - 77.08\ 3549) - 6.1(13221}{12-2}}$$

$= 48.95$

The sample size, n, is reduced by 2 because 2 variables 'a' & 'b' in the regression equation had to be estimated from the sample observations.

The calculation of the standard error is necessary because the least square line was calculated from sample data. Other samples would probably result in different estimates. Obtaining the least square calculation over all the possible observations that might occur would result in the calculation of the true least square line. The question is "How close does the sample estimate of least square line come to the true least square line.

Standard error is similar to standard deviation in normal probability analysis. It is a measure of variability around the regression line. The std error of estimates enables us to establish a range of values of the dependent variable within which we may have some degree of confidence that the true value lies. We can use the following equation to establish this range:

$$\hat{Y} - t_c S_e \leq Y \leq \hat{Y} + t_c S_e$$

From illustration 2.1, where $\hat{Y}_{34} = 284.48$, the 95% confidence interval can be calculated as follows:

$284.48 - 2.2281(48.95) \leq Y \leq 284.48 + 2.2281\ (48.95)$

$\leq Y \leq 393.6$

We are 95% confident that if X is estimated to be 34 units next period, the true labour cost will lie between 175.4 and 393.6. Note t_c from the student T tables, with 10 degrees of freedom and 5% significance level, is equal to 2.2281.

III. The F –test

The significance of the regression results can be tested by using the F- statistics. The F-statistics is a ratio which compares the explained sum of squares and the unexplained sum of squares.

$$\text{Therefore } F = \frac{\text{mean sum of squares due to regression}}{\text{Mean sum of squares due to residual}}$$

For calculation purposes:

$$F = \frac{r^2/K}{(1-r^2)/n\text{-}k\text{-}1}$$

F statistics can then be used to test the hypothesis that the relationship between the dependent variables and all the independent variables is not significant.

The Steps followed in the F- Test are:

State the hypothesis

H_0: Relationship between Y and all Xs is not significant.
H_A: Relationship between Y and all Xs is significant

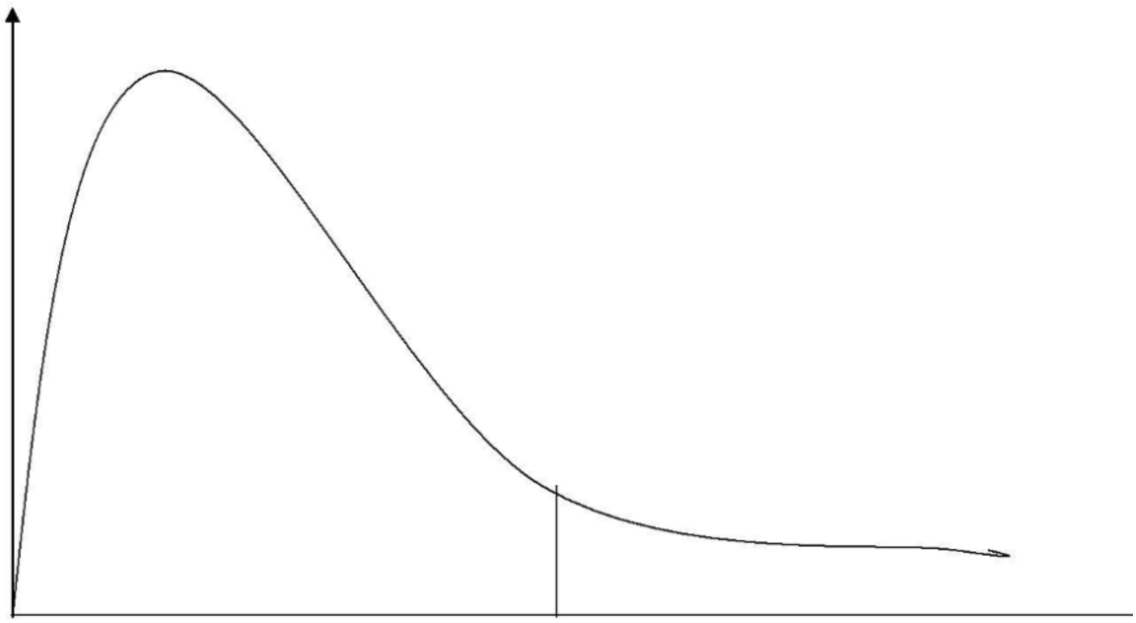
State the significant level

$\alpha = 5\%$

State the test statistics

$$F = \frac{r^2/K}{(1-r^2)/n\text{-}k\text{-}1}$$

State the decision rule



4.965

k = 1

$n-k-1 = 10$
$\alpha = 0.05$
$F_c = 4.965$

Computation of F statistics

$$F = \frac{0.565/1}{(1-0.565)/(12-1-1)}$$

$= 12.989$

Conclusion

Since the computed $F > F_c$ then we reject H_o. Therefore the relationship between the labour cost and the number of units is significant.

Testing the Slope

The strength of the relationship between the dependent variable and each of the independent variables can be determined using 3 methods:

Correlation coefficient (r)
Standard error of the slope (Sb)
Z or t statistics.

Correlation coefficient (r)

The correlation coefficient measures the degree of association between two variables such as the cost and the activity level.

$$r = \frac{n\Sigma xy - \Sigma x \,\Sigma y}{[n\Sigma x^2 - (\Sigma x)^2]\,[n\Sigma y^2 - (\Sigma y)^2]}$$

If the degree of association between two variables is very close then it would be almost possible to plot the observation on a straight line and r will be almost equal to one.

For illustration 2.1,

$r = \sqrt{r^2} = \sqrt{0.565} = 0.752$

$-1 \leq r \leq 1$

If r= -1, then the two variables are said to be perfectly negatively correlated
If r = +1, then the two variables are perfectly positively correlated
If r = 0, then there is no correlation between the two variables.

Std error of the slope (S_b)

The reliability of the estimate of the regression coefficient 'b' (i.e. the variable cost), is important since the analyst usually focuses on the rate of variability rather than the absolute level of prediction. This can be established by the use of the standard error of the slope.

The standard error of 'b' coefficient can be expressed as follows:

$$S_b = \frac{S_e}{\sqrt{\sum (x - \bar{x})^2}}$$

For calculation purposes,

$$S_b = \frac{S_e}{\sqrt{\Sigma x)^{2} \,/n}} = \frac{48.95}{\sqrt{16234 - \frac{430^{2}}{12}}}$$

$$= 1.70$$

We can then use S_b to construct confidence intervals using t distributions such that the true variable cost, B, will be:

$b - t_c S_b \leq B \leq b + t_c S_b$

where b is the estimated value of B

For illustration 1.2, the 95% Confidence Interval for the true variable cost will be:

$6.1 - 2.2281\ (1.7) \leq B \leq 6.1 + 2.2.281\ (1.7)$

$2.3122 \leq B \leq 9.8878$

We are 95% confident that the true variable cost, B, lies between 2.31 and 9.89

Z or t Statistics

If $n \geq 30$ we use Z, if, $n < 30$ we use t statistics. These statistics can be used to test the hypothesis:
Ho: B = O that is, there is no relationship between X and Y
HA: B$\neq$ O There is a significant relationship between X and Y

The level of significance is $\alpha = 0.05$

The degrees of freedom = n-k-1
For illustration 1.2 Df = 12-1 = 10

Test statistics $T = \frac{b}{S_b}$

The critical value of T for illustration 1.2 is 2.2281

Computation for illustration 2.1

$T = \frac{6.1}{1.7} = 3.59$

Conclusion

Reject Ho since computed $T > t_c$. This means that there is a significant relationship between the labour costs and number of units.

SPECIFICATION TESTS

These tests are used to test the validity of the regression assumptions. The necessary assumptions in linear regression are:

The underlying relationship between X and Y is linear.
The independent variable X is assumed to be known and is used to predict the dependent variable Y.

The errors or the residuals given by $\sum(Y - \hat{Y})^2$ are assumed to:

Be normally distributed.
Have and expected value (mean) of Zero (0).
Have a constant variance. This is referred to as homoscedasticity. If not constant we have heteroscedasticity.
Be independent i.e. they are not serially correlated or there is no autocorrelation.
The Specification tests can be done for illustration 2.1 as follows:

Week	Units	L. cost (Y)	Ŷ	e_i =y –Ŷ	e_i – e	$(ei– ei)^2$	e^2
1	34	340	28448	55.52	-	-	-
2	44	346	343.48	0.52	-55	3025	002704
3	31	287	266.18	20.82	-20.3	4409	433.4
4	36	262	296.68	-34.68	-55.5	44.09	433.4
5	30	220	260.08	-40.02	5.4	29.16	1606.41.
6	49	416	375.98	40.02	-80.1	6416.01	1601.60
7	39	337	314.98	22.02	18	324	484.88
8	21	180	205.18	-25.18	47.1	227.84	634.0
9	41	376	327.18	48.82	-4	5476	2383.39
10	47	295	363.78	-68.78	117.6	13829.76	4730.69
11	34	215	284.48	-69.48	1.2	0.44	4897.20
12	24	275	223.48	51.52	- 121.5	762.25	2654.31
				1.04		49461.6	23641.96

$\hat{Y} = 77.08 + 6.10\,x$

$E(e) = \frac{1.04}{12} = 0.08$

This is approximately equal to zero.

To test whether the observation is normally distributed we can construct a histogram of the observation.

Independence of observations

An important assumption for the simple linear regression model is the independence of errors. In many time series models, this assumption is violated because of the correlation of errors in successive observations. This is referred to as **autocorrelation.**

Autocorrelation occurs if a positive error is followed by another positive error and a negative error is followed by another negative error. If autocorrelation occurs then time should be considered as an important independent variable and therefore time varies analysis should be used.
We can use Durbin Watson 'D' statistics to determine whether observations are independent.

$$D = \frac{\Sigma (e_i - e_{i-1})^2}{\Sigma e_i^2}$$

where e_i is the error in time i
e_{i-I} is the error in time i-I

The Durbin Watson statistics provides a measure of association between successive values of the error term. The computed statistics is compared against two tabulated values d_u and d_l that depend on the desired confidence level of the test and the degrees of freedom of the data.

If the computed Durbin Watson "D" statistics is greater than D_u, then we can conclude that there's no positive correlation between error terms.

If $d_l \leq D \leq d_u$ then the test is inconclusive and therefore we can neither accept nor reject the null hypothesis.

Note

A rule of thumb, with uncorrelated errors then D approaches a value of 2. If errors are highly positively correlated, the D would be less than 1.5 and can be very near to zero (0).
For negatively correlated errors, the value of D will be above 2.5 with an upper limit of 4.

For illustration 2.1

$$D = \frac{\Sigma (e_i - e_{i-1})^2}{\Sigma e_i^2}$$

$$= \frac{49461.6}{23641.696} = 2.09$$

From the tables:
$d_l = 0.971$
$d_u = 1.331$

Since the calculated value of D is greater than d_u, then we can accept the null hypothesis, that there is no positive serial correlation.

The error of dependence is caused by:

The omission of an important variable such as the seasonal effect. (misspecification error).

The relationship is not linear.

A shift in production process which may be caused by change in equipment that has not been shown in the model.

2.5 MULTIPLE REGRESSION

The least square regression equation discussed above was based on the assumption that total cost was determined by only one activity based variable. However, other variables are likely to affect labour costs such as labour hours, material costs, machine hours, etc. These may have an effect on labour costs.

The equation for the simple regression can be expanded to include more than one independent variable as shown below:

$\hat{Y} = a + b_1X_1 + b_2 X_2 + b_3 X_3 + \ldots\ldots\ldots + b_n X_n$

For two independent variables, the function will be of the form:

$\hat{Y} = a + b_1X_1 + b_2 X_2$

The normal equations can be given by:

$\Sigma y = na + b_1 \Sigma X_1 + b_2 \Sigma x_2$
$\Sigma X_1Y = a\Sigma x_1 + b_1\Sigma X_1^2 + b_2 \Sigma X_1 \Sigma X_2 \Sigma$
$X_2 Y = a\Sigma x_2 + b_1\Sigma X 1\Sigma X_2 + b_2 \Sigma X_2$

Normally computers are used for the solution of multiple regression.

Multi-collinearity

Multiple regression analysis is based on the assumption that the independent variables are not correlated with each other, When the independent variables are highly correlated with each other then it is very difficult to isolate the effect of each one of these on the dependent variables. This occurs when there is a simultaneous movement of two or more independent variables in the same direction and almost at the same time. This condition is called multi-collinearity.

We can use the correlation matrix to determine whether 2 independent variables are highly correlated. If a correlation value of more than 0.8 exists between two independent variables, then the problem of multi-colinearity is bound to occur. Alternatively if the correlation coefficient between the two variables is greater than the multiple correlation coefficient, then multi-colinearity problem will occur. To remove the problem of multi-colinearlity, we drop one of the correlated variables. You can drop any of the variables.

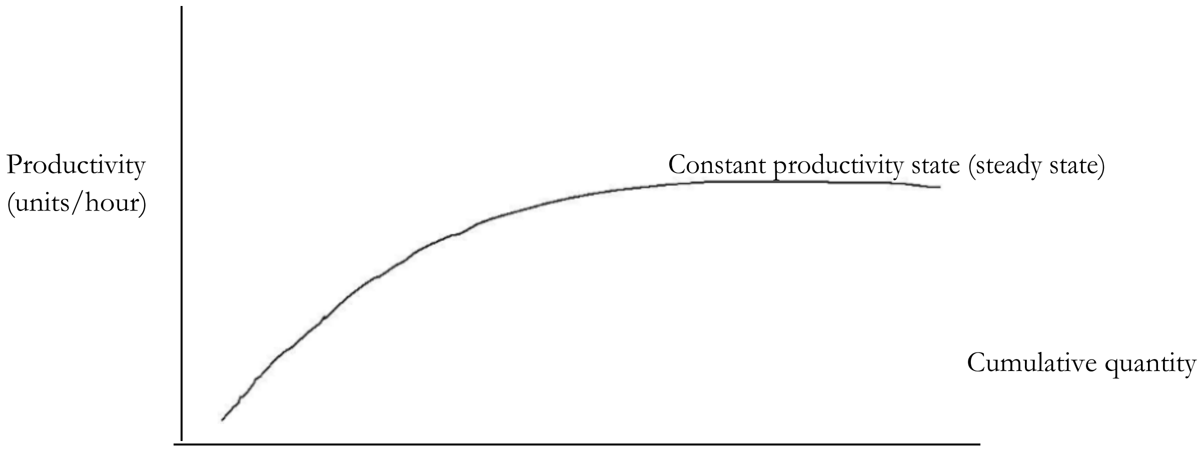
2.6 LEARNING CURVE THEORY

The first time a new operation is performed both workers and operating procedures are untried but as the operation is replaced the workers becomes more familiar with the work so that less hours are required. This phenomena is known as the learning curve effect.

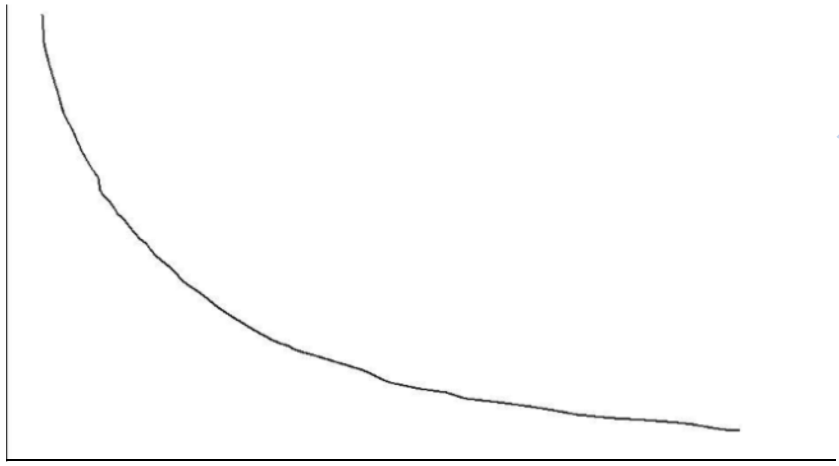
This is also referred to as improvement curve theory. It occurs when new production methods are introduced, new product s (either goods or services) are made or when new employees are hired. It is based on the proposition that as workers gain experience in a task, they need less time to complete the job and productivity increases.

The learning curve theory affects not only direct labour costs but also impacts direct labour related costs such as supervision, and direct material costs due to reduced spoilage and waste as experience is gained.

The time to perform many operations begins slowly and speeds up as employees become more skilled. Gradually, the time needed to complete an operation becomes progressively smaller at a constant percentage. Since this rate of improvement has a regular pattern, a learning curve can be drawn (see diagrams below) to estimate the labour hours required as workers become more familiar. These curves are also referred to as progress functions or experience curves.



A learning curve



The effect of experience on cost is summarised by a learning ratio (improvement ratio or learning rate) defined by the following;

$$\text{Learning ratio} = \frac{\text{Average labour cost for the first 2x units}}{\text{Average labour cost for the first x units}}$$

Example 1:

The first 500 units has an average labour cost of sh.12.50 and the average labour cost for the first 1000 units is sh.10.
Required: Calculate the learning ratio.

$$\text{Learning ratio} = \frac{\text{Sh.10}}{\text{Sh.12.50}} \times 100 = 80\%$$

Interpretation:
Every time cumulative output doubles, average cost declines to 80 percent of the previous amount. Since the average cost of the first 1000 units was sh.10, the average cost of the first 2010 units will be expected to be 20% or sh.8 per unit.

Learning curve equation:
The basic learning curve equation is

$$Y = ab^x$$

Where: a is the labour cost of the first unit
b is the cumulative production
x is the improvement exponent or an index learning given by:

x = <u>logarithm of the learning ratio</u> = <u>log (1- proportional</u>
<u>decrease)</u> logarithm of 2 log 2

x can take any value between -1 and zero.
Y is defined depending on whether a cumulative model or incremental model is being applied.

The above equation can be restated in the logarithmic form

Log Y = log a + x log b

Cumulative Total Cost
Each of the equations (i) and (ii) defines cumulative average cost. Either of them can be converted easily to a formula for the total labour cost of all units produced up to a given point. Total cost can always be calculated from a known average cost. Hence;

Total cost = bY = b (ab^x) = ab^{x+1}

Incremental cost
If producing a second 1000 units is to reduce cumulative average cost from sh.10 to sh.8, the cost of the second 1000 units will have to be only sh.6000, or sh.6 each. Hence;

	Total Cost (Sh.)	No. of Units	Average Cost (sh.)
First 1000 units	10,000	1,000	10
Second 1000 units	6,000	1,000	6
	16,000	2,000	8

Defining the learning curve in terms of this incremental relationship would be more useful but is more difficult to work with. As a result, learning curve improvement ratios are usually stated as percentage reductions in cumulative average labour cost.

Example 2:

A company makes an electronic navigational guidance system that is used for space craft, aircraft and submarines. The direct labour cost is subject to an 80% learning curve. The first unit is estimated to require 1250 direct labour hours.
Required:
Compute the average number of hours required for the first 2, 3, 4, 8 units.
Assume the company estimates the variable cost of producing each unit as shown;

Direct material cost	sh.40,000 per unit
Direct labour	sh.20 per hour
Variable production overhead	sh.1000 + 60% of direct labour cost

Required:

Estimate the total manufacturing cost of 1, 2, 3, 4 units of the product

Solution:

Number of labour hours

Units X	Average (y)	Total*	Marginal cost	Computations
1	1250	1250	1250	$Y = 1250 \times 1^{-0.322} = 1250$
2	1000	2010	750	$Y = 1250 \times 2^{-0.322} = 1000$
3	878	2634	634	$Y = 1250 \times 3^{-0.322} = 878$
4	800	3200	566	$Y = 1250 \times 4^{-0.322} = 800$
8	640	5120		$Y = 1250 \times 8^{-0.322} = 640$

*Total hours = Average labour hours x no. of units

(b) Unit no.	Direct materials	Direct labour	Variable manufacturing overhead	Total overheads
1	40,000	20x1250=25000	1000+(0.6x25000)	81,000
2	40,000	20x 750=15000	1000+(0.6x15000)	65,000
3	40,000	20x 634=12680	1000+(0.6x12680)	61,288
4	40,000	20x 566=11320	1000+(0.6x11320)	59,112

Applications of learning curves to accounting.
The learning phenomenon applies to time and it could thus affect any costs which are functions of time. Examples are hourly labour costs, indirect labour, supervision, etc.
Whenever costs are estimated, the potential impact of learning should be considered.
The phenomenon can also affect costs used in inventory valuation, costs used in decision making and costs used in performance evaluation. However, learning curves only apply to the early phases of production. After the steady state is achieved, costs tend to stabilise.

(i) In Inventory valuation- failing to recognise learning effects can have some unexpected consequences. (See example below).

Example 3:

Production of a new product starts in January and continues through the year. Direct material cost is sh.100 per unit through out the year. Because of the learning effect, the labour hours per unit drop from 1 hour (at sh.160 per hour) in January to 0.25 hour in December. Manufacturing overhead is all fixed at sh.80,000. If 1000 units will be produced in January the overhead application rate is sh.80 per hour. This rate is (mistakenly) applied throughout the year.

Unit inventory Value

	January Sh.	December Is (Sh.)	December Should be (Sh.)
Direct materials	100	100	100
Direct labour	160	40(0.25x160)	40
Overheads applied	80	20(0.25x80)	80
	340	160	220

(ii) Decision making – A product newly launched may at a glance appear to be unprofitable, however because of learning effect, the variable costs would drop by the end of the period making the product profitable.

(iii) Performance evaluation. A bank has developed labour time and cost standards of some of its clerical activities. These activities are subject to the learning curve effect. The management has also found that the time on these activities exceeded the standard. On investigation, it was found that there was high personnel turnover meaning the activities were done by inexperienced people. Changes were made in personnel policy and the personnel turnover was reduced. The time spent on clerical activities no longer exceeded standards.

Attempt the problem below before checking the solution given.

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average sh100 a unit. You also expect that a 90 percent learning curve will apply to the cumulative labour costs on this contract.

Required:

Prepare an estimate of the labour costs of fulfilling the contract.
Estimate the incremental labour cost of extending the production run to produce an additional 800 units. Estimate the incremental labour cost of extending the production run from 800 to 900 units.

Solution:

Average cost decreases by 10 percent every time the cumulative total production doubles.
Therefore: Average cost of first 200 units = 0.9 x Average cost of first 100 units
Average cost of first 400 units = 0.9 x Average cost of first 200 units
Average cost of first 800 units = 0.9 x Average cost of first 400 units

Combining these, we have;
Average cost of the first 800 units = 0.9 x 0.9 x 0.9 x sh.100 = sh72.90
Total cost = sh (72.90 x 800) = sh. 58,320

Average cost of the first 1600 units = 0.9 x 72.90 = sh.65.61
Total cost of 1600 units = 1600 x sh65.61 = sh.104,976
Additional cost of second 800 units = sh 104,976 – 58,320 = sh.46,656
Average cost = sh.58.32 /unit.

Because this increase will not increase cumulative production to twice some figure we already have, we need to
Use the formula.

Average cost = sh10,000 x 9^x

$x = \frac{-0.0458}{0.301} = -0.15216$

Hence;

Log av. cost = log 10,000 – 0.15216 log 9

Average cost = sh. 71.5833/unit

Total Cost = 900 x sh.71.5833 = sh.64,425
Incremental cost = sh.64,425 – 58,320
Average cost = sh.61. 05/unit

REINFORCING QUESTIONS

QUESTION ONE

CB plc produces a wide range of electronic components including its best selling item, the Laser Switch. The company is preparing the budgets for Year 5 and knows that the key element in the Master Budget is the contribution expected from the Laser Switch. The records for this component for the past four years are summarised below:

	Year 1	**Year 2**	**Year 3**	**Year 4**
Sale (unit)	150,000	180,000	200,000	230,000
	£	£	£	£
Sale revenue	292,820	346,060	363,000	448,800
Variable costs	131,080	161,706	178,604	201,160
Contribution	161,740	184,354	184,396	247,640

It has been estimated that sales in Year 5 will be 260,000 units.

Required:

As a starting point for forecasting Year 5 contribution, to project the trend, using linear regression;
To calculate the 95% confidence interval of the individual forecast for Year 5 if the standard error of the forecast
is £14,500 and the appropriate *t* value is 4,303, and to interpret the value calculated;
To comment on the advantages of using linear regression for forecasting and limitations of the technique.

QUESTION TWO

The theory of the experience curve is that an organisation may increase its profitability through obtaining greater familiarity with supplying its products or services to customers. This reflects the view that profitability is solely a function of market share.

Required:

Discuss the extent to which the application of experience curve theory can help an organisation to prolong the life cycle of its products or services.

QUESTION THREE

Savitt Ltd manufactures a variety of products at its industrial site in Ruratania. One of the products, the LT, is produced in a specially equipped factory in which no other production takes place. For technical reasons the company keeps no stocks of either LTs or the raw material used in their manufacture. The costs of producing LTs in the special factory during the past four years have been as follows:

YEAR	**1**	**2**	**3**	**4(Estimated)**
	Sh	Sh	Sh.	Sh.
Raw materials	70 000	100 000	130 000	132 000
Skilled labour	40 000	71 000	96 000	115 000
Unskilled labour	132 000	173 000	235 000	230 000
Power	25 000	33 000	47 000	44 000
Factory overheads	168 000	206 000	246 000	265 000
Total production costs	£435 000	583 000	££754 000	£786 000
				180 000
Output (units)	160 000	190 000	220 000	180,000

The costs of raw materials and skilled and unskilled labour have increased steadily during the past four years at an annual compound rate of 20% and the costs of factory overheads have increased at an annual compound rate of 15% during the same period. Power prices increased by 10% on 1 January Year 2 and by 25% on the 1 January of each subsequent year. All costs except power are expected to increase by a further 20% during Year 5. Power prices are due to rise by 25% on 1 January Year 5.

The directors of Savitt Ltd are now formulating the company's production plan for Year 5 and wish to estimate the costs of manufacturing the product LT. The finance director has expressed the view that 'the full relevant cost of producing LTs can be determined only if a fair share of general company overheads is allocated to them.' No such allocation is included in the table of costs above.

Required:

Use linear regression analysis to estimate the relationship of total production costs to volume for the product LT for Year 5 (ignore general company overheads and do not undertake a separate regression calculation for each item of cost),

Comment on the view expressed by the finance director.
Ignore taxation.

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

COMPREHENSIVE ASSIGNMENT NO.1

TO BE SUBMITTED AFTER LESSON 2

To be carried out under examination conditions and sent to the Distance Learning Administration for marking by the University.
TIME ALLOWED: 3HRS. ANSWER ALL QUESTIONS.

QUESTION ONE

The Management of Waity Ltd. is not happy with the company's current budgetary planning and control system and would wish to see it more effective than it is currently. For this purpose you have been brought into the company with the title, "Management Accountant (Special Duties)".

a. State what effectiveness for budgetary planning and control implies.
b. Prepare a plan of action showing how you would go about carrying out your assignment.
c. What is the meaning of "treasury management" within a group of companies and what activities does it normally cover?
d. What are the elements of a comprehensive scheme of cash forecasting and what steps can one take to adjust for uncertainty in cash forecasts? *(CPA Dec 2014)*

QUESTION TWO

a. Enumerate and comment on the benefits which may occur when using standard costing in conjunction with process accounts.
b. Explain how a profit centre approach may be applied in accounting within a processing industry and how standard costing may be useful in its implementation. *(ACCA Dec 1989)*

QUESTION THREE

a. Comment on factors likely to affect the accuracy of the analysis of costs into fixed and variable components.
b. Explain how the analysis of costs into fixed variable components is of use in planning, control and decision-making techniques used by the management accountant.
(ACCA June 1989)

QUESTION FOUR

. The Bozo-Dog Do-Da Ltd use an interlocking system of accounting. The financial Profit and Loss account for the year ended 30 September was:

	Sh		Sh
Purchases	25,210	Sales: 50,000 units at Shs 1.50 each	75,000
Less closing stock	4,080	Discounts received	260
	21,130	Profit on sale of land	2,340
Direct wages	10,500		
Work expenses	12,130		
Selling expenses	7,100		
Administration expenses	5,340		
Depreciation	1,100		
Net profit	20,300		
	77,600		77,600

The cost profit, however, was only Shs 19,770. Reconcile the financial and cost profits, using the following information:

a. Cost accounts value of closing stock: sh4,280
b. The works expenses in the cost accounts were taken as 100% direct wages.
c. Selling and administration expenses were charged in the cost accounts at 10% of sales and £0.10 per unit respectively.
d. Depreciation in the cost accounts was sh800.

QUESTION FIVE

The ABC corporation processes cooking oils from a type of nut grown in Western Kenya. The manufacturing process operates such that the nuts are initially processed into two products namely Mafuta and Karanga. The former is a high-grade cooking oil used for domestic purposes and also for specialised cooking. Karanga is subsequently processed into two final products, Chemsha, a low grade oil used for deep frying mainly in large scale establishments and Mlo, a low grade fat used in cattle feeds.

During the month of May, the company processed 100,000 killogrammes of nuts into Mafuta and Karanga at a total cost comprising direct materials—Shs 360,000, direct labour—Shs 216,000 and factory overheads—Shs 144,000. Karanga was subsequently processed into Chemsha and Mlo at an additional cost of Shs 36,000 for direct materials, Shs 84,000 for direct labour and Shs 48,000 for factory overheads.

In addition to the above costs each of the three products incurred entirely separate and variable costs of Shs 48,000 for Mafuta, Shs 384,000 for Chemsha and Shs 96,000 for Mlo up to final completion at which point 5,000 killogrammes of Mafuta, 10,000 kilogrammes of Chemsha and 25,000 killogrammes of Mlo were obtained. The final selling prices are Shs 192, Shs 48 and Shs 4.8 per kilogramme for Mafuta, Chemsha and Mlo respectively.

Required:

a. Prepare a statement to show the profitability of each product assuming that costs are allocated on the basis of net realisable value.

b. Management is considering further processing of Chemsha at an additional cost of Shs 9.60 per kilogramme. This would enable the company sell the product at a price of Shs 57.60 per kilogramme. The product mix however would change to 5,000 Kg., 9,000 kg. and 30,000 Kg. of Mafuta, Chemsha and Mlo respectively.

   Advise management on the suitability of adopting the proposed action. *(CPA June 1991)*

END OF COMPREHENSIVE ASSIGNMENT No.1

NOW SEND YOUR ANSWERS TO THE DISTANCE LEARNING CENTRE

LESSON THREE

SHORT TERM DECISIONS

OBJECTIVES

Examination of cost volume profit analysis and relevant cost decisions

INSTRUCTIONS

1. Read the Study Text and Chapter 8 and 9 of Management and Cost Accounting by Colin Drury
2. Attempt the reinforcing questions at the end of the lesson under examination conditions
3. Compare your answers with those given in Lesson 10

CONTENTS

3.1 COST-VOLUME PROFIT (C-V-P) ANALYSIS INTRODUCTION

In this section, you will learn to use cost-volume-profit analysis.
You can use cost-volume -profit analysis to analyze the natural relationship between cost, volume, and profit in pricing decisions. In cost-volume-profit analysis, you:

- Should consider only short-term operations. The short term may be defined as a period too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships.
- Assume that a straight line can reasonably be used in analysis. While actual price behavior may not follow a straight line, its use can closely approximate actual cost behavior in the short run.
- If purchase volume moves outside the relevant range of the available data, the straight-line assumption and the accuracy of estimates become questionable.
- If you know that product variable costs per unit are decreasing as quantity increases, consider using the log-linear improvement curve concept. Improvement curves are particularly useful in limited production situations where you can obtain cost/price information for all units sold.

In the short run, costs can be of three general types:

- **Fixed Cost.** Total fixed costs remain constant as volume varies in the relevant range of production. Fixed cost per unit decreases as the cost is spread over an increasing number of units. Examples include: Fire insurance, depreciation, facility rent, and property taxes.
- **Variable Cost.** Variable cost per unit remains constant no matter how many units are made in the relevant range of production. Total variable cost increases as the number of units increases. Examples include: Production material and labor. If no units are made, neither cost is necessary or incurred. However, each unit produced requires production material and labor.
- **Semivariable Cost.** Semivariable costs include both fixed and variable cost elements. Costs may increase in steps or increase relatively smoothly from a fixed base. Examples include**:** Supervision and utilities, such as electricity, gas, and telephone. Supervision costs tend to increase in steps as a supervisor's span of control is reached. Utilities typically have a minimum service fee, with costs increasing relatively smoothly as more of the utility is used.

Cost -volume-profit analysis is an estimating concept that can be used in a variety of pricing situations.
You can use the cost-volume relationship for:

- **Evaluating item price in price analysis.** Cost-volume-profit analysis assumes that total cost is composed of fixed and variable elements. This assumption can be used to explain price changes as well as cost changes. As the volume being acquired increases unit costs decline. As unit costs decline, the vendor can reduce prices and same make the same profit per unit.
- **Evaluating direct costs in pricing new contracts.** Quantity differences will often affect direct costs -- particularly direct material cost. Direct material requirements often include a fixed component for development or production operation set-up. As that direct cost is spread over an increasing volume unit costs should decline.
- **Evaluating direct costs in pricing contract changes.** How will an increase in contract effort increase contract price? Some costs will increase others will not. The concepts of cost-volume-profit analysis can be an invaluable aid in considering the effect of the change on contract price.
- **Evaluating indirect costs.** The principles of cost-volume-profit analysis can be used in indirect cost analysis. Many indirect costs are fixed or semivariable. As overall volume increases, indirect cost rates typically decline because fixed costs are spread over an increasing production volume.

The main assumptions required in C-V-P analysis are:

1. The relationship holds only within the relevant range. The relevant range is a band of activity within which a given cost behaviour is defined.
2. The behaviour of total cost and total revenue has reliably been determined and is lineal within the relevant range.
3. All costs can be divided into fixed and variable such that mixed costs are decomposed into their fixed and their variable components.
4. Selling prices are constant therefore we ignore quantity discounts.
5. Efficiency and productivity remain the same so that we therefore ignore the learning curve effect.
6. Prices of factors of production remain constant.
7. There are no limiting factors

3.2 ANALYZING THE COST-VOLUME RELATIONSHIP

This section examines algebraic and graphic analysis of the cost-volume relationship.

3.2.1 ALGEBRAIC ANALYSIS

The assumption of linear cost behavior permits use of straight-line graphs and simple linear algebra in cost-volume analysis.

Total cost is a semi-variable cost—some costs are fixed, some costs are variable, and others are semi-variable. In analysis, the fixed component of a semi-variable cost can be treated like any other fixed cost. The variable component can be treated like any other variable cost. As a result, we can say that:

Total Cost = Fixed Cost + Variable Cost

Using symbols:

$$C = F + V$$

Where:

C = Total cost

F = Fixed cost

V = Variable cost

Total variable cost depends on two elements:

Variable Cost = Variable Cost per Unit x Volume Produced

Using symbols:

$$V = V_u (Q)$$

Where:

V_U = Variable cost per unit

Q = Quantity (volume) produced

Substituting this variable cost information into the basic total cost equation, we have the equation used in cost-volume analysis:

$$C = F + V_U (Q)$$

Illustration 2.1

If you know that fixed costs are Sh.500, variable cost per unit is Sh.10, and the volume produced is 1,000 units, you can calculate the total cost of production.

$$C = F + V_u (Q)$$
$$= 500 + 10 (1000)$$
$$= \text{Sh.}10500$$

Given total cost and volume for two different levels of production, and using the straight-line assumption, you can calculate variable cost per unit.

Remember that:

- Fixed costs do NOT change no matter what the volume, as long as production remains within the relevant range of available cost information. Any change in total cost is the result of a change in total variable cost.
- Variable cost per unit does NOT change in the relevant range of production.

As a result, we can calculate variable cost per unit (V_U) using the following equation:

$$V_U = \frac{\text{Change in Total Cost}}{\text{Change in Volume}} = \frac{C_2 - C_1}{Q_2 - Q_1}$$

Where:

C_1 = Total cost for Quantity 1
C_2 = Total cost for Quantity 2
Q_1 = Quantity 1
Q_2 = Quantity 2

Illustration

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for Sh.60,000. The same quote offered 4,000 units for Sh.50,000. What is the apparent variable cost per unit?

$$V_u = \frac{C_2 - C_1}{Q_2 - Q_1} = \frac{60000 - 50000}{5000 - 4000} = \text{Sh. } 10$$

If you know total cost and variable cost per unit for any quantity, you can calculate fixed cost using the basic total cost equation.

3.2.2 GRAPHIC ANALYSIS

Introduction to Graphic Analysis.

When you only have two data points, you must generally assume a linear relationship. When you get more data, you can examine the data to determine if there is truly a linear relationship.

You should always graph the data before performing an algebraic analysis.

- Graphic analysis is the best way of developing an overall view of cost-volume relationship.
- Graphic analysis is useful in analyzing cost-volume relationships, particularly, when the cost and volume numbers involved are relatively small.
- Even when actual analysis is performed algebraically you can use graphs to demonstrate cost-volume analysis to others.

Steps of Graphic Analysis.

There are four steps in using graph paper to analyze cost-volume relationships:

Step 1. Determine the scale that you will use.

Volume is considered the independent variable and will be graphed on the horizontal axis. Cost is considered the dependent variable and will be graphed on the vertical axis. The scales on the two axes do not have to be the same. However, on each axis one block must represent the same amount of change as every other block of the same size on that axis. Each scale should be large enough to permit analysis, and small enough to permit the graphing of all available data and anticipated data estimates.

Step 2. Plot the available cost-volume data.

Find the volume given for one of the data points on the horizontal axis. Draw an imaginary vertical line from that point. Find the related cost on the vertical axis and draw an imaginary horizontal line from that point. The point where the two lines intersect represents the cost for the given volume. (If you do not feel comfortable with imaginary lines you may draw dotted lines to locate the intersection.) Repeat this step for each data point.

Step 3. Fit a straight line to the data.

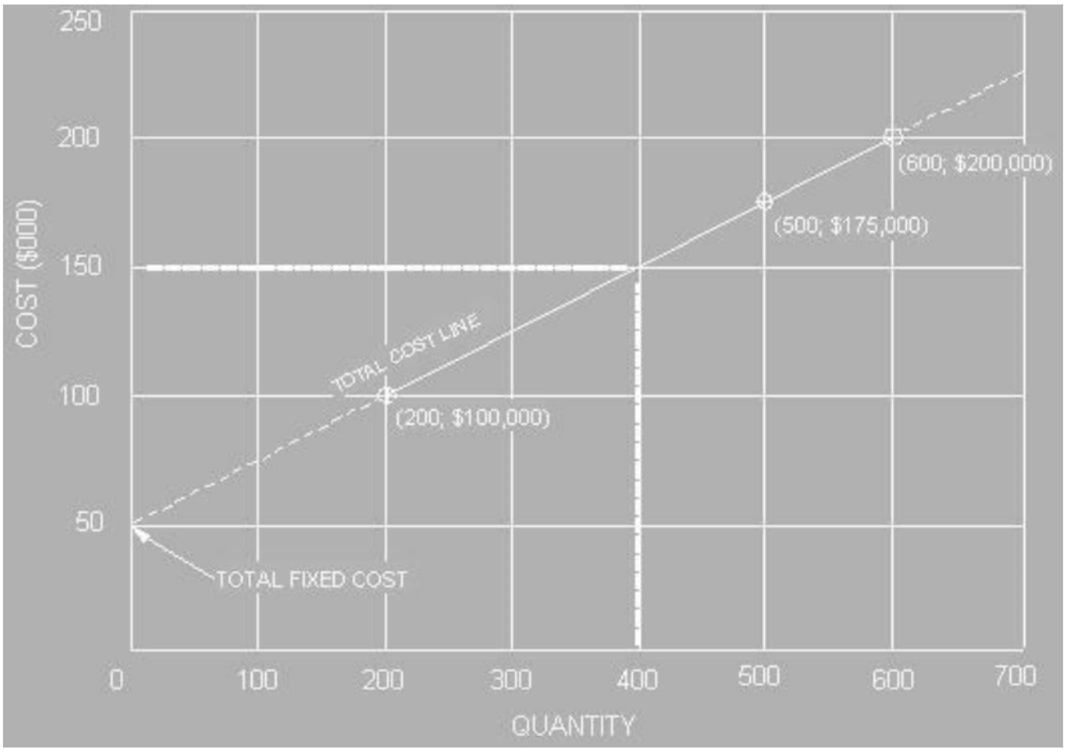
In this section of text, all data points will fall on a straight line. All that you have to do to fit a straight line is connect the data points. Most analysts use regression analysis to fit a straight line when all points do not fall on the line.

Step 4. Estimate the cost for a given volume.

Draw an imaginary vertical line from the given volume to the point where it intersects the straight line that you fit to the data points. Then move horizontally until you intersect the vertical axis. That point is the graphic estimate of the cost for the given volume of the item.
Example of Graphic Analysis. The four steps of cost-volume-profit analysis can be used to graph and analyze any cost-volume relationship. Assume that you have been asked to estimate the cost of 400 units given the following data:

Units	Cost
200	$100,000
500	$175,000
600	$200,000

Solution



The estimated cost will be $ 150.000.

3.3 BREAK EVEN ANALYSIS

Break even analysis is mainly used to explain the relationship between the cost incurred, the volume operated at and the profit earned. To compute the break even point we let

S be selling price per unit
V_u be variable cost per unit
Q be break-even quantities
F be total fixed costs

At Break even point:
Total revenue (TR) = Total Cost (TC)

Total revenue will be given by SQ while Total cost (TC) = $V_u Q + F$

At break-even point (BEP) therefore:

$$SQ = V_u Q + F$$

$$Q = \frac{F}{S - V_u}$$

$$B.E.P_{\text{(in units)}} = \frac{F}{S - V_u}$$

Illustration

Assume that you are planning to sell badges at the forthcoming Nairobi Show at Sh.9 each. The badges cost Sh.5 to produce and you incur Sh.2010 to rent a booth in the Show ground.

Required:

a) Compute the breakeven point
b) Compute the margin of safety
c) Compute the number of units that must be sold to earn a before tax profit of 20%
d) Compute the number of units that must be sold to earn an after tax profit of Sh.1640, assuming that the tax rate is 30%.

Solution

a) Break even point
BEP units = 2010/(9-5) = 500 units

BEP Sh. = 500 x 9 = 4500/-

b) Margin of safety

The margin of safety is the amount by which actual output or sales may fall short of the budget without the company incurring losses. It is a measure of the risk that the company might make a loss if it fails to achieve the target. A high margin of safety means high profit expectation even if the budget is not achieved. Margin of safety (MOS) can be computed as follows:

$$MOS = \frac{\text{Expected sales - Break even sales}}{\text{Expected sales}}$$

$$= \frac{600-500}{600} = 16.7\%$$

c) Target before tax profit (Y)

Let X be the number of units to produce

$$X = \frac{F + Y}{S - V_u}$$

$$X = \frac{2010 + 0.2\,(9X)}{9-5}$$

$$X = \frac{2010 + 1.8X}{4}$$

X = 909.09 approximately 910 units.

d) After Tax profit

Let Z be the after tax profit

$$Y = \frac{Z}{I - t}$$

Therefore

$$X = \frac{F + {}^{z}/_{1-t}}{S - V_u}$$

$$= \frac{2010 + \frac{1640}{1-0.3}}{9-5}$$

X = 1085.71

Approximately 1086 units.

3.4 C-V-P ANALYSIS – MULTIPLE PRODUCTS

The simple product CVP analysis can be extended to handle the more realistic situations where the firm produces more than one product. The objective in such a case is to produce a mix that maximises total contribution.

$$\text{Total BEP}_{\text{units}} = \frac{\text{Total fixed cost}}{\text{Average CM}}$$

$$\text{Average CM} = \sum_{t=1}^{n} (S_t - V_t)\alpha_t$$

where α_t is the sales mix of product t. S_t is the selling price of product t.
V_t is the variable cost of product t.
n is the number of units of product t sold

$BEP_{t\ \text{units}} = \alpha_t\,(\text{Total } BEP_{\text{units}})$

$BEP_{\text{tsh.}} = BEP_{t(\text{units})}\ x S_t$

Illustration

Assume that ABC Ltd produces two products, product A and B and the following budget has been prepared.

	A	B	Total
	120,000	40,000	160,000
Sales in units	Sh.	Sh.	Sh.
Sales @5/-, 10/-	600,000	400,000	100,000
Variable cost @ 4/-, 3/-	480,000	120,000	600,000
Contribution @ 1/- 7/-	120,000	280,000	400,000
Total fixed cost			300,000
Profit			100,000

Required:
a) Compute the break-even point in total and for each of the products.
b) The company proposes to change the sales mix in units to 1:1 for products A and B. Advice the Co. on whether this change is desirable.

Solution

	A	B	
Sales mix (units)	0.75	0.25	1
Sales mix (Shs)	0.60	0.40	1

Average CM $= \sum_{t=1}^{n}(S_t - V_t)\alpha_t$

= 0.75 (1) + 0.05(7)
= 2.5

Total BEP units $= \frac{\text{Total fixed cost}}{\text{Average CM}} = \frac{300000}{2.5}$

= 120,000 units

	BEP (units)	BEP(sh)	
A	120100 x 0.75 = 90,000	(90000x5)	= 450,000
B	120100 x 0.25 = 30,000	(30000 x 10)	= 300,000
	120,000		750,000

The above question can be solved by computing the BEPSh first and the using the Sales Mix in Shs.

Total BEP Sh. $= \frac{\text{Total fixed cost}}{\text{C/S sales ratio}}$

C/S ratio $= \frac{400,000}{1000,000} = 0.4$

Total BEP(sh) $= \frac{300000}{0.4} = 750,000$

			Sh.	Units
A	750000 x 0.6	=	450000	450000/5 = 90000
B	750000 x 0.4	=	300000	300000/10= 30000

	750000	120100

b) Changing sales mix in units to 1:1 ratio

The budget can be reproduced as follows:

	A	B	Total
Sales in units	80000	80000	160000
	sh	sh	sh
Sale @ 5/-, 10/-	400000	800000	1201000
V.c @ 4/-, 3/-	320100	240000	560000
Contribution	80,000	560,000	640,000
Total fixed cost			300,000
Net Profit			340,000

Sales mix in units is 80000/160000 = 0.5

Average CM = 0.5(1) + 0.5 (7) = 4

Total BEP units = $\frac{300,000}{4}$ = 75,000 units

	BEP_{units}	$BEP_{sh.}$
A (0.5 x 75000)	37500	187,500
B (0.5 x 75000)	37500	375,000
	75000	562,500

For manager of product line A, the change is good because he now breaks even at sh.187500 than on sh.450000. But for manager of product B, the change is not good because BEP has risen from sh.300000 to sh.375000.

3.5 C-V-P ANALYSIS UNDER UNCERTAINTY

A major limitation of the basic C.V.P analysis is the assumption that the unit variable cost, selling price and the fixed costs are constant and can be predicted with certainty. These factors however are variables with expected values and standard deviations that can be estimated by management.

There are various ways of dealing with uncertainty. Examples include:

- Sensitivity analysis
- Point estimate of probabilities
- Continuous probability distribution e.g. normal distribution
- Simulation analysis
- Margin of safety

3.5.1 POINT ESTIMATE OF PROBABILITIES

This approach requires a number of different values for each of the uncertain variables to be selected. These might be values that are reasonably expected to occur but usually 3 values are selected. These are:

The worst possible outcome
The most likely outcome
The best possible outcome

For each of these 3 values, a probability of occurrence will be estimated.

Illustration

Assume that a Management accountant of a Company that makes and sells product X has made the following estimate:

	Selling price Sh **Sales demand**		**Unit variable co** **Condition**		
<u>Condition</u>	<u>Unit</u>	<u>Prob.</u>		<u>Cost</u>	<u>Sh.</u>
Worst possible	45000	0.3	Best possible	3.5	0.30
Most likely	50000	0.6	Most likely	4.0	0.55
Best possible	55000	0.1	Worst possible	5.5	0.15

Fixed cost = Sh.240,000

Required:

a. Compute the expected profit
b. Compute the prob. that the company will fail to break even
c. If the Company has a profit target of Sh.60,000 what is the probability that the company will not achieve this target.

Solution

a)

E(Demand) = (45000 x 0.3) + (50000 x 0.6) + (55000 x 0.1) = 49000

E(variable cost) = (3.5 x 0.3) x (4 x 0.55) + (55 x 0.15) = Sh.4.075

E(Profit) = (10-4.075) 49000 – 240000 = <u>Sh.50325</u>

This can be worked out differently as shown below:

A **Demand**	B **Prob.**	C **Unit VC**	D **Prob.**	E **Contr**	F **Profit**	G **Joint Prob.**	(FxG) **weighted Profit**
45000	0.3	3.5	0.30	292500	52500	0.09	4725
		4.0	0.55	270000	30000	0.165	4950
		5.5	0.15	202500	(37500)	0.045	(1687.5)
50000	0.6	3.5	0.3	325000	85000	0.18	15300
		4.0	0.55	300000	60000	0.33	19800
		5.5	0.15	225000	(15000)	0.09	(1350)
55000	0.1	3.5	0.3	357500	117500	0.33	3525
		4.0	0.55	330000	90000	0.055	4950
		5.5	0.15	247500	7500	0.015	<u>112.5</u>
				Expected profit			<u>50325</u>

b) The P(Profit <0) = 0.045 + 0.09
= 0.135

Note:

This can be read from the above table

c) P(profit < 60000)

= 0.3 + 0.09 + 0.015
= 0.405

Worked example

Thunder manufacturing company produces a toxic product, 'coros' that must be sold in the month produced or else discarded. Thunder can manufacture 'coros' itself at a variable cost of Sh40 per unit or they can purchase it from an outside supplier at a cost of Sh70 per unit. Thunder can sell 'coros' at Sh80 per unit. Production levels must be set at the start of the period and cannot be changed during the period. The production process is such that at least 9,000 units must be produced during the period. Thunder management must decide whether to produce 'coros' or whether to purchase it from the outside supplier.

The possible sales of 'coros' and their probabilities are:

Demand (units)	Probability
4,000	0.4
7,000	0.5
11,000	0.1

Required:

a) Expected demand
b) Expected profit from purchasing 'coros' from an outside supplier and selling it
c) Expected profit from manufacturing and selling
d) Standard deviation of profits from purchasing and selling.
e) Standard deviation of profits from manufacturing and selling.
f) Coefficient of variation for each alternative

Solution

a) Expected demand is computed as follows:

Demand (units)	Probability	Expected demand(units)
4000	0.4	1600
7000	0.5	3500
11,000	0.1	1100
Expected demand		6200

b) The expected profit from purchasing and selling would be equal to the unit contribution times the expected quantity or

Sh (80 – 70) x 6200 = Sh62,000

c) Even though the production cost is stated as a variable cost, since a minimum of 9,000 units must be produced, the cost is really fixed up to that point because of the minimum production constraints. Units produced in excess of 9,000 could carry the variable cost of Sh40 each. The expected profit from manufacturing is:

Demand (units)	Probability	Manufacturing cost (Shs)	Profit	Expected profit (Shs)
4000	0.4	360,000	(40,000)	(16,000)
7000	0.5	360,000	200,000	100,000
11,000	0.1	440,000	440,000	44,000
				128,000

d) The standard deviation from purchasing and selling is:

$I - \bar{I}$	$(I - \bar{I})^2$ P (million)
(4,000 – 6200) Sh10	193.6
(7,000 – 6200) Sh10	32.0
(11,000 – 6200)Sh10	230.4
	456.0

∴ Standard deviation = $\sqrt{456m}$ = Sh21,354

e) The standard deviation from manufacturing and selling is

$I - \bar{I}$	$(I - \bar{I})^2$P (million)
-40,000 – 128,000	11,289.6
200,000 – 128,000	2,592.0
440,000 –128,000	9,734.4
Total	23,616.0

∴ Standard deviation =$\sqrt{23,616}$ million = Sh153,675

f) Coefficient of variation for purchasing and selling is (S/I)

i.e. $\frac{\text{Sh } 21,354}{\text{Sh } 62,000} = 0.344$

For manufacturing and selling is:

$\frac{\text{Sh } 153,675}{\text{Sh } 128,000} = 1.201$

Note:

The coefficient of variation is a measure of risk associated with each alternative.

3.5.2 CONTINUOUS PROBABILITY DISTRIBUTION (USE OF NORMAL DISTRIBUTION)

In reality the C-V-P variables might take any values in a continuous range. It could therefore be more appropriate to use a continuous probability distribution such as the normal distribution with an estimated mean and standard deviation. Estimates may be made of the expected sales volume, the expected selling prices, the expected variable cost and the expected fixed costs together with their probabilities.

It would therefore be possible to compute the expected profit and the likelihood that the company would break even or achieve a given target profit.

Illustration

Assume that the selling price of a product is estimated to be Sh.100, the variable cost Sh.60, and budgeted fixed cost is Sh.36000. The demand is normally distributed with a mean of 1000 units and a standard deviation of 90 units

Required

a. Compute the expected profit and standard deviation of profit
b. Compute the prob. that the company would not break even
c. Compute the prob. that a loss >Sh.1400 will occur

a) E(profit) = Contribution margin x E(D) - F.C

= (100-60) 1000 –36000
= Sh.4000

$\delta_{(\text{profit})} = \delta_{\text{ demand}}$ x CM = 90 x 40 =Sh.3600

b) P(profit <0)

$$z = \frac{x - u}{\delta} = \frac{0 - 4000}{3600} = -1.11$$

From the Z tables the value = 0.1335
Therefore P(profit<0) = 0.1335

c) P (profit < - 1400)

$$Z = \frac{-1400 - 4000}{3600} = -1.5$$

From the Z tables the value = 0.0668

Therefore P(profit <-1400) = 0.0668

3.6 CVP ANALYSIS AND COMPUTER APPLICATIONS

The output from a CVP model is only as good as the input. The analysis will include assumptions about sales mix, production efficiency, price loads, total fixed costs, variable costs and selling price per unit.

The CVP equation can be used to develop financial planning programs. These programs quickly calculate the effects of changes in price, costs and volume on an organisation's profits. They answer such "what-if" questions as:

i) How could a 5% increase in the sales price affect operating income?
ii) If Fast Food Co. increases its advertising budget by Sh1 million, how many hamburgers must it sell to cover the increase in fixed expenses?
iii) If the campus bookstore extends its hours, how much additional revenue must it earn to cover the increased operating expense?
iv) If variable production costs are reduced by 7%, how many units of product must be sold to earn Sh200,000 operating profit?

Such programs vary in complexity. Some simple programs can include only those variables discussed while other more complicated ones can include an organisation's complete budget.

Many firms use interactive programs of basic CVP equation on their microcomputers to analyse data they have collected and entered. These interactive capabilities allow managers to enter and change their inputs easily and also make the analysis of the financial effects of various alternatives simpler.

The computers' speed and accuracy in providing information from entered data improve the speed and accuracy with which the manager can select the most profitable actions.

Sensitivity analysis is one approach for coping with changes in the values of the variables. It focuses on how a result will be changed if the original estimates or the underlying assumptions change.

The widespread use of spreadsheet packages which do not require programming expertise, has enabled management accountants to develop CVP computerised models. The impact of alternative revised plans is quickly identified and changes only implemented when it is apparent that the original estimates are incorrect.

3.7 RELEVANT COSTS FOR NON-ROUTINE DECISIONS

A relevant cost is a cost that is appropriate to a specific management decision. To be relevant, a cost must be:

1. Future cost – A decision is usually about the future & management not what has already been done. A cost that has already been incurred is therefore irrelevant to any decision being made now e.g. costs already paid or costs committed by decisions made in the past.
2. Relevant costs are cash flows – It is assumed that decisions are taken which would maximize the satisfaction of the company owners & therefore such decisions must not be ignored. Such costs include depreciation, notional rent or notional interest or absorbed O/H.
3. Relevant costs arise as a direct consequence of making a decision. It should be an incremental cost i.e. the difference between the cost with the decision & the cost without the decision.

Assumptions

The key assumptions made in relevant costing are:

1. The cost behaviour is known.
2. The amount of fixed costs, unit variable costs, selling prices and sales demand are known with certainty.
3. The objective of the decision maker in the short-term is to maximize satisfaction which can be defined as maximization of short-term profit.
4. The information on which the decision is based is complete and reliable.

There are various types of decisions that can be considered in this section, Examples include:

a. Make or Buy decisions
b. Shut down problems
c. Extra shift decisions
d. Joint cost decisions

3.7.1 MAKE OR BUY DECISIONS (NO LIMITING FACTORS)

The choice between making or buying a given component is one which is likely to face all businesses at some time. It is often one of the most important decisions for management for the critical effect on profits that may ensue. The choice is critical, too, for the management accountant who provides the cost data on which the decision is ultimately based.

A make or buy problem involves a decision by an organisation about whether it should make a product or carry out an activity with its own internal resources or whether it should pay another organisation to carry out the activity. The make option gives management more direct control over the work, but the buy option may have benefits in that the external organisation has expertise and special skills in the work making it cheaper.

There are certain situations where the make or buy decision is not really a choice at all. There can be no alternative to making, where product design is confidential or the methods of processing are kept secret. On the other hand, patents held by suppliers may preclude the use of certain techniques and then there is no choice other than buying or going without. The supplier who has developed a special expertise or who uses highly specialized equipment may produce better-quality work which suggests buying rather than making. In other cases, the special qualities demanded in the product may not be available outside and so making becomes necessary.

Where technical considerations do not influence the make or buy decision, the choice becomes one of selecting the least-cost alternative in each decision situation. Comparative cost data are necessary, therefore, to determine whether it is cheaper to make or to buy. In general this requires a comparison of the respective marginal costs or, in some cases, the incremental costs of each alternative. Incremental costs are relevant in decisions which include capacity changes. For example, a certain component has always been bought out because the plant and equipment for its manufacture has not been installed in the factory. When considering the alternative to buying, the cost of making comprises all the incremental costs (including additional fixed expenditure) arising from the decision. The incremental cost also includes the opportunity cost of the investment in capital equipment, that is, the expected return from an alternative investment opportunity. A decision to buy a part which has previously been manufactured may release capacity for other uses or for disposal so that the incremental cost of the decision also includes the relevant fixed-cost savings.

Illustration

Assume that ABC Ltd makes four components with the following information:

	W	X	Y	Z
Production (units)	1000	2010	4000	3000
Unit marginal costs				
Direct material	4	5	2	4
Direct labour	8	9	4	6
Variable O/H	2	3	1	2
	14	17	7	12

Attribute Fixed Cost			sub contractor price	
		Sh.		Sh.
TO	W	1000	W	12
	X	5000	X	21
	Y	6000	Y	10
	Z	8000	Z	14

Committed Fixed Costs are Sh.30000

Required

Advice the company on the components to buy or make if any.

Solution

	W	X	Y	Z
Cost of buying per unit	12	21	10	14
Variable Cost of making	14	17	7	12
Extra variable cost of buying	(2)	4	3	2
No. of units	1 000	2 000	4000	3000
Total extra costs VC of buying	(2010)	8000	12010	6000
Less attributable FC	(1000)	(5000)	(6000)	(8000)
Net extra costs of buying	(3000)	3000	6000	(2010)

The decision is to Buy W and Z and Make X and Y

3.7.2 MAKE OR BUY DECISIONS UNDER LIMITING FACTORS.

One reason for buying products/services from another organisation is the scarcity of resources, so that the company may be unable to make all its components. In such a case the company should combine internal resources with buying externally to increase profitability. In situations where a company must sub-contract work to make up short fall in its in-house capability, then its cost will be minimized where the marginal cost of buying is least for each unit of scarce resource saved by buying externally.

Illustration

Assume that ABC Ltd makes four components with the following information:

	W	X	Y	Z
Production (units)	1000	2010	4000	3000
Unit marginal costs				
Direct material	4	5	2	4
Direct labour	8	9	4	6
Variable O/H	2	3	1	2
	14	17	7	12

Attribute Fixed Cost			sub contractor price	
		Sh.		**Sh.**
TO	W	1000	W	16
	X	5000	X	21
	Y	6000	Y	10
	Z	8000	Z	18

Committed Fixed Costs are Sh.30000

Assume that machine hours per unit required to produce the components are:

	Machine Hours
W	4
X	5
Y	3
Z	6

The total machine hours available is 27000 hours during the budget period.

Required:

Advice the company on which products to make and the ones to buy externally.

Solution

Required machine hours

W	4X1000 =	4000
X	5X2010 =	10000
Y	3X4000 =	12010
Z	6X3000 =	18000
I		44000
Available hours		27000
Shortfall		17000

Machine hours is therefore a limited resource

	W	X	Y	Z
Cost of buying per unit	16	21	10	18
Cost of making VC	14	17	7	12
Extra variable cost of buying	2	4	3	6
No. of units	1000	2010	4000	3000
Total extra V. Cost of buying	2010	8000	12010	18000
Less attributable F C	(1000)	(5000)	(6000)	(8000)
Net extra cost of buying	1000	3000	6000	10000
Divide the no. of mhrs saved	4000	10000	12010	18000
Net extra costs of buying per Machine hours saved	0.25	0.30	0.5	0.56
Priority for buying	1	2	3	4
Priority for making	4	3	2	1

3.7.3 ABANDONMENT DECISIONS

From time to time management will be faced with the problem of deciding to abandon an unprofitable activity. This is really a least-cost alternative decision and so made on the criterion of relative marginal costs.

Ceasing Production of Certain Products

It is sometimes suggested that, where a given product is apparently making a loss, manufacture and/or marketing of this product should cease, to improve the company's overall profit performance.

KENBAR CYCLES LIMITED
PROFIT AND LOSS STATEMENT FOR YEAR ENDED 31.12.19..

	Model A16	Model E35	Model N40	Total
	£'000	£'000	£'000	£'000
Direct materials	110	100	150	360
Direct labour	50	40	80	170
Variable overhead	65	60	100	225
Fixed overhead	45	120	220	385
TOTAL COSTS	270	320	550	1,140
Profit/(loss)	45	65	(50)	60
SALES VALUE	315	385	500	1,200

Model N40 is incurring losses of £50,000 per annum, which is ten per cent of its sales value. The implication of this profit and loss statement is that the withdrawal of Model N40 from the market will avoid losing £50,000 and (by inference) raise profits to £110,000. This is faulty reasoning, but a risk which is inherent in the total cost form of presentation. The marginal presentation of the year's results would avoid the risk and give a more meaningful report.

KENBAR CYCLES LIMITED

PROFIT AND LOSS STATEMENT FOR YEAR ENDED 31.12.19..

	Model A16	Model E35	Model N40	Total
	£'000	£'000	£'000	£'000
Sales value	315	385	500	1,200
Marginal cost	225	200	330	755
Contribution	90	185	170	445
Fixed overhead				385
Profit/Loss				60

Since Model N40 yields an annual contribution of £170,000, the abandonment of this product will lose this contribution and so turn the overall profit of £60,000 into a *loss* of £110,000. (The contribution from A16 and E35 is £275,000 towards the fixed costs of £385,000). The marginal presentation shows that it is better to continue production of Model N40 rather than lose its contribution. As a general proposition it can be postulated that it is more profitable to continue marketing a product which yields some contribution rather than abandon it. (If possible, it would be better still to replace it with another product having a higher P/V ratio).

3.7.4 TEMPORARY CLOSURE OF FACTORY OR DEPARTMENT

Here there is a similar situation to that of discontinuance of a product such as Model N40. A factory which is expected to earn some contribution should continue in operation rather than be shut down. However, if the factory is part of a group, the decision is quite different when the output from the closed factory is not lost but transferred to another factory in the group with spare capacity. For example, a temporary fall in the sales volume of a company's products may result in either of two factories being capable of satisfying the expected demand. In this situation the company can optimise its profits by concentrating production in that factory which has the lowest marginal costs. In reaching a decision, consideration should be given to predictable cost changes generated by the decision: such as additional distribution costs, care and maintenance of the closed premises, restarting costs, and any fixed cost savings such as salaries in the closed factory.

3.7.5 PERMANENT ABANDONMENT OF PREMISES

A company may find it more profitable to concentrate its output in some factories by closing down others. The decision, in this instance, is made on the basis of incremental costs and will depend on that combination of resources which yields the greater overall group profit. The permanent closure of a factory saves fixed cost expenditure and also frees capital (by the sale of assets) for alternative investment, as well as providing the opportunity to take advantage of low marginal costs elsewhere. It is possible that the sale of freehold land and buildings could provide considerable investment funds free of interest which would make the abandonment particularly attractive. This has been demonstrated effectively by **asset stripping** following a successful takeover.

There may be a high social cost in a factory closure which is difficult to evaluate, but in any case it will be borne by the whole community rather than the individual manufacturer. A growing awareness of the social consequences which follow factory closures may persuade politicians that the cost to the community represents a hidden subsidy to the profits of an individual company. A tax or other deterrent for such cases in the future would be an additional cost of abandonment decisions and so make it relatively less profitable to close a factory.

3.7.6 EXTRA SHIFT DECISION

These decisions are concerned with whether or not a company should work for 8 hrs, 16hrs, or 24 hrs a day or week day's only or weekends also. The factors to consider are:

i. Whether the work force would be willing to work extra shifts & if so what overtime or shift premium they would accept.
ii. Whether extra hours have to be worked just to remain competitive
iii. Whether extra hours would resort in extra revenue or whether there would be in demand pattern from customers.

Illustration

XYZ currently operates a single production shift which incurs costs and earns revenue stated below:

		£	
Sales (10000 units)		360000	
Direct material	120100		
Direct labour	100000		
Variable O/Hs	20100		(240000)
Contribution Fixed		120100	
Cost		(90000)	
Profit		30000	
Profit margin		8.36%	

Sales demand exists for an extra 6000 units which can be made in a 2nd shift at current selling price. The labour in the 2 nd shift will be paid at time & ¼. Additional fixed cost of £10000 will be incurred but due to the increase in purchase of materials a quantity discount of 5% will be given on all materials purchased.

Required:

Advice the company on whether to operate the 2nd shift.

Solution: Analysis of Second shift

			£
Sales (6000x36)			216000
Direct labour (1.25x10000)		125,000	
Variable O/Hs (2x6000)		12010	
Direct material			
Purchase 12x6500	72010		
Less discount 5% x 192010	9600	62400	
Additional Fixed Cost		10000	209400
Incremental total			6600
Profit margin			3.1%

Decision

Operate the second shift since it results in incremental profits.

3.7.7 JOINT PRODUCT DECISIONS

When a manufacturing Company carries out a process operation in which 2 or more joint products are made from a common process a number of decision problems can arise. These are-

(1) If the joint product can be sold at existing condition at the split-off point or after further separate processing, then a decision should be made on whether to process further.
(2) If extra demand for a joint product exists and not others then it is necessary to know whether it is worth making more output of the joint product so as to make a profit on one and dispose off the other.
(3) If it is possible to change the input so as to change the product mix, then product mix decisions should be made.

Joint Product further processing decisions

In these decisions the relevant costs are the additional costs of further processing, which should be compared with the incremental revenue of further processing. The joint costs incurred before the split-off points are irreverent.

Illustration.

ABC Ltd produces product A&B from the same process. Joint processing costs of $150,000 are incurred up to the split off point where 100,000 units of A and 50,000 units of B are produced. The selling prices for products A and B at the split-off point are $1.25 per unit and $2.00 per unit respectively.
Units of A can be processed further to produce 60,000 units of A^+ which will incur a fixed cost $20,000 and variable cost of $0.3 per unit.

Required

Advice the Company whether to sell product A or product A+

Solution

Incremental revenue $3.25x60,000 -1.25x100,000		$70,000
Further processing costs		
Fixed cost	20,000	
variable cost $0.3 x 100,000	30,000	50,000
Incremental profit from further processing		20,000

Decision

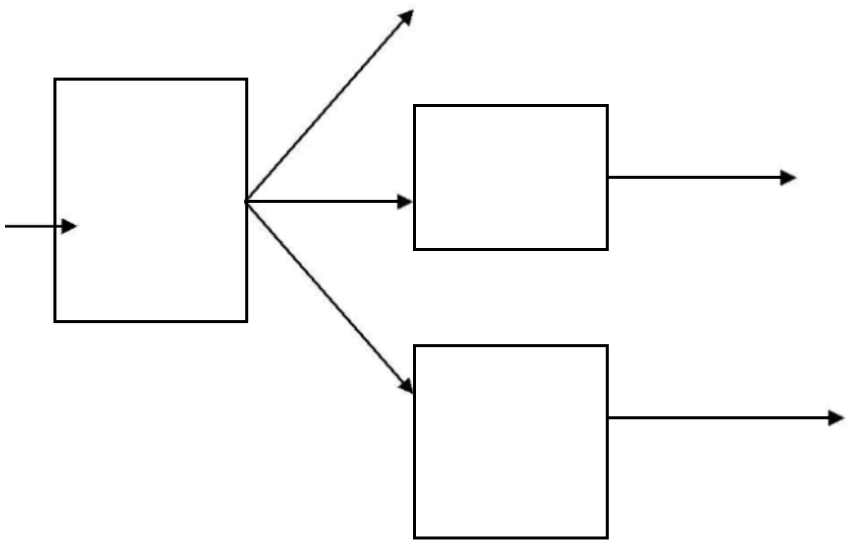
Process further since incremental profit is positive

Joint product Break-even point of extra Output.

If more output of one joint product is required it would require production of additional units of other joint products. The incremental costs of extra output should include the costs of producing the non-required joint product unless there is revenue generated by disposing of those products.

Illustration

ABC Ltd manufactures 3 products in a series of process as shown below,
Raw materials



Cost	Process 1	process 2	process 3
Raw materials	$40,000	-	-
Variable overheads	16,000	3,000	5,000
Fixed overheads	10,000	7000	10,000

Selling prices.

A ----- $3		BX	------$12
B ------ $10		CX	------$10
C ------ $6			

Assume all the fixed costs of process 2 $ 3 are avoidable

Required

(a) Determine whether the Co. is maximising its profit by further processing product B to BX and C to CX.

(b) Calculate the break even selling price if the Co. was to receive an order for an extra 1000Kgs of product CX, which would incur extra delivery costs of $1800

i. Assume that the extra output of A $ B would be disposed of at scrap value which covers their disposal cost.

ii. Assume that there would be extra demand at the current prices for product A$B.

Solution

Further processing of Product B to BX and C to CX

Incremental selling price- B= 12- 10 =$2
C= 10- 6 =$4
Therefore total sales increase = 2 x 4000 = $8000
4 x 5000 = $20,000

	B	C
Incremental revenue	$8,000	20,000
Incremental cost		
Variable costs	3,000	5,000
Fixed costs	7,000	10,000
	10,000	15,000
Incremental profit/loss	(2,000)	5,000

Decision

The Co. is making a good decision to further process C to CX since incremental profit is positive but it is not making a good decision to further process product B to BX because incremental profit is negative.

(b) (i) Assumption 1

Incremental units = $\frac{1000}{5000}$ x100=20%

Extra Variable Cost of 1000kg of CX.

process 1 –material (20% x 40000)	$8000
Variable overheads(20%x16000)	3200
Process 2 –variable overheads (20%x 5000)	1000
	12200
Extra fixed costs of delivery	1800
Total extra costs	14000

Break even price $=\frac{14000}{1000}$ = 14 per kg.

b) (ii) Assumption 2.

Extra cost of A&B

Total extra costs (as in (i) above)		14,000
Less revenue of:		
A: 200kg @$3	600	
B: 8000kg @ $10	8000	8600
Net extra costs		5400

Break-even price $= \frac{5400}{1000}$ = $5.40 per Kg.

Product mix decision

A manufacturing Company may be faced with a decision about whether to change the product mix in its process so as to produce a greater proportion of one product and less of another e.g. if a process produces product X and Y in the ratio of 2.1 it may be possible to change the ratio to 3:2 but such a decision requires consideration of the relevant costs and relevant revenue of the change.

Illustration

XYZ Ltd produces 2 joint products P&Q in the ratio of 2:1.After the split off point the products can be sold for industrial use or taken to mixing plant for blending and refining. (the 2nd option usually followed). The following information is given for a specific week:

Sales	**P**	**Q**
	2010 litres	1000litres
Price per litre	\$35	\$60
Sales revenue	\$70,000	\$60,000
Joint process cost	\$30,000	\$15,000
Blending & refining	\$25,000	\$25000
Other separable cost	\$5,000	\$1000
	60,000	41,000
Profits	\$10,000	\$19000

Joint process costs (which are allocated on volume) are 75% fixed and 25%variable, whereas the mixing plants costs are 40% fixed and 60% variable. There are only 40 hours available in the mixing plant (usually 30hrs are taken up to processing of product P&Q equally and 10 hrs are used for other work that generates a contribution of \$2010 per hour)

It has been suggested that it might be possible to change the mix of the joint process to 3:2 for P&Q respectively at a cost of \$5 for each additional litre of Q produced by the process.

Required

Advice the Co. on whether to change the mix

Solution

Proposed mix
P = 3/5 x 3000 = 1800 litres
Q =2/5 x 3000 = 1200 litres

Cost Benefit Analysis

Incremental revenue of Q 200@ \$60			12010
loss of revenue of P 200 @\$35			7000
Net incremental revenue			5,000
Incremental Costs			
Joint processing costs	200@ \$ 5		1000
Blending and refining			
Extra costs of Q	25000/1000*.6*200	3000	
Savings of P	25000/2010*.6*200	(1500)	1500
Other separable costs	200(1-2.5)	(300)	
Opportunity costs	(3-1.5)x2010	3000	(5700)
Net Incremental profit			(200)

Decision

The Company should not change the mix because it results in an incremental loss of \$200.

REIFORCING QUESTIONS

QUESTION ONE

Sniwe plc intends to launch a commemorative product for the 2014 Olympic games onto the UK market commencing 1 August 1990. The product will have variable costs of £16 per unit. Production capacity available for the product is sufficient for 2,000 units per annum. Sniwe plc has made a policy decision to produce to the maximum available capacity during the year to 31 July 1991. Demand for the product during the year 31 July 1991 is expected to be price dependant as follows:

Selling price per unit	Annual sales
£	units
20	2,000
30	1,600
40	1,200
50	1,100
60	1,000
70	700
80	400

It is anticipated that in the year to 31 July 2014, the availability of similar competitor products will lead to a market price of £40 per unit for the product during that year.
During the year to 31 July 2014, Sniwe plc intend to produce only at the activity level required to enable them to satisfy demand with stocks being run down to zero if possible. The policy is intended as a precaution against a sudden collapse of the market for the product by 31 July 2014.

Required:

(ignoring tax and the time value of money)

a) Determine the launch price at 1 August 1990 which will maximise the net benefit to Sniwe plc during the two year period to 31 July 2014 where the net demand potential for the year to 31 July 2014 is estimated as (i) 3,600 units and (ii) 1,000 units.

b) Identify which of the launch strategies detailed in (a)(i) and (a) (ii) above will result in unsold stock remaining at 31 July 2014. Advise management of the minimum price at which such unsold stock should be sold in order to alter the initial launch price strategy which will maximise the net benefit to Sniwe plc over the life of the product.

c) Comment on any other factors which might influence the initial launch price strategy where the demand in the year to 31 July 2014 is estimated at 1,000 units.

QUESTION TWO

A sports good manufacturer in conjunction with a software house, is considering the launch of a new sporting simulator based on video tapes linked to a personal computer enabling much greater realism to be achieved. Two proposals are being considered. Both use the same production facilities and as these are limited, only one product can be launched .
The following date are the best estimates the firm has been able to obtain:

	Foot ball simulator	Cricket simulator
Annual volume(units)	40,000	30,000
Selling price	£130 per unit	£200 per unit
Variable production costs	£80 per unit	£100 per unit
Fixed production costs	£600,000	£600,000
Fixed selling and Administrative costs	£450,000	£1,350,000

The higher selling and administrative costs for the cricket simulator reflect the additional advertising and promotion costs expected to be necessary to sell the more expensive cricket system.

The firm has a minimum target of £200,000 profit per year for the new products. The management recognises the uncertainty in the above estimates and wishes to explore the sensitivity of the profit on each product to changes in the value of the variables (volume, price, variable cost per unit, fixed costs).

Required

a) To calculate the expected profit from each product;
b) To calculate the critical value for each variable (ie the value at which the firm will earn £200,000), assuming that all other variables are expected (express this as an absolute value and as a percentage change from the expected value.);
c) To discuss the factors which should be considered in making a choice between the two products.

QUESTION THREE

Multiple CVP

A company sells two products A and B with contribution margin ratios of 40 and 30 per cent and selling prices of sh.5 and sh.2.50 a unit. Fixed costs amount to sh.72,000 a month. Monthly sales average 30,000 units of product and 40,000 units of product B.

Required:

(a) (i) Assuming that three units of product A are sold for every four units of product B, calculate the sales volume necessary to breakeven, in shillings and in units.
(ii) Calculate the margin of safety in sales shillings
(b) If the company spends an additional sh.9,700 on advertising, sales of product A can be increased to 40,000 units a month. Sales of product B will fall to 32,000 units a month if this is done. Should this proposal be accepted?
c) Recalculate the breakeven point in shillings based on the figures in (b)
d) State the condition that would have to hold true for the company to earn a zero profit at the breakeven volume you calculated in (c)

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

LESSON FOUR

INVENTORY CONTROL AND QUEUING THEORY

OBJECTIVES
Examination in depth of Inventory Control and Planning and control techniques

INSTRUCTIONS

1. Read study text below and Chapter 25 of Management and Cost Accounting, by Colin Drury 5th Edition

2. Attempt the reinforcing questions at the end of the lesson under examination conditions

3. Compare your answers with those given in Lesson 10

CONTENTS

1.1 INVENTORY CONTROL

Introduction

The activities of a business during a financial year combine investment projects in progress with new projects commencing and others terminating within the year. it would appear reasonable to presume, therefore, that business financial reports are presented in the cash-flow mode used to appraise investments, to facilitate comparison of actual with planned cash flows.

Some businesses do make such comparisons as part of their retrospective monitoring of investment decisions, but there is no obligation to do so. Cash-flow accounting, as it is called, has its supporters, but its introduction is frustrated by statutory and non-statutory regulations.

The Companies Act requires limited companies to produce profit and loss accounts and balance sheets in prescribed form. The Inland Revenue assumes that taxable profit has been computed by applying recognized accounting principles. The Accounting Standards Committee recommends the application of standard practices in the measurement of profit and portrayal of a company's financial position in its balance sheet. More compellingly, profit and loss reporting is compatible with the investors' objectives of stable and growing earnings.

Profit is measured conventionally by setting against the sales revenue for a period the costs expired in earning that revenue. That is, sales are *matched* against their relevant costs. Profit is therefore more evenly reported than it would be if all cash receipts and payments, capital and revenue, were fully reflected in the accounts of the period in which they are received and paid.

The management accountant also adopts the *matching principle* when preparing control information in both actual and budgeted form, and also ascertains full product cost as a starting point for setting selling prices.

This outlines the systems and methods used to control the flow of resources through production and service cost centres, for their eventual inclusion in product and period costs.

MATERIAL CONTROL

It is said that "any fool can sell"—it is buying at the right price that is more critical to the achievement of a satisfactory return on capital employed. Buying price is important of course, but buying the right materials, are equally important if production targets are to be achieved and investment in inventories to be minimized.

What to order

This is governed by product specifications, but an efficient buyer will always have his ear to the ground to discover new and substitute materials and components of advantageous quality and price. Other economies can be realized by reducing the variety of materials purchased by *standardization*, e.g. reducing the variety of colours of paint stocked, or by introducing value analysis into the decision process.

Value analysis

Is a formalized technique involving a rigorous analysis of products at the design stage or at any time during the saleable lives, to determine their value characteristics. These are the attributes that a customer looks for in a product and include its use value (functional qualities), appeal value (colour, style etc.) and second-hand value (e.g. trade-in-price). The object of value analysis is to build into the product the optimum of desired value at minimum cost, by introducing the most up-to-date designs, materials and methods of manufacture. No more value need be built into the product than is desired by the customer. For example, moulded plastic bumper bars are now fitted to many cars, because they are cheaper and equally as functional as chromium-plated steel ones.

How much to order

Supposing the estimated annual usage of a component by Harambee Agricultural Machinery Ltd is 20,000 units. Usage is even throughout the year and only one order per annum is placed with the supplier. Because only one delivery is made, average stock will be high, i.e. 20,000 | 2 = 10,000 and consequently stockholding costs will be very high. On the other hand, the costs of ordering will be negligible. If two orders are placed there will be less in stock (i.e. average 5,000), which will reduce holding costs, but ordering costs will increase. Thus, the higher the number of orders placed, the lower are stockholding costs, but the higher are ordering costs.

Stockholding costs include interest on the capital invested in stocks, storage, insurance, rates, security, building maintenance, heating, etc. Ordering costs include buying-department staff costs, receiving and handling.

Assuming that the cost of each Harambee component is £10, that holding cost is 10% of stock value and the cost of placing an order is £1, the total annual cost of stockholding and ordering when different numbers of orders are placed, is as follows:

Number of orders	4	20	50	100	200	400
Size of order	5,000	1,000	400	200	100	50
Average Stock (50% order)	2,500	500	200	100	50	25
Holding cost	£2,500	£500	£200	£100	£50	£25
Ordering cost (£1 per order)	£4	£20	£50	£100	£200	£400
Total Annual Cost	£2,504	£520	£250	£200	£250	£425

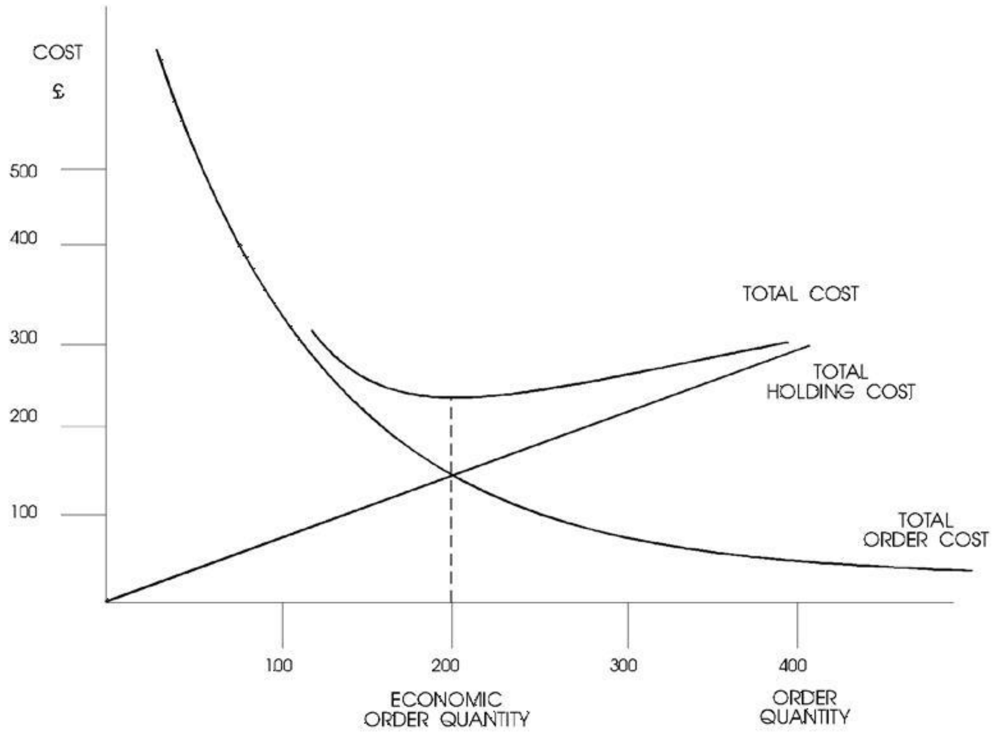


Figure 6 Economic Order Quantity

Placing 100 orders a year results in the lowest of ordering and holding cost of £200, therefore the economic order quantity is 200 units.
The same information is graphed in *Figure 1* above, showing that the *economic order quantity* (EOQ) is the point where ordering and holding costs are equal, and total £200.

As costs of ordering and holding stock are equal at the EOQ point, we can build a simple mathematical model to solve the problem, as follows:

$$\frac{Q}{2} \times H = \frac{A}{Q} \times P$$

Where Q = EOQ
H = holding cost per unit
A = annual demand
P = cost of placing an order

$$\textit{finally } Q = \sqrt{\frac{2AP}{H}}$$

Using the data in the previous example:

$$EOQ = \sqrt{\frac{2 \times 20{,}000 \times 1}{1}}$$

$$= \sqrt{40{,}000}$$
$$= 200$$

Although the model assumes that holding and ordering costs are fixed, this simplification is acceptable given a relatively unchanging level of production activity. In addition, because the total cost curve in the *Figure 1* is relatively flat either side of the EOQ, minor errors and approximation in the variables used in the calculation may not affect the end result significantly.

Practical constraints on the use of the model include restrictions on the available storage space, the availability of quantity discounts (though the model can be modified in this respect), the seasonal nature of supplies, the shelf-life of products and delivery schedules imposed by suppliers.

When to order

If deliveries from suppliers normally take two weeks to arrive, then replenishment orders should be placed with them when the level of stocks represents two weeks' supply. For example, if usage is 200 units a week, an order (the EOQ) will be placed when the stock level falls to 400 units. Figure 6(a) illustrates that, with *certain* knowledge of usage and lead time, delivery takes place just as stock is exhausted.

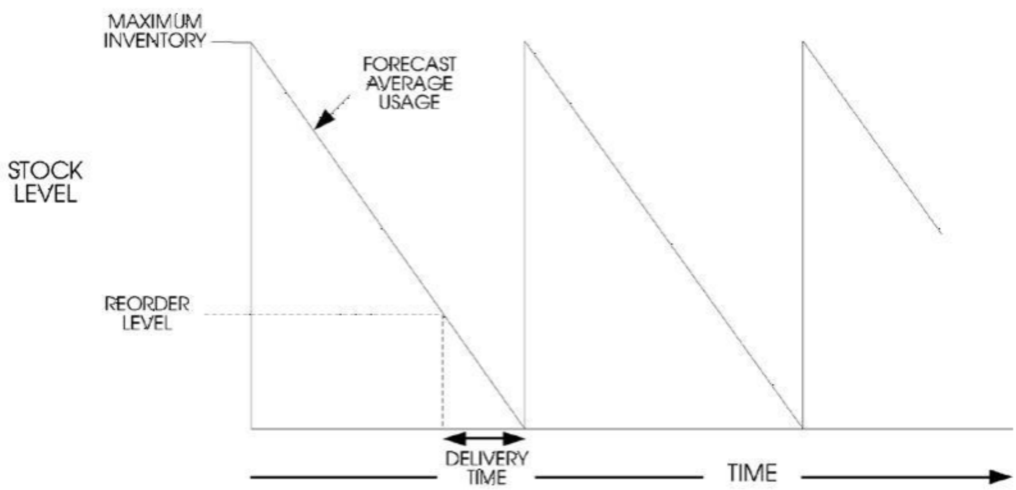


Figure 7(a) Stock levels when usage and replenishment times are known

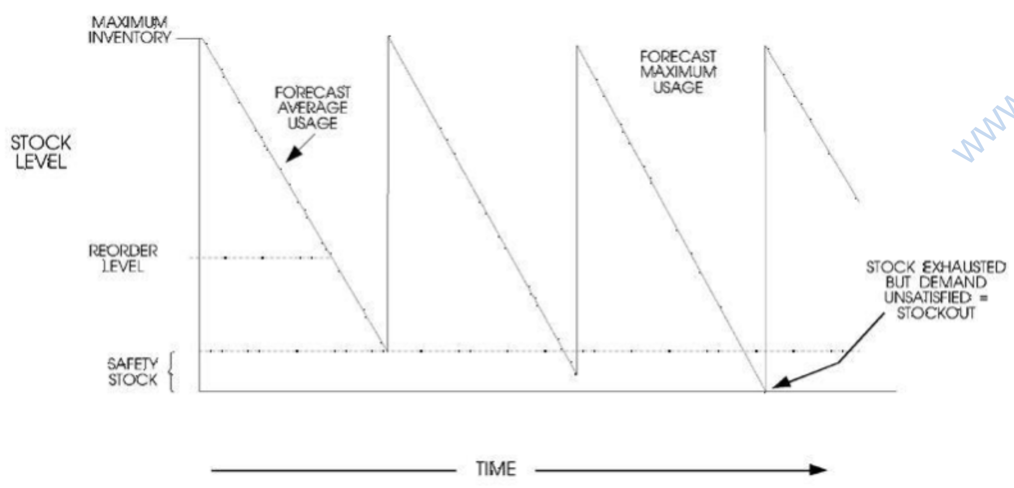


Figure 2(b) Stock levels when usage and replenishment are uncertain

Lead times and usage may not be stable and provision against running out of stock the becomes necessary *(Figure 2 (b))*. *Safety stocks* have a cost, however, and this has to be balanced against the cost of running out of stock. `Stock outs' may cause loss of customers and the probability of this happening at various levels of safety stock must be estimated. The point at which the cost of carrying safety stocks plus the cost of `stock outs' is lowest, indicates the safety stock level. Notice that uncertainty causes the reorder level to be at a higher level to include the required safety stock.

Controlling material flow

Figure 3 outlines the progressive stages in purchasing, issuing and recording materials in a manufacturing concern. An efficient system of documenting and recording is vitally necessary, not only for accounting purposes, but to ensure that the right materials arrive at the right place at the right time.

The *purchase requisition* submitted to the buyer may be triggered automatically if the system is computerised, by a message from the stores ledger that the reorder level has been reached. Other requests to purchase may be raised by the production planning department for new product materials not yet carried in stock, and also by any departmental head for supplies and equipment of any kind.

The buyer, ideally after making *enquiries* of several suppliers, sends a *purchase order*, and eventually the material is received, checked by the good-inwards department as to quality and quantity, and is detailed on a *goods-received note* (GRN). One copy of the GRN goes to the buyer to write off the outstanding order record; one to the accounts department for checking against the order and invoice—the latter authorising payment to the supplier; and one to the stores department with the materials.

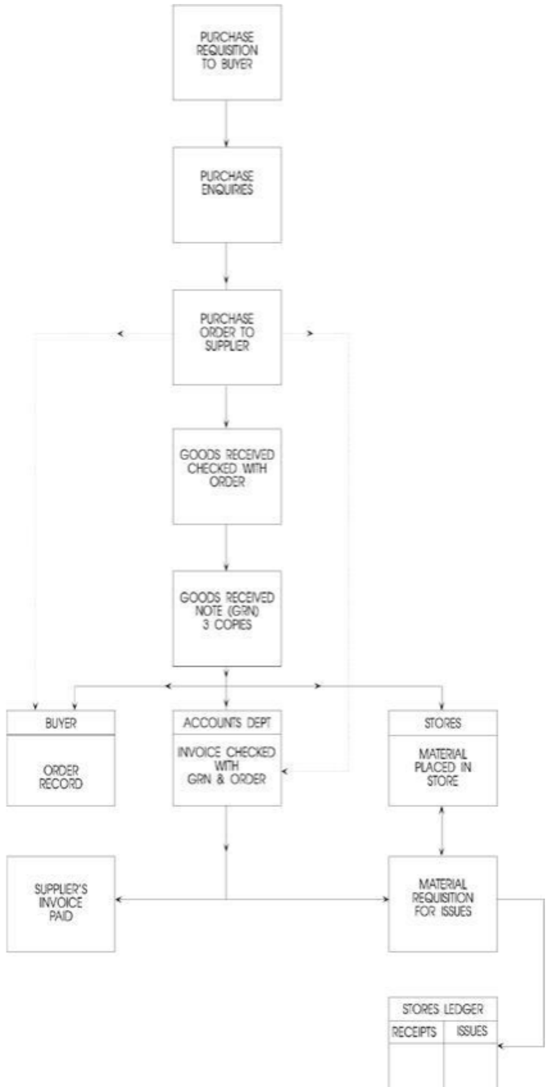


Figure 3 Procedure of material acquisition, recording and issue

A *stores record* is maintained into which the quantity and value of materials received is entered. Issues of materials to production are made by authorised materials requisitions which are also entered into the stores ledger to keep that record up to date continuously, and also into the appropriate job or process const record.

As already indicated, all the above procedures may be integrated into a computerised stock record which can provide information at the press of a button to the storekeeper, buyer, production planner, financial manager or any other person authorized to key into it. For example, information on slow-moving stock items can be obtained automatically and without delay.

Material storage

Sophisticated mathematical models to control economic buying, and systems control the flow of material may all be for nought if the obvious—efficient storekeeping—ignored. Good practice in this respect implies:

- the employment of a well-trained stores staff
- use of the most efficient equipment—for storage and handling
- easy access to items—stored in logical order
- siting of stores convenient to users
- security against theft and fire
- protection against deterioration
- a system of continuously checking physical with recorded stocks.

FURTHER CONSIDERATIONS OF INVENTORY CONTROL DECISIONS

INTRODUCTION

Factories, workshops, engineering departments handle raw materials used in the manufacture of products. The main objectives in handling these materials are:

a. Maximum customer's service
b. Minimum possible investment on materials, handling costs etc.
c. Avoid shortages as far as possible so that production is not stopped or customer's goodwill is not lost.

Some of these objectives are basically in conflict and require a scientific approach to get an optimal solution in order to earn maximum profit for a given investment. Inventory control is the study involving `Material Management' and the associated costs in such a way that the total cost is kept minimum for a given investment.

INVENTORY PLANNING AND CONTROL

The major goal of "inventory control" is to discover and maintain the optimum level of investment in all types of inventories, from raw materials and supplies to finished goods that helps to maximize long-run profits.

Two limits must be imposed in controlling inventory levels, because there are two danger points that management usually wants to avoid. They are:

i. That inadequate inventories, disrupts production and may lose sales.
ii. That excessive inventories, introduces unnecessary carrying costs and obsolescence risks.

MOTIVES FOR HOLDING INVENTORIES

If production and delivery of goods were instantaneous, there would be no need for inventories. However in reality, the manufacturing and purchasing processes do not function quickly enough to avoid the need for having inventories. Hence inventories must be maintained so that he or she does not turn to another source of supply. In turn, production operations cannot flow smoothly without having inventories of direct materials, work-in-progress, supplies, etc. In other words inventories must be viewed as cushions:

i. To absorb planning errors and unforeseen fluctuations in supply and demand. (i.e. precautionary motive).

ii. To facilitate smooth production and marketing operations.
Note: The fundamental questions to which answers are required are:

 a. How much should the company order (when stocks are re-ordered)?
 b. When should the company reorder?
 c. How much should the company produce, and when (if internal production is involved).

Accordingly regardless of their complexity, all inventory planning models focus on the twin problem of size and timing.

1. CHOOSING ORDER QUANTITY (SIZE—PROBLEM)

The objective of inventory decisions is usually to minimize total inventory costs to the company. Costs are ascribed to all elements which are of interest in reaching its inventory decisions (e.g. purchasing costs, stock out costs etc.), and solutions, are derived based on these costs.

Several inventory planning models exist. These models can be classified into two basic classified into two basic classifications:

i. Deterministic Models:—whereby all parameters are known with certainty, e.g. lead-time, annual demand, etc.

ii. Stochastic Models:—in which parameters (particularly demand and lead time) are not known with certainty, but follow known probability distributions (i.e. risks)

A. THE DETERMINISTIC MODELS

1. THE BASIC EOQ MODEL

This is the most simple of all the models discussed. In addition to the general assumptions which relate to all deterministic models (i.e. certainty of all parameters) it is further assumed that:

a. Demand is continuous, and constant over time.
b. That suppliers lead time is zero i.e. stocks are delivered immediately on the day the order is made.
c. That stock-outs are not allowed.
d. There are *no* bulk quantity discounts.
e. Holding costs per unit, ordering costs per order and costs per unit are constant.

Relevant costs of basic EOQ model

The relevant costs that should be considered when determining optimum inventory levels can be classified into two categories:
i. Ordering costs.
ii. Holding (Carrying) Costs.
i.e. TC = Ordering Costs + Holding Costs

i. Ordering Costs
These are incurred in getting purchased items into the company's inventory or stores, and usually consist of clerical costs of:

1. Making the purchase requisition.
2. Issuing of a purchase order
3. Follow-up action
4. Receiving the goods
5. Inspection for quality control
6. Placing goods in stores
7. Paying vendors (Suppliers)

Note:

The basic EOQ model assumes that these costs are fixed constant for each order made.

ii. Carrying costs of inventory

These are costs incurred because the firm has decided to maintain inventories. They usually consist of:

1. Stock-out costs
2. Insurance costs
3. Warehouse and storage costs
4. Material handling costs
5. costs of obsolescence

$$\text{Total Ordering Cost} = \frac{\text{Total demand for period}}{\text{Quantity Ordering}} \times \text{Ordering Costs per period}$$

$$= \frac{DO}{Q}$$

$$\text{Total Holding Costs} = \frac{\text{Quantity Ordered}}{2} \times \text{Holding Costs per unit}$$

$$= \frac{Q\,H}{2}$$

Therefore total relevant costs (TC) for any order quantity can be expressed as:

$$TC = \frac{DO}{Q} + \frac{Q\,H}{2}$$

We can determine a minimum of this total cost function by:

i. Differentiating the above formula with respect to Q and setting the derivative (1st) equal to zero.

$$\frac{dTC}{dQ} = \frac{-DO}{Q^2} + \frac{H}{2} = 0$$

$$\frac{H}{2} = \frac{DO}{Q^2}$$

$$Q^2 = \frac{2DO}{H}$$

$$Q = \sqrt{\frac{2DO}{H}}$$

ii. Equating ordering costs to holding costs.

$$\frac{DO}{Q} = \frac{QH}{2}$$

$$Q^2 = \frac{2DO}{H}$$

$$Q = \sqrt{\frac{2DO}{H}}$$

Illustration

1. Assume a wholesaler has to supply his customer with 40,000 units of a given product every year.
2. Assume that demand is fixed and known
3. Assume the cost of placing each order is Shs 2.00 while the holding cost per unit is Shs 1.00.

Required:

a. Determine the optimum order size using the basic EOQ model.
b. Determine total costs incurred at optimum order size.

Solution

a.

$$Q^2 = \sqrt{\frac{2 \times 40{,}000 \times 2}{1}}$$

$$Q^2 = \sqrt{160{,}000} = \sqrt{16 \times 10^4}$$

$$Q = \underline{400} \text{ units}$$

b.

$$TC = \frac{40{,}000\,(2)}{400} + \frac{400\,(1)}{2}$$

$$= 200 + 200$$

$$TC = \text{Shs } \underline{400}$$

1. EOQ Model with quantity discounts

Circumstances frequently occur where firms are able to obtain quantity discounts for large purchase orders. Buying in bulk has some advantages and disadvantages.

Advantages

i. (A saving in purchase rule) Decreases in unit cost, which consists of the total amount of discount for the period.

ii. A reduction in the total ordering costs because fewer orders are placed to take advantages of the discounts.

Disadvantages

Increased holding cost arising from higher stock levels when large quantities are purchased.

Such as:

i. Stock out cost
ii. Insurance.
iii. Deterioration
iv. Security etc.

Broadly there are two types of discount structures:

i. Fixed discount on "all units" when the order placed is for a minimum quantity.
ii. Variable discounts for given ranges.

1. The case of a fixed discount

When evaluating inventory decisions when a fixed discount rate exists, the appropriate procedure is to compare the total costs of the EOQ with the total costs when discounts are taken. The option giving lower costs is then chosen.

Note:

The Unit (variable) cost (i.e. Purchase Price) behave in the following manner.

$$C = \begin{cases} C_o & \text{if } 0 \leq Q \leq Q_b \\ C_o(1 - P) & \text{if } Q \geq Q_b \end{cases}$$

Where C_o = basic unit cost without a discount
P = Discount rate allowed.
Q_b = Break-point (Quantity)—where discounts become operational.

In order to determine the optimal ordering quantity, it is necessary to include the costs of the inventory with the carrying ordering costs.

Total costs of Inventory = Total Purchase cost + Total order cost + Total carrying cost

$$TC = DC_o + \frac{Q^*}{2}H + \frac{D}{Q^2}o \qquad \text{If } 0 \leq Q \leq Q_b \qquad \text{(i)}$$

$$TC = DC.(1 - P) + \frac{Q}{2}H + \frac{D}{Q}o \qquad \text{If } Q \geq Q_b \qquad \text{(ii)}$$

Note:

Equation (ii) i.e. with discounts will give a lower TC than equation (i) for the same. The decision whether to go for the discount lies on a trade-off between extra carrying costs vs a reduction in acquisition costs.

Illustration

1. Assume X Ltd purchases a raw material from an outside supplier at cost of Shs 70 per unit.
2. Assume total annual demand for the product is 9,000 units.
3. Assume the holding cost is Shs 40 per unit and the ordering cost is Shs 50 per order.

4. Assume a quantity discount of 3% of the purchase price is available for orders in excess of 1,000 units.

Required:

a. Calculate the EOQ and the associated costs.

b. Calculate the total costs if the company purchased in batches of 1,000 units (N.B. It is not wise to buy in batches with more than 1,000 units because of the increase rate in carrying exceed the rate at which ordering costs decline).

c. Advise the management on the appropriate inventory policy.

Solution

a. i. $$EOQ = \sqrt{\frac{2 \times 9{,}000 \times 50}{40}}$$

$$= \sqrt{22{,}500} = \sqrt{225 \times 10^2}$$

$$= \underline{150} \text{ units}$$

ii. $$TC = \frac{9{,}000}{150}(50) + \frac{150}{2}(40) + 9{,}000\,(70)$$

$$= 3{,}000 + 3{,}000 + 630{,}000$$

$$= \text{Shs } \underline{636{,}000}$$

c. $$Q = 1{,}000$$

$$TC = \frac{9{,}000}{1{,}000}(50) + \frac{1{,}000}{2}(40) + 9{,}000\,(70)(1 - 0.03)$$

$$= 450 + 20{,}000 + 611{,}100$$

$$= \text{Shs } \underline{631{,}500}$$

Note: Discount price = 70 (1 - 0.03) = Shs <u>67.90</u>

d. Decline in inventory costs = 631,550 - 636,000

= Shs <u>4,450</u>

Decision

The firm saves Shs 4,450 by taking the quantity discount.

THE CASE OF VARIABLE QUANTITY DISCOUNTS

In practice, suppliers may offer different discounts for different quantities purchased. For example:

Segment	Quantity Purchased	Unit Price
1	0 — 500	Shs 100
2	501 — 1,000	Shs 90
3	1001 — 1,500	Shs 80
4	over — 1,500	Shs 70

The best approach to the solution in this case is to apply the price-breaks theorem. This works as follows:

1. For each segment an EOQ is calculated. There are two possible requests:

   i. The EOQ is within the quantity segment (i.e. valid)
   In this case, the EOQ is used as the minimum cost quantity for that segment.

   ii. The EOQ is outside the quantity segment (i.e. invalid)
   In this case the minimum cost quantity will be the quantity within the segment closest to the EOQ as calculated.

2. Select the quantity that leads to the lowest total inventory costs (i.e. Purchase, Ordering & Carrying).

Illustration:

1. Assume a manufacturer uses 3,300 drums of a certain chemical per year.
2. Assume delivery costs incurred per order are Shs 40 and inventory carrying costs are estimated to be 30% of stock value.
3. Assume the normal cost per drum is Shs 22 but the supplier offers discount of 1.5% on orders for 500 drums or more, and 3% on orders for 1,000 drums or more.

Required:

Determine the order quantity the manufacturer should adopt to minimise total costs.

Solution:

Note:

There are 3 discount levels (0, 1.5% & 3%) and hence 3 segments.

Steps

1. Calculating the EOQ for each segment.

a. For Segment 1 (0 - 499 drums)
C = Shs 22

$$EOQ = \frac{200}{7} \text{ drums}$$

$$EOQ = \sqrt{\frac{2 \times 3,300 \times 40}{0.3\ (22)}}$$

$$= \sqrt{\frac{264,000}{6.6}} = \sqrt{40,000}$$

$$= \underline{200}$$

b. For Segment 2 (500 - 999 drums)

C = 22(1 - 0.015)
C = 22(0.985)
C = Shs $\underline{21.67}$

$$EOQ = \sqrt{\frac{2 \times 3{,}000 \times 40}{0.3\ (21.67)}}$$

$$= \sqrt{\frac{264{,}000}{6.50}} = \sqrt{40{,}615}$$

= $\underline{201.5}$ drums (can round to 202

drums) c. For Segment 3 (1,000 or more)

C = 22(1 - 0.03)
C = Shs $\underline{21.34}$

$$EOQ = \sqrt{\frac{264{,}000}{0.3\ (21.34)}}$$

$$= \sqrt{\frac{264{,}000}{6.40}} = \sqrt{41{,}237}$$

= $\underline{203}$ drums

Segment	Price Break Quantity	Unit Price	EOQ	Valid Quantity	
1	0 – 499	Shs 22	200	200	(Valid)
2	500 - 999	Shs 21.67	202	500	(Invalid)
3	over 1000	Shs 21.34	203	1,000	(Invalid)

2. Calculate total costs

Segment	**Units ordered**	**Total costs**	
1	200	(3,300 x 22) + (200 |2 x 6.6) + (3,300 |200 x 40)	= Shs 73.920
2	500	(3,300 x 21.67) + (500 |2 x 6.5) + (3,300 |500 x 40)	= Shs 73.400
3	1,000	(3,300 x 21.34) + (1,000 |2 x 6.4) + (3,300 |1000 x 40	= Shs 73.754

Decision:

The firm should order 500 drums per order per annum and incur Shs 73.400 total costs.

NON-ZERO LEAD TIME (DETERMINING REORDER POINT)

This basic EOQ model assumes that the suppliers lead time is zero (i.e. goods are delivered immediately on the day the order was made). In reality, however, supplies are rarely ordered and received on the same day. Accordingly, orders must be placed some time before stocks reach zero. In world of certainty (when demand is continuous and constant) the reorder point will be the number of days/weeks lead time multiplied by the daily/weekly usage during the period.

i.e. Reorder point = Average daily usage x Lead time in days.

Note:

The reorder point has no cost implications, since it does not affect the EOQ.

Illustration:

1. Assume X Ltd uses 50,000 kg of a raw material annually.
2. Assume ordering costs are Shs 160 per order and stock holding costs are Shs 0.25 per kg per annum.
3. Assume the purchase price is Shs 20 per kg and no quantity discounts are offered.
4. Assume lead time for delivery of orders is 4 weeks.
5. Assume working time is 50 weeks a year.

Required:

a. Calculate the EOQ
b. Calculate the inventory reorder level.
c. Calculate the total costs per annum.

Solution

a. $$EOQ = \sqrt{\frac{2 \text{ x } 50,000 \text{ x } 160}{0.25}}$$

$$= \sqrt{\frac{16,000,000}{0.25}} = \sqrt{64,000,000}$$

EOQ = <u>8000</u> kgs

Make 6.25 orders (50,000 |8,000) per annum.

b. Reorder level = Demand per week x Lead time.

$= \frac{50,000}{50} \times 4$

= 1,000 x 4

= <u>4,000</u> units.

c. Total costs = (50,000 x 20) + (8,000 |2 x 0.25) + 50,000 |8,000 (160)

= 1,000,000 + 1,000 + 1,000

= Shs <u>1,002,000</u>

INVENTORY PLANNING & CONTROL UNDER UNCERTAINTY

The basic EOQ model assumes that all the parameters (elements) in the model are certain (i.e. can be predicted accurately in advance). These parameters are:

i. Demand or usage of stocks
ii. Lead times.
iii. Holding costs per unit, ordering costs per order and costs per unit.

In reality however, stock demand, supplies lead times and cost date are not known with certainty. Accordingly to make the models applicable to real situations we must consider uncertainty when planning for inventory levels.

To protect itself from conditions of uncertainty, a firm will maintain a level of safety stocks for raw materials, work-in-progress and finished goods stocks. Thus safety stocks are the amount of stocks that are carried in excess of the expected use during the lead time to provide a cushion against running out of stocks. Thus the reorder point is computed as safety stock plus the average usage during the lead time i.e. reorder point = Average usage during lead time + safety (buffer) stock.

DETERMINING THE SAFETY STOCKS LEVEL

1. Uncertainty of demand

Demand is the most troublesome variable to predict accurately. Actually, demand may fluctuate from day to day, from week to week or from month to month. Thus, the firm takes the risk of running out of stocks if there are sudden increases in demand. Hence safety stock is the extra inventory held as a buffer of protection against the possibility of stock due to higher demand.

However, a larger inventory of safety stock will involve a higher inventory carrying costs, and on the other hand, the higher safety stock will decrease stock-out costs. Therefore one has to make a balance between these two costs in order to find out an optimal safety costs.

Note:

The optimum safety-stock level exists where the costs of carrying an extra unit are exactly counter balanced by the expected stock-out costs. This would be the level that minimises the annual total stock-out and carrying costs.

Stock-out costs

These are the opportunity costs of running out of stock. They include:

i. The costs of lost customer sales, and therefore lost contribution to fixed costs.
ii. Potential loss of goodwill with customers whose demand cannot be net.
iii. Acquiring emergency supplies at higher prices to meet demand.
iv. Cost production of finished goods, where raw material stock-outs occur.

The computation of safety stocks lingers on demand forecasts. The manager will have some notion (usually based on past experience) of the range of daily demand. That is the probability that exists for usage of various quantities.

Hence total inventory costs will be as follows:

Total inventory costs = Purchase price cost + carrying costs + stock-out cost + order costs.

= Purchase price costs + "normal~ carrying costs (Q |2 H) + Buffer Stock holding costs (B x H) + Stock-out costs + order costs.

Total inventory costs = D·C + Q |2·H + (B x H) + stock-out costs + D |Q · Co

Where: D = Total annual demand
H = Holding costs per unit
B = Buffer stock
Q = EOQ

Note:

1. The normal EOQ formula is used to compute order quantity Q. Hence purchase costs, carrying costs, and ordering costs remain unchanged. Only buffer stock holding costs and stock-out costs change. Accordingly, the minimisation of these two costs will also mean that total/overall costs will be minimized.

2. Stock-out costs = Number of units short x Probability of being short.

= {(Number of units short x Stock-out costs per unit Probability of being short)—Stock-out costs for every usage duration} x Number of orders per year.

3. Buffer Stock-holding costs = Bx H

Illustration

A company has an annual demand for material X of 250 tonnes per annum. Order lead time is 4 days and usage during lead time as shown by past record is

Usage	Probability
0	0
1	0.01
2	0.05
3	0.15
4	0.25
5	0.30
6	0.10
7	0.09
8	0.05

The cost per tonne is £20 and stock holding cost is 25% per annum of the stock value. Delivery cost per batch is £4. The costs of stock out also estimated to be £4.

You are required to:

a. Calculate the economic batch quantity and the expected number of orders per annum
b. Ascertain the re-order level taking the information given above into consideration.

Solution:

a. EBQ or EOQ where stock-outs are permitted

$$EBQ \text{ or } EOQ = \sqrt{\frac{2DK}{h}}\sqrt{\frac{h + C_s}{C_s}}$$

Where D is annual demand
K is order cost

h is holding cost

and C_s is stock out cost

$$EBQ = \sqrt{\frac{2 \times 250 \times 4}{5}}\sqrt{\frac{5 + 4}{4}}$$

$$= \sqrt{400}\sqrt{\frac{9}{4}}$$

$$= 30$$

No. of orders $= \frac{250}{30}$

(*Note:* Stock holding cost b = 25% of £20 = £5)

We are required to find out expected demand during lead time.

Usage x	Prob $p(x)$	$x\,p(x)$
0	0	0.00
1	0.01	0.01
2	0.05	0.10
3	0.15	0.45
4	0.25	1.00
5	0.30	1.50
6	0.10	0.60
7	0.09	0.63
8	0.05	0.40
	$\Sigma xp(x)$	4.69

Hence normal usage during lead time is 4.69 tonnes but maximum usage during lead time can be as large as 8 tonnes with a probability of 0.05.

Average usage in four days = 4.69 tonnes and at this level there is no buffer stock. The problem is how much buffer stock we should have, so that the cost of holding stock together with the cost of expected stock-out cost is minimum.

Let $\propto_1$ = 4.69 tonnes.
B = Buffer stock
S = Re-order level (for lead time) = $\propto_1 + B$

Hence B = $S - \propto_1$
Here B = S - 4.69

1	2	3	4	5	6	7
Re-order Levels S	**Buffer Stock B**	**Expected Stock-out per order**	**Expected Annual Shortage**	**Expected stock-out cost (£)**	**Holding Cost (£)**	**Total cost (£)**
4.69	0	0.5974	4.978	19.912	0	19.912
5	0.31	0.43	3.58	14.32	1.55	15.87
6	1.31	0.19	1.583	6.332	6.55	12.882 ← minimum cost
7	2.31	0.05	0.4166	1.664	11.55	13.214
8	3.31	0	0	0	16.55	16.55
			Working column 3 x $\frac{250}{30}$	Column 4 x 4	Column 2 x 5 i.e. B x h	

Hence for minimum cost S = 6

Buffer stock S = 6 - 4.69
= 1.31 tones

and minimum cost = £12.88

Workings and explanations

1. When Re-order Level S = 4.69
B = 0

Hence possible shortages will occur when demand is 5, 6, 7 or 8 units.

Demand	Shortage x	Prob p(x)	Expected Value xp(x)
5	5 - 4.69	0.30	0.31 x 0.30 = 0.93
6	6 - 4.69	0.10	1.31 x 0.10 = 0.131
7	7 - 4.69	0.09	2.31 x 0.09 = 0.2079
8	8 - 4.69	0.05	3.31 x 0.05 = 0.1655
		Σxp(x)	0.5974

This is expected stock out per order when demand is 5, 6, 7 or 8

2. Similarly if Re-order level is 5 Shortages will occur if demand is 6, 7, 8

Working in the way as above

Demand	Shortage x	Prob p(x)	Expected Stock-out cost
6	1	0.10	0.10
7	2	0.09	0.18
8	3	0.05	0.15
		Σxp(x)	0.43

3. If re-order level is 6, shortage will occur if demand is 7 or 8

Demand	Shortage x	Prob $p(x)$	Expected Stock-out cost
7	1	0.09	0.09
8	2	0.05	0.10
		$\Sigma xp(x)$	0.19

4. If re-order level is 7, shortage if demand is 8
Expected shortage cost = (8 - 7) x 0.05 = 0.05

5 If re-order level is 8, there is no probability of shortage

a. Expected Annual Shortage = (Expected Stock-out per order) x No. of orders

i.e (Expected stock-out per order) x $\frac{250}{3}$

3

b. Expected stock-out cost = (Expected annual shortage) x Shortage cost per item

c. Holding cost = Buffer stock x cost of holding per item

Important Note:

If shortage C_s is not taken into account

$$EOQ = \sqrt{\frac{2 \times 250 \times 4}{5}}$$

$$= \sqrt{400} = 20$$

and number of orders = $\frac{250}{20}$ = 12.5

Using this model, we can still develop a table similar to the table worked out in this problem.

Graph of total cost against re-order level can be drawn using the table. The minimum cost and the re-order level can then read from the graph. Once re-order level is known, safety stock can be worked out.

In the next example, the initial EOQ value has been taken using the formula

$$EOQ = \sqrt{\frac{2\,D\,K}{h}}$$

Illustration

1. Assume a manufacturer has experienced trouble from stock shortages (stock-outs) of raw materials X which is required in a manufacturing process. Usage of X averages 6,000 units per year where a year consists of 50 weeks.

2. Assume the costs of ordering each batch of X is Shs 30 and the lead time is 2 weeks (known for certain). The annual holding costs amount to Shs 1 per unit of X held. The cost of a stock-out has been estimated to be Shs 5 per unit short.

3. Assume the demand (usage) is unknown. However, the total usage of raw material X over the 2 week lead time is expected to be as follows:

Usage in units	Probability
60	0.07
120	0.08
180	0.20
240	0.30
300	0.20
360	0.08
420	0.07

Required:

a. Calculate the EOQ, and the expected number of orders per annum
b. Calculate the average usage per 2 weeks (Lead-time)

Solution

Annual demand = 6,000 units
Order cost = Shs 30
Holding cost = Shs 1
Shortage cost = Shs 5 per unit short

$$EOQ = \sqrt{\frac{2 \times 6,000 \times 30}{1}} = 600 \text{ units}$$

$$\text{Number orders} = \frac{6,000}{600} = 10$$

Average usage for two weeks = 60 x 0.07 + 120 x 0.08 + 180 x 0.20 + 240 x 0.30 + 300 x 0.20 + 360 x 0.08 + 420 x 0.07

= 240 units

Re-order Levels	*Buffer Stock*	*Expected Stock-out per order*	*Expected Annual Shortage*	*Expected stock-out cost*	*Holding Cost at Shs 1*	*Total cost*
240	0	34.2	342	1,710	0	1,710
300	60	13.2	132	660	60	720
360	120	4.2	42	210	120	330
420	180	0	0	0	180	180

i. Expected Stock out per order where re-order level is 240

(300 - 240) x 0.2 + (360 - 240) x 0.08 + (420 - 240) x 0.07 = 34.2

ii. similarly for re-order level = 300

(360 - 300) x 0.08 + (420 - 300) x 0.07 = 13.2

iii. and for re-order level = 360

(420 = 360) x 0.07 = 4.2

iv. No shortage when re-order level is 420
Hence total cost is minimum when re-order level is 420

units. Best policy is to have safety stock of 180 units.

SENSITIVITY ANALYSIS OF EOQ MODEL

Sensitivity Analysis is concerned with the way in which those results of solutions change in response to change in model parameters.

$$EOQ = \sqrt{\frac{2\,D\,O}{H}}$$

Note:

It is important to appreciate that in formulating our inventory models, we have really been performing a planning exercise. Thus we have made certain assumptions and estimates (e.g. annual demand D, holding costs H, & ordering costs O) and out solutions have obviously been affected by these. When for example calculating a deterministic EOQ value with maximisation of total inventory costs as the objective, expected annual demand (D) is taken into account. If we subsequently find that annual demand has differed from that expected then we will find that the EOQ we selected was not the optimum and, as a result, the total inventory cost was not actually minimum.

Illustration:

1. Assume X Ltd expected annual demand for 1991 for 62,500 units of raw material X per annum.

2. Assume holding costs are Shs 15 per unit per annum and each order costs Shs 10.

3. Assume that the end of the year actual demand has been found to have been for 90,000 units, not the 62,500 expected.

Required:

Calculate the additional costs borne by X Ltd through basing the size and frequency of orders on expected figures.

Solution:

a. Calculating EOQ based on expected figures, and the associated inventory costs.

$$EOQ = \sqrt{\frac{2 \times 62{,}500 \times 10}{5}}$$

$$EOQ = \sqrt{\frac{1,250,000}{5}} = \sqrt{250,000}$$

$= \underline{500}$ units

Total inventory costs $= \frac{62,500}{500}(10) + \frac{500}{2}(5)$

$= 1,250 + 1,250$

$=$ Shs $\underline{2,500}$

b. Calculating the total inventory costs based on actual demand.

TC $= \frac{90,000}{500}(10) + \frac{500}{2}(5)$

$= 1,800 + 1,250$

$=$ Shs $\underline{3,050}$

c. Calculating the total inventory costs had the company forecasted the actual demand accurately.

$$EOQ = \sqrt{\frac{2 \times 90,000 \times 10}{5}}$$

$$EOQ = \sqrt{\frac{1,800,000}{5}} = \sqrt{360,000}$$

$=\underline{600}$ units

TC $= \frac{90,000}{500}(10) + \frac{600}{2}(5)$

$= 1,500 + 1,500$

$=$ Shs $\underline{3,000}$

Therefore, X Ltd total inventory costs are actually Shs 50 (3,050 - 3,000) higher than they would have been if the EOQ had been set in accordance with perfect information.

Observation:

Hence the change in demand by 44% $\frac{(99,000 - 62,500)}{62,000}$ has result in a 20% $\frac{(600 - 500)}{500}$

Increase in EOQ but only 91.64% $\frac{(3,050 - 3,000)}{3,000}$ increase in total costs.

Hence we can conclude that total cost is relatively insensitive to changes in demand.

Selective Inventory Management

The inventory of an industrial firm generally comprises thousands of items with diverse prices, usage and lead time, as well as procurement and/or technical problems. It is neither desirable nor possible to exercise the same degree of control over all those items. The organisation should pay more attention and care to those items whose usage value is high and less attention to those whose usage and consumption value is low. The organisation has, therefore, to be selective in its approach to control its investment in various types of stocks and inventories. Such a system is known as *`selective inventory control'* system.

ABC Analysis (Pareto Analysis)

In ordinary parlance, *ABC analysis* can be best compared with our class society where the population is categorised into Top, Middle and Lower classes. In the case of inventories also, it has been noticed that out of a large number of items (in a million-tonne capacity steel plant there would be usually about 50,000 items of inventory of various types) that are generally held in stock, some of the items are quite significant whereas the others are not that important. Through *ABC* plan which is in fact an analytical approach based on common statistical techniques, the relative importance of the various items is established for the purpose of individual scrutiny and subsequent control. Through this technique `VIP' or the `privileged few' and the `trivial many' are distinguished and treated as such.

ABC analysis contemplates to classify all the inventory items in a number of categories, generally in three categories based on their values. Items of high value but small in number are classified as `A' items which would be under a strict control. `C' items represent relatively small value items and would be under simple control. Items of moderate value and size are classified as `B' items and would attract reasonable attention of the management. Since this plan concentrates attention on the basis of the relative importance of the various items of inventory, it is also known as `control by importance and exception'. As items are classified in order of their relative importance in terms of value, it is also known as the `proportional value analysis'.

It has been found that normal inventory items in most organisations show the following distribution pattern:

A — 5 to 10% of total number of items account for about 70% of the total consumption value. These items may be called "A" items.

B — 10 to 20% account for 20% of total consumption value.

C — The remaining large number of items account for the balance of 15% of the consumption value.

Remark When a detailed scrutiny was conducted in respect of inventories held by the Ford Motor Company, the following results were obtained.

1. 9% of the total items (in number) were accounting for 57% of the total value of the inventory. These were classed as `A' items.
2. 10% of the items (in number) were found to be accounting for 18% of the total value. These were categorised as `B' items.
3. 81% of the items (in number) were found to be accounting for only 25% of the total value. These were classified as `C' items.

Similarly, when ABC ANALYSIS was done in the case of G.E.C., the results obtained were as under:

`A' Category—Items accounting for 8% of the total number but 75% of total value.

`B' Category—Items accounting for 25% of the total number and 20% of total value.

`C' Category—Items accounting for 67% of the total number and only 5% of the total value.

It would thus be observed that substantial and effective controls are made possible if greater attention is focussed on `A' category items since these would be covering quite a substantial part of the inventory in terms of shilling value.

Some Remarks:

1. There is no hard and fast rule that all the inventory items should be classified only in these three categories. There can be a large number of classification based on the requirements of the company and the nature of the items. For example `A' items may be further sub-classified as $A_1$, $A_2$, $A_3$, etc. The same principle may be extended to `B' items also or alternatively all the inventory items may be classified into A, B, C, D, E, F, etc.

2. All items that the company consumes must be considered together while classifying into ABC classes. Separate classification of raw materials, spares and consumable is not really meaningful.

While classifying as ABC items, what counts is the consumption shillings and not the unit price of an item or its consumptions in terms of units. Thus of the three items given below, the last one is most important since its annual consumption in terms of value is more than the other two.

Item	*Price/Unit Shs*	*Annual Consumption in units*	*Annual Consumption in Shs*
X	20,000	2	40,000
Y	0.02	100,000,000	200,000
Z	1,000	500	500,000

4. Even though, so far we have referred to annual consumption, it is not at all necessary that the consumption figures should be taken only for one year. It can be for 6 months or even 3 months. But the period should be so selected that the consumption figures would be representative. However, annual figures are far more convenient and are universally followed.

5. If a firm follows ABC analysis, it will devote much time and effort on the control of `A' items. For example, extra care will be taken in the determination of minimum, maximum, reorder level, etc. of the `A' items, whereas so much control may not be exercised on `C' items. `A' items may be purchased only once in a year. For `A' items perpetual inventory system may be applied whereas in the case of `C' items, only a bin card may be maintained. In the same way an appropriate accounting method for `B' items may be devised. However, in the classification of items into ABC categories if there are some critical items which are of small value whose non-availability may hamper the production, may in the normal situation, be classified as `C' items but, due to the critical nature of these items extra care may be taken so that these may not go out of stock.

6. The objective of classifying inventory items into `A', `B' and `C' categories is to develop policy guidelines for selective control. Such a policy can be designed in a variety of ways. In general `A' items merit a tightly controlled inventory system with periodic attention, and `C' items to subject to loose control with casual attention.

7. TABLE SHOWING FEATURES OF ABC ANALYSIS

Nature	A items (High value)	B items (Moderate value)	C items (Low value)
1. Extent of control	Rigid control (close day to day control)	Moderate control (Regular review)	Loose control (infrequent review)
2. Safety stock coverage	Low safety stocks	Medium safety stock	Large safety stocks
3. Frequency of order	Frequently	Less frequently	Bulk ordering
4. Degree of posting	Individual posting	Small group postings	Group postings
5. Period of review	Every fortnight	Quarterly	Yearly
6. Sources of supplies	Good number of sources	Few reliable sources	One or two sources
7. Follow up	Vigorous	Periodic	Occasional
8. Control statements	Weekly control statements	Monthly control reports	Quarterly control reports
9. Forecasting	Emphasis on accurate forecast	Focus on past trend	Rough estimate
10. Level of management	Senior management	Middle management	Stores supervisor
11. Lead time	Maximum efforts to reduce lead time	Moderate efforts	Minimum clerical efforts
12. Value % and item percentage (Approximation)	80% of the value in 20% of the items	15% of the value in 30% of the items	5% of the value in 50% of the items

ADVANTAGES OF "ABC ANALYSIS"

The benefits derived from this analysis and its subsequent follow up are summarised below:

- Facilities selective control and thereby saves valuable time of busy executives.
- Eliminates lot of unnecessary paper work involved in various other control procedures. Tangible savings can be effected in this behalf by following Two-Bin System which is very closely related with this technique.
- Facilitates Inventory Control and control over usage of stores materials which ultimately results in cost control.

LIMITATIONS OF ABC ANALYSIS

Although ABC analysis is a fundamental tool for exercising selective control over numerous inventory items, it does not, in its present form, permit precise consideration of all relevant problems of inventory management. For instance, a never-ending problem in inventory management is that of adequately handling thousands of low-value `C' items. Low-value purchases frequently require more items and thereby reduce the time allowance available to purchasing personnel for value analysis, vendor investigation, and other `B' items.

Besides, if ABC analysis is not periodically reviewed and updated, the very approach of control may be defeated. For example, `C' items like diesel oil in a firm, will become most high-value items during power crisis should, therefore, deserve more attention, but this point may be overlooked if classification of items is not reviewed and updated.

The following steps are involved in implementing the ABC analysis:

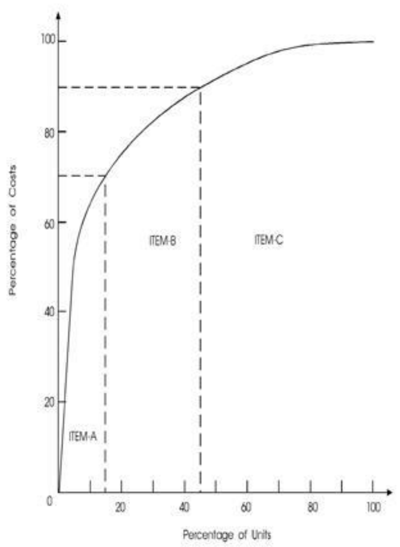
1. Classify the items of inventories, determining the expected use in units and price per unit for each item.
2. Determine the total value of each item by multiplying the expected units by its unit price.
3. Rank the item in accordance with the total value, giving first rank to the item with highest total value and so on.
4. Compute the ratios (percentage) of number of units of each item to total units of all items and the ratio of total value of each item total value of all items.

Combine items on the basis of their relative value to form three categories—A, B and C. The data in the table below illustrates the ABC analysis.

TABLE: ABC ANALYSIS

Item	Units	% of total		Cumulative	Unit price	Total cost	% of total		Cumulative
1	10,000	10			30.4	304,000	38.00		
			15	15				70	70
2	5,000	5			51.20	256,000	32.00		
3	16,000	16			5.50	88,000	11.00		
			30	45				20	90
4	14,000	14			5.14	72,000	9.00		
5	30,000	30			1.70	51,000	6.38		
			55	100				10	100
6	15,000	15			1.50	22,500	2.81		
7	10,000	10			0.65	6,500	0.81		
TOTAL	100,000					800,000			

The tabular and graphic representation indicate that "Item A" forms a minimum proportion, 15 per cent of total units of all items, but represents the higher value, 70 per cent. On the other hand, "Item C" represents 55 per cent of the total units and only 10 per cent of the total value. "Item B" occupies the middle place. Items A and B jointly represent 45 per cent of the total units and 90 per cent of the investment. More than half of the total units are item C, representing merely 10 per cent of the investment. Thus, a tighter control should be exercised on "Item A" in order to maximise profitability on its investment. In case of "Item C" simple controls will be sufficient.



Just-in Time (JIT) Inventory management

JIT is a system whose objective is to produce or to purchase products or components as they are required by customers or for use rather than for stock.
A JIT system is a pull system which responds to demand as opposed to a push system in which stocks acts as buyers* between the different element of the system such as purchasing, production and sales

JIT production

Is a production system, which is driven by demand for the finished products whereby each component on the production line is produced only when needed for the next stage.

JIT purchasing

_On the other hand is a purchasing system in which material purchased are contracted so as that the receipt and usage of materials to the maximum extent possible, coincide.
JIT concept can be traced back to the Japanese company whose success in the international market generates interest among many western companies as t6o how this success was achieved.
The implementation of JIT production methods was considered to be pursuit of excellence in all phase of manufacturing systems design and operations.
The JIT are to produce the required items at the required quality and in the required quantities, at the precise time that they are required.

JIT seeks to achieve the following goals-

(1). Elimination of non-value adding activities.
(2). Zero inventory
(3). Zero defects
(4). Batch size of one.
(5). Zero break-downs
(6). 100% on time delivery services.

The above goals represent perfection and are most unlikely to be archived in practice. They do however offer targets and create a climate for continuous improvement and excellence.

Major features of JIT.

(1). Elimination of non-value added activity

JIT manufacturing can be described as a philosophy of management, dedicate to the elimination of waste. Waste is defined as anything that does not add value to a product.
The cycle time involved in manufacturing and selling a product consist of –

- Process time-add values
- Inspection time
- More time
- Queue time
- Storage time

Of these 5 steps only process time actually adds value to the products. All the other activities add cost and No value to the production and therefore are deemed as non-value within the JIT philosophy.
Usually in many companies, process time is less than 10% of total manufacturing, lead and cycle time. Therefore 90% of the manufacturing lead time disassociated with the product, adds cost but no-value to the product by adapting a JIT philosophy and forecasting, on reducing lead time, it is claimed that total cost can be significantly reduced.
The ultimate products with lead-time = processing time, and eliminating all non-value adding activities.

(2). Factory Layout

The first stage of implementing the JIT manufacturing techniques is to rearrange the factory floor away from the batch production functional layout towards a production layout using low lines with a functional plant layout production through a No. of special departments that normally contain a group of similar machines.
Products are processed in stage batches so as to minimize the set times when machine settings are changed between processing batches of different products. Batches move via different and complex routes through the various departments, traveling over much of the factory floor before they are complete.
Each process normally involves a considerable amount of waiting time and which much time is taken transporting items from one process to another.

A further problem is that it is not easy at any point in time to determine what progress has been made on individual batches and therefore detailed cost accumulation records are necessary to track work in progress. This results in long manufacturing cycles and high work in progress levels.

The JIT solution is to reorganize the production process by dividing the many different products that an organization makes into families of similar products or component. All the products is a in a particular group will have similar production requirement and routing. Production is necessary used* so that each production family is manufactured in a well-defined production cell based on flow line principles. In a production how lines specialist department containing similar markets no longer exist. Instead groups of dissimilar markets are organised into products or component family flow lines that function like an assembly line.

For each production lines the market are placed close together in the order in which they are required by the group of products to be processed. Items in each product family can now move one at time from process to process more easily, thereby reducing <u>wip,</u> and <u>lead time.</u>
The aim is to produce products or component from start to finish without <u>returning to the stock room or stores</u>

(3). Batch size of one.

Set up time is the amount of time required to adjust equipment and to retool for a different product. Long set ups a change over time make the production of batches with a small no. of units uneconomical.
However, the creation of large batches lead to substantial lead time delays and the creation of high inventory levels. The JIT philosophy is to reduce and eventually eliminate set-up times.E.g. by investing in advanced manufacturing technology some machines setting can be adjusted automatically instead of manually.
Alternatively some set up times can be eliminated entirely by reducing products, so that markets do not have to be reset each time a different product has to be made.
If the set up times are approaching zero, then there's no advantage production in batches and therefore the optimal batch size can be one. With a batch size of one, the work can flow smoothly to the next stage without the need for and to schedule the next machine to accept this item.

JIT purchasing arrangements

JIT philosophy also extends to adapting JIT purchasing techniques whereby delivery of material immediately precedes their use. By arranging with suppliers for more frequent deliveries stocks can be out to a **minimum**
Considerable savings in material handling expenses can be obtained by requiring suppliers to inspect materials before their delivery and guarantee their quality.
This improved services is obtained by giving more business to fewer suppliers and **placing long tern purchasing orders,** therefore the suppliers has an assurance of long term sales and can plan to meet this demand.

Companies that have implemented JIT purchasing techniques have claimed to substantially reduce **their investment in raw materials and work in progress stocks.**

Other advantages include

1. Substantial savings in factory space.
2. Large quantity discount.
3. Savings in time from negotiating with fewer suppliers
4. Reduction in paper work arising from issuing (long term orders) to a few suppliers rather than individual purchase order to many suppliers.

JIT and Management Accounting.

Management accountants in many organisations have been criticised because of their failure to change their managing accounting system to reflect the mode from a traditional manufacturing to a JIT manufacturing system.
Conventional management accounting systems can encourage behaviour that is inconsistent with JIT phylosopy, management, accounting must support JIT manufacturing by monitoring, identifying, and communicating to decision makers any delays errors and waste in the system.

Modern management accounting systems are now placing greater emphasis on providing information on suppliers relialibility set up times cycle times, percentage of deliveries that are on time and defect rates All these measures are critical in supporting JIT manufacturing philosophy.

FINANCIAL MANAGER'S ROLE IN INVENTORY MANAGEMENT

The techniques of inventory management, discussed above are very useful in determining the optimum level of inventory and finding answers to the problems of the economic order quantity, the re-order point and the safety stock. The techniques are very essential to economise the use of resources by minimising the total inventory cost. Although out treatment of inventory management has been simple, it indicates the broad framework of managing inventories. Many sophisticated techniques have been evolved to handle inventory management problems more efficiently and effectively and the improvements are still continuing. For the majority of the companies, inventory represents a substantial investment. Thus, the goal of the wealth maximisation is related to the efficiency with which inventory is managed. Consequently, the financial manager has an important role to play in the management of inventory, although it is not his operating responsibility to control inventory. The financial manager should see that only an optimum amount is invested in inventory. He should be familiar with the inventory control techniques and ensure that inventory is managed well. He should introduce the policies which reduce the lead time, regulate usage and thus, minimise safety stock. The net effect would be to reduce inventory investment and increase the firm's prospects of making more profits.

QUEUING THEORY

When limited facilities fail/delays to satisfy demands made upon them, problems occur which generate queues or waiting lines. Examples are:

- Customers waiting at cash desks in a supermarket or bank
- A stock of items in a warehouse awaiting storage
- Letters in an ‘in’ tray waiting to be opened
- Telephone calls to a switchboard waiting to be answered.

In business, queues have certain ‘economic’ or ‘cost’ implications;
It may be too costly for a company to allow long queues to develop. Eg if employees queue up at a store counter for new material supplies, the company will incur the cost of idle time and lost production due to their time spent waiting.

It will probably be costly to speed up service, and thus reduce waiting time and queue lengths because it would be necessary to employ extra service assistants, service counters or service equipment. Customers may expect to be served within a certain length of time, otherwise they may take their custom elsewhere.

Queuing problems are therefore concerned with

- Average waiting times
- The average length of queues (ie. the average number of people in a queue or service system)
- The cost of a servicing system.

The management may therefore wish to provide a servicing system which either:
minimise the joint costs of: a) servicing customers b) (idle) time waste by customers in the queue or balance the requirement to provide a satisfactory service time with the interests of economy i.e. to provide a reasonably quick service but at a relatively low cost.

Queuing problems:

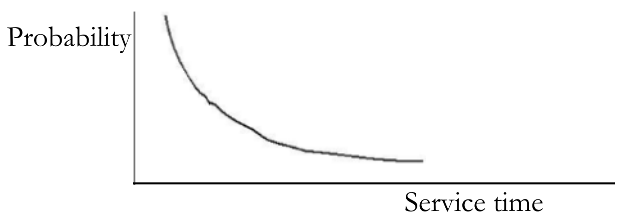
There are two main approaches to queuing problems:

- simulation
- queuing theory formula

Where simple situations apply, queuing theory should be used in preference to simulation. This is because; it is quicker to use and therefore cheaper
the results obtained are more definite as they do not depend on the selection of random numbers.

In complex situations a solution can only be found through
simulation. Features of a queue:
A calling population – refers to the number of potential customers. This number may be considered finite or infinite. An infinite calling population is normally assumed as it provides convenient simplification of the analysis. This is a reasonably accurate assumption provided that the rate of arrival of future customers is not affected by the number of customers already in the queue.
An arrival pattern – refer to the way in which customers arrive in a queue. They may arrive in large groups or as isolated individuals, may arrive at regular intervals or at irregular intervals, or randomly. A common arrival pattern is for customers to arrive at queues individually and in a random manner. When this occurs, the number of arrivals per unit of time generally follows a paisson at probability distribution.
The servicing pattern – this may also vary with customers served individually one at a time or in 'bulk'. When customers arrive randomly, it is usual for the service pattern also to be random. It is commonly assumed that the probability distribution of the time to be served is the same for all customers, and that this service time is independent of the time of day or the size of a queue (eg even if the queue is very long, the server will not speed up his service times in order to reduce its length.

A common example is that service times follow an exponential probability distribution i.e. $y=^{e-x}$



Service channels - these refer to the number of service points and queues;
If there is only one queue, but several service counters, the customer at the head of the queue will move to the first free counter when it becomes available. Where this system operates, there is a basic multiple channel or multi-channel system.

If there are several service counters each with its own queue, there is a more complex multiple channel or multichannel system.
Traffic intensity- this is the ratio of the average arrival rate to the average service rate. Unless a service rate is faster than the rate of new customers arriving in the queue the queue gets longer. An important assumption in queuing theory is that the traffic intensity must be less than one.
Queue, discipline – refers to the way in which customers behave in a queue, and to the order in which they are served. For example;
A customer may arrive at a queue and decide that it is too long and go away;
The queue may be at a maximum length so that a new customer arriving cannot join it. A customer may be kept waiting so long in the queue that he/she decides to leave it.
In a complex channel system, a customer may cross over from one queue to another because it seems shorter.

A steady state or a transient state – customers may be served in several ways depending on either the server or the customers;
Customer may be given priority treatment
An official may deal with his in-tray by taking the top documents out first ie. LIFO system. There may be a first come first served queue discipline ie FIFO.

A queue or a service system is in a transient state when it is dependent on time. (e.g. a workers canteen may be kept open all day, but there will be times of the day when the system is busy and times when it is relatively empty.

A system is in a steady state or in equilibrium when it is not dependent on time. Steady state systems are the ones commonly considered in queuing theory.

A system may be in a transient state for a short time (e.g. when a service counter opens there might be an initial rush of customers) but then it may settle down into a steady state (until it closes for the day.)

ARRIVAL RATES, SERVICE RATES, AND TRAFFIC INTENSITY

The (average) arrival rate is the rate of arrival of customers at a queue, and is often denoted by x. If 10 customers arrive at a queue each hour:

x = 10 and $\frac{1}{x}$ x 60 minutes = every 6 minutes is the inter-arrival time.

The (average) service rate is the rate at which customers could be served, and is often denoted by the letter μ.
If on average, 20 customers could be served every hour, μ = 20 and the average service time would be

$$\frac{1}{\mu} \times 60 \text{ minutes} = 3 \text{ minutes.}$$

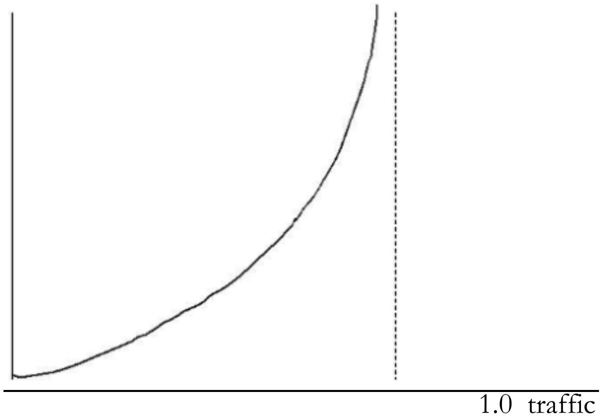
The traffic intensity is the ratio or the average arrival rate to the average service rate. It is denoted by *P*.

Thus $P = \frac{x}{\mu}$

If the arrival rate is 10 per hour and the average service rate is 20 per hour, the traffic intensity would be:

$$P = \frac{x}{\mu} = \frac{10}{20} = 0.5$$

If a queuing system is to work x must have a lower value than μ and the traffic intensity P , must be less than one. As x approaches μ in size, the average length of the queue and the average time spent in the system will grow as the service personnel find it increasingly hard to keep up with the demand. This point is illustrated below.



When the traffic intensity is equal to or greater than 1, the average time spent by each customer in the system would theoretically be infinite.

Multiple channel systems
Where there is a multiple channel system, with c channels, and a service rate at each separate Channel of μ , $P = \frac{x}{\mu c}$

QUEUES AND SYSTEMS

A queue refers to the customers who are waiting to be served.
A system refers to the customers in the queue waiting to be served and also to the customers being served.

SIMPLE QUEUES.

A simple queue has the following characteristics;
There is a simple service channel
There are 'discrete' customers e.g. customers in a bank, or aircraft waiting to take off as opposed to storm water waiting to drain away.
There is an infinite population of potential customers and an infinite maximum queue. There are no simultaneous arrivals of customers at a queue.
The queue discipline is on a first come first served basis i.e. FIFO with no priorities or queue barging. No customer leaves before being served.
The queue is in a steady state of operation.
The number of arrivals (or demands) in a unit of time follow the poisson distribution Service times follow a negative exponential distribution
Traffic intensity is less than 1.

Formula-simple queues.

The probability of having no queue on arriving i.e. the probability that the service point is busy at any moment chosen at random = P This is $\frac{\lambda}{\mu}$ as there is only one channel.

From (a) it follows that the probability that a customer is served immediately = 1- P. This is the same as the probability that there is no one in the system.

The average number of people in system is

$$\frac{P}{1-P} \text{ or } \frac{\lambda}{\mu-\lambda}$$

The average number of people in the queue $= \frac{P^2}{1-P}$

e. The average time spent in the queue $= \frac{P}{\mu(1-P)}$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

f. From (e) it follows that the average time spent in the system is

$$\frac{P}{\mu(1-P)} + \frac{1}{\mu} = \frac{1}{\mu(1-P)} \text{ or } \frac{1}{\mu-\lambda}$$

Example:

For a single queue, single service point system with Poisson arrivals and exponential service time, show how the average number of people in the system could vary if an average of 12 people needed the service every hour and the average service rate took alternative values of 24, 18 and 15 per hour. What is the probability that a person entering the system would have to queue?

a. $\lambda = 12 \quad \mu = 24$

Probability of queuing $P = \frac{12}{24} = 0.5$

Average number of customers in the system $= \frac{P}{1-P} = \frac{0.5}{1-0.5} = 1$ person

b. $\lambda = 12 \quad \mu = 18$

Probability of queuing $= P = \frac{12}{18} = \frac{2}{3}$

Average number of customers in system $= \frac{2/3}{1-2/3} = 2$ people

c. $\lambda = 12 \quad \mu = 15$

Probability of queuing $= P = \frac{12}{15} = \frac{4}{5}$

Average number of customers in system $= \frac{4/5}{1-4/5} = 4$ people

Cost of serving and waiting

Costs of a system might include:

The labour cost of service plus any items of expenditure incurred as a direct consequence of providing service at a given rate or by a given number of channels.

The opportunity cost of customer's waiting time, eg the loss of contribution/profit from production and sales as a consequence of employees being kept waiting in a queue for service instead of being busy and productive.

Example:

A team of 15 men is employed to unload lorries at a terminal. The team works a 6 hour day during which 36 lorries arrive (i.e. 6 per hour) and it takes 7 ½ minutes to unload one lorry with the team acting as a single unit. Lorries are

Served on a FIFO basis.

It has been estimated that the cost of keeping lorries waiting is sh 6 per hour. Members of the team are each paid
Sh 2.50 per hour. It is also estimated that if the size of the team increased to 20 men, the average service time would fall to 5 minutes.

Required:

Calculate the cost of the present system and the cost of the proposed system, and determine whether an increase in the size of the team would be justified on grounds of cost.

Solution:

The cost of service with
15 man team = 15 x 2.50 x 6 = sh. 225 per day
20 man team = 20 x 2.50 x 6 = sh. 300 per day

The daily cost of lorry waiting time, at sh.6 per hour may be calculated in either of 2 ways.
by calculating the average number of lorries in the system and multiplying this number by (sh 6 per hour x 6 hours per day) Sh. 36 per day or
by calculating the average waiting time in the system, and multiplying this time by sh.6 per hour and by the number of lorries in a 6 hour day i.e. 36.

Average number of customers in the system = $\frac{\lambda}{\mu - \lambda}$ or $\frac{P}{1 - P}$

15 man team
$\lambda = 6$ $\mu = 60/7.5 = 8$ $P = 0.75$

Average number in system = $\frac{6}{8-6} = 3$

Cost per day per customer = 36
Total daily cost of waiting time 3 x 36 = 108

20 man team
$\lambda = 6$ $\mu = 60/5 = 12$ $P = 0.5$

Average number in system = $\frac{6}{12-6} = 1$

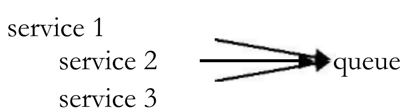
Cost/per day per customer = 36
Total average daily cost of waiting time = 1x 36 = 36

Summary	Sh	Summary	Sh
Cost of service per day	225	Cost of service per day	300
Cost of waiting time per day	108	Cost of waiting time per day	36
Cost of system, per day	333	Cost of system per day	336

The 15 man team is marginally more economical by sh 3 per day.

Multichannel system
Within the framework of the basic assumptions above, it is possible to examine more complicated situations. These include:

a. A single queue and several service points



b. Several queues and a single service point

queue 3

c. Several queues and several service points service 1 ⟷ queue
service 2 ⟷ queue
service 3 ⟷ queue

The multichannel formulae are normally expressed in terms of another function *Po* which is the probability that there are no customers in the system, and is given by the formula:

$$Po = \frac{C!(1-P)}{(Pc)^c + C!\,(1-P)\left(\sum_{n=2}^{c-1} \frac{1}{n!}(pc)^n\right)}$$

Example

In a three channel system, the rate of service at each channel is 5 customers per hour and customers arrive at the rate of 12 per hour. What is the probability that there are no customers in the system at a given point in time?

C= 3, $\lambda = 12$, $\eta = 5$, $\rho = 12/3 \times 5 = 0.8$

$$P_0 = \frac{3!\,(1-0.8)}{(0.8 \times 3)^3 + 3!\,(1-0.8\ (x)}$$

$$\text{Where } x = \sum_{o}^{c-1} \frac{1}{n!} (\rho c)^n$$

$$= \frac{3 \times 2 \times 1\ (0.2)}{(2.4)^3 + 3 \times 2 \times 1)\ (0.2)\ (x)}$$

x is the sum of 3 figures giving $\frac{1}{}$ $((\rho c)^n$

where $n = 0;\ \frac{1}{0!}(0.8 \times 3)^o = 1.0$

$n = 1;\ \frac{1}{1!}\ (0.83 \times 3)^1 = 2.4$

$n = 2;\ \frac{1}{2!}\ (0.8 \times 3)^2 = ½\,(2.4)^2 = \underline{2.88}$

$\therefore X\ \ \underline{6.28}$

$$P_o = \frac{1.2}{13.824 + 1.2\ (6.28)} = 0.056$$

If the service rate were increased from 5 to 8 per channel per hour, we would expect the queue sizes to shorten and the probability that there is no one in system to increase.

Thus if $\eta = 8$, c = 3 $\lambda = 12$, $\eta = 5$, $\rho = \frac{12}{8 \times 3} = 0.5$

$$P_o = \frac{3!\,(1.05)}{(0.5 \times 3)^3 + 3!\,(1-0.5)\,(x).}$$

(x) is the sum of the values;

$n = 0;\ \frac{1}{0!}\ (0.5 \times 3)^{0} = 1$

$n = 1;\ \frac{1}{1!}\ (.5 \times 3)^{1} = 1.5$

$n = 2;\ \frac{1}{2!}\ (05 \times 3)^{2} = \underline{1.125}$

(x) = $\underline{3.625}$

$$P = \frac{6\ (0.5)}{3.375 + 6(0.5)\ (3.625)} = \frac{3}{14.25} = 0.211$$

Multi-channel queuing formulae;

The following formula can be derived using ρ, ρ_o and η, the average rate of service for each channel;

$$\text{Average number of customers in the system} = \frac{\rho\ (\rho c)^c\ \rho_o}{c!\ (1 - \rho)^2} + \rho c$$

$$\text{Average number of customers in the queue} = \frac{(\rho c)^c\ \rho_0}{c!\ (1 - \rho)^2\ c\eta} + \frac{1}{\eta}$$

$$\text{Average time a customer spends in the queue} = = \frac{(\rho c)^c\ \rho_0}{c!\ (1 - \rho)^2\ c\eta}$$

These formula would also apply to a simple queue with only one service channel.

SIMULATION MODELS

Simulation is a method of analyzing a system by experimentally duplicating its behavior. Management accountants can be able to make meaningful inferences concerning the operation of some real-world system by constructing a model of the system and then experimenting with the model. If the model is an adequate representation of the real system, a study of the responses of the model to various decisions will give considerable insight into the effects of implementing such decisions in the real system.

Simulation is used where analytical techniques are not available or would be very complex. E.g. in inventory control problems, production planning problems, corporate planning, and queuing problems etc.

Model Construction

The success of a simulation exercise is related to the predictive quality of the underlying model, so that considerable care should be taken with model construction. Important factors in model development are:

Object oriented: The model should be constructed with some definite purpose and the model results must be directly related to this purpose.

Critical variables and relationships: Model building is an iterative, creative process with the aim of identifying those variables and relationships which must be included in the model.

Simplicity: The best model is the simplest that has adequate predictive qualities.

Management involvement: To construct good models these must be a thorough understanding of actual operations. Only management here these knowledge so that they must be involved.

Types of Simulation

1 Operational Gaining Method

This refers to those situations involving conflict of interest among players or decision makers within the framework of a simulated environment. The two most widely used forms of operational gaming are military games and business management games (mostly computer oriented)

Military gaming is essentially a training device for military leaders, enabling them to test alternative strategies under simulated war conditions.

Management games have gained wide acceptance in business and education. The primary use of business games is to help the participants be they executive in the industry or students in business develop their ability to

Make difficult interdependent business decisions
Evaluate new ideas and
Introduce new techniques of decision making, (all in a simulated environment)
Simulation of business environment provides valuable experience in conceptualizing ideas and in logical thinking.

2 Monte Carlo Method

In Monte Carlo simulation the behavior of at least some components of the model are probabilistically determined. It can be used to solve several different classes of problems. The first are problems that involve some kind of stochastic process while the second are deterministic methods.

To carry out a realistic simulation involving probabilistic elements, it is necessary to avoid bias in the selection of the values which vary. This is done by selecting randomly using one of the following methods.

Random nos. generated by
computers Random no. tables

Variables in a Simulation Model

A business model usually consists of linked series of equations and formulae arranged so that they behave in a similar manner to the real system being investigated. The formulae and equations use a no. of factors or variables which can be classified into 4 groups.

Input or exogenous variables

These are variables of two types:

1 Controlled variables:

These are variables that can be controlled by management. Changing the input values of the controlled values, and noting the change in the output results is the prime activity of simulation. E.g. re-order level and re-order quantity.

2 Non-Controlled Variables

These are input variables which are not under management control. Typically these are probabilistic or stochastic variables. E.g. in pdn simulation the no. of breakdowns, in inventory-demand and lead-time.

b) Parameters

These are also input variables which for a given simulation have a constant value. They are factors which help specify the relationship between other variables e.g. in pdn simulation the time taken for routine maintenance, the cost of stock-out, etc.

c) Status Variables

Status variables may be used to specify the day's and seasons to be used in a simulation e.g. demand may be affected by the day of the week

d) Output Variables

These are the results of the simulation. They arise from the calculations and tests performed in the model. The output variables must be carefully chosen to reflect the factors which are critical to the real system being simulated and they must relate to the objectives of the real system. E.g. the output variables for an inventory simulation would include: Cost of holding stock, no. of stock-outs, no. of unsatisfied orders, no. of replenishment orders, cost of the re-ordering e.t.c.

Constructing the Model

Steps

Identify the objectives of the simulation (A detailed listing of the results expected will help to clarify the output variables.
Identify the input variables. Distinguish between controlled and non-controlled variables.
Where necessary determine the probability distribution
Identify any parameters and status
variables Identify the output variables
Determine the logic of the model

Note:

The key questions are how are the input variables changed into output results, what formulae/decision rules are required? How will the probabilistic elements be dealt with? How should the results be presented?

Inventory Model Illustration

XYZ Ltd has set the re- ordering point at 15 units and order quantity (Q) of 20 units. The holding cost have been computed to be Shs 10 per unit per week, and the cost of placing an order is Shs 25. Also the stock-out cost is Shs 100 per unit short. Assume the inventory on hand at the beginning of first week is 20 units. The demand and lead time have been shown to be explained by the following prob. distribution.

Demand (units)	Probability	Cum. Prob.	Distribution of Random Nos.
0	0.02	0.02	00 – 01
1	0.08	0.10	02 – 09
2	0.22	0.32	10 – 31
3	0.34	0.66	32 – 65
4	0.18	0.84	66 – 83
5	0.09	0.93	84 – 92
6	0.07	1.00	93 – 99

Demand (weeks)	Prob	Cum Prob.	Distribution of Random Nos.
1	0.23	0.23	00 – 22
2	0.45	0.68	23 – 67
3	0.17	0.85	68 – 84
4	0.09	0.94	85 – 93
5	0.06	1.00	94 – 99

Required

Simulate the problem for 14 weeks, and determine the average weekly cost using the following random nos.
68 52 50 90 59 08 72 44 95 85 81 93 28 89 15 60 03

Steps:

Set up a probability distribution for each relevant variable
Build a cumulative probability distribution for each variable
Establish interval of random numbers for each variable and therefore allocate the random numbers Generate the RNs using a table, or computer
Perform the simulation

Demand			Delivery			Inventory				
Wks	Rand. No.	DD units	Rand. No.	Del. (wks)	Units recvd.	Bal. Hand	Inv hold	Ord. cost	Stock cost	Total cost
0						20				
1	68	4				16	160			160
2	52	3				13	130			
3	90	5	50	2		8	80	25		155
4	59	3			20	25	250			240
5	08	1				24	240			200
6	72	4				20	200			170
7	44	3				17	170			
8	95	6				11	110			
			85	4				25		135
9	81	4				7	70			70
10	93	6				1	10			10
11	28	2				0	0		100	100
12	89	5			20	15	150			
			15	1				25		175
13	60	3	15	1	20	32	320			320
14	3	1				31	310			310
							2200	Total		2375

Average Shs. 169.94

Illustration 2 (C – V analysis)

ABC Ltd is considering marketing a new product. The investment is £5, 000. There are three uncertain factors. Selling price, variable cost, and annual sales volume. The product has a life of only one year and the various possible levels of these factors together with estimated probabilities are given below.

Selling Price	**Prob.**	**Cum. Prob.**	**Distr. Random No.**
£4	0.3	0.3	0 – 2
5	0.5	0.8	3 – 7
6	0.2	1.0	8 – 9

Variable costs Prob. Cum. Prob Distr. Of Random Nos.

£2	0.1	0.1	0
3	0.6	0.7	1 – 6
4	0.3	1.0	7 – 9

Assume that the three factors are statistically independent

Required

Simulate the problem and determine the average profits (use twenty five trials)

Note

Profit = (Price – Cost) x volume – 5, 000
Use the following random nos. 80 60 43 63 21 40 360 569 167 386 283 161 573 996 497 726 953 050 118 064 886 924 077 008 401 658 401

Trial	**Rand. No.**	**Price £**	**Random No.**	**Cost**	**Rand. No.**	**Vol. '000**	**Profits £**
1	8	6	0	2	6	5	15
2	0	4	4	3	3	4	-1
3	6	5	3	3	2	4	3
4	1	4	4	3	0	3	-2
5	3	5	6	3	0	3	1
6	5	5	6	3	9	5	5
7	1	4	6	3	7	5	0
8	3	5	8	4	6	5	0
9	2	4	8	4	8	4	-5
10	1	4	6	3	1	3	-2
11	5	5	7	4	3	4	-1
12	9	6	9	4	6	5	5
13	4	5	9	4	7	5	0
14	7	5	2	3	6	5	5
15	9	6	5	3	3	4	7
16	0	4	5	3	0	3	-2
17	1	4	1	3	8	5	0
18	0	4	6	3	4	4	-1
19	8	6	8	4	6	5	5
20	9	6	2	3	4	4	7
21	0	4	7	4	7	5	-5
22	0	4	0	2	8	5	5
23	4	5	0	2	1	3	4
24	6	5	5	3	8	5	5
25	4	5	0	2	1	3	4
						Total	52
						Average 2.08	

Average Profit = £2080

Note: The expected profit = (5 – 3) 4 000 – 5 000 = £3 000. This significantly overstates the expected profitability of the investment.

The Role of Computers in Simulation

Computers can be used to:

To generate the random numbers
To simulate thousands of trials. This is done extremely fast, accurately and reliably.
Several combinations of the decision variables can be simulated very fast.
Provide management with timely reports, enhancing the competitive edge of the firm.

Advantages of Simulation

1 It can be used in areas where analytical techniques are not available or would be too complex.
Constructing the model inevitably must involve management and this may enable a deeper insight to be obtained into a problem.
A well-constructed model enables the results of various policies and decisions to be examined without any irreversible commitments being made.
Simulation is a cheaper and less risky than altering the real system.
Can be used for training decision makers.

Disadvantages of Simulation

Although all models are simplification of reality, they may still be complex and require a substantial amount of managerial and technical time.
Practical simulation inevitably involves the use of computers which may be a handicap to firms without computer facilities or easy access to a computer.
Simulation don not produce optimal results. The manager makes the decision after testing a no. of alternative policies. There is always the possibility that the optimum policy is not selected.
Simulation can be expensive in terms of design personnel, facilities e.g computers.
Each simulation model is unique and therefore cannot be generalized to other environments.

REINFORCING QUESTIONS

QUESTION ONE

Explain the advantages and disadvantages of the Just-In-Tie (JIT) inventory system. (6 marks)
A company has determined that the EOQ for its only raw material is 2010 units every 30 days. The company knows with certainty that a four-day lead time is required for ordering. The following is the probability distribution of estimated usage of the raw material for the month of December 2012.

Usage (units)	Probability
1800	0.06
1900	0.14
2010	0.30
2100	0.16
2200	0.13
2300	0.10
2400	0.07
2500	0.04

Stock-outs will cost the company Sh.100 per unit and the average monthly holding cost will be Sh.10 per unit

Required

Determine the optimal safety stock (12 marks)
Compute the probability of being out of stock. (2 marks)
(Total: 20 marks)

QUESTION TWO

Samaki Ltd., a company based in Mombasa, exports vital fishing hooks to Madagascar. The demand for the hooks is constant and Samaki Ltd., is able to predict the annual demand with considerable accuracy. The predicted demand for the next couple of year is 200,000 hooks per year.

Samaki Ltd. purchases its hooks from a manufacturer in Mombasa at a price of Sh.400 per hook. In order to transport the purchases from Mombasa to Madagascar, Samaki Ltd. must charter a ship. The charter services usually charge Sh.20,000 per trip plus Sh.40 per hook (this includes the cost of loading the ship). The ships have a capacity of 10,000 hooks. The placing of each order including arranging for the ship requires 5 h ours of employee time. It takes about a week for an order to arrive at the Samaki Ltd. warehouse in Madagascar. The warehouse has a capacity of 15,000 hooks.

When a ship arrives at the Samaki warehouse, the hooks can be unloaded at a rate of 25 hooks per hour per employee. The unloading equipment used by each employee is rented from a local supplier at a rate equivalent to Sh.100 per hour. Supervisory time for each shipload is about 4 hours. The employees working in the warehouse have several tasks:

Placing the hooks into storage, after they are unloaded which can be done at the rate of about 40 per hour.
Checking, cleaning etc. of the hooks in inventory requires about one-half hour per hook per year.
Removing a hook from inventory and preparing it for shipments to a customer requires about one-eighth of an hour.
Security guards general maintenance, etc. require about 10,000 hours per year.

The average cost per hour of labour is equivalent to Sh.200 (including fringe benefits). Samaki Ltd. has developed the following prediction equation for its general overhead (excluding shipping materials, fringe benefits, and equipment rental):

Predicted overhead for the year = Sh.20,000,000 + (Sh.160 x Total labour hours)
The materials used to ship one hook to a customer costs Sh.20 and the delivery costs average out to about Sh.40 per hook.
The company requires a before-tax rate of return of 20 per cent on its investment.
The ordering policy from the manufacturers by Samaki Ltd., is based on an EOQ. Model, which is determined by the demand for hooks in Madagascar.

Required

Determine the quantity that should be ordered each time and the re-order level (15 marks)
If the true overhead prediction equation is:
Sh.16,000,000 + (Sh.240 x Total labour hours), what is the cost of the prediction error? (10 marks)
(Total: 25 marks)

QUESTION THREE

Peter Oloo is a fishmonger in Kisumu. As a result of adverse business changes in the region, the supply and demand for fish are subject to random variations making it difficult to project the next day's business.

Management accounts in relation to the previous 300 days reveal the following mode of behaviour:

Number of fish purchased from fishermen	Number of days	Number of fish Sold to customers	Number of days
100	30	100	45
200	60	200	60
300	90	300	90
400	90	400	75
500	30	500	30
	300		300

Peter Oloo buys each fish at Sh.40 and sells it for Sh.60 if sold on the same day; if the fish is sold the following day it will fetch only Sh.20. If not sold during the second day its value drops to zero and Peter Oloo donates it to children's home. Peter Oloo's Policy is to satisfy the days demand from the fresh fish first; and any further demand will be satisfied from the stock of fish from previous day. Failure to satisfy demand costs Peter Oloo Sh.20 for every fish supplied to the customer. There are no backorders in the business.

Required:

Simulate Peter Oloo's operations for 8 days clearly indicating profits made each day. (16 marks) What are the average daily profits for Peter Oloo?
Use the following random numbers
573423709751483681320931644925928345 (4 marks)
(Total: 20 marks)

COMPREHENSIVE ASSIGNMENT NO.2

TO BE SUBMITTED AFTER LESSON 4

To be carried out under examination conditions and sent to the Distance Learning Administrator for marking by the University.

EXAMINATION PAPER. **TIME ALLOWED: THREE HOURS.**

ANSWER ALL QUESTIONS

QUESTION ONE

You are required to prepare:

a. the standard cost of 1,000 X 1 lb tins of product M sold;

b. a production operating statement for the ingredients section of direct materials for the month of April 1991 to show direct material cost variance analysed into materials price, materials mixture and materials yield variances.

Standard cost information to produce one ton of product M is as follows:

Direct materials		**Quantity lbs**	**Price per lb**
			£
Ingredients	A	800	0.07
	B	600	0.04
	C	1,000	0.06

Direct labour:	**hour's**	**Rate per hour**
		£
Blending	24	0.06
Filing	50	0.70
Packing	54	0.50
Overhead		0.40

Packing materials:	**Price per gross**
	£
Tins—1 lb. each	3.60
Outer cartons to hold 5 X 1 lb tins	2.40
Outer carton labels	1.20
Labels for tins	1.80

An allowance of 5% should be included in the standard cost of packing materials for spoilage before and during usage.

For every 105 tins packed and delivered from production ready for sale, 5 tins deteriorate in finished stock and are disposed of with no salvage value.

The following information is given for the month of April 1991:

Actual production:
42,000 X 1 lb tins

Actual materials used		lbs.	Price per lb.
			£
Ingredients	A	17,000	0.080
	B	10,000	0.036
	C	21,000	0.050

Answers should be given correct to two decimals of £1.

(CIMA)

QUESTION TWO

A company's forecasts indicate a net profit of 5% on annual sales of 20,000 units at £20 each and with variable costs estimated at £15 per unit.

This is regarded by the managing director to be unsatisfactory and so four proposals are put forward to improve the situation.

a. Calculate for each proposal separately:

   i. The new net profit achieved;
   ii. The percentage return on sales value;
   iii. The new break-even sales volume in units.

b. State very briefly the key problems likely to be encountered in achieving each proposal. The four proposals are:

1. Administration now on a regional basis to be centralised and so reduce fixed costs by £12,000.
2. Institute a cost reduction programme to save £1 per unit of variable cost.
3. Increase number of units sold by 20% by means of a 5% reduction of selling prices.
4. Increase selling price by 10% although this would reduce the number of units sold by 5%.

(CIMA)

QUESTION THREE

A manufacturing company operates an integrated standard costing system.

You are required to:

a. record the information given below in the books of the company for the three months ended 30 April, 1991;
b. prepare for presentation to management the interim:

   i. profit and loss account for the three months ended 30 April, 1991 showing clearly variances from standard;
   ii. Balance sheet as at 30th April 1991.

The balances in the integrated ledger at the beginning of the current financial year on 1st February, 1991 were as follows:

	£000's
Finished goods stock, at standard	95
Work-in-progress, at standard	45

Raw materials stocks, at standard	110
Debtors	160
Trade creditors	105
Other creditors, including tax and dividends	130
Cash	65
Investments	300
Fixed assets	825
Provision for depreciation of fixed assets	115
Ordinary share capital	800
8% Debentures	300
Reserves and profit and loss account	150

The budget for the year commencing 1st February, 1991 included the following:

Production in standard hours 720,000

	£000's
Factory overhead	288
Administration, selling and distribution expenses	110
Sales, at standard selling prices	1,500

The following information is given for the three months ended 30th April 1991:

	£000's
Sales, actual quantities at:	
Actual selling prices	400
Standard selling prices	380
Income from investments	5

At actual cost

	£000's
Payments:	
Received from debtors	395
Made to trade creditors	200
Made to other creditors	33
Made for wages	120
Credit transactions:	
Purchases of raw materials	154
Indirect materials and expenses	37
Administration, selling and distribution expenses	30
Costs:	
Direct wages, 180,000 actual hours	94
Indirect wages	26
Provision for depreciation of fixed assets, to be charged to factory overhead	15
Debenture interest accrued	6

At standard prices

Stores transactions:	**£000's**
Purchases of raw materials on credit	144
Raw materials issued to production	129
At standard cost	
Stocks at 30th April, 1991	**£000's**
Finished goods	120
Work-in-progress	41
Raw materials	125
Standard content of production input:	
Raw materials	130
Direct wages, 190,000 standard hours @ £0.5 per hour	95
Factory overhead, 190,000 standard hours @ £0.4 per hour	76

(CIMA)

QUESTION FOUR

Because of the inadequacy of profit measurement on an absorption costing basis for the individual products made by your company, you have turned your attention to the presentation of product marginal costs, the determination of unit contributions, and the expression of these contributions in terms of the limiting factor within the business.
Your are required to report to your managing director on the relevance this approach might have in cost-estimating and selling price-fixing.

(CIMA)

QUESTIONFIVE

As management accountant you are required to prepare a report for your board of directors on measures which could be used to assist in controlling the level of stocks and total working capital on a regular monthly basis.

To illustrate the measures you propose and their purpose all the following information from the latest available accounts should be used in your report:

Annual net cash flow generated by operations	130
Annual cost of sales	1,000
Annual purchases of raw materials 330	
Current liabilities (creditors, tax and dividends)	200
Current assets:	
Debtors	125
Stock of raw materials	55
Stock of finished goods	60
Cash	10

(CIMA)

END OF COMPREHENSIVE ASSIGNMENT No.2

NOW SEND YOUR ANSWERS TO THE DISTANCE LEARNING CENTRE FOR MARKING

LESSON FIVE

RESOURCE ALLOCATION DECISIONS

OBJECTIVES
Further studies in Linear Programming , Transportation and Assignment modelling.

INSTRUCTIONS

1. Read the Study Text and Chapter 26 of Management and Cost Accounting by Colin Drury
2. Attempt the reinforcing questions at the end of the lesson under examination conditions
3. Compare your answers with those given in Lesson 10

CONTENTS

1. Linear Programming: Simplex Method
2. Sensitivity Analysis in Linear Programming
3. Transportation problem as a particular form of the general linear programming model
4. Assignment as a special form of the Transportation Problem

1. LINEAR PROGRAMMING

Algebraic Solution: The Simplex Method

This chapter introduces the general method called the simplex algorithm, which is designed to solve any linear programme. The information that can be secured from the simplex method goes beyond determining the optimum values of the variables. Indeed, it provides important economic interpretations of the problem and shows how sensitivity analyses can be carried out algebraically.

The simplex method solves linear programming in iterations where the same computational steps are repeated a number of times before the optimum is reached.

The Standard Form of the LP Model

An LP model may include constraints of the types ≤, =, and ≥. Moreover, the variables may be non-negative or unrestricted in sign. In order to develop a general solution method, the LP problem must be put in a common format, which we call the standard form. The properties of the standard LP form are:

1) All the constraints are equations with non-negative right-hand side.
2) All the variables are non-negative.
3) The objective function may be maximization or minimization.

We now show how any LP model can be put in the standard format.

Constraints

1) A constraint of the type ≤ (≥) can be converted to an equation by adding a slack variable to (subtracting a surplus variable form) the left side of the constraint.

   For example, in the constraint: $X_1 + 2X_2 \leq 6$ we add a slack $S_1 \geq 0$ to the left side to obtain the equation: $X_1 + 2X_2 + S_1 = 6$, $S_1 \geq 0$.
   If the constraint represents the limit on the usage of a resource, S_1 will represent the slack or unused amount of the resource.

   Next consider the constraint: $3X_1 + 2X_2 - 3X_3 \geq 5$, we subtract a surplus variable $S_2 \geq 0$ from the left side to obtain the equation.
   $3X_1 + 2X_2 - 3X_3 - S_2 = 5$, $S_2 \geq 0$

2) The right side of an equation can always be made non-negative by multiplying both sides by -1.

   For example, $2X_1 + 3X_2 - 7X_3 = -5$ is mathematically equivalent to $-2X_1 - 3X_2 + 7X_3 = +5$.

3) The direction of an equation is reversed when both sides are multiplied by -1. For example, whereas $2 < 4$, $-2 > -4$. Thus the inequality $2X_1 - X_2 \leq -5$ can be replaced by $-2X_1 + X_2 \geq 5$.

Variables

Unrestricted variable Yi can be expressed in terms of two non-negative variables by using the substitution: $Y_i = Y_i' - Y_i''$, $Y_i', Y_i'' \geq 0$

The substitution must be effected throughout all the constraints and in the objective function.

The LP problem is normally solved in terms of Yi′ and Yi″, from which Yi is determined by the reverse substitution.

Objective Function

Although the standard LP model can be either the maximization or the minimization type, it is sometimes useful to convert one form to the other.

The maximization of a function is equivalent to the minimization of the negative of the same function, and vice versa.

For example: Max. $Z = 5X_1 + 2X_2 + 3X_3$ is mathematically equivalent to
Min. $(-Z) = -5X_1 - 2X_2 - 3X_3$

Equivalence means that for the same set of constraints the optimum values of X1, X2 and X3 are the same in both cases. The only difference is that of the values of the objective functions, although equal numerically, will appear with opposite signs. Example: Write the following LP model in the standard form:

Minimum:	$Z = 2X_1 + 3X_2$ ←→	Minimum:	$Z = X_1' - 2X_1'' + 3X_2$
Subject to:	$X_1 + X_2 = 10$	Subject to:	$X'_1 - 2X''_1 + 3X_2 = 10$
	$-2X_1' + 3X_2 \leq -5$		$2X'_1 - 2X''_1 - 3X_2 - S_2 = 5$
	$7X_1 - 4X_2 \leq 6$		$7X'_1 - 7X''_1 - 4X_2 + S_3 = 6$
	X_1 Unrestricted		$X_1', X_1'', X_2, S_2, S_3 \geq 0$
	$X_2 \geq 0$		

The Simplex Method

In the graphical solution the optimum solution is always associated with a corner (or extreme) point of the solution space. The simplex method is based fundamentally on this idea. It employs an iterative process that starts at a feasible corner point normally the origin, and systematically moves from one feasible extreme point to another until the optimum point is eventually reached.

There are two rules that govern the selection of the next extreme point in the simplex method.

1) The next corner point must be adjacent to the current one.
2) The solution can never go back to a previously considered extreme point.

Representation of the solution space by the standard form

We shall use the example previously stated for the graphical solution. The standard form of the model is given by:

Maximise: $Z = 3X_E + 2X_I + OS_1 + OS_2 + OS_3 + OS_4$

Subject to:

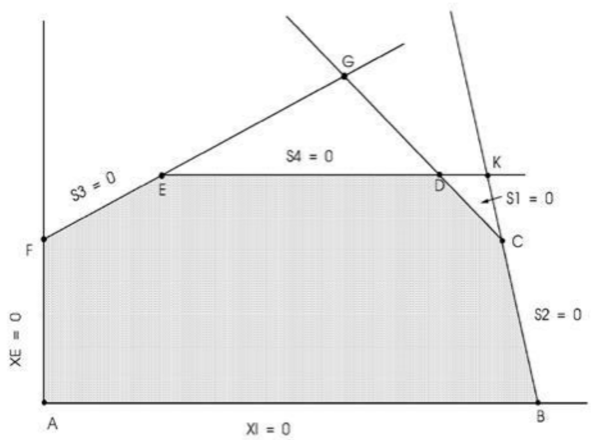
$$
\begin{aligned}
X_E + 2X_I + S_1 &= 6 \\
2X_E + X_I + S_2 &= 8 \\
-X_E + X_I + S_3 &= 1 \\
X_I + S_4 &= 2
\end{aligned}
$$

$$X_E, X_I, S_1, S_2, S_3, S_4 \geq 0$$

The figure depicts the solution space. Every point in this space can be represented in terms of the variables X_E, X_I, S_1, S_2, S_3 and S_4 of the standard form. (To show this point, observe that Si = 0, i = 1,2,3 and 4 reduces the associated equation into an edge of the solution space. For example: Si = 0 is equivalent to XE + 2XI = 6, which represents edge CD. Having S1 > 0 will move the feasible points from edges toward the interior of the solution space).

Our main interest is to identify the extreme points algebraically. When we examine the figure we note that the values of X_E, X_1, S_1, S_2, S_3 and S_4 associated with the extreme points A, B, C, D, E, and F have the following definite pattern with regard to whether or not their values are equal to zero.

Extreme Point	*Zero Variables*	*Non-zero Variables*
A	X_E, X_I	S_1, S_2, S_3, S_4
B	S_2, X_I	S_1, X_E, S_3, S_4
C	S_2, S_I	X_1, X_E, S_3, S_4
D	S_4, S_I	X_1, X_E, S_3, S_2
E	S_4, S_3	X_1, X_E, S_1, S_2
F	S_4, X_E	X_1, S_3, S_1, S_2



Two observations are immediately evident:

Since the standard form has four equations and six unknowns, each extreme point must have two (= 6 - 4) variables at zero level.

2) Adjacent extreme points differ in only one variable.

The first observation indicates that we can identify the extreme points of the solution space algebraically by setting zero as many variables as the difference between the number of unknowns and the number of equations. This is a unique property of the extreme points.

The unique property of the extreme points yields the following general procedure for determining the extreme points algebraically. Assume that the standard form has m equations and n variables ($m \leq n$) together with the non-negativity restrictions. All the feasible extreme points are determined by considering all the unique non-negative solutions of the m equations in which exactly n - m variables are set equal to zero.

Mathematically, the unique solutions resulting from setting n - m variables equal to zero are called basic solutions. If a basic solution satisfies the non-negativity restrictions, it is called a feasible basic solution. The variables set equal to zero are called non basic variables, the remaining ones are called basic variables.

The general conclusion is that the algebraic definition of basic solutions in the simplex method now takes the place of the extreme point in the graphical solution space.

The second of the two observations is very useful computationally because the simplex method moves from a current extreme point to an adjacent one. Since adjacent extreme points differ only in one variable we can determine the next (adjacent) extreme point by interchanging a current nonbasic (zero) variable with a current basic valuable. This idea greatly simplifies the simplex method computations.

The basic-nonbasic interchange process gives rise to two suggestive names. The entering variable is a current nonbasic variable that will "enter" the set of basic variables at the next (adjacent extreme point) iteration. The leaving variable is a current basic variable that will "leave" the basic solution in the next iteration.

Computational details of the Simplex Algorithm

The steps of the simplex algorithm

follow: Step 0:

Using the standard form, determine a starting basic feasible solution by setting n - m appropriate (nonbasic) variables at zero level.

Step 1:

Select an entering variable from among the current (zero) non-basic variables which, when increased above zero, can improve the value of the objective function. If none exists, stop; the current basic solution is optimal. Otherwise go to step two.

Step 2:

Select a leaving variable from among the current basic variables that must be set to zero (become nonbasic) when the entering variable becomes basic.

Step 3:

Determine the new basic solution by making the entering variable basic and the leaving variables nonbasic. Go to step 1.

The details of the simplex method will be explained by using the same example. This will require expressing the objective function and all the constraints of the standard form as:

$$\left.\begin{array}{r} X_E + 2X_I + S_1 \\ 2X_E + X_I + S_2 \\ -X_E + X_I + S_3 \\ X_I + S_4 \end{array}\right\} \begin{array}{l} = 6 \\ = 8 \\ = 1 \\ = 2 \end{array} \qquad \text{CONSTRAINT EQUATIONS}$$

$$Z - 3X_E - 2X_I = 0 \qquad - \qquad \text{OBJECTIVE FUNCTION}$$

As mentioned above, the starting solution is determined from the constraint equations by setting two (= 6 - 4) variables equal to zero, provided that the resulting solution is unique and feasible. It is evident that by putting $X_E = X_I = 0$, we immediately obtain $S_1 = 6$, $S_2 = 8$, $S_3 = 1$ and $S_4 = 2$. We can thus use this point as a starting feasible solution. The corresponding value of Z is zero, since both X_E and X_I are zero. As a result, by changing the objective equation so that its right side is equal to zero, we can see that the right sides of the objective and constraint equations will automatically yield the complete starting position. This is always the case when the starting solution consists of all slack variables.

We can summarize the foregoing information in a convenient tableau form as follows:

Basic	Z	X_E	X_I	S_1	S_2	S_3	S_4	Solution	(RHS)
S_1	0	1	2	1	0	0	0	6	S_1—equation
S_2	0	2	1	0	1	0	0	8	S_2—equation
S_3	0	-1	1	0	0	1	0	1	S_3—equation
S_4	0	0	1	0	0	0	1	2	S_4—equation
Z	1	-3	-2	0	0	0	0	0	Z—equation

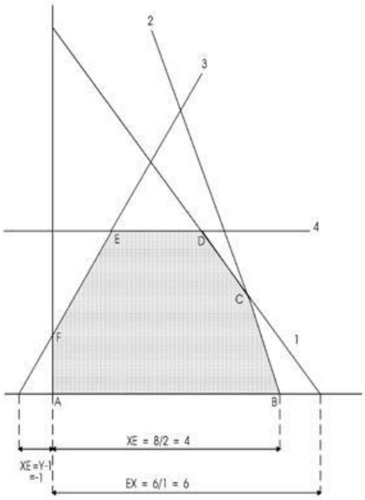
The information in the tableau reads as follows. The "basic" column identifies the current basic variables S_1, S_2, S_3, and S_4 whose values are given in the "solution" column. This implicitly assumes that the nonbasic variables X_E and X_I (those not present in the "basic" column) are at zero level. The value of the objective function is Z = 0, as shown in the solution column.

How do we know if the current solution is the best (optimum)? By inspecting the Z-equation, we note that the current zero variables, X_E and X_I, both have negative coefficients, which is equivalent to having positive coefficients in the original objective function. Since we are maximizing, the value of Z can thus be improved by increasing either X_E or X_I above zero level. However, we always select the variable with the most negative objective coefficient because computational experience has shown that such a selection is more likely to lead to the optimum solution rapidly.

The above observation is the basis for what we call the optimality condition of the simplex method. It states that, in the case of maximization, if all the nonbasic variables have non-negative coefficients in the Z— equation of the current tableau, the current solution is optimal. Otherwise, the non-basic variable with the most negative coefficient is selected as the entering variable.

Applying the optimality condition to the starting tableau, we select X_E as the entering variable. At this point, the leaving variable must be one of the current basic variables S_1, S_2, S_3 and S_4. This is achieved by using the feasibility condition which selects the leaving variable as the current basic variable that will be the first to reach zero level when the entering variable X reaches its maximum value at the adjacent extreme point. Of course, we need to do this without the use of the graphical solution. However, the graphical solution can assist us to develop the feasibility condition algebraically.

Considering our example. The maximum feasible value of X_E equals the smallest positive intercept of the constraints with X_E axis. Algebraically, each of these intercepts is equal to the "ratio" of the right-hand side of the constraint equation to the associated positive coefficient of the entering variable X_E. If the coefficient of X_E is negative or zero, the associated constraint does not intersect X_E in the positive direction. This is evident in the figure, where constraint (3) with a negative X_E coefficient (= -1) intersects the X_E axis in the negative direction and constraint (4) with zero X_E coefficient is parallel to the X_E axis. On the other hand, the intercepts of constraints (1) and (2) are given by $X_E = 6/1 = 6$ and $X_E = 8/2 = 4$. Thus X_E reaches its maximum value of 4 at B, at which point S_2 will be leaving variable.



The ratios (intercepts) defined above and the leaving variable can be determined directly from the simplex tableau. First, identify the column under the entering variable X_E and cross out all its negative and zero elements in the constraint equations. Then, excluding the objective equation, take the ratios of the right-side elements of the equations to the uncrossed-out elements under the entering variable. The leaving variable is the current basic variable associated with the minimum ratio.

The starting tableau is repeated below after applying the feasibility condition (i.e computing ratios and identifying the leaving variable). For the purpose of computing the next iteration, we identify the column under the entering variable as the entering column. The row associated with the leaving variable will be called the pivot equation and the element at the intersection of the entering column and the pivot equation will be called the pivot element.

Basic	*Z*	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*	*Ratio*
S_1	0	1	2	1	0	0	0	6	6
S_2	0	2	1	0	1	0	0	8	4
S_3	0	-1	1	0	0	1	0	1	—
S_4	0	0	1	0	0	0	1	2	—
Z	1	-3	-2	0	0	0	0	6	

Pivot Row or Pivot Equation

Entering Column

After determining the entering and leaving variables (by applying the optimality and feasibility conditions), the next iteration (new basic solution) is determined by applying the Gauss-Jordan method. The method effects a change in basis by using two types of computations.

1. Type 1 (pivot equation): New pivot equation = Old pivot equation/pivot element.
2. Type 2 (All other equations including Z): New equation = Old equation - (Its entering columns coefficient) x (New pivot equation).

Type 1 computations make the pivot element equal to 1 in the new pivot equation, whereas type 2 computations create zero coefficients everywhere else in the entering column.

Applying type 1 to the starting tableau, we divide the S_2 equation by the pivot element 2. Since X_E takes the place of S_2 in the basic column, type 1 will lead to the following starting tableau.

Basic	*Z*	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*	*Ratio*
S_1	0	0	{3/2}	1	-½	0	0	2	2/(3/2) = {4/3}
[X_E	0	1	½	0	½	0	0	8/2 = 4]	4/½ = 8
S_3	0	0	3/2	0	½	1	0	5	5/(3/2) = 10/3
S_4	0	0	1	0	0	0	1	2	2/1 = 2
Z	1	0	-½	0	3/2	0	0	12	

Notice that the "solution" column yields the new value of X_E (=4), which equals the minimum ratio of the feasibility condition.

To complete the tableau we carry out the following type 2 computations.

1. Z equation: Old Z equation — (-3) X New pivot equation = New Z equation
2. S_1 equation: Old S_1 equation — (1) X New pivot equation = New S_1 equation
3. S_3 equation: Old S_3 equation — (-1) X New pivot equation= New S_3 equation
4. S_4 equation: The new S_4 equation is the same as the old S_4 equation because its entering column coefficient is zero.

We can see this above in the new tableau. The new solution gives X_E = 4 and X_1 = 0 (point B in the graph). The value of Z has increased from 0 to 12. The increase follows because each unit increase in X_E contributes 3 to the value of Z; thus total increase in Z is 3 x 4 = 12.

Notice that the new tableau has the same properties as the preceding one: namely once the non basic variables X_1 and S_2 are set equal to zero, the values of the basic variables are immediately given in the solution column.
This is precisely what the Gauss-Jordan method accomplishes.
Examining the last tableau, the optimality condition selects X_1 as the entering variable because its Z coefficient is -1/2. The feasibility condition then shows that S_1 is the leaving variable. The ratios shown in the last tableau indicate that X_1 enters the basic solution at the value of 4/3 (= minimum ratio), thus improving the value of the objective function by (4/3) x (1/2) = 2/3.

The following Gauss-Jordan operations will produce the new tableau.

i.	New pivot (S_1) equation	=	Old S_1 equation divided by (3/2)
ii.	New Z equation	=	Old Z equation — (-½) x New pivot equation
iii.	New X_E equation	=	Old X_E equation — (½) x New pivot equation
iv.	New S_3 equation	=	Old S_3 equation — (3/2) x New pivot equation
v.	New S_4 equation	=	Old S_4 equation — (1) x New pivot equation

The computations lead to the following tableau:

Basic	*Z*	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*
X_1	0	0	1	2/3	-1/3	0	0	4/3 = 1 1/3
X_E	0	1	0	-1/3	2/3	0	0	10/3 = 3 1/3
S_3	0	0	0	-1	1	1	0	3
S_4	0	0	0	-2/3	1/3	0	1	2/3
Z	1	0	0	1/3	4/3	0	0	12 2/3

The solution yields X_E = 3 1/3 and X_1 = 1 1/3 (point C). The value of Z is 12 2/3. This increase (12 2/3 - 12) = 2/3 is the result of X_1 increasing from 0 to 4/3, with each unit increase accounting for 1/2 in the objective function. The total increase in Z thus equals (4/3) x 1/2 = 2/3.

The last tableau is optimal because none of the non-basic variables has a negative coefficient on the Z equation. This completes the simplex method computations.

The simplex method is applied above to a maximization problem. In considering a minimization problem, we need only change the optimality condition so that the entering variable is selected as the variable having the most positive coefficient in the Z equation. The feasibility condition is the same for both problems. We summarize the two conditions here for convenience.

Optimality Condition

The entering variable in maximization (minimization) is the non-basic variable with the most negative (positive) coefficient in the Z equation. When all the non-basic coefficients in the Z equation are non-negative (non-positive), the optimum is reached.

Feasibility Condition

For both the maximization and minimization problems, the leaving variable is the basic variable having the smallest ratio (with positive denominator).

3. INTERPRETING THE SIMPLEX TABLEAU - SENSITIVITY ANALYSIS:

We can now see that our attention must be directed to reading, interpreting and analysing the (simplex) results. It is erroneous, however, to assume that only one can interpret the simplex tableau without having adequate knowledge of how and why the simplex method works.
It will be disappointing to you to think that all we can get out of the optimum simplex tableau is a list of variables and their optimum values. The fact is that the simplex tableau is "loaded" with important information, the least of which are the optimum values of the variables. The following list summarizes the information that can be obtained from the simplex tableau, either directly or with simple additional computations.

1. The optimum solution
2. The status of resources
3. The unit worth of each resource
4. The sensitivity of the optimum solution to changes in availability of resources, coefficients of the objective function, and usage of resources by activities.

The first three items are readily available in the optimum simplex tableau. The fourth item requires additional computations that are based on the information in the optimum solution.

To demonstrate these items we use the same example, which we repeat here for convenience. The optimum tableau is given as:

Basic	*Z*	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*
X_1	0	0	1	2/3	-1/3	0	0	4/3 = 1 1/3
X_E	0	1	0	-1/3	2/3	0	0	10/3 = 3 1/3
S_3	0	0	0	-1	1	1	0	3
S_4	0	0	0	-2/3	1/3	0	1	2/3
Z	1	0	0	1/3	4/3	0	0	12 2/3

Maximise: $Z = 3X_E + 2X_I$

Subject to:

$$X_E + 2X_I + S_1 = 6$$
$$2X_E + 2X_I + S_2 = 8$$
$$-X_E + 2X_I + S_3 = 1$$
$$X_I + S_4 = 2$$

$$X_1, X_E, S_1, S_2, S_3, S_4 \geq 0$$

Optimum Solution

From the stand point of implementing the LP solution, the mathematical classification of the variables as basic and non-basic is of no importance and should be totally ignored in reading the optimum solution. The variables not listed in the "basic" column necessarily have zero values. The rest have their values in the solution column. In terms of the optimum solution of our example, we are interested primarily in the product mix of exterior and interior paint, that is, the decision variables X_E and X_I. From the optimum tableau we have the following summary:

Decision Variable	Optimum Value	Decision
X_E	3 1/3	Produce 3 1/3 tons of exterior paint
X_I	1 1/3	Produce 1 1/3 tons of interior paint
Z	12 1/3	Resulting profit is 12 2/3 thousand Shs

Status Resources

We had classified constraints as scarce and abundant, depending respectively on whether or not the optimum solution "consumes" the entire available amount of the associated resource. Our objective is to secure the information from the optimum tableau. First we must clarify one point. Speaking of resources implies that there is a maximum limit on its availability, which means that the constraint must originally be of type ≤. Thus constraints of the type ≥ cannot physically represent a resource restriction; rather, they imply that the solution must meet certain requirements, such as satisfying minimum demand or minimum specifications.

In our problem we have four constraints of the type ≤. The first two (representing raw materials usage) are "authentic" resource restrictions. The third and fourth constraints deal with demand limitations imposed by the market conditions. We can think of these constraints as limited "resources", since increasing demand limits is equivalent to expanding the company's share in the market. Monetarily, this has the same effect as increasing availability of physical resources (such as raw materials) through allocation of additional funds. Following the discussion above, the status of resources (abundant or scarce) in any LP model can be secured directly from the optimum tableau by observing the values of the slack variables. In our problem we have the following summary:

Resource	Slack	Status of Resource
Raw material A	$S_1 = 0$	Scarce
Raw material B	$S_2 = 0$	Scarce
Limit on excess of interior over exterior paint	$S_3 = 3$	Abundant
Limit on demand for interior paint	$S_4 = 2/3$	Abundant

Positive slack means that the resource is not used completely, thus is abundant whereas a zero slack indicates that the entire amount of the resource is consumed by the activities of the model. In the summary above we see that demand limitations (resource 3 and 4) are "abundant"; hence any increase in their maximum limit will simply make them "more" abundant without affecting the optimum solution.

The resources that can be increased for the purpose of improving the solution (increasing profit) are raw materials A and B, since the optimum tableau shows they are scarce. A logical question would naturally arise: which of the scarce resources should be given priority in the allocation of additional funds in order to improve profit most advantageously? We answer this question below when we consider the per unit worth of the different resources.

Unit Worth of a Resource

A unit worth of a resource is the rate of improvement in the optimum value of Z as a result of increasing the variable amount of that resource. This point was analyzed graphically. The graphical analysis shows that the unit worth of resources 1,2,3 and 4 are:

$Y_1 = 1/3$ thousand Shillings/additional ton of material A
$Y_2 = 4/3$ thousand Shillings/additional ton of material B
$Y_3 = Y_4 = 0$

This information is readily available in the optimum simplex tableau. Look at the coefficient in the Z equation under the starting basic variables S_1, S_2, S_3, and S_4, which we reproduce here for convenience.

Basic	X_E	X_I	S_1	S_2	S_3	S_4	*Solution*
Z	0	0	1/3	4/3	0	0	12 2/3

These coefficients exactly equal Y_1, Y_2, Y_3 and Y_4. The theory of LP tells us that it is always possible to secure the unit worth of a resource from coefficients of the starting basic variables in the optimum Z equation. There should be no confusion as to which coefficient applies to which resource, since Si is uniquely with resource i. Consider the optimal Z equation of the model.

$$Z = 12\ 2/3 - (1/3\ S_1 + 4/3\ S_2 + 0\ S_3 + 0\ S_4)$$

If we increase S_1 from its current zero level to a positive level, the value of Z will decrease at the rate of 1/3 thousand shillings/ton. But an increase in S_1 is actually equivalent to reducing resource 1 (raw material A), as could be seen from the first constraint: $XE + 2XI + S1 = 6$.

We thus conclude that a reduction in the first resource will reduce the objective function at the rate of 1/3 thousand shillings/ton. Since we are dealing with linear functions, we can reciprocate the argument by concluding that an increase in the first resource will increase Z at the rate of 1/3 thousand shilling/ton. A similar argument applies to resource 2.

Turning to resource 3 and 4, we find that their unit worth is zero ($Y3 = Y4 = 0$). This should be expected, since the two resources are already abundant. This will always be the case when the slack variables are positive. In spite of the fact that we have associated monetary values with the unit worth variables yi, we should not think of them in the same terms as, for example, the real price we may pay to buy the resource. Instead, they are economic measures that quantify the unit worth of a resource from the viewpoint of the optimal objective value. The value of these economic measures will vary as we change the constraints even though we may be utilizing the same physical resources. For this reason, economists prefer to use the apt terms shadow price, imputed price, or more technically, dual price to describe the unit worth of resource.

Observe that the definition of shadow prices (unit worth of a resource) gives us the rate of improvement in optimum Z. It does not specify the amount by which a resource can be increased while maintaining the same rate of improvement. Logically, in most situations we would expect an upper limit beyond which any increase of the resource would make its constraint redundant, with the result that a new basic solution and hence, new shadow prices, must be sought. The presentation below addresses the point of determining the maximum change in the availability of a resource before its constraint becomes redundant.

Maximum change in resource availability

We normally use the shadow prices to describe which resources should be expanded. Our goal is to determine the range of variation in the availability of a resource that will yield the unit worth (shadow price) encountered in the optimum tableau. To achieve this, we need to perform additional computations. We shall first demonstrate how the procedure works and then show how the same information can be secured from the optimum tableau.

Suppose that we consider the first resource in our model by the amount Δ_1 , meaning that available raw material A is 6 + Δ_1 tons. If Δ_1 is positive, the resource increases; if it is negative, the resource decreases. Although we normally shall be interested in the case where the resource increases ($\Delta_1 > 0$), we shall present both cases for the sake of generalization. How is the simplex tableau changed by effecting the change Δ_1. The simplest way to answer the question is to increase Δ_1 to the right side of the first constraint in the starting tableau and then apply the same arithmetic operations that were used to develop the successive iterations. If we keep in mind that the right side constraints are never used as pivot elements, it is evident that the change Δ_1 will affect only the right side of each iteration.

Basic	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*
X_1	0	1	2/3	-1/3	0	0	1 1/3 + 2/3 Δ_1
X_E	1	0	-1/3	2/3	0	0	3 1/3 - 1/3 Δ_1
S_3	0	0	-1	1	1	0	3 - 1 Δ_1
S_4	0	0	-2/3	1/3	0	1	2/3 - 2/3 Δ_1
Z	0	0	1/3	4/3	0	0	12 2/3 + 1/3 Δ_1

In the optimum iteration the constraints (1 1/3, 3 1/3, 3, 2/3, 12 2/3) represent the right side of the optimum tableau before Δ_1 is effected, and (2/3, -1/3, -1, -2/3, 1/3) are the coefficients under S_1 in the same tableau. Why S_1?
Because it is uniquely associated with the first constraint. In other words, for the right side changes in the second, third and fourth constraints, we should use the coefficients under S_2, S_3 and S_4, respectively.

What does this information mean?

Since we have concluded that the change Δ_1 will affect only the right side of the tableau, it means that such a change can only affect the feasibility of the solution. Thus Δ_1 may not be changed in a manner that will make any of the (basic) variables negative. This means that Δ_1 must be restricted to the range that will maintain the non-negativity of the right side of the constraint equations in the optimum tableau.

That is

$X_I = 4/3 + 2/3\,\Delta_1 \geq 0$ (1)
$X_E = 10/3 - 1/3\,\Delta_1 \geq 0$ (2)
$S_3 = 3 - \Delta_1 \geq 0$ (3)
$S_4 = 2/3 - 2/3\,\Delta_1 \geq 0$ (4)

To determine the admissible range for Δ_1, we consider two cases:

Case 1

$\Delta_1 > 0$. Relation (1) is always satisfied for $\Delta_1 > 0$. Relations (2) and (3) and (4), on the other hand, produce the following respective limits.

$\Delta_1 \leq 10$, $\Delta_1 \leq$, 3 and $\Delta_1 \leq 1$

Case 2

$\Delta_1 < 0$. Relations (2), (3) and (4) are always satisfied for $\Delta_1 < 0$, whereas relation (1) yields the limit $\Delta_1 \geq -2$

By combining cases 1 and 2, we see that $-2 \leq \Delta_1 \leq 1$ will always result in a feasibility solution. Any change outside this range (i.e decreasing raw material A by more than 2 tons or increasing it by more than 1 ton) will lead to infeasibility and a new set of basic variables.

Maximum change in marginal Profit/Cost

Just as we did in studying the permissible ranges for changes in resources, we are also interested in studying the permissible ranges for changes in marginal profits (or costs). We have seen graphically that the objective function coefficients can change within limits without affecting the optimum values of the variables (the optimum value of Z will change, though).
In this presentation, we show how this information can be obtained from the optimum tableau.

In the present situation, as in the case of resource changes, the objective equation is never used as a pivot equation. Thus, any changes in the coefficients of the objective function will affect only the objective equation in the optimum tableau. This means that such changes can have the effect of making the solution non-optimal. Our goal is to determine the range of variation for the object coefficients (one at a time) for which the current optimum (solution) remains unchanged.

To illustrate the computations, suppose that the marginal profit or X_E in our model is changed from 3 to 3 + δ_1, where δ represents either positive or negative change. This means that the objective function reads as: $Z = (3 + \delta_1) X_E + 2X_I$.

If we use this information in the starting tableau and carry out the same arithmetic operations used to produce the optimum tableau, the optimum Z equation will appear as:

Basic	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*
Z	0	0	$\frac{1}{3} - \frac{1\delta_1}{3}$	$\frac{4}{3} + \frac{2\delta_1}{3}$	0	0	$12\frac{2}{3} + \frac{10\delta_1}{3}$

This equation is the same as the optimum Z equation before the change δ_1 is effected, modified by terms of δ_1. The coefficients of δ_1 are essentially those in the X_E equation of the optimum tableau, which are
Pivot Element

X_E	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	0	$\frac{10}{3}$

We choose the X_E equation because X_E is the variable whose objective coefficient is being changed by δ_1.

The change δ_1 will not affect the optimality of the problem as long as all the Z equation coefficients of the non-basic variables remain non-negative (maximization) that is:

$1/3 - \delta_1/3 \geq 0$ (1)
$4/3 + 2\delta 1/3 \geq 0$ (2)

Relationship (1) shows that $\delta_1 \geq 1$ and the relation (2) yields $\delta_1 \geq -2$. Both relations limit δ_1 by $-2 \leq \delta_1 \leq 1$. This means that the coefficient of X_E can be as small as 3 + (-2) = 1 or as large as 3 + 1 = 4 without causing any change in the optimal values of the variables. The optimal value of Z, however, will change according to the expression 12 2/3 + 10/3 δ_1 where $-2 \le \delta \le 1$.

The foregoing discussion assumed that the variable whose coefficient is being changed has an equation in the constraints. This is true only if the variable is basic (such as X_E and X_I above). If it is non-basic, it will not appear in the basic column.

The treatment of non-basic variable is straight forward. Any change in the objective coefficient of a non-basic variable will affect only that coefficient in the optimal tableau. To illustrate this point, consider changing the coefficient of S_I (the first slack variable) from 0 to 0 + δ3. If you carry out the arithmetic operations leading to the optimum tableau, the resulting Z equation becomes:

Basic	*X_E*	*X_I*	*S_1*	*S_2*	*S_3*	*S_4*	*Solution*
Z	0	0	$\frac{1}{3} - \delta_3$	$\frac{4}{3}$	0	0	12 2/3

It shows that the only change occurs in the coefficient of S_I, where it is decreased by δ_3. As a general rule, then, all we have to do in the case of a non-basic variable is to decrease the Z coefficient of the non-basic variable by the amount by which the original coefficient of the variable is increased.

Extensions to Linear Programming

In many real situations the solutions to linear programming models make sense only if they have integer values. Rounding off the linear programming solution to the nearest integer sometimes does not produce a feasible solution. In this particular case, simplex method (or graphical method) can be a limitation. We use linear programming techniques to solve such problems.

Integer Programming

It is a technique for solving a linear programming model with an added constraint that the decision variables must only be non-negative integers.

In the case of linear programming there is no single, method that can be used for solving all types of integer linear programming problems.

Two widely used techniques are:

i. The method of integer forms
ii. Branch and bound techniques

Simplex method (or graphical method) is still valid if the required answer gives integer values.

Dynamic programming

It is an extension which finds solutions to problems involving a number of decisions which have to be made sequentially. For example, the amount of a product to be made next month may depend on the amount sold this month and so on.

Thus dynamic programming is a quantitative technique which divides a given problem into stages (or sub-problems which are interrelated). Here we attempt to find a combination of decisions which will maximise overall effectiveness.

Usually, we work backwards from the natural end of the problem until the initial problem is finally solved (as in the decision trees).

The decision made at each stage influences the next stage. This method is also termed as recursive approach.

Some applications of dynamic programming

1. Production and distribution problems.
2. Scheduling inventory control.
3. Resource allocation.
4. Replacement and maintenance problems.

Advantages and Limitations of Dynamic Programming

Advantages

1. In certain types of problems such as inventory control management, Chemical Engineering design, dynamic programming may be the only technique that can solve the problems.
2. Most problems requiring multistage, multiperiod or sequential decision process are solved using this type of programming.
3. Because of its wide range, it is applicable to linear or non-linear problems, discrete or continuous variables, deterministic or stochastic problems.
4. The mathematical techniques used can be adopted to the computer.

Limitations

1. Each problem has to be modelled according to its own constraints and requirements. This requires great experience and ingenuity.
2. The number of state variables have to be kept low to prevent complicated calculations.
1. Where applicable, methods such as simplex are more efficient than general programming approach.

USE OF COMPUTER SYSTEMS IN LINEAR PROGRAMMING

When a computer is to be used for linear programming there are a number of steps:

i. Development of the equations which describe the system;
ii. Set up the matrix and some packages include a matrix generator which allows the data to be specified in a form suitable for the use with the package, converting it to the form required for the application of the mathematical techniques. The input program should include routines for checking the data and provide comprehensive and understandable error messages.
iii. production of the solution; some packages can run in either batch or interactive mode, the former being more appropriate with large problems where the time required to reach a solution is fairly long. Since the size of the matrix grows rapidly as the number of constraints and variables increases, potential users need to investigate that the limitations imposed by the package and the hardware on which it runs are not such as to restrict their problem formulation.
iv. Output of the results in a form suitable for the user; again many packages include a report writer. Users may require the facility to examine the results and possibly modify some of the constraints prior to selecting a set of printed reports.

A package should also:

i. Include facilities for saving program files for use in later runs
ii. Allow sensitivity analysis to be carried out.

4. LINEAR PROGRAMMING: TRANSPORTATION MODEL

In the obvious sense, the model deals with the determination of a minimum cost plan for transporting a single commodity from a number of sources (e.g. factories) to a number of destinations (e.g. warehouses).

The model can be extended in a direct manner to cover practical situations in the area of inventory control, employment scheduling, personnel assignment, cash flow, scheduling, dam reservoir levels, and many others. The model also can be modified to account for multiple commodities. It is basically a LP that can be solved by the simplex method. However, its special structure allows the development of a solution procedure called the transportation technique, that is computationally more efficient. It must be emphasized that the "new" technique essentially follows the exact steps of the simplex method. It can be extended to cover a number of important applications, including the assignment model, the transhipment model. However, the transportation model and its extensions are also special cases of network models.

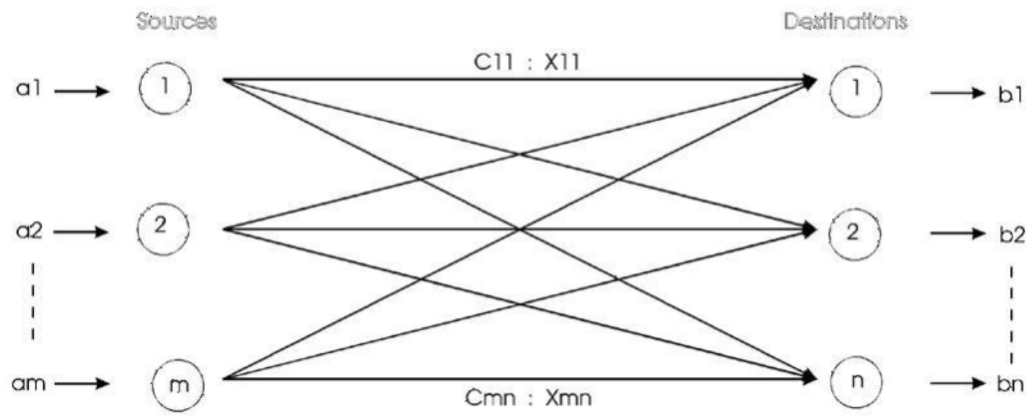
Definition and Application of the Transportation Model:

In the direct sense, the transportation model seeks the determination of a transportation plan of a single commodity from a number of sources to a number of destinations. The data of the model include:

1. level of supply at each source and amount of demand at each destination.
2. The unit transportation cost of the commodity from each source to each destination.

Since there is only one commodity, a destination can receive its demand from one or more sources. The objective of the model is to determine the amount to be shipped from each source to each destination such that the total transportation cost is minimized.

The basic assumption of the model is that the transportation cost on a given route is directly proportional to the number of units transported. The "unit of transportation" will vary depending on the "commodity" transported i.e. a truck load may be a better unit than single product units.



m = Source
n = Destination

A source or destination is represented by a node. The line joining a source with a destination represents the route through which the commodity is transported.

The amount of supply at source i is ai and the demand at destination j is bj. The "unit" transportation cost between source i and destination j is cij

Let xij represent the amount transported from i to destination j.

The LP Model representing the transportation problem is given generally as:

$$minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} cij\ xij\ subject\ to$$

$$(1) \quad \sum_{j=1}^{n} xij \leq ai \qquad i = 1, 2 \ldots m$$

$$(2) \quad \sum_{i=1}^{m} xij \geq bj \qquad j = 1, 2 \ldots n$$

$xij \geq 0$ *for all i and j*

(1) It stipulates that the sum of the shipments <u>from</u> a source cannot exceed its supply.
(2) It requires that the sum of the shipments <u>to</u> a destination must satisfy its demand.

$$\sum_{i=1}^{n} ai$$

The model described above implies that the total supply

$$\sum_{j=1}^{m} bj$$

must at least equal the total demand if the total supply equal the total demand then it is called a balanced transportation model. It differs from the above model in the fact that all constraints are equations.

$$\sum_{j=1}^{n} xij = ai \quad \sum_{i=1}^{m} xij = bj \quad \sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj \quad \begin{matrix} i = 1, 2 \ldots m \\ j = 1, 2 \ldots n \end{matrix}.$$

In real life, this is not necessarily true. However, a transportation model can always be balanced. The balancing, in addition to its usefulness in modelling certain practical situations is important for the development of a solution method that fully exploits the special structure of the transportation model.

A more compact method for representing the transportation model than the linear equations is to use what we call the transportation tableau. It is a matrix form with its rows representing the sources and its columns the destinations. The cost element cij are summarized in the northeast corner of the matrix cell (i,j). This will be the basis for the development of a special simplex-based method for solving the transportation problem.

In some cases, the problem could be unbalanced because the total supply does not equal the total demand.

Where demand exceeds supply, a fictitious or dummy source (plant) can be added with its capacity equal to the difference. The cost of shipping will be zero. Physically, the amount shipped to a destination from a dummy plant will represent the shortage quantity at that destination. We may look at the situation differently, however, by saying that a penalty cost is incurred for every unsatisfied demand unit at the destination centres. In this case the unit transportation cost will equal the unit penalty costs at the various destinations.

In a similar manner, if the supply exceeds the demand, we can add a fictitious or dummy destination that will absorb the difference. Any quantity shipped from a plant to a dummy destination represents a surplus quantity at that plant. The associated unit transportation cost is zero. However, we can charge a storage cost for holding these commodities at the plant, in which case the unit transportation cost will equal the unit storage cost.

At times we can find cells where we cannot allocate any unit due to physical constraints. In that case the transportation cost should be shown as M—being M a very high cost.

Solution of the Transportation Problem:

The basic steps of the transportation technique are:

Step 1:

Determine a starting basic feasible solution.

Step 2:

Determine an entering variable from among the non-basic variables. If all such variables satisfy the optimality condition (of the simplex method), Stop; otherwise, go to Step 3.

Step 3:

Determine a leaving variable (using the feasibility condition) from among the variables of the current basic solution; then find the new basic solution. Return to Step 2

The vehicle of explanation is the table shown below. The unit transportation cost cij is in shillings. The supply and demand are given in number of units.

Destination: 1

Destination:		1	2	3	4	Supply
	1	10 x11	0 x12	20 x13	11 x14	15
Source	2	12 x21	7 x22	9 x23	20 x24	25
	3	0 x31	14 x32	16 x33	18 x34	5
Demand		5	15	15	10 =	45

Determination of the Starting Solution

The general definition of the transportation model requires that $\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$

This requirement results in one dependant equation, which means that the transportation problem has only m + n - 1 independent equations.

Thus, as in the simplex method, a starting basic feasible solution must include m + n - 1 basic variables.

There are three procedures to secure a starting basic feasible solution without utilising artificial variables as in the simplex tableau. This starting basic feasible solution can be obtained easily and directly.

The procedures are:

(A) The northwest-corner rule;
(B) The least-cost method;
(C) Vogel's approximation.

(B) and (C) usually provide better starting solutions in the sense that the associated values of the objective function are smaller.

(A) The northwest-corner method starts by allocating the maximum amount allowable by the supply and demand to the variable x11 (the one in the northwest corner of the tableau). The satisfied column (row) is then crossed out, indicating that the remaining variables in the crossed-out column (row) equal zero. If a column and a row are satisfied simultaneously only one (either one) may be crossed out. (This condition guarantees locating zero basic variables, if any, automatically). After adjusting the amounts of supply and demand for all uncrossed-out rows and columns, the maximum feasible amount is allocated to the first uncrossed-out element in the new column (row).

The process is completed when exactly one row or one column is left uncrossed-out. This procedure is applied to the example.

The resulting starting solution is therefore:

Basic variables: x11 = 5; x12 = 10; x22 = 5; x23 = 15; x24 = 5; x34 = 5
The remaining variables are non-basic at zero level.

Associated transportation cost = Shs 410.

	1	2	3	4	
1	5	10			15
2		5	15	5	25
3				5	5
	5	15	15	10	

Determination of entering variable (Method of Multipliers):

The entering variable is determined by using the optimality condition of the simplex method. The computations of the objective equation coefficients are based on the primal-dual relationships. Another method, called the stepping-stone procedure, is also available for determining the entering variable. Although the computations in the two methods are exactly equivalent, the stepping-stone method gives the impression that the procedure is completely unrelated to the simplex method.

In the method of multipliers we associate the multipliers ui and vj with row i and column j of the transportation tableau.

For each basic variable xij in the current solution, the multipliers ui and vj must satisfy the following equation:

ui + vj = cij for each basic variable xij

These equations yield m + n - 1 equations (because there are only m + n - 1 basic variables) in m + n unknowns. The values of the multipliers can be determined from these equations by assuming an arbitrary value for any one of the multipliers (usually U1 is set equal to zero) and then solving the m + n -1 equations in the remaining m + n - 1 unknown multipliers.

Once this is done, the evaluation of each non-basic variable Xpq is given by:
Cpq = Up + vq - Cpq for each non-basic variable Xpq (these values will be the same regardless of the arbitrary choice of the value of U1).

The entering variable is then selected as the non-basic variable with the most positive Cpq (compare with the minimization optimality condition of the simplex method).

If we apply this procedure to the non-basic variables in the current solution of our example, the equations associated with the basic variables are given as:

X11: U1 + V1	=	C11	=	10	Let U1 = 0	=>	V1 = 10
X12: U1 + V2	=	C12	=	0			V2 = 0
X22: U2 + V2	=	C22	=	7			U2 = 7
X23: U2 + V3	=	C23	=	9			V3 = 2
X24: U2 + V4	=	C24	=	20			V4 = 13
X34: U3 + V4	=	C34	=	18			U3 = 5

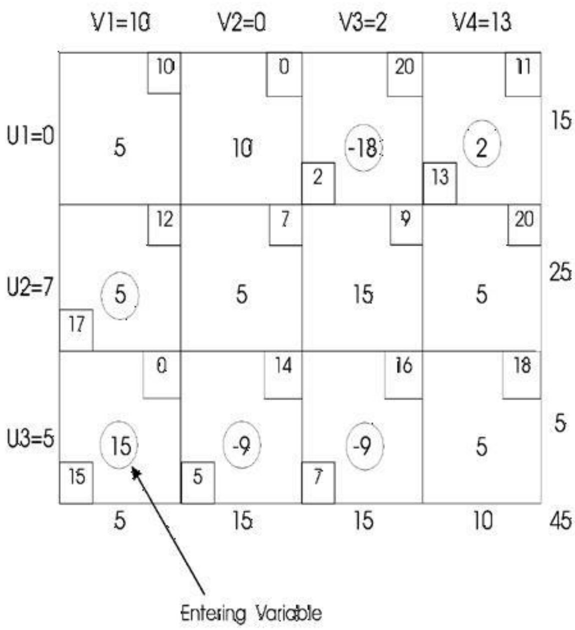
The evaluations of non-basic variables are thus given as follows:

X13:	C13	=	U1 + V3 - C13	=	0 + 2 - 20	=	-18	
X14:	C14	=	U1 + V4 - C14	=	0 + 13 - 11	=	2	
X21:	C21	=	U2 + V1 - C21	=	7 + 10 - 12	=	5	
X31:	C31	=	U3 + V1 - C31	=	5 + 10 - 0	=	15	←*Most positive*
X32:	C32	=	U3 + V2 - C32	=	5 + 0 - 14	=	-9	
X33:	C33	=	U3 + V3 - C33	=	5 + 2 - 16	=	-9	

Since X31 has the most positive Cpq, it is selected as the entering variable.

The equation Ui + Vj = Cij which we use for determining the multipliers, have such a simple structure that is really unnecessary to write them explicitly. It is usually much simpler to determine the multipliers directly from the transportation tableau by noting the Ui of row i and Vj of column j add up to cij when row i and column j intersect in a cell containing the basic variable Xij.

Once Ui and Vj are determined, we can compute Cpq for all non-basic Xpq by adding Up of row p and Vq of column q (this addition may be written in the corner of the cell opposite to the cost) and then subtracting Cpq in the cell at the intersection of row p and column q (and writing it in the middle of the cell).



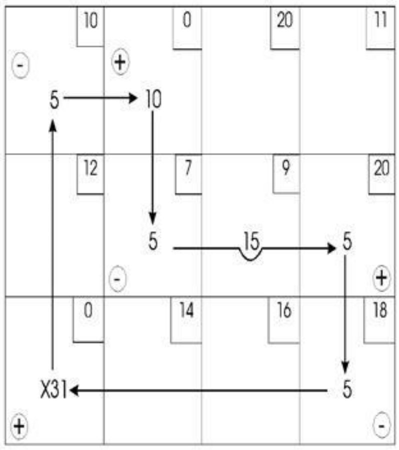
Determination of leaving variable (Loop Construction):

This step is equivalent to applying the feasibility condition in the simplex method. However, since all the constraint coefficients in the original transportation model are either zero or one, the ratios of the feasibility condition will always have their denominator equal to 1. Thus the values of the basic variables will give the associated ratios directly.

For the purpose of determining the minimum ratio, we construct a closed loop for the current entering variable. The loop starts and ends at the designated non-basic variable. It consists of successive horizontal and vertical (connected) segments whose end points must be basic variables, except for the end points that are associated with the entering variable. This means that every corner of the loop must be a cell containing a basic variable. It is immaterial whether the loop is traced clockwise or counterclockwise. Observe that for a given basic solution, only one unique loop can be constructed for each non-basic variables. The loop, in our example, may be defined in terms of the basic variable as follows:

$$X31 \rightarrow X11 \rightarrow X12 \rightarrow X22 \rightarrow X24 \rightarrow X34 \rightarrow X31$$

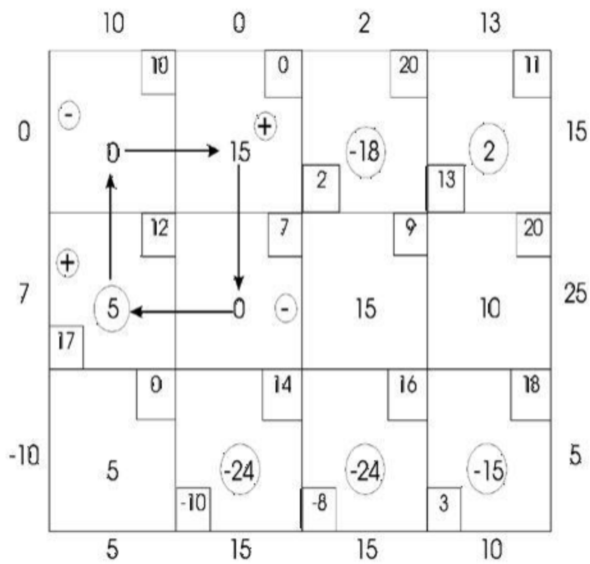
We can see that if X31 (the entering variable) is increased by 1 unit, then, to maintain the feasibility of the solution, the corner basic variables of the X31 loop must be adjusted as follows: Decrease X11 by one unit, increase X12 by one unit, decrease X22 by one unit, increase X24 by plus (+) and minus (-) signs in the appropriate corners.



The leaving variable is selected from among the corner variables of the loop that will decrease when the entering variable X31 increases above zero level. These are indicated in the table by the variables in the square labelled by a minus sign (-). X11, X22 and X34 are the basic variables that will decrease when X31 increases. The leaving variable is then selected as the one having the smallest value, since it will be the first to reach zero value and any further decrease will cause it to be negative (compare the feasibility condition of the simplex method, where the leaving variable is associated with the minimum ratio). In this example the three (-) variables have the same value (=5), in which case any one of them can be selected as the leaving variable. Suppose that X34 is taken as the leaving variable; the new solution is given in the next table.

Its new cost is: 0x10 + 15x10 + 0x7 + 15x9 + 10x20 + 5x0 = Shs 335

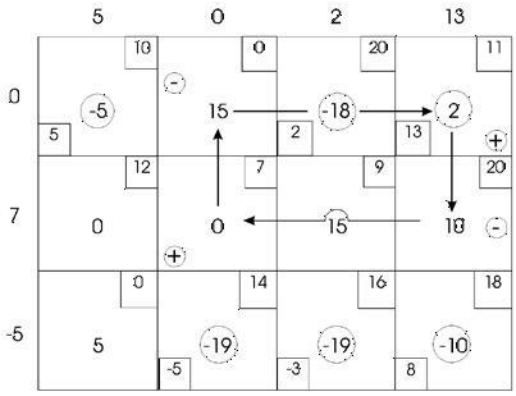
This cost differs from the one associated with the starting solution by 410 - 335 = 75, which is equal to the number of units assigned to X31 (=5) multiplied by C31 (=15)

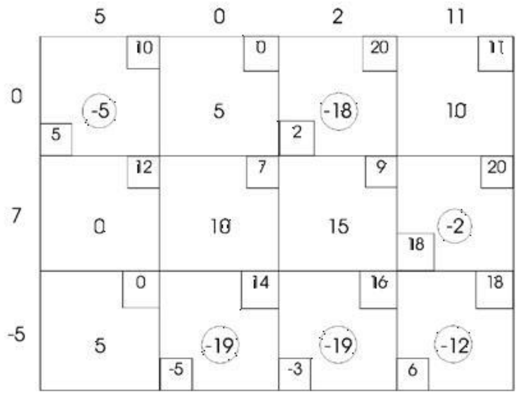


The new basic solution is degenerate, since the basic variables X11 and X22 are zero. Degeneracy, however, needs no special provisions, and the zero basic variables are treated as any other positive basic variables.

The new basic solution is now checked for optimality by computing the new multipliers. The non-basic variable X21 with the largest positive Cpq thus enters the solution. The closed loop associated with X21 shows that either X11 or X22 can be leaving variable. We arbitrarily select X11 to leave the solution.

The new basic solution is shown in the table below. The new values of Ui and Vj and Cpq are computed anew. The new entering and leaving variables are X14 and X24, respectively.





By effecting this change we obtain the new table above. Since all Cpq are non-positive, the optimum solution has been attained (compare with the minimization optimality condition of the simplex method). The optimal solution is summarized as follows:

Ship	5 units from (source) 1 to (destination) 2 at	5 X 0	=	Shs 0
Ship	10 units from (source) 1 to (destination) 4 at	10 X 11=		Shs 110
Ship	10 units from (source) 2 to (destination) 2 at	10 X 7	=	Shs 70
Ship	15 units from (source) 2 to (destination) 3 at	15 X 9	=	Shs 135
Ship	5 units from (source) 3 to (destination) 1 at	5 X 0	=	Shs 0

The total transportation cost of the schedule is Shs 315.

The relationship between the method of multipliers and the simplex method can be established by showing that Cpq as defined above, directly equals the coefficients of the objective equation in the simplex tableau associated with the current iteration (shadow prices).

Improved starting solution:

The northwest corner method does not necessarily produce a "good" starting solution for the transportation model. Here we present two procedures that determine the starting solution by selecting the "cheap" routes of the model.

The least-cost method:

The procedure is as follows: Assign as much as possible to the variable with the smallest unit cost in the entire tableau. (Ties are broken arbitrarily). Cross out the satisfied row or column. (As in the northwest-corner method, if both a column and a row are satisfied simultaneously, only one may be crossed out). After adjusting the supply and demand for all uncrossed-out rows and columns, repeat the process by assigning as much as possible to the variable with the smallest uncrossed-out unit cost. The procedure is complete when exactly one row or one column is left uncrossed-out.

The same example is used to illustrate this method.

The total cost associated with this solution is Shs 335 which is better (smaller) than the one provided by the northwest-corner method.

[10] 0	[0] 15	[20]	[11] 0	~~15~~	~~0~~
[12]	[7]	[9] 15	[20] 10	~~25~~	~~10~~
[0] 5	[14]	[16]	[18]	~~5~~	
~~5~~	~~15~~	~~15~~	10	45	
~~0~~			0		

Vogel's Approximation Method (VAM):

This method is a heuristic and usually provides a better starting solution than the two methods described above. In fact, VAM generally yields an optimum, or close to optimum, starting solution.

The steps of the procedure are as follows:

Step 1

Evaluate a penalty for each row (column) by subtracting the smallest cost element in the row (column) from the next smallest cost element in the same row (column).

Step 2

Identify the row or column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least cost in the selected row or column. Adjust the supply and demand and cross-out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of them is crossed out and the remaining row (column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties—(in Step 3)

Step 3

a. If exactly one row or one column remains uncrossed out, stop.
b. If only one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least-cost method.
c. If all uncrossed-out rows and columns have (assigned) zero supply and demand, determine the zero basic variables the least-cost method. Stop.
d. Otherwise, recompute the penalties for the uncrossed-out rows and columns, then go to step 2.
e. (Notice that the rows and columns with assigned zero supply and demand should not be used in computing these penalties).

Row penalties	1	2	3	4	Supply	
Column penalties	- - 10	7 7 7	- - 7	9 9 7		
11 11 11 10	(10)	(0) 5	(20)	(11) 10	15	
- 13 2 2	(12)	(7) 10	(9) 15	(20)	25	10
- - - 14	(0) 5	(14)	(16)	(18) 0	5	0
	5	15	15	10		

The cost of the program is Shs 315 which happens to be optimal. The given version of VAM breaks ties between penalties arbitrarily. However, breaking of ties may be crucial in rendering a good starting solution.

5. THE ASSIGNMENT MODEL:

Consider the situation of assigning m jobs (or workers) to n machines. A job i(= 1,2,3 ...m) when assigned to machine j(= 1,2,3 ...n) incurs a cost Cij. The objective is to assign the jobs to the machines (on job per machine) at the least total cost. The situation is known as the assignment problem.

The formulation of this problem may be regarded as a special case of the transportation model. These jobs represent "sources" and machines represent "destinations". The supply available at each source is 1; that is
ai = 1 for all i. Similarly, the demand required at each destination is 1, that is bj = 1 for all j. The cost of "transporting" (assigning) job i to machine j is Cij. If a job cannot be assigned to a certain machine, the corresponding Cij is taken equal to M, a very high cost. We represent in the table the general ideal of the model.

Machine

		1	2	3	—	n	
	1	C11	C12	C13	—	Cln	1
	2	C21	C22	C23	—	C2n	1
Job	3	C31	C32	C33	—	C3n	1
	m	Cm1	Cm2	Cm3		Cmn	1
		1	1	1		1	

Before the model can be solved by the transportation technique, it is necessary to balance the problem by adding fictitious jobs or machines, depending on whether m < n or m > n. It will be assumed that m = n without loss of generality.

The assignment model can be expressed mathematically as follows:

Let Xij = 0 if the j^{th} job is not assigned to the i^{th} machine.
1 if the j^{th} job is assigned to the i^{th} machine.

The model is thus given by: Minimize

$$Z = \sum_{i=j}^{n} \sum_{j=1}^{n} Cij\ Xij$$

$$Subject\ to: \sum_{j=1}^{n} Xij = 1 \quad i = 1, 2, \ldots n$$

Xij = 0 or 1

$$\sum_{i=1}^{n} Xij = 1 \qquad j = 1, 2, \ldots n$$

To illustrate the assignment method, let us consider the following problem with 3 jobs and 3 machines. The initial solution (using the northwest corner rule) is obviously degenerate. This will always be the case in the assignment model regardless of the method used to obtain the starting basis. In fact, the solution will continue to be degenerated at every iteration.

The special structure of the assignment model allows the development of an efficient method of solution. This method will be illustrated with the example:

Job \ Machine	1	2	3	
1	$\boxed{5}$ 1	$\boxed{7}$ 0	$\boxed{9}$	1
2	$\boxed{14}$	$\boxed{10}$ 1	$\boxed{12}$ 0	1
3	$\boxed{15}$	$\boxed{13}$	$\boxed{16}$ 1	1
	1	1	1	

The optimal solution of the assignment model remains the same if a constant is added or subtracted to any row or column of the cost matrix. This is proved as follows: If Pi and qj are subtracted from the i^{th} and row j^{th} column, the new cost elements becomes: C'ij = Cij - Pi - qj.

This yields the new objective function:

$$Z' = \sum_i \sum_j C'ij = \sum_i \sum_j (Cij - Pi - q)\ Xij =$$

$$Z' = \sum_i \sum_j C'ij\,Xij - \sum_i Pi \sum_j Xij - \sum_j qj \sum_i Xj$$

$$Since\ \sum_i Xij = 1 \quad , \quad \sum_j Xij = 1\ \ We\ get: Z' = Z - constant$$

This shows that the minimization of the original objective function yields the same solution as the minimization of Z.

This idea indicates that if one can create a new C'ij matrix with zero entries, and if these zero elements or a subset thereof constitute a feasible solution, this feasible solution is optimal, because the cost cannot be negative.

In the table the zero elements are created by subtracting the smallest element in each row (column) from the corresponding row (column). If one considers the row first, the new C'ij matrix is :

		1	2	3	
	1	0	2	4	P1=5
// Cij//	=2	4	0	2	P2=10
	3	2	0	3	P3=13

The last matrix can be made to include more zeros by subtracting q3 = 2 from the third column. This yields the following table:

		1	2	3
	1	0	2	2
// Cij//	=2	4	0	0
	3	2	0	1
				q3=2

The squares give the feasible (and hence optimal) assignment: (1,1), (2,3) and (3,2); costing 5 + 13 + 12 = 30. Notice that this cost is equal to P1 + P2 + P3 + q3.

Unfortunately, it is not always possible to obtain a feasible assignment as in the example above. Further rules are thus required to find the optimal solution. These rules are illustrated with the following example:

(1)

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

(2)

0	3	5	2
2	0	3	2
0	1	7	3
3	2	3	0

(3)

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

After carrying out the same initial steps as in the previous example, one gets the table (3). A feasible assignment to the zero elements is not possible in this case. The procedure then is to draw a minimum number of lines through some of the rows and columns such that all the zeros are crossed out (as shown in table (3)).
The next step is to select the smallest uncrossed-out element and add it to every element at the intercession of two lines. This yields table (4), which gives the optimal assignment (1,1) , (2,3) , (3,2) and (4,4). The corresponding total cost is: 1 + 10 + 5 + 5 = 21. It should be noted that if the optimal solution was not obtained in the step above, the given procedure of drawing lines should be repeated until a feasible assignment is achieved.
(4)

0	2	1	1
3	0	0	2
0	0	3	2
4	2	0	0

REINFORCING QUESTIONS

QUESTION ONE

Boots Ltd. manufactures a range of five similar products, A, B, C, D and E. the table below shows the quantity of each of the required inputs necessary to produce one unit of each product, together with the weekly inputs available and selling prices of each product.

Inputs	A	**B**	**C**	**D**	**E**	**Weekly inputs available**
Raw materials (Kg)	6.0	6.5	6.1	6.1	6.4	35,000 Kgs
Forming (hours)	1.00	0.75	1.25	1.00	1.00	6,000 hours
Firing (hours)	3.00	4.50	6.00	6.00	4.50	30,000 hours
Packing (hours)	0.50	0.50	0.50	0.75	1.00	4,000 hours
Selling price (Sh.)	40	42	44	48	52	

The costs of each input is as follows:

Material	Sh.2.10 per Kg
Forming	Sh.3.00 per hour
Firing	Sh.1.30 per hour
Packing	Sh.8.00 per hour

Required:

a) Formulate this problem as a Linear Programming problem. (7 marks)

b) The problem has been solved using a computer package and the following final tableau of a simplex solution has been produced:

Basis	**A**	**B**	**C**	**D**	**E**	**X**	**S**	**T**	**U**	**Value**
A	1	1.18	1.04	0.46	0	0.36	0	0	-2.29	3,357
B	0	-0.34	0.23	0.02	0	-0.18	1	0	0.14	321
T	0	1.37	2.97	2.28	0	-0.27	0	1	-2.79	9,482
E	0	-0.09	-0.02	0.52	0	-0.18	0	0	2.14	2,321
Zj	0	1.26	1.06	0.51	0	2.02	0	0	8.81	105,791

Where A, B, C, D and E are the weekly production levels for the five products; X is the amount of raw material that falls short of the maximum available; S, T an U are the respective number of hours short of maximum weekly input of forming, firing and packing time.

i Use this tableau to find the optimum weekly production plan. (4 marks)

ii Describe the implications of using this plan in terms of unused resources and overall contribution to profit. (3 marks)

iii In the context of this problem explain the meaning of "The dual or shadow price of a resource" (3 marks)

iv There is a proposition that the company manufactures an additional product which would sell at Sh.50 per unit. Each unit will need 6 kg of raw material, one hour of forming time, five hours of firing time and one hour of packing time. Is it s worthwhile proposition? (3 marks)

(Total: 20 marks)

QUESTION TWO

1. Highlight how the transportation algorithm can be modified for profit maximization rather than minimization of costs. (3 marks)
2. The Executive Furnitures Ltd. (EFL) produces a unique type of computer desks. Four of EFL's main outlets are S_1, S_2, S_3, and S_4. These outlets already have requirements in excess of the combined capacity of its three production plants P_1,P_2, and P_3. The company needs to know how to allocate its production capacity to maximize profits.
   Distribution costs (in Sh.) per unit from each production plant to each outlet are given in the following table:

		To			
		S_1	S_2	S_3	S_4
		Sh.	**Sh.**	**Sh.**	**Sh.**
	P1	220	240	220	360
From	P2	240	200	180	280
	P3	260	200	260	240

Since the four outlets are in different parts of the country and as there are differing transportation costs between the production plants and the outlets along with slightly different production costs at different production plants there is a pricing structure which enables different prices to be charged at the four outlets. Currently, the price per unit charged is Sh.2,300 at S_1, Sh.2,350 at S_2, Sh.2,250 at S_3, and Sh.2,400 at S_4. The variable unit production costs are Sh.1,500 at plants P_1 and P_3 and Sh.1,550 at plant P_2. The demand at S_1,S_2, S_3 and S_4 are 850, 640, 380 and 230 desks respectively while the plant capacity at plant P_1, P_2 and $P_3$are 625, 825 and 450 desks respectively.

Required:

Using the transportation algorithm, determine the contribution to profit for the optimal allocation. (17 marks)

(Total: 20 marks)

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

LESSON SIX

BUDGETARY CONTROLS AND ADVANCE VARIANCES

OBJECTIVES

- To consider advanced issues in budgeting
- To explain variance investigation decisions

INSTRUCTIONS

- Read Chapters 15 to 19 of Management and Cost Accounting by Colin Drury 5th Edition
- Read Study Text
- Complete reinforcing questions under examination conditions and compare your answers with those given in Lesson 10.

CONTENTS

BUDGETARY CONTROLS

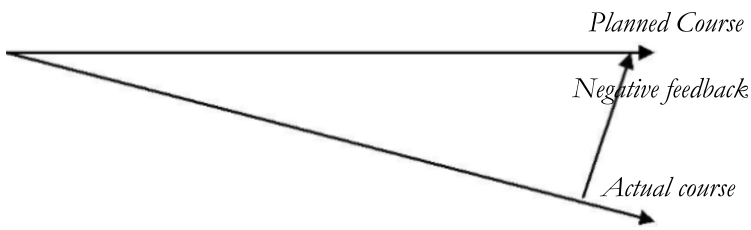
Control in a business is the process of guiding organisation into viable patterns of activity in an environment. The main objective of a control system is to make sure that the right things get done. A system such as a big organisation must be controlled to keep it steady or to enable it to change safely. Control is therefore required because unpredictable disturbances might enter the system so that actual results deviate from the expected results or goals. Examples of such disturbances are entry of a powerful competitor in the market, unexpected increase in cost, a decline in quality standards, failure of a supplier to deliver promised raw material, or the tendency of employees to stop working in order gossip.

To have an efficient control process there has to be a plan, a budget, or a target towards which the system as a whole will be aiming. Control is dependent on the receipt and the processing of information both to plan and to compare actual results to the plan so as to judge what control measures, if any, are needed.

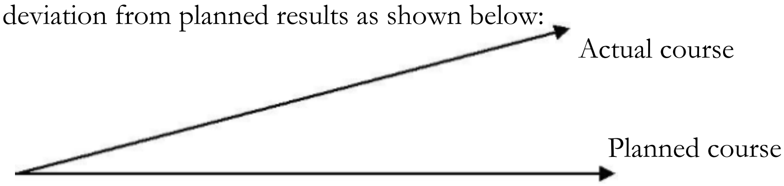
There are two types of control systems:
Feedback control system Feed-forward
control system Feedback Control
System
Feedback is information about actual achievements or actual results produced within the organisation (e.g. management control reports) with the purpose of helping with the control decisions. Feedback is internal in that, it is concerned with measuring the output of the system itself. Most common types of control systems in a business such as budgetary control, stock control and the production control system are based on feedback control cycles. Feedback can be either positive or negative.

Negative feedback

This is information which indicates that the system is deviating from its planned or prescribed course and that some re-adjustment is necessary to bring it back on course. It is referred to as negative since the control action would need to reverse the direction/movement of the system towards its planned course. This can be shown as follows:



Positive feedback
Positive feedback results in control action that causes actual results to maintain or increase their path of deviation from planned results as shown below:



Feed-forward control system
Feed-forward control system describes a system in which deviations in the system are anticipated in a forecast of future results, so that corrective actions can be taken in advance of any deviation actually occurring. Examples of feed forward control systems include:
Critical Path Analysis
Cash budget

CONDITIONS NECESSARY IN A CONTROL CYCLE

There are four necessary conditions that must be satisfied before any system can be said to be controlled. These are:
Objective of the process being controlled must exist
The output of the process must be measurable in terms of the dimensions defined by the objective.
A predictive model of the process being controlled is required so that causes for the non-attainment of objective can be identified and proposed corrective action analysed.
There must be capability of taking action so that deviation of attainment from objective can be reduced.

IMPORTANCE OF A BUDGET

A Budget is a plan expressed in monetary terms. It is prepared prior to the budget period and may show income, expenditure and the capital to be employed i.e. a budget is what the company wants to happen as opposed to a forecast which is what is likely to happen.

The budget may be prepared for the following
reasons: As a planning aid
To communicate ideas and plans to everyone affected by them
To co-ordinate the activities of different departments or sub-units in the organisation
To provide the framework for responsibility accounting whereby managers of budget centres are made responsible for the achievement of budget targets for the operations under their personal control.
To establish a system of controls by setting up targets against which actual results will be
compared. To motivate employees to improve performance.

Budgetary control is the establishment of budgets relating the responsibility of executives to the requirements of a policy and the continuous comparison of actual and budgeted results either to service by individual action the objectives of that policy or to provide a basis for its revision.

Budgetary controls are used for the following reasons:
To define the objectives of the organisation as a whole
To reveal the extent by which actual results have exceeded or fallen short of the budget.
Budgetary control helps in indicating why actual results differ from the budgeted results.
It is important as a basis for the revision of the current budget or the preparation of future budgets.
To ensure that resources are used as efficiently as possible.
To see how well the activities of the organisation have been co-ordinated To
provide some central control especially where activities are decentralized.

HUMAN BEHAVIOUR AND BUDGETARY CONTROL

An important feature of control in business is that control is exercised by managers over people. Their attitudes and response to budgetary planning and control will affect the way in which it operates.

In 1953 Chris Argyris identified the following four perspectives of budgetary control:
The budgets are seen as a pressure device used by management to force lazy employees to work harder. The intention of such pressure is to improve performance but the unfavourable reaction of sub-ordinates against it seems to be at the core of the budget problem.
Budget men want to see failure. The accounting department is usually responsible for recording actual achievements and comparing this against a budget. Accountants are therefore budget men and their success is to find significant adverse variances and identify the managers responsible. The success of a budget man is the failure of another manager and this failure causes loss of interest and declining performance.
The accountant, on the other hand, fearful of having his budget criticized by management deliberately makes it hard to understand.

Target and goal congruence. The budget usually sets targets for each department. Achieving the departmental targets becomes of paramount importance regardless of the effect this may have on other departments and the overall company performance. This is the problem of goal congruence. Management style. Budgets are used by managers to express their character and patterns of leadership on sub-ordinates. Sub-ordinates resentful of their leader's styles blame the budget rather than the leader.

BUDGETARY STYLES

Budgets can be set from the top down (imposed) or from the bottom up (participatory).
Imposed budgets
In this approach to budgeting, top management prepares a budget with little or no help from operating personnel, which is then imposed upon the employees who have to work to the budgeted figures.

Imposed budgets are effective in the following conditions:

- In a newly formed organisation
- In a very small business
- During periods of economic hardship
- When operational managers lack budgeting skills
- When the organisation's different units require precise co-ordination

Advantages of Imposed budgets

They increase the probability that the organisation strategic plans are incorporated into the planned activities

- They enhance the co-ordination between the plans and objectives of divisions.
- They use senior management awareness of total resource availability.
- They decrease the possibility of inputs from inexperience or uniformed lower level employees.
- They decrease the period of time taken to draw up the budget.

Disadvantages of Imposed budgets

- It may result in dissatisfaction, defensiveness and low morale among employees, who must implement the budget.
- The feeling of team spirit may disappear.
- The acceptance of organisations goals and objectives could be limited.
- The feeling of the budget as a punitive device could arise.
- Unachievable budget for overseas divisions may be imposed on local divisions if consideration is not given to the local operating and political environment.

Participative Style of Budgeting

In this approach to budgeting, budgets are developed by lower level managers who then submit them to their superiors. The budgets are based on the lower level manager perception of what is achievable and the associated necessary resources. The degree to which lower level managers are allowed to participate in the budgeting process depends on:
Senior managers awareness of participatory budgeting
advantages Their agreement with those advantages

Participatory budgets are effective in the following areas:

- In a well established organisation
- In a very large business
- During periods of economic boom
- When operational managers have strong budgeting skills
- When the organisation's different units act autonomously

Advantages of participatory budgets

- Information from employees most familiar with each unit's needs and constraints is included.
- Knowledge spread among several levels of management is pooled together.
- Morale and motivation is improved
- Acceptance and commitment to organisation's goals and objectives by operational managers is increased
- Co-ordination between divisions is improved
- Operating management are able to develop plans which tie in within organisation goals and objectives
- Specific resource requirement are included
- Senior managers overview of the organisation is mixed with operational level details
- An expression of the expectation of both senior management and subordinate is provided.

Disadvantages of participatory budgets

- They consume more time and therefore are more expensive
- The advantage of management participation may be negated by failure to implement the budget by senior management leading to a dissatisfaction similar to that experienced with imposed budgets.
- Such budgets may cause managers to introduce budgetary slacks (A budgetary slack is a margin set between what can be achieved and what managers state will be achieved)
- Managers may be unqualified to participate and therefore the budgets may be unachievable
- Participatory budgets may support empire building by sub-ordinates.
- An earlier start to the budgeting process maybe required when there is uncertainty about the future.

CONTIGENCY THEORY

Some researchers have argued that the context in which budgetary control is used is as important as the style in which it is implemented and used. This is known as the contingency theory. The contingency approach to management accounting is based on the assumption that there is no universally appropriate accounting system applicable to organisations in all circumstances. Rather contingency theory attempts to specific aspects of an accounting system that are associated with certain defined circumstances and demonstrate an appropriate matching.

Major factors identified are:

Environmental factors such as:

- Its degree of predictability
- The degree of competition faced in the market
- The number of different product in the markets
- The degree of hostility exhibited by competition

Organisational Structure Factor including:

- Size of the organisation
- Interdependence of the parts or sub-units
- The degree of decentralization
- Availability of resources

Technological Factors such as:

- The nature of the production process
- The routineness or complexity of the production process
- How well the relationship between inputs and outputs is understood
- The amount of variety in each task that has to be performed

ALTERNATIVE APPROACHES TO BUDGETING

There are two main approaches to budgeting. These are:

Incremental budgeting

This is used to describe an incremental cost approach to budgeting where the next period budget is based on the current years results plus an extra amount (an increment) for estimated growth or inflation next period.
Incremental budgeting is sufficient only if current operations are effective, efficient and economical without an alternative course of action available to the organisation. Although incremental budgeting is easy to prepare, it encourages slacks and wasteful spending to creep into budgets and become a normal feature of actual spending. It does not encourage performance to be improved or looking for alternative ways of carrying out the production.

Zero-Base Budgeting

Zero-Base Budgeting (ZBB) was first developed and introduced for business by Peter A. Pyhrr. During the development of ZBB, Mr. Pyhrr was the Manager, Staff Control, at Texas Instruments, Inc. Dallas. From this beginning ZBB has been explored and adopted by many other businesses. The principle behind ZBB is that each cost centre budget should be made from 'scratch' (a zero base). It starts from the basic assumption that the budget for the next year is zero and every process/expenditure must then be justified fully in order to be included in next year's budget. ZBB is useful for discretionary costs. In ZBB there should be a positive attempt to eliminate inefficiency and slacks from current operations.

The development and implementation of the ZBB model requires managers and others in the organization to engage in several major planning, analytic and decision-making processes. These major processes of ZBB include the following:

Definition of the Mission and Goals of the Organization

Usually the organization has already established mission and goal statements. However, it may be necessary to redefine the ones that are already in existence and/or create new ones. This redefinition is particularly useful when there have been major changes in the internal and external environment.

Identification of the Organization's Decision Units and Decision Packages

A ZBB decision unit is an operating division for which decision packages are to be developed and analyzed. It can also be described as a cost or a budget center. Managers of each decision unit are responsible for developing a description of each program to be operated in the next fiscal year or years. In ZBB these programs are referred to as decision packages and each decision package usually will have three or more alternative ways of achieving the decision package's objectives. Briefly, each decision package alternative must contain, as a minimum, goals and/or objectives, activities, resources and their dollar costs. Also, the decision package should contain a description of how it contributes to the mission and goals of the organization.

Analysis of Each Decision Package

This analytic process allows the manager of the decision package and its alternatives to assess and justify its operation. Several questions should be asked and answered during the analytical process.
Does this decision package support and contribute to the goals of the organization?
What would be the result to the organization if the decision package is eliminated?
Can this decision package's objectives be accomplished more effectively and/or efficiently? This question will require creative planning by the person(s) developing the decision package.

Ranking of Decision Packages

The ranking process is used to establish a rank priority of decision packages within the organization. During the ranking process managers and their staff will analyze each of the several decision package alternatives. The analysis allows the manager to select the one alternative that has the greatest potential for achieving the objective(s) of the decision package. Ranking is a way of evaluating all decision packages in relation to each other. Since, there are any number of ways to rank decision packages, managers will no doubt employ various methods of ranking. The main point is that the ranking of decision packages is an important process of ZBB.

Acceptance and Allocation of Resources

Managers, following a review and analysis of all decision packages, will determine the level of resources to be allocated to each decision package. Managers at different levels of responsibility in the organization usually perform the review and analysis. Sometimes, the executive levels of management may require the managers of the decision packages to revise and resubmit their decision packages for additional review and analysis.

Budget Preparation

The organization's budget is prepared following the acceptance and approval of the decision packages. Once the organization's budget has been approved managers of the decision units will place in operation all approved decision packages during the next fiscal year.

Monitoring and Evaluation

The last major process of ZBB is monitoring and evaluation. The processes of planning, analysis, selection and budgeting of decision packages prepare the organization for operation during the next year. However, what managers plan to happen in the next fiscal year may or may not occur. Adjustments may be essential during the year in order to achieve the decision package objectives. Also, there is a need to know whether or not the organization did accomplish what it set out to achieve and what level of achievement was obtained. The monitoring and evaluation process of ZBB requires that the following be included in the overall design and implementation of decision packages.

Decision package content should include:

- Measurable performance objectives
- Appropriate activities as means for achieving the performance objectives
- Resource allocation essential for conducting the activities
- Methods for carrying out the activities as planned
- Evaluation of objective achievement during and at the end of the program of activities
- Procedures for reporting objective achievement to managers of the organization

FLEXIBLE BUDGETING

Flexible budget may be used in one of two ways: Planning and Control

At the planning stage when budgets are set, to reduce the effect of uncertainty. For example, suppose that a company expect to sell 10,000 units of output during the next year. A master budget (the fixed budget) would be prepared on the basis of this expected volume. However, if the company thinks that output and sales might be as low as 8,000 units or as high as 12,000 units, it may prepare contingency flexible budgets, at volumes of say, 8,000, 9,000, 11,000, 12,000 units. The advantages of planning with flexible budgets include:

Finding out well in advance the costs of layoff pay, idle time and so on if output falls short of budget;
Deciding whether it would be possible to find alternative uses for spare capacity if output falls short of budget (for example, whether employees could be asked to overhaul their own machines instead of paying for an outside contractor);
Estimating the costs of overtime, sub-contracting work on extra machine hire if sales volume exceeds the fixed budget estimate, and finding out if there is a limiting factor which would prevent high volumes of output and sales being achieved.

It has been suggested, however, that since many cost items in modern industry are fixed costs the value of flexible budget in planning is dwindling. For example:
In many manufacturing industries, plant cost (depreciation, rent and so on) are a very large proportion of total costs, and these tend to be fixed costs;
Wages costs also tend to be fixed, because employees are generally guaranteed a basic wage for a working week of an agreed number of hours.
With the growth of service industries, fixed salaries and overheads will account for most of the costs of a business, and direct materials will be relatively small portion of total costs.

Flexible budgets are also used retrospectively at the end of each month (control period) or year, to compare actual results achieved with the result that would have been expected if the actual circumstances had been known in advance. Flexible budgets are an essential factor in budgetary control and variance analysis

STANDARD COSTING AND BUDGETARY CONTROL

In practice, the terms standard cost and budgeted cost might be used interchangeably. Whereas it is possible to have budgeting without standard costs, it is not possible to have a standard cost system without a total cost budgeting system.

Standard costing and budgetary control are interlinked items. Once standard cost have been determined it is relatively easy to compute budgets for production costs and sales and, when actual figures differ from expected standards, to calculate variances, to provide a basis for control reporting.

A standard cost is an average expected unit cost. It is set using the best available estimates and cannot be expected in practice that actual results will conform to standard. Variances should therefore be expected to fluctuate randomly within normal limits. Such random fluctuations need no investigation and tolerance limits are set (investigate only those variances which exceeds Sh.x or y% of the standard cost).

Standard costing is appropriate in any situation where the same resources are used over and over again in the same way. It is therefore particularly appropriate for manufacturing businesses producing large numbers of identical items, especially where the same operations are combined in different ways to produce different products. It also has applications in service businesses that involve repetitive operations.

Installing a standard costing system entails designing an information system that can collect and analyse details about activities in such a way that the standards can be set and applied. In effects this means collecting quantitative data about the use of resources.

The advantages and disadvantages of standard costing

The advantages for controlling having a standard costing system in operation can be summarized as follows;

- Carefully planned standards are an aid to more accurate budgeting
- Standard costs provide a yardstick against which actual costs can be measured.
- The setting of standards involves determining the best materials and methods which may lead to economies.
- A target of efficiency is set for employees to reach and cost-consciousness is stimulated
- Variances can be calculated which enable the principle of management by exception to be operated. Only the variances which exceed acceptable tolerance limits need to be investigated by management with a view to control action.

The disadvantages of standard costing include the following.

- It is difficult to set accurate standards
- The collection and analysis of data to run a standard costing system may be very time-consuming
- Standards may be seen as a pressure device

VARIANCES ANALYSIS

Variances are the differences between actual results and expected results. Expected results are the standard costs and standard revenues.

Price, rate and expenditure variances measure the difference between the actual amount of money paid and the amount of money that should have been paid for the actual quantity of materials or the actual number of hours of labour or variance overheads. Usage and efficiency variances measure the difference between the actual physical quantity of materials used or hours taken and the quantities that should have been used or taken for the actual volume of production. These physical differences are then converted to money values by applying the appropriate standard cost.

Basic variance analysis such as this should be well understood by this stage since they were covered in section 2 costing. However it may be important to remind you of the following:

Knowledge brought forward from Costing
The selling price variance is a measure of the effect on expected profit of a different selling price to the standard selling price. It is calculated as the difference between the standard revenue from the actual quantity of goods sold and the actual revenue.
The sales volume variance is the difference between the actual units sold and the budgeted quantity, valued at the standard profit or contribution per unit. In other words, it measures the increase or decrease between standard and actual profit or contribution as a result of the sales volume being higher or lower than budgeted.
Price, rate and expenditure variances measure the difference between the actual amount of money paid and the amount of money that should have been paid for the actual quantity of materials or the actual number of hours of labour or variable overheads used. Note that if materials are valued at standard cost, the materials price variance is calculated on purchases in the period but if they are valued at actual cost the variance is calculated on materials used in production in the period.
Usage and efficiency variances are quantity variances. They measure the difference between the actual physical quantity of materials used or hours taken and the quantities that should have been used or taken for the actual volume of production. These physical differences are then converted into money values by applying the appropriate standard cost.
The idle time variance is simply the number of hours of idle time valued at the standard rate per hour.

Illustration 1

The following standard costs apply in a business that manufactures a single product.

Standard weight to produce one unit	12kgs
Standard price per kg	Sh.9
Standard hours to produce one unit	10
Standard rate per hour	Sh.4

Actual production and costs for one accounting period were as follows.

Materials used	3,770kgs
Materials costs	Sh.35,815
Hours actually worked	2,755
Hours paid for	2,900
Wages paid	Sh.11,571

The actual output was 290 units.

Required

Calculate relevant material and labour variances

Answer

	Sh.
3,770kgs should cost (xSh.9)	33,930
But did cost	35,815
Material cost variance	1,885 (A)
290 units should use (x 12kgs)	3,480kgs
But did use	3,770kgs
Material usage variance in kg	290kgs (A)
x standard cost per Kg	x Sh.9
Material usage variance in Sh.	Sh.2,610 (A)

	Sh.
2,900 hours should cost (xSh.4)	11,600
But did cost	11,571
Labour rate variance	29 (F)
290 units should take (x10hrs)	2,900 hrs
But did take	2,755 hrs
Labour efficiency variance in hrs	145 hrs (F)
X standard rate per hour	x Sh.4
Labour efficiency variance in Sh.	Sh.580 (F)
Idle time variance:	Sh.580 (A)

Illustration 2

Basic analysis ltd produces and sells one product only, the BBT, the standard cost for one unit being as follows:

	Sh.
Direct material A- 10 kg at Sh.20 per kg	200
Direct material B- 5 litres at Sh.6 per litre	30
Direct wages- 5hrs at Sh.6 per hour	30
Fixed production overhead	50
Total standard cost	310

The fixed overhead included in the standard cost is based on an expected monthly output of 900 units During April Year 1 the actual results were as follows.

Production	800 units
Material A	7,800 kgs used, costing Sh.159,900
Material B	4,300 units used costing Sh.23,650
Direct wages	4,200 hrs worked for Sh.24,150
Fixed production overhead	Sh.47,000

Required

Calculate price and usage variances for each material
Calculate labour rate and efficiency variances
Calculate fixed production overhead expenditure and volume variances

Solution

Price variance – Material A

	Sh.
7,800kgs should have cost	156,000
But did cost	159,000
Price variance	3,900 (A)

Usage variance- Material A

800 units should have used (x 10kgs)	8,000kgs
But did use	7,800kgs
Usage variance in kgs	200kgs (F)
X standard cost per kilogram	x Sh.20
Usage variance in Sh.	Sh.4,000 (F)

Price variance- Material B

	Sh.
4,300 units should have cost (x Sh.6)	25,800
But did cost	23,650
Price variance	2,150 (F)

Usage variance-Material B	
800 units should have used (x 5litres)	4,000
But did use	4,300
Usage variance in litres	300
X standard cost per litre	x Sh.6
Usage variance in Sh.	Sh.1,800 (A)

Labour rate variance	Sh.
4,200 hrs should have cost (x Sh.6)	25,200
But did cost	24,150
Rate variance	1,050 (F)

Labour efficiency variance	
800 units should have taken (x5hrs)	4,000 hrs
But did take	4,200 hrs
Efficiency variance in hours	200 hrs (A)
x standard rate per hour	xSh.6
Efficiency variance in Sh.	Sh.1,200 (A)

fixed overhead expenditure variance	
	Sh.
Budgeted expenditure (Sh.50 x 900)	45,000
Actual expenditure	47,000
Expenditure variance	2,000 (A)

Fixed overhead volume variance	
	Sh.
Budgeted production at standard rate (900 x Sh.50)	45,000
Actual production at standard rate (800 x Sh.50)	40,000
Volume variance	5,000 (A)

INTERPRATATION OF VARIANCE

Trend, materiality and controllability

The point of comparing flexed budget and actual figures is to see what corrective action, if any, is needed to ensure that the plan will be successfully completed. Thus every variance needs to be considered to see whether it should prompt control action.

Three important points should be kept in mind.

Materiality

Small variations in a single period are bound to occur occasionally and are unlikely to be significant. Obtaining an explanation is likely to be time consuming and irritating for the manager concerned. The explanation will often be 'chance', which is not helpful in any case. For such variations further investigation is not worthwhile;

Trend

However, small variations that occur consistently may need more attention. Variance trend is more important than a single set of variances for one accounting period. Trend analysis provides information which gives an indication as to whether a variance is fluctuating within acceptable control limits or is moving into an out of control situation'. Trend is discussed further below.

Controllability

Controllability must also influence the decision whether to investigate further. If there is a general worldwide price increase in the price of an important raw material there is nothing that can be done internally to control the effect of this. If a central decision is made to award all employees a 10% increase in salary, staff costs in a division will increase by this amount and the variance is not controllable by a division's manager. Uncontrollable variances call for a change in the plan, not an investigation into the past

Variance trend

If, say, an efficiency variance is Sh.1,000 adverse in month 1, the obvious conclusion is that the process is out of control and that corrective action must be taken. This may be correct, but what if the same variance is Sh.1,000 adverse every month? The trend indicates that the process is in control and the standard has been wrongly set.

Suppose, though, that the same variance is consistently Sh.1,000 adverse for each of the first six months of the year but that production has steadily fallen from 100 units in month 1 to 65 units by month 6. The variance trend in absolute terms is constant, but relative to the number of units produced, efficiency has got steadily worse.

Illustration

Assume that one unit takes ten hours to produce. The standard labour cost is Sh.5 per hour. In period one 100 units are produced in 1,200 hours. In period six 85 units are produced in 850 hours. What is the best way of presenting this information to management?

SOLUTION

The labour efficiency variance can be calculated in the normal way.

	Period 1		Period 6
	Hours		Hours
100 units should take	1,000	65 units should take	650
But did take	1,200	but did take	850
Efficiency variance in hours	200	Efficiency variance in hours	200
x standard rate per hour	xSh.5	x standard rate per hour	xSh.5
	Sh.1,000		Sh.1,000

The absolute measures, whether in hours or shillings, do not convey what is happening at all. What is needed is a relative measure.

In physical terms, one unit takes 12hrs to make in period one, but more than 13 hours (850/65) in period six.

In monetary terms the variance can be related to standard cost and expressed as a percentage. In period one Sh.1,000 represent 20% of the standard cost for 100 units of Sh.5,000. In period six Sh.1,000 represents over 30% of the standard cost for 65 units of Sh.3,250

The conclusions that may be drawn from this are as follows.

Single period variances are not necessarily a good indication of whether or not a process is in control

Absolute measurement may disguise some of the significance of variance. It is helpful to supplement this information by measurement over time in % terms against an appropriate base.

Management signals

Variance analysis is a means of assessing performance, but is only a method of signalling to management areas of possible weakness where control action might be necessary. It does not provide a ready-made diagnosis of faults, nor does it provide management with a ready-made indication of what action needs to be taken. It merely highlights items for possible investigation.

Individual variances should not be looked at in isolation. As an obvious example, a favourable sales price variance is likely to be accompanied by an adverse sales volume variance: the increase in price has caused a fall in demand. We now know in addition that sets of variances should be scrutinized for a number of successive periods if their full significance is to be appreciated.

Here are some of the signals that may be extracted from variance trend information.
Material price variances may be favourable for a few months, then shift to adverse variances for the next few months and so on. This could indicate that prices are seasonal and perhaps stock could be built up in cheap seasons, if not inconsistent with JIT policy.
Regular, perhaps fairly slight, increase in adverse price variances usually indicates the workings of general inflation. If desired, allowances could be made for general inflation when flexing the budget.
Rapid large increases in adverse price variances may suggest a sudden scarcity of a resource. It may soon be necessary to seek out cheaper substitutes.
Gradually improving labour efficiency variances may signal the existence of a learning curve, or the motivational success of a productivity bonus scheme. In either case opportunities should be sought to encourage the trend.
Worsening trends in machine running expenses may show up that equipment is deteriorating and will soon need repair or even replacement.
Uncertainty in variance analysis
Horngren identifies seven principal sources of variances, most of which ultimately derive from the fact that it is not possible to know what is going to happen in advance These sources are:
Inefficiencies in Operations: such problems as spoilage and idle time will be very familiar from typical examination questions on variances.

Inappropriate standards (or targets):

This is a problem arising from deficiencies in *planning*. If not enough time and resources are devoted to setting accurate standards in the first place, and if they are not kept up to-date, subsequent performance is highly likely to deviate from what was expected.

Mis-measurement of actual results:

Scales may be misread, the pilfering of wastage or materials may go unrecorded, items may be wrongly classified (as material X3 say, when material X8 was used in reality), or employees may make 'cosmetic' adjustments to their records to make their own performance look better than it really was.

Implementation breakdown:

This means that for a variety of causes employees will not always implement the plan in the way that was intended. The classic example is the purchase of raw materials at a lower than budgeted price, causing quality problems for production. Such problems may arise whether employees act with the best intentions or whether they deliberately take their own course because they do not agree with the plan. They may also be caused by poor communications or inadequate training.

Parameter prediction error:

This is another aspect of faulty *planning*. As Hongren says, 'planning decisions are based on predictions of future costs, future selling price, future demands and so on. In many cases there will be a difference between the actual value and the predicted value'. Such differences are not only due to uncertainty about the future: the predictions may not have taken proper accounts of conditions existing at the time when it was made, like a recently agreed pay rise, or an agreement to increase wages in three months time.

Inappropriate decision models:

Variance can arise when chosen decision model fails to capture important aspects affecting the decision. The solution to a linear programming model can be used when setting standards for direct material purchase prices. These standards, however, may be inappropriate if the LP solution is not feasible because the LP models fail to recognize a constraint on labour availability or storage capacity'. (It is the relationship between the variables that causes the problem here, not the failure to predict accurately.)

Randomness of operating processes:

A standard is an *average* figure: really it represents the mid-point of a range of possible values and therefore individual measurements taken at specific times will deviate *un*predictably within this predicable range.

STANDARD COSTING IN THE MODERN ENVIROMENT

Standard costing has traditionally been associated with labour-intensive operations, but it can be applied to capital-intensive production too. With the shift to an 'advanced manufacturing technology' environment we have seen the following.

- The introduction of robotics
- The introduction of flexible manufacturing systems (FMS).
- Computer aided design/computer aided manufacture (CADCAM) systems.
- Job flexibility, with workers capable of being moved from one aspect of work to another. The traditional one-man-one-machine manufacturing system does not apply.

It is quite possible that with manufacturing technology variable overheads are incurred in relation to machine time rather labour time, and standard costs should reflect this where appropriate.

With CADCAM systems, the planning of manufacturing requirements can be computerized, with the useful spin-off that standard costs can be constructed by computer, thus saving administrative time and expense while providing far more accurate standards.

However, as mentioned earlier, it has been argued that traditional variance analysis is unhelpful and potentially misleading in the modern organization, and can make managers focus their attention on the wrong issues, for example over-producing and stockpiling finished goods, because higher production volumes mean that overheads are spread over more units. Standard costing concentrates on quantity and ignores other factors contributing to effectiveness In a total quality environment, for instance, quantity is not an issue, however; quality is. Effectiveness is such an environment therefore centers on high quality output (produced as a result of high quality input and the elimination of non-value adding activities) and the cost of failing to achieve the required level of effectiveness is measured not in variances, but in terms of internal and external failure costs, neither of which would be identified by a traditional standard costing analysis.

Standard costing system might measure, say, labour efficiency in terms of individual tasks and level of output. In a total quality environment, labour is more likely to be viewed as a number of multi-task teams who are responsible for the completion of a part of the production process. The effectiveness of such a team is more appropriately measured in terms of re-working required, returns from customers, defects identified in subsequent stages of production and so on.

REINFORCING QUESTIONS

QUESTION ONE

a. Describe the extent to which standard costing variance may be interrelated
b. Explain how you would determine whether or not a standard variance should be investigated
c. Should variances occur in a TQM environment?
d. Why may variance analysis give rise to poor decisions in a JIT manufacturing environment?

QUESTION TWO

Budgeted and standard data for a product include the following. Direct labour:
Ten employees work a 45 hour week. Standard rate of pay is £4 per hour. Output per hour is 40 kg of product.

Direct materials;

Material	**Quantity (kg)**	**Price per Kg (£)**
X	60	2.00
Y	40	1.00
Z	100	1.40

From this standard mix, 180 kg of product are expected.
Actual data for the first week in April, were as follows.

Hours worked	45
Rate of pay	£4 per hour
Overhead incurred	£5400
Output	1980 kg

Production and consumption of materials were as follows:

Material	**Quantity (kg)**
X	700
Y	440
Z	1120

Required:

a) Calculate the following direct material variances for each material; i. Total ii. Price iii. Usage iv. Mix v. Yield
b) Calculate the direct labour efficiency variance

QUESTION THREE

Tardis Ltd. manufactures three products the Dalek, the Yeti and the Cyberman. The budget relating to period 1 is given below.

	Unit sales Price (£)	**Unit full cost of product** (£)	**Profit per unit** (£)	**Budgeted sales unit**	**Standard mix (%)**
Dalek	5	3	2	500	50
Yeti	7	4	3	300	30
Cyberman	10	6	4	200	20

Actual sales in period 1 were as follows:

	Units	%
Dalek	700	46.7
Yeti	300	20.0
Cyberman	500	33.3
	1,500	100.0

Required:

Calculate
a) Sales quantity variance
b) Sales mix variance
c) Sales volume variance by product and in total.

QUESTION FOUR

Energy product Co. produces a gasoline additive "gas gain." This product increases engine efficiency and improves gasoline mileage by creating a more complete burn in the combustion process. Careful controls are required during the production process to ensure the proper mix of input chemicals and to control evaporation. If the controls are ineffective, a loss in output and efficiency results.

The standard cost of producing a 500 litre batch of "gas gain" is sh.1350. The standard materials mix and related standard cost of each chemical used in a 500*l* batch are as follows;

Chemical	**Standard** Input quantity (*l*)	**Standard** Price per litre (Sh)	**Total** Costs
Echol	200	2.00	400
Protex	100	4.25	425
Benz	250	1.50	375
CT-40	50	3.00	150
Total	600		1350

The quantities of chemicals purchased and used in the past production period are shown in the schedule below.

A total of 140 batches of "gas gain" were manufactured. Energy products determine cost and chemical use variation at the end of each production period.

Chemical	**Quantity Purchased (*l*)**	**Total Purchase price (sh)**	**Quantity Used (*l*)**
Echol	25,000	53,650	26,600
Protex	13,000	62,400	12,880
Benz	40,000	58,400	37,800
CT-40	7,500	22,200	7,140
Total	85500	196,650	84,420

Required:

a) Calculate price variances by chemical
b) Calculate the materials (i) Mix variance (ii) Yield variance (iii) Usage (quantity)variance for each chemical used.

QUESTION FIVE

The chief accountant of Gannet Ltd. is interested in developing rudimentary financial models of aspects of the company's activities. Among those ideas under consideration is the budgeting of a simple cash budgeting model for use in forecasting the monthly cash flow from routine operations in any month.

Sales in the month of June 2010 were £100,000 the gross profit margin earned was 33 1/3% on cost of sales. It is confidently expected that the value of sales will increase by 1% per month for the foreseeable future.

20% of sales are made for cash, and the remainder on credit. Of the cash due form credit sale customers it is expected that 20% will be collected in the month after sale, 60% in the second month after sale, and 20% in the third month after sale. No bad debts are expected.
All purchases for resale are paid for in full in the month after purchase. All inventory purchased in the month **t** is sold in month **t+2**. Similarly, all sales in month **t+2** are made from inventory purchased in month **t.**

Monthly expenses (other than cost of sales) are expected to be as follows:

Payroll	5% of the preceding month's sales
Utilities	£3000
Depreciation	£1000
Other expenses	£10,000

No expenses are accrued.
It has been decided that, at this stage, no other factors, will be included in the model. The effects of dividends, taxation, capital expenditure and any other matter not already specified may therefore be ignored.

Required:

(a) Set out in the form of equations a model of Gannet's cash flows arising in any month from normal trading activities suitable for use as a formula for cash budgeting. (11 marks)

(b) Apply your model to produce a statement of the relevant cash flows arising in September 2010. (7 marks)

(c) Suggest how you could incorporate uncertainty in to your model if the chief accountant's staff were uncertain about the rate of sales growth, the relative proportion of cash and credit sales, and the speed of payment by credit customers. (7marks)

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

COMPREHENSIVE ASSIGNMENT No.3

TO BE SUBMITTED AFTER LESSON 6

To be carried out under examination conditions and sent to the Distance Learning Administrator for marking by the University

EXAMINATION PAPER. **TIME ALLOWED: THREE HOURS.**
ANSWER ALL QUESTIONS

QUESTION ONE

(a) What is meant by the term "integrated accounting" and what advantages does it present compared to other systems of cost and financial accounting? (8 marks)

(b) The following summary of Trading and Profit and Loss Account appears in the financial accounts of XYZ Company for the year ended 30 November 1994.

	Sh	Sh		Sh
To opening stock	175,000		By Sales	11,007,500
Purchases	4,725,000			
	4,900,000			
Less closing stock	297,500	4,602,500		
Direct wages		1,470,000		
Works expenses:				
Indirect wages	86,100			
Heat/light/power	239,400			
Sundry	42,000			
Depreciation	427,000	794,500		
		6,867,000		
Gross profit c/d		4,140,500		
		11,007,500		11,007,500
To Selling Expenses:			By Gross profit b/d	4,140,500
Advertising	224,700		Gain on sale of	
Salaries	576,100		freehold land	127,750
Travelling	114,100		Investment income	3,500
Show room	169,050			
Bad debts	29,750			
Miscellaneous	84,000	1,197,700		
Administration Expenses:				
Salaries	575,750			
Printing/stationery	87,150			
Audit fee	17,500			
General	82,250	762,650		
Financial Expenses:				
Debenture interest	61,250			
Bank interest/charges	114,625			
Hire Purchase Interest	15,575	191,450		
Net Profit		2,119,950		
		4,271,750		4,271,750

Notes:

1. In the cost accounts:

i. Works on cost, excluding depreciation, is charged at a rate of Sh.1.75 per product produced.
ii. Selling on cost is charged at a rate of 12% of sales turnover.
iii. Administration expenses are charged at a fixed sum of Sh 875,000 per annum. Included in this is an allowance of Sh 105,000 to cover all financial income and expenditure.

2. Depreciation in the financial accounts is calculated on a straight line basis, but in the cost accounts there is a fixed annual charge equivalent to half of the straight line calculation plus a running charge of Sh 0.875 per unit produced.
3. The company owns the business premises but in the cost accounts, in addition to the items mentioned above, there is a notional charge of Sh 17,500 per annum for rent.
4. Items of a non-revenue nature are not included in the cost accounts.
5. In the cost accounts stock is valued at direct cost (material and labour) but in the financial accounts it includes an allowance for overheads.
6. Opening stock consists of 5,000 units valued as follows:

	Sh
Materials	108,500
Labour	38,500
Overheads	28,000
	175,000

Closing stock consists of 8000 units valued as follows:

	Sh
Materials	185,500
Labour	59,500
Overheads	52,500
	297,500

Sales were 210,000 units at Sh 52.50 per unit

7. The profit per cost accounts is given as Sh 1,922,375.

Required:

Prepare a detailed statement reconciling the profit of Sh 2,119,950 as disclosed by the financial accounts with the figure of Sh 1,922,375 as disclosed in the cost accounts. (12 marks)

(Total: 20 marks)
(CPA DEC `94)

QUESTION TWO

(a) Two opposing political parties are nominating candidates for presidency in the Peoples Republic of Yanke in separate conventions that are being held simultaneously. The following probabilities apply for the respective party winning the election for the indicated nominee pair. Naturally each party wishes to maximise its probability of winning the presidency.

Peoples Party	Republican Party	Probability for the Peoples Party winning	Probability for the Republican Party winning
Maina	Salat	0.75	0.25
Maina	Simiyu	0.25	0.75
Onyango	Salat	0.30	0.70
Onyango	Simiyu	0.60	0.40

Subtracting the Republican's win probability from the People's win probability gives the Peoples payoff measure so that the structure is converted into a zero-sum game. By carrying out this operation, construct the appropriate payoff table and then, using the minimax criterion, determine the value of this political game. (10 marks)

(b) Two suspects are taken into police custody and separated. The prosecutor is certain that they are guilty of a specific crime, but he does not have adequate evidence to convict them at a trial. He points out to each suspect that each has two alternatives: to confess to the crime the police are sure they have committed, or not to confess. If they both do not confess then the prosecutor states that he will book them on some minor charge and they will both serve one year in prison. If they both confess, they will be jailed each for 8 years. If one confesses and the other does not the "co-operative" one will receive lenient treatment (i.e. three months jail term) while the other will receive the maximum jail term allowed by the law (i.e fourteen years) for this crime.

Required:

i. Convert this situation into a non-zero sum game and determine how long each suspect will serve in jail. (5 marks)

ii. Discuss some of the managerial applications of this game. (5 marks)

(Total: 20 marks)

(CPA DEC `92)

QUESTION THREE

Fatuma Ltd. plans to manufacture a new product called Beauty 92 which requires a substantial amount of direct labour on each unit. Based on the company's experience with other products which required similar amounts of direct labour, the management of Fatuma Ltd. believes that there is a learning factor in the production process used to manufacture Beauty 92.

Each unit of Beauty 92 requires 50 square centimetres of raw material at a cost of Sh.30 per square centimetre. The standard direct labour rate is Sh 25 per direct labour hour. Variable manufacturing overhead is assigned to products at a rate of Sh 40 per direct labour hour. The company adds a markup of 30% on variable manufacturing costs in determining an initial price offer for all its products.

Data on the production of the first two lots (16 units) of Beauty 92 is as follows:

i. The first lot of 8 units required a total of 3,200 direct labour hours.

ii. The second lot of 8 units required a total of 2,240 direct labour hours.

Based on prior production experience, Fatuma Ltd, anticipates that there will be no significant improvement in production time after the first 32 units. Therefore, a standard for direct labour hours will be established based on the average hours per unit for units 17 - 32.

Required:

(a) Based on the data presented above for the first 16 units, calculate the learning rate applicable to the direct labour required to produce Beauty 92. (5 marks)

(b) Calculate the standard for direct labour hours which Fatuma Ltd should establish for each unit of Beauty 92. (5 marks)

(c) After 32 units have been manufactured, Fatuma Ltd. receives an order of 96 units. What price per unit should Fatuma Ltd. charge on this order? (5 marks)

(d) Knowledge of the learning curve phenomenon is a valuable Management tool. Explain how this knowledge can be used in planning and controlling business operations. (5 marks)

(Total: 20 marks)
(CPA JUN `92)

QUESTION FOUR

Kensoup produces and markets soup products. The company's normal production volume is 3,000 kg per month. The product is packed in packages of 3 kg per month. The product is packed in packages of 3 kg. with each pack containing 12 packets of 250 grams each. The standard cost of producing a 3 kg. soup pack is as follows:

		Sh
Direct materials -	5 Kg. at Sh 40 per Kg	200
Direct labour -	0.5 hrs at Sh 100 per hour	50
Variable overheads -	based on direct labour hours	40
Fixed overheads -	based on direct labour hours	30
		320

In the month of September, the company produced and sold 1,200 packages of 3 Kg. soup at a total cost of Sh 408,200 comprised of the following amounts:

		Sh
Direct materials -	6,800 kg	261,800
Direct labour -	540 hrs	56,700
Variable overheads		52,800
Fixed overheads		36,900
		408,200

There were no beginning or ending inventories of either raw materials, work-in-progress or finished goods.

Required:

(a) Prepare the following accounts assuming that variances are written off to cost of sales;

i. Work-in-progress;
ii. Finished goods;
iii. Cost of Sales;
iv. Variances control account (14 marks)

(b) Comment on the appropriateness of the method used above with regard to stock valuation. (6 marks)

(Total: 20 marks)
(CPA DEC `89)

QUESTION FIVE

Trans-Africa Airways has five types of aircraft which it has to assign to five different routes. Because of variations among the routes (namely distances, number of passengers, weather conditions and airport facilities) the aircraft are not all equally adapted to each route.

The cost (in millions of shillings) over each of the routes for each aircraft is shown below:

Route/ Aircraft	Nairobi - Harare	Nairobi - Lagos	Nairobi - Bombay	Nairobi - London	Nairobi - New York
DC - 7	10	15	17	40	45
DC - 10	8	12	20	38	43
F - 27	7	14	18	34	37
B - 727	6	13	12	30	32
Airbus	9	12	21	42	41

Required:
Determine how the aircraft should be assigned to the various routes so as to determine the total costs. (10 marks)

(b) The Audit Manager of Jasho, Certified Public Accountants, estimates that it will take 30 hours to complete the audit of the fixed assets of a medium size manufacturing company.

The following figures (in hours) have been obtained from recently completed assignments of medium size manufacturing companies:

26, 27, 28, 28, 29, 29, 30, 30, 31, 32.

Test the validity of the manager's estimate at the five per cent level of significance.

Note. At 9 degrees of freedom, to .05 = 1.83 (10 marks)

(Total: 20 marks)
(CPA JUN `92)

END OF COMPREHENSIVE ASSIGNMENT No.3

NOW SEND YOUR ANSWERS TO THE DISTANCE LEARNING CENTRE FOR MARKING

LESSON SEVEN

GAME THEORY AND MARKOV ANALYSIS

OBJECTIVES

A study of:

Classification of Games:

Zero and Non-zero sum games.

Markov analysis:

The use of Markov analysis in the formulation of strategic moves.

CONTENTS

Game theory and Markov analysis have been adequately covered in the Study Text. Management and Cost Accounting by Colin Drury does not cover these topics. Particular attention should be paid to solved examples in the Study Text.

Attempt the reinforcing questions at the end of this lesson and check your answers with those given in Lesson 10.

- Introduction
- Game Theory
- Non-zero Sum Games
- Markov Analysis

1. INTRODUCTION

THE GAMES ECONOMISTS PLAY

It sounds like a sports fan's dream. In Stockholm on October 11th, three men share a $1m prize for their skill at analysing games. They are not television pundits, or armchair critics of Manchester United or the Miami Dolphins, but economists. Two Americans, John Harsanyi and John Nash, and a German, Reinhard Selten, have won 1994 Nobel prize for economics for their studies of "game theory".

Game theory may sound trivial. It is not. In the past 20 years or so it has revolutionised the economics of industrial organization and has influenced many other branches of the subject, notably the theories of monetary policy and international trade. These days, no economics student can hope to graduate without knowing the rudiments of it.

Odd though it may seem, until game theory came along most economists assumed that firms could ignore the effects of their behaviour on the actions of others. That is fine when markets are perfectly competitive: what one firm or consumer does can make no difference to the overall picture. Fine, too, when unchallenged monopolists hold sway: they have no rivals to worry about.

But in many instances this assumption is wrong. Many industries are dominated by a few firms: by building a new plant or cutting prices (or threatening to cut them), a firm can affect how its rivals behave. And it is not just in industrial economics that such calculations matter. Some countries may impose (or threaten) trade sanctions against others in an attempt to prise open protected markets. A government may put up short-term rates when inflation is low to convince financial markets that it is serious about fighting inflation; with luck, the markets will then require lower long-term interest rates.

These examples are, in a way, just like games: no football coach plans an attack without taking into account the defenders' likely response. Modern game theory was fathered by Johm von Neumann, a mathematician, and Oskar Morgenstern, an economist, who published "Theory of Games and Economic Behaviour" in 1944 (an anniversary which was not lost on 1994 Nobel-prize givers). Messrs Harsanyi, Nash and Selten have honed it into the sharp tool economists use today.

In the early 1950's Mr Nash produced a compelling way of working out how games will end up when players cannot commit themselves, or do not want to collude with each other. A "Nash equilibrium" occurs when no player wants to change his strategy, given full knowledge of other players' strategies.

A familiar dilemma

In each box, firm 1's profit is on the left and firm 2's on the right

		Firm 2's price: High		Firm 2's price: Low	
Firm 1's price	High	3	3	1	4
	Low	4	1	2	2

The chart shows one famous example of a Nash equilibrium. An industry has two firms (1 and 2). Each of them can choose a "high" or a "low" price. If they both choose high prices, they make hefty profits, of $3m apiece. With low prices they make only $2m each. But if one sets a high price and the other a low one, the low-price firm makes $4m; the high price firms gets a mere $1m. Although the firm would do best if they both set high prices, they will not. If firm 1 sets a high price, firm 2's best choice is to undercut it; it would then make $4m, rather than $3m. And if firm 1 set a low price, firm 2 should do the same: it would earn $2m instead of $1m. The same goes for firm 1: it too sets a low price. So they both choose low prices—and they make only $2m each.

The incredible game theorist

But Mr Nash's work needed refining. First, it applies to games played only once, or in which players move simultaneously. But virtually all interesting economic games involve continual interaction between players. Mr Selten extended the Nash equilibrium to such settings. From that emerges the importance of credibility: there is no point in one player following a plan which other players know will have to be changed at some point.

For example, a monopolist might try to keep a would-be rival out of its market by threatening a price war if the rival steps in. Such a war might mean that the entrant would make a loss. But it would be costly for the monopolist, too. If the price war costs a lot, the monopolist would do better to share the market with the new entrant. In that case, the threat of a price war is not credible: the entrant can go into the market, knowing the incumbent would be foolish to fight back.

Second, it is not realistic to assume that players know what is in each other's minds. As Adam Brandenburger of Harvard Business School puts it, "games are played in a fog," Mr Harsanyi cut through the mist, by showing that games in which players are not well informed about each other can be analysed in almost the same way as ordinary games.

When some players have information that others do not, their strategies can alter their reputations to their advantage. A government that shoves interest rates up to signal its anti-inflation credentials is one example. Bond traders do not know whether it truly wants to curb inflation, but hope that it might; the longer interest rates stay up, the more hopeful they get. Or take a monopolist that would like to stop entry into its market. It can do so if would-be rivals fear that it likes fighting price wars, in spite of the cost. By fighting anyone who does enter, the monopolist can build a reputation as a price warrior and put other entrants off. An oligopolist may be able to keep profits up by setting high prices and nurturing reputations for being friendly to their rivals.

Yet some economists are still wary of game theory, despite the insights it has brought. That is partly because the theory is difficult: it demands lots of tricky mathematics. That, however, is merely a reflection of the complexity of the world. But there is a more telling objection—namely, that game theory is just that: theory. So far there have been precious few real-life applications. Game theorists have been good at explaining the intricacies underlying strategic interdependence and producing ever more refined concepts of equilibrium, but less adept at giving governments and firms practical advice.

Even that criticism, though, is weakening. Central bankers now know (or should know) all about reputation and credibility. America's Federal Communications Commission used game theorists to design 1994 auction of radio spectrum. And businesses, too, are beginning to learn about game theory: Harvard's Mr Brandenburger, for example, teaches it to MBA students and executives. The time is coming, he says, when self-respecting MBA'S, as well as economics graduates with a taste for maths, will not leave school without it.

2. GAME THEORY

Game theory was developed for the purpose of analyzing competitive situation involving conflicting interests. In game theory, there are assumed to be two or more persons with different objectives, each of whole action influences, but does not completely determine the outcome of the game. Each person is assumed to know his opponents objectives. Game theory provides solutions to such games assuming that each of the players is to maximize his minimum expected profit or minimize his maximum expected loss. This criterion (which is based on a conservative view of the problem) is referred to as Minimax or maximini criterion.

Types of games:

Four basic ways in which competitive situations (or games) can be classified

are: **a) Number of Competitors**

In game theory a competitor is characterized as a distinct set of interests and is usually referred to as a person. Competitors could be individuals, group of individuals, corporation, and an army e.t.c. The smallest no. of competitors are 2 and the situation is referred to as a two-person game. If there are more than two competitors, the resulting many-person competitive situation is called and N-person game.

b) Nature of the payoff.

Games are also classified with respect to the nature of the payoff, that is, what happens at the end of the game. The distinction in this respect is between zero-sum games and non zero-sum games. If the sum of the payoffs to all players of a game is zero, counting winnings as positive and losses as negative, then, the game is zero-sum otherwise it is non zero sum. Zero-sum games are strictly competitive games. In a non-zero sum game, the interests of competitors may best be served if they corporate with each other.

c) The amount of information the competitors have:

There are three basic aspects of the game about which the players need some information in order to play

(i) Who their competitors are
(ii) What their competitors can do
(iii) How the outcome of the game will be affected by the actions taken by participants.

Games in which each participant knows the payoff for winning, knows who the competitors are, and knows all the moves the competitors make as soon as they make them are referred to as games with perfect information. Games lacking full information on what competitors can do or on what the outcome of the game will be in certain situations are said to be games with incomplete information. Games with complete but imperfect information may also exist.

d) Strategies

In game theory a strategy for a particular player is a plan which specifies his action for every possible action of his opponent. It is a complete plan for playing the game in every possible eventuality. Games can be categorized according to the number of strategies available to each player. If player 1 has M possible strategies and player 2 has N possible strategies, then the game is M x N. If the greatest no. of strategies available to any player is finite, then the game is finite and if at least one player has an infinite no. of available strategies, then the game is infinite.

Game theory is used to determine the optimum strategy in a competitive situation. When two or more competitors are engaged in making decisions, it may involve conflict of interests. In such a case the outcome depends not only upon an individual's action but also upon the actions of the others. Both (competing) sides face a similar problem. Hence game theory is a science of conflict. Game theory does not concern itself with finding an optimum strategy but it helps to improve the decision process.

Game theory has been used in business and industry to develop bidding tactics, pricing policies, advertising strategies, timing of the introduction of new models into market, etc.

RULES OF GAME THEORY

i. The number of competitors is finite.
ii. There is a conflict of interests between the participants.
iii. Each of these participants has available to him a finite set of available courses of action i.e. choices.
iv. The rules governing these choices are specified and known to all the players.
While playing each player chooses a single course of action from the list of choices available to him.
v. The outcome of the game is affected by choices made by all of the players. The choices are to be made simultaneously so that no competitor knows his opponent's choices until he is already committed to his own.
vi. The outcome for all specific choices by all the players is known in advance and numerically defined.
vii. The players act rationally and intelligently.

When a competitive situation meets all these criteria above, we call it a game.

Note:

Only in a few real competitive situations can game theory be applied because all the rules are difficult to apply at the same time to a given situation.

Two-person, zero-sum games

Illustration

Two players X & Y have two alternatives. They show their choices by pressing two types of buttons in front of them but they cannot see the opponents move. It is assumed that both players have equal intelligence and both intend to win the game.

This sort of simple game can be illustrated in tabular form as follows:

	PLAYER Y		
		Button r	**Button t**
PLAYER X	Button m	X wins 2 points	X wins 3 points
	Button n	Y wins 3 points	Y wins 2 points

The game is biased against Y, because if player X presses button m, he will always win. Hence Y will be forced to press button r, to cut down his losses.

Alternative Example

	PLAYER Y		
		Button r	**Button t**
PLAYER X	Button m	X wins 3 points	Y wins 4 points
	Button n	Y wins 2 points	X wins 1 points

In this case X will not be able to press button m all the time in order to win (or button n). Similarly Y will not be able to press button r or button t all the time in order to win. In such a situation, each player will exercise his choice for part of the time based on probability.

STANDARD CONVENTIONS IN GAME THEORY
Consider the following table

	Y	
X	3	-4
	-2	1

X plays row I, Y plays Column I, X wins 3 points
X plays row I, Y plays Column II, X loses 4 points
X plays row II, Y plays Column I, X loses 2 points

X plays row II, Y plays Column II, X wins 1 points

3, -4, -2, 1 are the known pay offs to X (X takes precedence over Y)

Here the game has been represented in the form of a matrix. When the games are expressed in this fashion the resulting matrix is commonly known as **PAYOFF MATRIX.**

Strategy

It refers to a total pattern of choices employed by any player. Strategy could be **pure** or a **mixed** one. In a **pure strategy**, Player X will play one row all of the time or player Y will also play one of his columns all the time.

In a **mixed strategy**, Player X will play each of his rows a certain portion of the time and player Y will play each of his columns a certain portion of the time.

Value of the Game

The value of the game refers to the average pay off per play of the game over an extended period of time.

Example

$$\begin{array}{cc} & \textbf{Player Y} \\ \textbf{Player X} & \begin{bmatrix} 3 & 4 \\ -6 & 2 \end{bmatrix} \end{array}$$

In this game Player X will play his first row on each play of the game. Player Y will have to play first column on each play of the game in order to minimise his losses.

So this game is in favour of X and he wins 3 points on each play of the game.

This game is a game of pure strategy and the value of the game is 3 points in favour of X.

Example

Determine the optimum strategies for the two players X and Y and find the value of the game from the following pay off matrix.

$$\begin{array}{cc} & \textbf{Player Y} \\ \textbf{Player X} & \begin{bmatrix} 3 & -1 & 4 & 2 \\ -1 & -3 & -7 & 0 \\ 4 & -7 & 3 & -9 \end{bmatrix} \end{array}$$

Strategy:

Assume the worst and act accordingly.

If X plays first:

If X plays with his first row then Y will play with his 2nd column to win 1 point.
Similarly, if X plays with his second row, then Y will play his 3rd column to win 7 points. And if X plays with his third row, the Y will play his 4th column to win 9 points.

In this game, X cannot win so he should adopt first row strategy in order to minimise his losses.

This decision rule is known as "**Maximin Strategy**", i.e. X chooses the highest of these minimum payoffs.

Using the same reasoning from the point of view of Y

If Y plays with his first column, the X will play his third row to win 4 points.
If Y plays with his second column, the X will play his first row to lose 1 point.
If Y plays with his third column, the X will play his first row to win 4 points.
If Y plays with his forth column, the X will play his first row to win 2 points.

Thus the player Y will make the best of the situation by playing his 2nd column which is a "**Minimax strategy**".
This game is also a game of pure strategy and the value of the game is -1 (win of 1 point per game to Y). Using matrix notation, the solution is shown below:

$$
\begin{array}{lc}
 & \textbf{Player Y} \\
\textbf{Player X} & \begin{bmatrix} 3 & -1 & 4 & 2 \\ -1 & -3 & -7 & 0 \\ 4 & -7 & 3 & -9 \end{bmatrix} \\
\textbf{Column maximum} & \begin{matrix} 4 & -1 & 4 & 2 \end{matrix}
\end{array}
\qquad
\begin{array}{c}
\textbf{Row minimum} \\ -1 \\ -7 \\ -9
\end{array}
$$

In this case value of the game is -1
Minimum of the column maximum's is -1
Maximum of the row minimum's is also -1

i.e. X's strategy is Maximin Strategy
Y's strategy is Minimax Strategy

SADDLE POINT

The saddle point in a payoff matrix is one which is the smallest value in its row and the largest value in its column. It is also known as equilibrium point in the theory of games.

Saddle point also gives the value of such a game. In a game having a saddle point, the optimum strategy for both players is to play the row or column containing the saddle point.

Note:

If in a game there is no saddle point, the players will resort to what is known as mixed strategies.

MIXED STRATEGIES

Example:

Find the optimum strategies and the value of the game from the following pay off matrix concerning two person game.

$$
\begin{array}{lc}
 & \textbf{Player Y} \\
\textbf{Player X} & \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix}
\end{array}
$$

In this game there is no saddle point.

Let Q be the proportion of time player X spends playing his 1st row; and 1-Q be the proportion of time player X spends playing his 2nd row;

Similarly

Let R be the proportion of time player Y spends playing his 1st column and
1-R be the proportion of time player Y spends playing his 2nd column.
The following matrix shows this strategy.

		Player Y	
		R	**1-R**
	Q	1	4
Player X	**1-Q**	5	3

X's Strategy

X will like to divide his play between his rows in such a way that his expected winnings or losses when Y plays the first column will be equal to his expected winnings or losses when Y plays the second column.

COLUMN I

Points	Proportion Played	Expected Winnings
1	Q	Q
5	1-Q	5(1-Q)

Total =Q + 5(1-Q)

COLUMN II

Points	Proportion Played	Expected Winnings
4	Q	4Q
3	1-Q	3(1-Q)

Total = 4Q + 3(1-Q)

$\therefore$ Q + (1-Q)5 = 4Q + 3(1-Q)
Giving Q = 2/5 and (1-Q) = 3/5

This means that player X should play his first row 2/5th of the time and his second row 3/5 of the time.

Using the same reasoning Y' Strategy;

1 X R + 4(1-R) = 5R + 3(1-R)
Giving R = 1/5 and (1-R) =4/5

This means that player Y should divide his time between his first and second column in the ratio 1:4

This is shown in the following matrix

$$\begin{array}{ccc} & & \textbf{Player Y} \\ & & \begin{array}{cc} 1/5 & 4/5 \end{array} \\ & \begin{array}{c} 2/5 \\ 3/5 \end{array} & \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} \\ \textbf{Player X} & & \end{array}$$

Short-cut Method of determining Mixed Strategies

$$\begin{array}{cc} & \textbf{Player Y} \\ \textbf{Player X} & \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} \end{array}$$

STEP I

Subtract the smaller payoff in each row from the larger one and smaller payoff in each column from the larger one.

$$\begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} \qquad \begin{array}{l} 4-1 = 3 \\ 5-3 = 2 \end{array}$$

5-1 = 44-3 = 1

STEP II

Interchange each of these pairs of subtracted numbers found in Step I

$$\begin{array}{ccc} & Y & \\ X & \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \\ & \begin{array}{cc} 1 & 4 \end{array} & \end{array}$$

Thus player X, plays his two rows in the ratio 2:3
and player Y, plays his columns in the ratio 1:4

This is the same result as calculated before.

To Determine the Value of the Game in Mixed Strategies

In a simple 2 X 2 game without a saddle point, each player's strategy consists of two probabilities denoting the portion of the time he spends on each of his rows or columns. Since each player plays a random pattern, the probabilities are listed as under:

Pay Off	Strategies which produce this pay off	Joint Probability
1	Row I Column I	2/5 X 1/5 = 2/25
4	Row I Column II	2/5 X 4/5 = 8/25
5	Row II Column I	3/5 X 1/5 = 3/25
3	Row II Column II	3/5 X 4/5 = 12/25

Expected Value (or value of the game)

Pay Off	Probability p(x)	Expected Value x p(x)
1	2/25	2/25
4	8/25	32/25
5	3/25	15/25
3	12/25	36/25

$$\Sigma x\, p(x) = \frac{85}{25} = \frac{17}{5} = 3.4$$

3.4 is the value of the game.

DOMINANCE

Dominance strategy is useful for reducing the size of the payoff table.

Rules of Dominance

i. If all the elements in a column are greater than or equal to the corresponding elements in another column, then the column is dominated.

ii. Similarly, if all the elements in a row are less than or equal to the corresponding elements in another row, then the tow is dominated.

Dominated rows and columns may be deleted which reduces the size of the game.

Note:

Always look for dominance and saddle point when solving a game.

Example:

Determine the optimum strategies and the value of the game from the following 2 X m payoff matrix game for X and Y.

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 6 & 3 & -1 & 0 & -3 \\ 3 & 2 & -4 & 2 & -1 \end{bmatrix} \end{array}$$

In this case columns I, II and IV are dominated by columns III and V. Hence Y will not pay these columns.

So the game is reduced to 2 X 2 matrix
Hence this game can be solved using
Method already discussed

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} -1 & -3 \\ -4 & -1 \end{bmatrix} \end{array}$$

GRAPHICAL METHOD

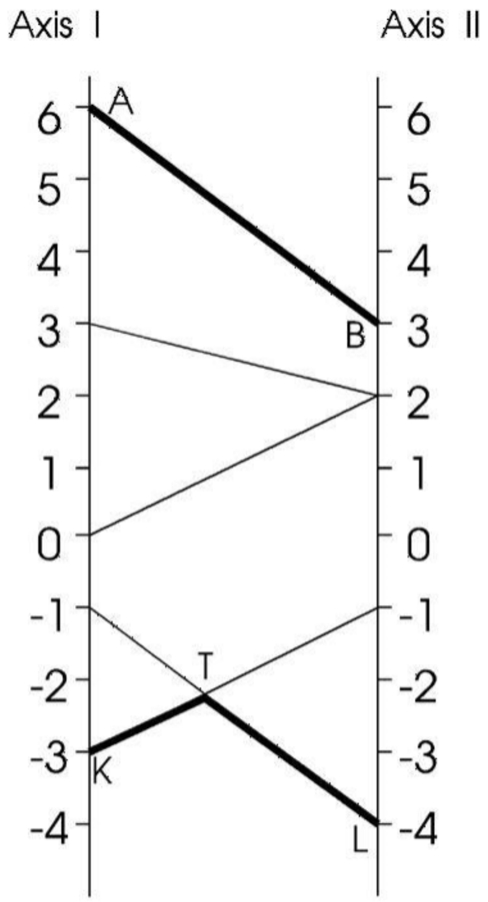
Graphical methods can be used in games with no saddle points and having pay off *m X 2* or *2 X n* matrix. The aim is to substitute a much simpler 2 X 2 matrix for the original m X 2 or 2 X m matrix.

Example I

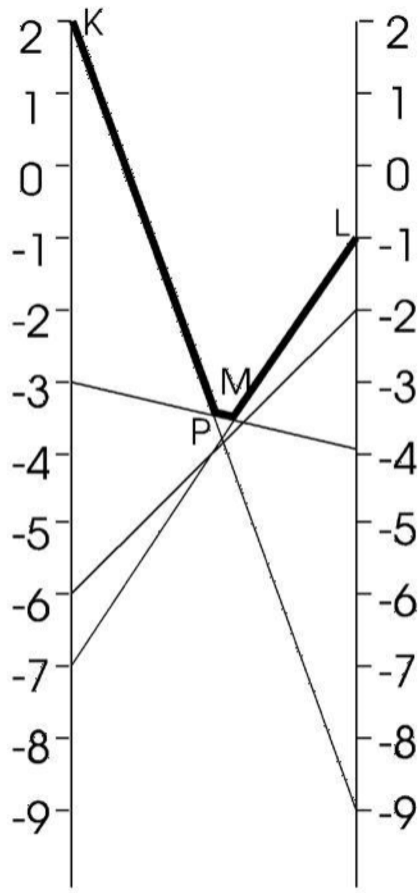
Determine the optimum strategies and the value of the game from the following payoff matrix game.

$$\begin{array}{cc} & \mathbf{Y} \\ \mathbf{X} & \begin{bmatrix} 6 & 3 & -1 & 0 & -3 \\ 3 & 2 & -4 & 2 & -1 \end{bmatrix} \end{array}$$

Draw two vertical axes and plot two payoffs corresponding to each of the five columns. The pay off numbers in the first row are plotted on Axis I and those in second row on Axis II.



Example I



Example II

Thus the two pay off numbers 6 and 3 in the first column are shown respectively by point A on Axis I and point B on Axis II.

Join the corresponding pay off numbers on Axis I and Axis II by straight lines.

On the two intersecting lines at the very bottom, thicken them from below up to the point of intersection, i.e. highest point on the boundary.

The thick lines on the graph KT and LT meet at T

The two lines passing through T, identify the two critical moves of Y which combined with X, yield the following 2 X 2 matrix.

$$\begin{array}{cc} & \mathbf{Y} \\ \mathbf{X} & \begin{bmatrix} -1 & -3 \\ -4 & -1 \end{bmatrix} \end{array}$$

The value of the game and the optimum strategies can be calculated using the methods described earlier.

Example II

Determine the optimum strategies and the value of the game from the following pay-off matrix concerning a two person 4 X 2 game.

$$
\begin{array}{cc}
 & Y \\
X & \begin{bmatrix} -6 & -2 \\ -3 & -4 \\ 2 & -9 \\ -7 & -1 \end{bmatrix}
\end{array}
$$

This method is similar to the previous example, except we thicken the line segments which bind the figure from the top and take the lowest point on the boundary.
The segments KP, PM and ML drawn in thick lines bind the figure from the top and their lowest intersection M, through which the two lines pass defines the following 2 X 2 matrix relevant to our purpose.

$$
\begin{array}{cc}
 & Y \\
X & \begin{bmatrix} -3 & -4 \\ -7 & -1 \end{bmatrix}
\end{array}
$$

The optimum strategies and the value of the game can now be calculated.

3. NON-ZERO SUM GAMES

Within very vast situations of possible non-zero games, varying degrees of co-operation exist between the participants. Games theory has been sufficiently developed to deal with two extreme forms—the co-operative game and non-cooperative game.

In a co-operative game, the players have complete freedom to communicate with each other. They can make threats, enter into agreements favourable to them. They can freely negotiate and enter into binding agreements. In a non-cooperative game there is no communication between the participants and no way of enforcing agreements.

Prisoner's dilemma games, battle of sexes games, chicken and Hawk (dove game) are all examples of non-zero sum games.

1. Prisoner's Dilemma

Story

Two persons are arrested for a crime. The police lack sufficient evidence to convict either suspect and consequently need them to give testimony against each other.

The police tell each suspect that if he testifies against the other (or does not cooperate with the other), he will be rewarded for testifying and hence will be released, provided the other suspect does not testify against him. If neither testifies, the prosecutor will be unable to prove the crime and each suspect can only receive minor sentence. If both suspects confess and testify against each other, then both will receive an intermediate prison sentence. Hence the conflict of interest, the tendency to double cross and lose the confidence of the other.

Each suspect must decide under the circumstances, whether or not to confess.

It is possible to translate such a situation in the form of a table and a payoff matrix as follows:

		Suspect Two	
		Not confess	Confess and testify
Suspect one	Not confess	One year prison term for each	10 year prison term for suspect one, suspect two released
	Confess and testify	Suspect one released and suspect two gets 10 year prison term	Both suspects get 4 years prison term

Payoff Table

		Suspect Two	
		Not confess	Confess and testify
Suspect one	Not confess	-1, -1	-10, 0
	Confess and testify	0, -10	-4, -4

This is an example of non-zero sum, non-cooperative game.

2. Battle of the Sexes

game Story

Two players (a couple) wish to go to an event together but disagree about whether to go to a football game or the variety show. Each player gets a utility of 2 if both go to his or her preferred event, a utility of 1 if both go to the other's preferred event and zero if both are unable to agree and stay at home or go out individually. The pay-off matrix can be represented as follows:

		Her	
		Football (F)	Variety Show (V)
His	Football (F)	2, 1	0, 0
	Variety Show (V)	0, 0	1, 2

3. Chicken and Hawk (dove game)

Two players meet at a one-lane bridge and each must choose whether to cross first or wait for the other. If both play Tough (T), they crash in the middle of the bridge and get -1 each, if both play Weak (W), they keep on waiting and get 0, if one player chooses Tough (T) and the other Weak (W), then the tough player crosses first receiving 2, and the other weak one receives 1.

		Player II	
		(T)	(W)
Player I	(T)	-1, -1	2, 1
	(W)	1, 2	0, 0

All these games are similar and have similar types of Nash Equilibrium points.

1. For instance in battle of sexes games, there are three Nash equilibrium points. Two are pure strategies with payoffs (2,1) and (1,2) and third one can be a mixed strategy depending upon sex, (who is more dominant) i.e.

   Mixed Strategy:

   Player `His' plays F with probability 2/3 and player `Her' plays F with probability 1/3. Similarly player `His' plays `V' with 1/3 of the probability and player `Her' with probability 2/3.

2. In the Chicken and Hawk game, if both players act tough, then the probability is 1/2, 1/2.

Note:

This is only possible if both players act rationally and from long experience are able to coordinate their games. Another assumption is, the players study the moves of each other at every stage and try to discover the pattern of the opponent's play.

4. Each individual farmer can maximise his own income by maximising the amount of crops that he produces. When all farmers follow this policy, the supply exceeds demand and the prices fall. On-the-other-hand, they can agree to reduce the production and keep the prices high.

This creates a dilemma to the farmer.

This is an example of a non-zero sum game.

Similarly, marketing problems are non-zero sum games, as elements of advertising come in. In such cases, the market may be split in proportion to the money spent on advertising multiplied by an effectiveness factor.

5. The table given below is a pay off matrix for two large corporations A and B. Initially they both have the same prices. Each considers cutting their prices to gain market share and hence improve profit.

		CORPORATION B	
		Maintain Prices	**Decrease Prices**
CORPORATION A	Maintain Prices	3,3 Status quo	1,4 B gets market share and profit
	Decrease Prices	4,1 A gains market share and profit	(2,2) Both retain market share but lose profit

The entries in the pay off matrix indicates the order of preferences of the players i.e. first A and then B.

We may suppose that if both corporations study the situation, they will both decide to play row I, column I (3,3).

However:

Suppose A's reasoning is as follows:
If B plays column I, then I should play column II because I will increase my gain to 4.

In the same way B's reasoning may be as follows:

If A plays row I, then I should play column 2, to get pay off 4 per play.

If both play 2 (row two and column two), each receives a pay-off of 2 only.

In the long run, pay-off (2,2) forms a new equilibrium point because if either party departs from it without the other doing so he will be worse off before he departed from it.

Game theory seems to indicate that they should play (2,2) because it is an equilibrium point but this is not intuitively satisfying. On the other hand (3,3) is satisfying but does not appear to provide stability. Hence, the dilemma.

THEORY OF METAGAMES

This theory appears to describe how most people play non-zero sum games involving any number of persons.
Prisoner's dilemma is an example of this, The aim is to identify points at which players actually tend to stabilise their play in non-zero sum games.
This theory not only identifies equilibrium points missed by traditional game theory in games that have one or more such points but also does so in games in which traditional theory finds no such points.

Its main aim is that each player is trying to maximise the minimum gain of his opponent.

ADVANTAGES AND LIMITATIONS OF GAME

THEORY Advantage:

Game theory helps us to learn how to approach and understand a conflict situation and to improve the decision making process.

Limitations:

1. Businessmen do not have all the knowledge required by the theory of games. Most often they do not know all the strategies available to them, nor do they know all the strategies available to their rivals.
2. There is a great deal of uncertainty. Hence we usually restrict ourselves to those games with known outcomes.
3. The implications of the minimax strategy is that the businessman minimises the chance of maximum loss. For an ambitious businessman, this strategy is very conservative.
4. The techniques of solving games involving mixed strategies where pay-off matrices are rather large, is very complicated.
5. In non-zero sum games, mathematical solutions are not always possible. For example, a reduction in the price of commodity may increase overall demand. It is also not necessary that demand units will shift from one firm to another.

PRACTICE QUESTIONS

QUESTION ONE

A has two ammunition stores, one of which is twice as valuable as the other. B is an attacker who can destroy an undefended store but he can only attack one of them. A can only successfully defend one of them.

What would A do so as to maximise his return from the situation no matter what B may do?

QUESTION TWO

Determine the optimum strategies and the value of the game for the following pay-off matrix.

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \end{array}$$

QUESTION THREE

(a) For the following pay-off matrix for firm A determine the optimal strategies for both the firms and the value of the game (You may use maximin—minimax principle).

$$\begin{array}{cc} & \text{Firm B} \\ \text{Firm A} & \begin{bmatrix} 3 & -1 & 4 & 6 & 17 \\ -1 & 8 & 2 & 4 & 12 \\ 16 & 8 & 6 & 14 & 12 \\ 1 & 11 & -4 & 2 & 1 \end{bmatrix} \end{array}$$

(b) Explain the principle of dominance in Game theory and solve the following game

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & 3 & 2 & 7 & 4 \\ 3 & 4 & 1 & 5 & 6 \\ 6 & 5 & 7 & 6 & 5 \\ 2 & 0 & 6 & 3 & 1 \end{bmatrix} \end{array}$$

QUESTION FOUR

(a) Define pure and mixed strategies. What is a fair game?

(b) Solve the following game graphically and find the value of the game.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 8 & 4 & -2 \\ -2 & -1 & 3 \end{bmatrix} \end{array}$$

QUESTION FIVE

A party X sends two bombers I and II, to bomb an installation of the opponent Y. Bomber II follows bomber I at a distance. One of the bombers carries a bomb and the other acts as an escort. Y has single fighter to attack the two bombers. The bombers are equipped with guns of different calibres to engage the fighter. If the fighter attacks bomber II, it can be engaged by the gun of this bomber only whereas if it attacks bomber I it can be engaged by the guns of both the bombers. The probability of the fighter being shot down in the first case is 0.3 and in the second 0.7. If the fighter is not shot down it destroys the bombers it attacks with a probability of 0.6. It is required to analyse the game and determine

(a) For X, which bomber to carry the bomb
(b) For Y, which bomber to attack

QUESTION SIX

There are two competing department stores A and B in a city. Both stores have equal reputation and the total number of customers is equally divided between the two. Both the stores plan to run annual discount sales in the last week of December. For this they want to attract more number of customers by using advertisements through newspapers, radio and television. By seeing the market trend, the store A constructed the following pay-off matrix where the numbers in the matrix indicate a gain or loss of customers. Find optimal strategies for stores A and B using any method.

$$
\begin{array}{ccc}
 & & \begin{array}{ccc} \text{Newspaper} & \text{Radio} & \text{Television} \\ N & R & T \end{array} \\
 & \begin{array}{c} N \\ R \\ T \end{array} & \begin{bmatrix} 40 & 50 & -70 \\ 10 & 25 & -10 \\ 100 & 30 & 60 \end{bmatrix}
\end{array}
$$

with A labelling the rows.

QUESTION SEVEN

A steel company is negotiating with its union for revision of wages to its employees. The management with the help of a mediator has prepared a pay-off matrix shown below. Plus sign represents wage increase, while –ve (negative) sign is for wage decrease. Union has also constructed a table which is comparable to that developed by management. The management does not have the specific knowledge of game theory to select the best strategy for the firm. You have been called to assist the management on the problem. What game value and strategies you suggest would be acceptable to both parties.

Nairobi Breweries

Union Strategies

		U1	U2	U3	U4
	B1	+2.50	+2.70	+3.50	-0.20
Breweries	B2	+2.00	+1.60	+0.80	+0.80
Strategies	B3	+1.40	+1.20	+1.50	+1.30
	B4	+3.00	+1.40	+1.90	0

CHECK YOUR ANSWERS WITH THOSE GIVEN ON THE FOLLOWING PAGES

ANSWERS TO PRACTISE QUESTIONS—GAME THEORY

QUESTION ONE

Let the value of the small store be = 1
and the value of the large store be = 2

If both survive, A loses nothing, if only large store survives, A loses 1 and if smaller store survives, A will lose 2.

Payoff matrix

		Attacker B	
Defender A		Attack the smaller store 1	Attack the larger store II
	Defend the smaller store 1	Both survive 0	The larger store destroyed -2
	Defend the larger store II	The smaller store destroyed -1	Both survive 0

Payoff matrix

		B		
		1	2	Row minimum
A	1	0	-2	-2
	2	-1	0	-1
	Column maximum	0	0	

There is no saddle point.

Hence this is a problem of mixed strategy.

sing the method as given in text.

		B I	B II	
		I	II	Row minimum
A	I	0	-2	0-(-2) = 2
	II	-1	0	0-(-1) = 1
	Column maximum	0-(-1) = 1	0-(-2) = 2	

The final strategy is given by the matrix

		B I	B II	
		I	II	Probability
A	I	0	-2	1/3
	II	-1	0	2/3
	Probability	2/3	1/3	

Conclusion

A plays his first row 1/3 rd of the time (randomly)

A plays his second row 2/3 rd of the time

Similarly:

B plays his first column 2/3 rd of the time

B plays his second row 1/3 rd of the time

The value of the game is

$0 \times (1/3 \times 2/3) + (-2) \times (1/3 \times 1/3) + (-1)\ (2/3 \times 2/3) + 0 \times 1/3 \times 2/3)$

$= 0 - 2/9 - 4/9 + 0$

$= -6/9$

$= -2/3$

QUESTION TWO

	Y			Row minimum
	1	2	-1	-1
X	-2	1	1	-2
	2	0	1	0
Column maximum	2	2	1	

There is no saddle point

Let the three probabilities of Y be p, q, r

The three payoffs to Y corresponding to each of the three moves of his opponent X must all be equal to the optimal value of V of the game.

Y's payoffs against the three moves of X are

1p + 2q + (-1r)-2p + 1q + 1r   2p + 0q + 1r

We obtain three equations by equating each of these payoffs to V

1p + 2q - 1r = -2p + 1q + 1r = 2p + oq + 1r = V

Also p + q + r = 1 (Total probability)

1p + 2q - 1r = -2p + 1q + 1r [1]
1p + 2q - 1r = 2p + oq + 1r [2]
p + q + r = 1 [3]

Solving these three equations simultaneously we get

p = $\frac{2}{17}$ q = $\frac{8}{17}$ and r = $\frac{7}{17}$

Similarly using the same reasoning as before, let the three probabilities of x be p',q',r',

We get 1p' 2q' + 2r' = 2p' + 1q' + or' = -1p' + 1q' + 1r'

Also p' + q' + r' = 1

Solving them simultaneously we get

p' = $\frac{3}{17}$ q' = $\frac{5}{17}$ r' = $\frac{9}{17}$

Hence X should play his rows in the ratio 3:5:9 (randomly)
Y should play his columns in the ratio 2:8:7

Payoff

1x (2/17 x 3/17) + 2 x (8/17 x 3/17) + (-1)(7/17 x 3/17)

+ 6 other values calculated in the same way as before which amount to 11/17

Alternatively

Value of the game is

$$\frac{1 \times 3 + (-2) \times 5 + 2 \times 9}{3 + 5 + 9} = \frac{11}{17}$$

QUESTION THREE

(a)

	Firm B					Row minimum
Firm A	3	-1	4	6	17	-1
	-1	8	2	4	12	-1
	16	8	[6]	14	12	6←
	1	11	-4	2	1	-4
Column maximum	16	11	6 ↑	14	17	

There is a saddle point

Strategy Firm A plays row three all the time
Firm B plays column three all the time

It is a game of pure strategy and the value of the game is 6

(b) Dominance: Study Text

	Player B					Row minimum
Player A	1	3	2	7	4	1
	3	4	1	5	6	1
	6	[5]	7	6	5	5←
	2	0	6	3	1	0
Column maximum	6	5 ↑	7	7	6	

As above, it is a game of pure strategy

Player A Row 3
Player B Column 2

Value of the game 5

Alternatively

Reduce the size of the game using `dominance'. For instance row 4 is dominated, column 4 is dominated. Afterwards use standard method, to solve the problem.

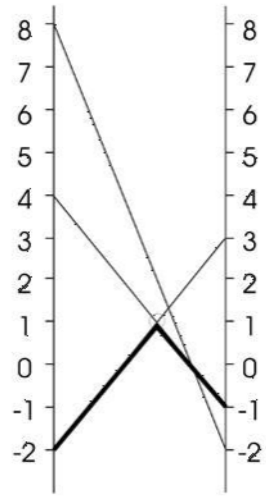
QUESTION FOUR

Pure and mixed strategies - Study Text

Fair game: it is a game which gives an equilibrium point (either by using pure strategy or mixed strategy). Intelligent and rational players will accept this `payoff' as fair to the players. This is also known as `Nash Equilibrium' sometimes.

A Nash equilibrium is a profile of strategies such that each players strategy is an optimal response to the other players' strategies.

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 8 & 4 & -2 \\ -2 & -1 & 3 \end{bmatrix} \end{array}$$



Using graph, payoff matrix is reduced to

$$\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

Using standard methods, the solution is

$$\begin{array}{ccc} & & B \\ & & \begin{array}{cc} 1/2 & 1/2 \end{array} \\ & 2/5 & \\ A & 3/5 & \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \end{array}$$

i. A plays his first row 2/5th of the time randomly A plays his second row 3/5th of the time randomly

ii. B plays his second column half the time
B plays his third column half the time

B does not play his first column at all.

x	p(x)	x(px)
4	1/2 x 2/5	4/5
-2	1/2 x 2/5	-2/5
-1	1/2 x 3/5	-3/10
3	1/2 x 3/5	9/10
		Total = 1

$4/5 + (-2/5) + (-3/10) + (9/10) = 1$

QUESTION FIVE

In this case we have a simple case of a 2 x 2 game, the yield to x is the probability the bomb carrier is not hit

The strategies are:

X_1 Bomber I carries the bomb
X_2 Bomber II carries the bomb

The strategies of the opponent are

Y_1 To attack Bomber I
Y_2 To attack Bomber II

Payoff Matrix

		Party Y	
		Y_1	Y_2
Party X	X_1	0.7 + 0.3 x 0.4 = 0.82	1
	X2	1	0.3 + 0.7 x 0.4 = 0.58

Construct the matrix of the game by determining the average yield for every combination of strategies as follows:

Now $p(0.82) + 1(1 - p) = p \times 1 + (1 - p) \times 0.58$

which gives $p = 0.7$

(x plays x_1, pth position of the time and x_2, 1 - p position of the time.

Also $0.82q + 1(1-q) = 1q + 0.58 (1 - q)$

which gives $q = 0.7$

		Y	
		q	1-q
X	p	0.82	1
	1-p	1	0.58

X's strategy 1st row:2nd row = 7:3
Y's strategy 1st column:2nd column = 7:3

QUESTION SIX

$$\begin{array}{ccc} & & \text{B} & \\ & & \text{Newspaper} & \text{Radio} & \text{Television} \\ & & \text{N} & \text{R} & \text{T} \\ & \text{N} & \\ \text{A} & \text{R} & \\ & \text{T} & \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{N} & \text{R} & \text{T} \end{array} \\ \text{A} \begin{array}{c} \text{N} \\ \text{R} \\ \text{T} \end{array} & \begin{bmatrix} 40 & 50 & -70 \\ 10 & 25 & -10 \\ 100 & 30 & 60 \end{bmatrix} \end{array}$$

				Row minimum
	40	50	-70	-70
	10	25	-10	-10
	100	30	60	30
Column maximum	100	50	60	

There is no saddle point

Looking at the pay-off matrix row two is dominated and column one is dominated, hence size of the matrix can be reduced to

$$\begin{array}{cc} & \begin{array}{cc} \text{R} & \text{T} \end{array} \\ \begin{array}{c} \text{N} \\ \text{T} \end{array} & \begin{bmatrix} 50 & -70 \\ 30 & 60 \end{bmatrix} \end{array}$$

Using standard techniques, the probabilities are

$$\begin{array}{cc} \begin{array}{c} 1/5 \\ 4/5 \end{array} & \begin{bmatrix} 50 & -70 \\ 30 & 60 \end{bmatrix} \\ & \begin{array}{cc} \frac{13}{15} & \frac{2}{15} \end{array} \end{array}$$

Hence A's strategy is [1/5, 0, 4/5] i.e N:R:T = 1/5:0:4/5

and B's strategy is [0, 13/15, 2/15] i.e N:R:T = 0:13/15:2/15

The value of the game is 50 x (1/5 x 13/15) + (-70) x (1/5 x 2/15) + 30 x (4/5 x 13/15) + (60) x (4/5 x 2/15)

= 34

QUESTION SEVEN

	U_1	U_2	U_3	U_4	Row minimum
B_1	+2.50	+2.70	+3.50	-0.20	-0.20
B_2	+2.00	+1.60	+0.80	+0.80	+0.80
B_3	+1.40	+1.20	+1.50	+1.30	+1.20
B_4	+3.00	+1.40	+1.90	0	0
Column maximum	+3.00	+2.70	+3.50	+1.30	

There is no saddle point

Using dominance, column U_1 and column U_3 are dominated. Hence the game can be reduced to

	U_2	U_4
B_1	+2.70	-0.20
B_2	+1.60	+0.80
B_3	+1.20	+1.30
B_4	+1.40	0

At this stage we can use the graphical method or reduce the size of the game using dominance again (row four is dominated by row two).

Using graphical method, the final matrix is reduced to

$$\begin{bmatrix} +2.70 & -0.20 \\ +1.20 & +1.30 \end{bmatrix}$$

Solving this, we get

$$\begin{array}{cc} 1/30 & \multirow{2}{*}{$\begin{bmatrix} +2.70 & -0.20 \\ +1.20 & +1.30 \end{bmatrix}$} \\ 29/30 & \\ & \begin{matrix} 1/2 & 1/2 \end{matrix} \end{array}$$

Hence Breweries strategies B1:B2:B3:B4 = 1/30:0:29/30:0

i.e. row B1 : row B3 = 1 : 29

Union strategies U1:U2:U3:U4 = 0:1/2:0:1/2

Column U2 : U4 = 1 : 1

Value of the game

2.70 x (1/30 x 1/2) + (-0.20) x (1/30 x 1/2) + (1.20) x (29/30 x 1/2) + (1.30) x (29/30 x 1/2)

= +1.25

Note: Interpret all the results in the solutions given, according to the language of the questions.

4. MARKOV ANALYSIS

Markov Chains

Markov Chains are named after the Russian statistician A.A Markov who developed probabilistic models that are often applicable to decision making problems in business and industry associated with dynamic systems.

Markov Chains are a special case of the more general probabilistic models known as **stochastic processes**, in which the current state of a system depends upon all previous states. The successive future states of the Markov process are referred to as Chains—hence the name Markov Chains.

Markov Processes

A Markov process is stochastic process in which the current state of the system depends only on the immediately preceding state of the system.

Markov Analysis

It is a way of analysing the current movement of some system in an effort to predict the future movement of the same system.

There are two elements that must be determined in the process of constructing a Markov model in the system. These elements are the possible states of the system and the probabilities of moving between states (also called transition probabilities). A system state is a status of the system at a particular point in time, such as whether or not a machine is operating, whether an account is paid or not paid etc.

Transition probabilities

Represent the probability of the system moving from one state to another during a particular period. We can organise the transition probabilities in the form of a table or matrix.

Markov Properties

1. Transition probabilities are dependent only on the current state of the system i.e. given that the present state is known, the conditional probability of the next state is independent of the states, prior to the present state. (This is known as property of no memory).

2. The transition probabilities are constant over time.

3. The transition probabilities of moving to alternative states in the next time period, given a state in the current time period, must sum to one.

   These properties are quite restrictive and hence the application of Markov Analysis is limited to few real-world problems.

   Given that a set of possible states in Markov Chain is finite, a square matrix, P, made up of all Pij's of Markov Chain can be formed.

The following table represents the commonly used transition probability matrix.

$$
\begin{array}{ccc}
 & & \text{To State} \\
 & & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5\ldots\ldots\ldots & n \end{array} \\
P & \begin{array}{r} 1 \\ \text{From } 2 \\ \text{State } 3 \\ 4 \\ 5 \\ : \\ n \end{array} & \begin{pmatrix}
P_{11} & P_{12} & P_{13} & P_{14} & P_{15}\ldots\ldots & P_{1n} \\
P_{21} & P_{22} & P_{23} & P_{24} & P_{25}\ldots\ldots & P_{2n} \\
P_{31} & P_{32} & P_{33} & P_{34} & P_{35}\ldots\ldots & P_{3n} \\
P_{41} & P_{42} & P_{43} & P_{44} & P_{45}\ldots\ldots & P_{4n} \\
P_{51} & P_{52} & P_{53} & P_{54} & P_{55}\ldots\ldots & P_{5n} \\
: & : & : & : & : & : \\
P_{n1} & P_{n2} & P_{n3} & P_{n4} & P_{n5}\ldots\ldots & P_{nn}
\end{pmatrix}
\end{array}
$$

Where n is the number of exhaustive and mutually exclusive states.

Pij is the transition probability of going from the present i th state to the next j th state.

Thus the rows represent the possible present states (i's) and columns, represent the possible future states (j's).

By definition $P_{11} + P_{12} + P_{13} +\ldots\ldots P_{1n} = 1$

Similarly $P_{21} + P_{22} + P_{23} +\ldots\ldots P_{2n} = 1$

etc.

Applications of Markov Chains

They are a particular class of probabilistic models and their applications include analysis of:

i. Inventory systems
ii. Replacement and maintenance policies for machines
iii. Brand loyalty in marketing
iv. Time series of economic data such as movement of stocks
v. Accounts receivable in accounting
vi. Expected payouts of life insurance policies etc.

Steady state condition

In many cases, the Markov process will converge to a steady state or equilibrium.
In general, as number of transitions `n' increase, the state values tend to stabilize at steady state.
This is a logical occurrence, since the present state tends to lose significance.

Steady State vector

A steady vector is given in terms of decimal proportions as

$\begin{bmatrix} X & Y & Z \end{bmatrix}$ such that

$$\begin{bmatrix} X & Y & Z \end{bmatrix} \begin{bmatrix} \text{Transition matrix} \end{bmatrix} = \begin{bmatrix} X & YZ \end{bmatrix}$$

subject to X + Y + Z = 1
i.e. once a steady state is reached, multiplication of a state condition by the transition probabilities does not change the state condition.

Some standard terms as used in the context of Markovian processes.

Transition probabilities

These are the probabilities of moving from one state to another in the next time period. Usually they are written in the form of a probability matrix.
Example

	To	
From	0.6	0.4
	0.8	0.2

Sum of probabilities in any row equals one **OR**

	From	
To	0.6	0.8
	0.4	0.2

Sum of probabilities in any column equals one

Transient Analysis

A state is said to be transient if it is impossible to move to that state from any other state except itself. This state is temporary and eventually a steady state is reached. This analysis can be performed using usual probability transition matrices.

Absorbing State

A state is said to be an absorbing (or Trapping state) state if it is impossible to leave the state.

This will occur if any P_n is equal to 1.0

Example

		To		
		A	B	C
	A	1	0	0
From	B	1/2	1/4	1/4
	C	1/3	1/3	1/3

Since p_{11} = 1.0, state A is an absorbing state. This state represents the conditions, under which the modelled process can terminate. All other states of the model are transient states.

Cyclic Chains

In Markov Chains the current state of the system depends on all previous states. It is a stochastic process. Sometimes transition probability matrices are different from State I to State II, State II to State III etc and from cyclic chains which repeat in the same order.

Recurrent State

A state is recurrent if it is certain to occur again, given that it has occurred at least once, otherwise it is said to be **transient**. In other words, a transient state will occur only a limited number of times and then go away for ever whereas the recurrent state is permanent. If a process is finite (i.e. has a finite number of states) and is irreducible, then all states are recurrent. This is the most common form of Markov Chain in applications.

PRACTICE EXAMPLES — Dec.1986 (ACCA)

QUESTION ONE

There are three types of breakfast meal available in supermarkets known as brand BM1, brand BM2, and brand BM3. In order to assess the market, a survey was carried out by one of the manufacturers. After the first month the survey revealed that 20% of the customers purchasing brand BM1 switched to BM2 and 10% of the customers purchasing brand BM1 switched to BM3. Similarly after the first month of the customers purchasing brand BM2, 25% switched to BM1 and 10% switched to BM3 and of the customers purchasing brand BM3 5% switched to BM1 and 15% switched to BM2.

Required:

i. Display in a matrix S, the patterns of retention's and transfers of customers from the first to the second month, expressing percentages in decimal form.
ii. Multiply matrix S by itself (that is form S^2).
iii. Interpret the results you obtain in part (ii) with regard to customer brand loyalty.

Answer

i. The matrix showing the pattern of retention and transfer from the first to the second month is

$$S = \begin{array}{c} \begin{array}{ccc} BM1 & BM2 & BM3 \end{array} \\ \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} \end{array} \begin{array}{c} \\ BM1 \\ BM2 \\ BM3 \end{array}$$

(The second element in the first row shows the 20% movement from BM1 to BM2 and so on.)

ii. The product of matrix S with itself is demonstrated as follows

$$\begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} \times \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.25 & 0.65 & 0.10 \\ 0.05 & 0.15 & 0.80 \end{bmatrix} = \begin{bmatrix} 0.5450 & 0.2850 & 0.1700 \\ 0.3425 & 0.4875 & 0.1700 \\ 0.1125 & 0.2275 & 0.6600 \end{bmatrix}$$

where for example the second element in the first row, that is 0.2850 is the result of multiplying the corresponding elements of the first row of S by the second column of S and summing the products.

$$\begin{aligned} 0.2850 &= 0.70 \times 0.20 \text{ x} + 0.20 \times 0.65 + 0.10 \times 0.15 \\ &= 0.14 \quad +0.13 \quad +0.015 \text{ etc.} \end{aligned}$$

iii. The resulting matrix may be interpreted in the following way:

Of the original customers who buy BM1, 54.5% will remain loyal to the brand in month three, 28.5% will have switched to BM2 and 17% will have switched to BM3.

Of the original customers who buy BM2, 48.75% will remain loyal to the brand in month three, 34.25% will have switched to Bm2 and 17% will have switched to BM3.

Of the original customers who buy BM3, 66% will remain loyal to the brand in month three, 11.25% will have switched to BM1 and 22.75% will have switched to BM2.

QUESTION TWO

TRANSITIONS LIMITED

Transitions Limited allows customers one month's credit for the settlement of their accounts, but in practice many customers take longer to pay. Each outstanding account is classified each week in one of the following four ways:

A account less than 1 month old, payment still due;
B account more than one month old, but less than 3 months old - payment overdue;
C the account was settled during the previous 7 days;
D the account is over 3 months old, and should be written off as a bad debt, or the customer has given some other reason to suppose that the debt should be written off.

The situation changes from week to week, and during the course of any week, some accounts in category A may move into category B or category C, and some accounts in category B may move to category C or category D.

The probability of these changes occurring from any one week (week n) to the next week (week n + 1) may be given by a probability transition matrix.

Category in week (n + 1)

$$\text{Category in week } n \quad \begin{array}{c} \\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{array} \\ \begin{bmatrix} 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Required:

(a) What is the probability that an account which is overdue for payment will:

i. Become a bad debt next week?
ii. Be paid during next week? (2 marks)

(b) What is the probability that an account in category A in week 1 will be paid by week 5 i.e 4 weeks later? Use matrix multiplication to determine your answer. What is the probability that an account in category B in week 1 will be paid by week 5? **(18 marks)**

Answer
TRANSITIONS LIMITED

(a) The probability that an account in category B will:

i. become a bad debt next week is 0.1
ii. be paid next week is 0.4

(a) The probability that an account category A in week 1 will be paid in week 3 is

$$\text{Week 1} \overset{\text{Week 2}}{\begin{pmatrix} 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}} \quad \underset{\text{Week 2}}{\text{x}} \overset{\text{Week 3}}{\begin{pmatrix} 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Week 3

$$= \text{Week 1} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0.04 & 0.42 & 0.48 & 0.06 \\ 0 & 0.25 & 0.60 & 0.15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

(Workings not shown)

The probability that an account in category A in week 1 will be paid in week 4 is:

$$\begin{pmatrix} 0.04 & 0.42 & 0.48 & 0.06 \\ 0 & 0.25 & 0.6 & 0.15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{x} \quad \begin{pmatrix} 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Week 4

$$= \text{Week 1} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0.008 & 0.234 & 0.656 & 0.102 \\ 0 & 0.125 & 0.7 & 0.175 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The probability that an account in category A in week 1 will be paid by week 5 can be given by the following:

$$\begin{pmatrix} 0.008 & 0.234 & 0.656 & 0.102 \\ 0 & 0.125 & 0.7 & 0.175 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{x} \quad \begin{pmatrix} 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Week 5

A B C D

$$= \text{Week 1} \begin{array}{c} \textbf{A} \\ \textbf{B} \\ \textbf{C} \\ \textbf{D} \end{array} \begin{pmatrix} 0.0016 & 0.1218 & 0.7512 & 0.1254 \\ 0 & 0.0625 & 0.8 & 0.1875 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution:

An account in category A in week 1 has a 0.7512 probability of being paid by week 5.

An account in category B in week 1 has a 0.8 probability of being paid by week 5.

QUESTION THREE

Two industries A and B share the market for a particular product. Industry A is an old and well established industry, industry B is new. A is alarmed at the progress being made by B and has asked its market-research department for a forecast of future market shares for the two industries, assuming that the same market conditions prevail. The current market shares held by the two industries are as follows:

Industry	Market Share
A	0.8
B	0.2

The market research department has determined that customers switch between the two firms according to the following probabilities.

		To A	To B
From	A	0.5	0.5
	B	0.6	0.4

(a) Determine the market research department's market share forecast for one period in the future, two periods in the future, three periods in the future.

(b) Based on results in part (a), estimate the equilibrium market share for each industry rounded to the nearest whole percentage.

(c) Illustrate the customer switching between the two industries as a Markov probability diagram.

(d) Use a tree diagram to illustrate the three-period transition beginning with firm A. Prepare the same illustration beginning with firm B.

(e) Describe the four probability values.

(f) Insert the four probabilities in the appropriate cells of the following matrix.

		Ending Industry A	Ending Industry B
Beginning Industry	A		
	B		

Answer

(a) Period One

$$\begin{array}{cc} A & B \\ \end{array} \quad \begin{bmatrix} 0.80 & 0.2 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.5 \\ 0.6 & 0.4 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.52 & 0.48 \end{bmatrix} \end{array}$$

OR

From

$$\text{To} \quad \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.80 \\ 0.20 \end{bmatrix} = \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix}$$

Period 2

$$\begin{bmatrix} A \\ B \end{bmatrix} \quad \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix} = \begin{bmatrix} 0.548 \\ 0.452 \end{bmatrix}$$

Period 3

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.548 \\ 0.452 \end{bmatrix} = \begin{bmatrix} 0.5452 \\ 0.4548 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$0.5A + 0.6B = A$$

$$0.6B = 0.5A$$

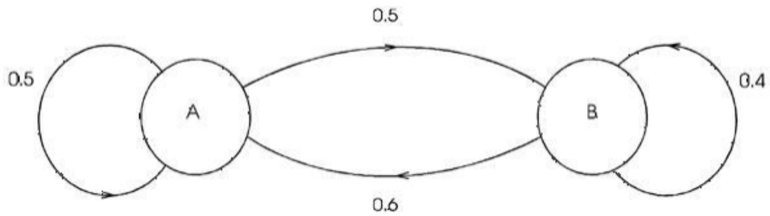
$$\frac{B}{A} = \frac{0.5}{0.6} = \frac{5}{6}$$

$$A:B = 6:5$$

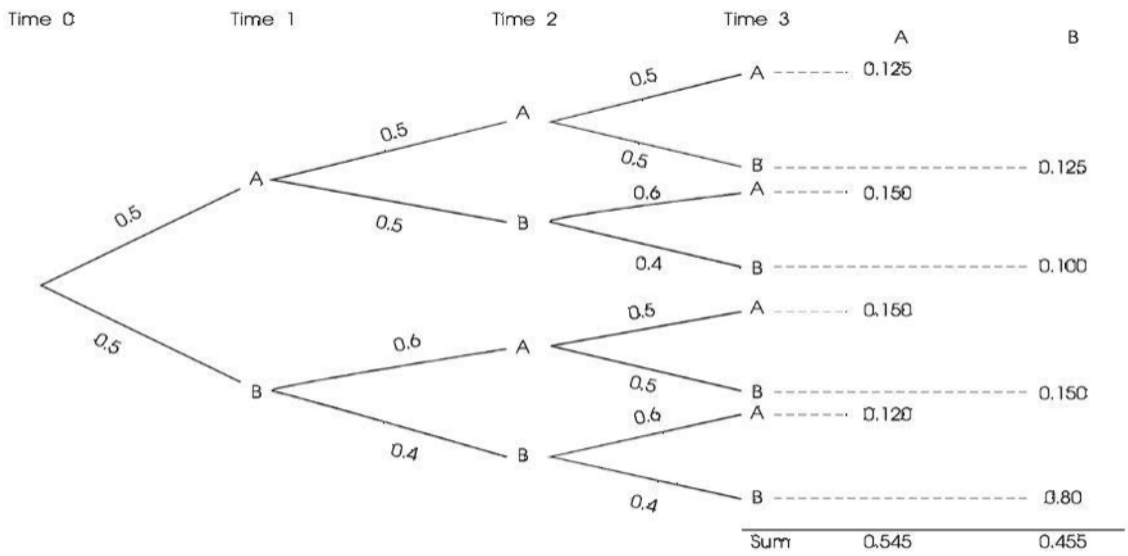
$$A = 6/11 \text{ X } 100\% \approx 55\%$$
$$b = 5/11 \text{ X } 100\% \approx 45\%$$

This trend can be easily seen from period I, to period II, to period III.

(c)



(d)



(e) They are the state probabilities for A and B at time 3, given the status for A and B at time 0.

(f)

$$\text{From} \quad \overset{\text{To}}{\begin{bmatrix} 0.545 & 0.455 \\ 0.546 & 0.454 \end{bmatrix}}$$

Note: We can get period three value from

$$\begin{bmatrix} 0.8 & 0.2 \end{bmatrix} \quad x \quad \begin{bmatrix} 0.545 & 0.455 \\ 0.546 & 0.454 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5452 & 0.4548 \end{bmatrix}$$

REINFORCING QUESTIONS

QUESTION ONE

A department store is interested in predicting the behaviour of customers for which accounts receivable are outstanding. Its credit department has been asked to analyze its records and predict payments probabilities. Historical records have yielded the following payment patterns of credit customers.

	Month n + 1	
Month n	Paid bill	Did not pay bill
Paid bill	0.90	0.10
Did not pay bill	0.80	0.20

Assume that these probabilities are used to predict the behaviour of a credit customer with regard to bill payment.

a. If a credit customer did not pay his bill in month *n*, what is the probability he will not pay it in any of the next three months?

b. If a customer did not pay his bill in month *n*, what are the probabilities he will pay his bill in month *n* + 1, in month *n* + 2, and in month *n* + 3?
(Note: Assume that monthly bills include purchases made in that month plus any outstanding balance from previously unpaid bills).

c. Determine the steady-state conditions—that is, the probability the customer will pay a bill or not pay a bill in month *n* + 1, regardless of whether he paid in month *n*.

QUESTION TWO

Students switch among the various colleges of a university according to the following probability transition matrix.

	To		
From	Engineering	Liberal Arts	Business
Engineering	0.5	0.3	0.2
Liberal Arts	0.1	0.7	0.2
Business	0.1	0.1	0.8

Assume that the number of students in each college of the university at the beginning of the fall quarter is as follows:

Engineering 3000
Liberal Arts 5000
Business 2010

a. Forecast the number of students in each college after the end of the third quarter, based on a four-quarter system. Determine by first computing P^*.

b. Determine the steady-state conditions for the university.

QUESTIONTHREE

It has been said that stock market prices have a tendency to move in opposite direction from day to day. Assume that a stock market analyst has determined from historical data that a particular stock price will move up or down according to the following probabilities:

	Day n + 1	
Day n	Increase	Decrease
Increase	0.3	0.7
Decrease	0.8	0.2

Determine the steady-state conditions.

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

LESSON EIGHT

PERFORMANCE EVALUATION

OBJECTIVES

- The assessment of Divisional, Departmental and Corporate Management
- Determining appropriate rewards and appropriate motivation

INSTRUCTIONS

- Read Chapters 20 and 21 of Management and Cost Accounting by Colin Drury 5th Edition
- Read Study Text
- Complete reinforcing questions under examination conditions and compare your answers with those given in Lesson 10.

CONTENTS

- Introduction
- Responsibility Accounting
- Steps Of Chosing An Accounting Based Performance Measure
- Alternative Performance Measures
- Transfer Pricing And Performance Evaluation

INTRODUCTION

Performance evaluation deals with the area of MA that is concerned with:
Holding individual managers responsible for certain aspects of the organisational performance.
Making them accountable by producing regular performance reports relating to matters for which they are responsible.
Motivating managers to achieve better results by setting targets for performance, judging actual results against targets and rewarding good performers.
Giving managers control information to enable them to make decisions about improving their performance.

There are two interrelated aspects involved and these are:
Giving managers authority to make decisions and holding them responsible for the exercise of that authority. This is referred to as decentralization.
Defining a system of accountability by which to judge how the authority has been used and the responsibility carried out. This is referred to as responsibility accounting.

RESPONSIBILITY ACCOUNTING

This is a term used to define the measuring of performance of decentralized units, using account results. Responsibility accounting recognizes various decision centres throughout an organisation and trace costs (revenues, assets and liabilities) to the individual managers who are primarily responsible for making the decisions about the costs in question. A responsibility centre is a unit in an organisation headed by a manager having direct responsibility for its performance. Examples of responsibility centres include cost centre, profit centre, and investment centre. These centres are defined below:

Cost Centre

Cost centre is a production service location activity or item of equipment whose costs maybe attributed to cost units. It is therefore any unit of the organisation to which cost can be attributed.
Managers in the cost centre have control over various controllable costs (That is costs incurred in the centre) but may have no control for any alterations apportioned from other cost centres.

Performance measurement in a cost centre can be accomplished through variance analysis or through efficiency measures such as output/input ratio.

Profit Centre

A profit centre is a subunit of an organisation such as a division of a company to which both revenue and costs are assigned so that the profitability of that subunit can be measured. It is also referred to as a strategic business unit.

Managers in an profit centre have control over costs, and revenue decisions. Performance measurement in a profit centre can be accomplished through the use of profit margin or contribution/sales ratio.

Investment Centre

An investment centre is a subunit of the organisation where managers have control over cost, revenue and some investment decisions. Managers can buy some assets so as to increase profitability.

Performance in an investment centre is measured by ratios such as return on investment which relates the profit earned to the amount of capital invested. Performance can also be measured from absolute measures such as residual income.

STEPS OF CHOSING AN ACCOUNTING BASED PERFORMANCE MEASURE

Consider the overall goal of the organization as a whole. It is important to choose a measure of accomplishment that represents top management goals. Such measures include operating income, net income, Return on investment (ROI), sales, etc. Determine whether the measure should be maximized or minimized.
Select definitions for such items as income and investments (i.e. should income be based on variable or absorption costing? should central overheads be allocated? should investments consist of total assets, net assets or net worth?)
How should items such as income and investments be measured (i.e. should we use historical costs, replacement costs, realisable or current values?)
Determine the standards that should be applied (i.e. Should all divisions be required to earn the same rate of return on all investments?)
What timing of feedback is needed? Should it be monthly or quarterly.

ALTERNATIVE PERFORMANCE MEASURES

There are various measures that can be used to measure performance of a decentralised company.
The main ones are:
Return on Investment
Residual income
Discounted cash flow methods such as the NPV method
Value added statements
Bench marking
Balanced score card

RETURN ON INVESTMENT AND RESIDUAL INCOME

This is a traditional approach to performance measurement given by:

$$ROI = \frac{\text{Income}}{\text{Invested Capital}}$$

(method) of Liability analysis

ROI can provide more insight to performance when it is divided into the Dupont components.
The Dupont method states that:

ROI = Capital turnover X profit margin

$$= \frac{\text{Revenue}}{\text{Invested capital}} \times \frac{\text{Income}}{\text{Revenue}}$$

Dupont method leads to the generalization that ROI can be increased by any action
that: decreases costs
Increases revenue
Decreases invested capital

Return on investment highlights the benefits that managers can obtain by decreasing investment in both current and fixed assets. Investment in cash, inventory, accounts receivable and fixed assets should be minimized for any level of effective performance. This requires that idle cash is invested, proper inventory levels are kept, credit is managed judiciously and fixed assets are invested in carefully.

However, return on investment may induce managers of a highly profitable division to reject projects, which from the view point of the organisation as a whole should be accepted. ROI encourages managers to make decisions which may increase short-term profit without considering their effect on the future of the company.

ILLUSTRATION

Assume that a company has 3 subsidiaries A, B, and C and that the company does not allocate corporate headquarters' costs or interest on long-term debt to the subsidiaries. Summary of the results are as follows.

	£'000' A	£'000' B	£'000' C	£'000' H	£'000' Total
Operating y income	240	300	480		1020
Variable cost of H				80	
Fixed costs of H				120	
Interest on L.T debt				400	(600)
Income before taxation					420
Taxes					150
Income after taxation					270
Average book values					
Current assets	400	500	600	200	1700
Fixed assets	600	1500	2400	300	4800
					6500

Required:

Compute the return on investment.
Compute residual income assuming that the company requires a 10% interest on total assets of each subsidiary.
Assume that there is an asset available to subsidiary A which costs £100,000 but which has an annual profit of £20,000. Advice the manager of A on whether to undertake the project and comment on whether this decision is in line with the overall objective of the organisation.

Solution

a)

		A	B	C	Total
ROI = Income / Capital	240 / 1000	300 / 2010	480 / 3000	820 / 6500	
		0.24	0.15	0.16	0.126

The best performer is subsidiary A, followed by C, while B is the worst performer.

b) RI = Income – Imputed interest charge

$RI_A = 240 - (0.1 \times 1000) = 140$
$RI_B = 300 - (0.1 \times 2010) = 100$
$RI_C = 480 - (0.1 \times 3000) = 180$
Based on RI, the best performer is C, followed by A and B in that order.

c) With the new project ROI of A will be:

$$ROI = \frac{260}{1100}$$
$$= 23.6\%$$

The overall ROI will be

$$ROI = \frac{860}{6600} = 12.7\%$$

Based on ROI, the manager of A should not invest in project A since ROI decreases from 24% to 23.6%. However, from the overall point of view the project is viable as it increases ROI BY 0.1%

The Residual Income with the new project will be:

$RI_A = 260 - (0.1 \times 1100) = 150$

The manager of A should take project as it increases residual income from 140 to 150.

Note

The objective of maximizing residual income assumes that as long as the division earns a rate in excess of the imputed charge on the investment, then it should expand. Residual income is a short-term measure and therefore, contradicts the going concern concept of the firm.

DISCOUNTED CASH FLOW TECHNIQUES (eg. NPV)

The NPV method can be used to determine the profitability of the projects undertaken by the division. The NPV method is familiar at this point. However, an illustration will help in revision.

Illustration

Division X of Harvest agricultural machinery ltd was considering adding a small weeding implement to their product range. Sufficient capacity was currently available to cope with the additional production, but an extra special-purpose machine costing £40,000 would have to be acquired and paid for immediately prior to commencement of production. The machine could be traded in at the end of year five for £5,000.

A new sales promotion programme would be mounted to market the new product and this would cost £10,000 at the commencement of production and £5,000 at the end of the first year.

The management accountant produced the following forecast figures relating to the proposed net product:

Year	Sales	Variable Cost	Fixed Cost
	£	£	£
1	52,000	30,000	23,000
2	78,000	47,000	28,000
3	100,000	65,000	30,000
4	60,000	35,000	24,000
5	36,000	19,000	20,000

Fixed costs include £5,000 per annum for an additional part time supervisor, £ 7,000 per annum depreciation on the machine and the balance an appointment of existing costs. It can be assumed that no credit is given to customers or received from suppliers.

If the new project is accepted, it would take up facilities that could be used for another purpose to generate a net cash flow of £8,000 per annum.

The divisional manager has asked the management accountant to submit an appraisal of the project covering a five year period. Assume an imputed interest charge by the headquarter of 10%.

Solution

Year	**0**	**1**	**2**	**3**	**4**	**5**	**Total**
Marketing expense	(10,000)	(5,000)					
New machine	(40,000)						
Net cash earnings		17,000	26,000	30,000	20,000	12,000	
Opportunity cost	______	(8,000)	(8,000)	(8,000)	(8,000)	(8,000)	
Net cash flow	(50,000)	4,000	18,000	22,000	12,000	9,000	
PV factor at 10%	1.000	0.909	0.826	0.751	0.683	0.621	
Net present value	(50,000)	3,626	14,868	16,522	8,196	5,589	(1,189)

Notes

1. As the project has a negative NPV of £1,189, it should be rejected (but see later where the same project is adjusted for taxation and inflation)
2. Net cash earning is sales less variable cost and £5000 additional supervision.
3. Project cash flows might have to be adjusted for any credit allowed to customers or received from suppliers
4. Apportioned fixed costs are irrelevant because they will be incurred whether this project goes ahead or not (depreciation is not cash flow), but the increment expense of £5000 for the part time supervisors is relevant.
5. Value of alternative use of exiting facilities, £8000, is foregone and therefore treated as an opportunity cost.

VALUE ADDED STATEMENTS

Are intended to show how much wealth or value has been created by the company's operations and how the wealth has been shared out to interested groups e.g. shareholders, investors in debt capital, employees, govt and the amount retained for re-investment. The value added statement shows:

How much value has been created by a firm's own effort

How this wealth has been shared out among stakeholders

The difference between the purchase cost of external material and services and the selling prices of the company goods and services is the value/wealth created by the company itself. This is referred to as value added. Value added statements can provide additional information to senior managers to help them in comparing performance of different divisions.

Illustration

ABC Ltd makes 2 products each in a different division and revenues and cost for each division are given below:

		Product A £ '000'			**Product B £ '000'**
Direct material		60			24
Direct labour		10			10
Variable – mat	1		1		
- lab	4	5	4		5
Fixed - mat	5			4	
- lab	5			5	
- Depn	5	15		2	11
Total costs		90			50
Sales		100			60

	10	10
Profit Margin	10%	16.7%

Required

Prepare a value added statement and compute the following ratios.
Profit to value added
Contribution to value added
Value added to every £1 of labour

Comment on the better product

Solution

Value Added Statement

		A		B
		£ 000		**£ 000**
Sales		100		60
Materials – Direct	60		24	
- Indirect	6	66	5	29
Value added:		34		31
Shared between				
Lab – Direct	10		10	
- Indirect	9	19	9	19
Depreciation		5		2
profit		10		10
		34		31

	A	B
Profit to value added	$= \frac{10}{34} = 29\%$	$\frac{10}{31} = 32\%$
Contribution to value added	25/34 = 74%	21/31 = 68%
Value added per £1 of labour	34/19 £1.79	31/19 £1.63

Comment

Although product B has a higher profit to value added ratio, A makes more value added per every £ of labour cost suggesting better wealth creation by labour efforts and has a higher ratio of contribution to value added. This analysis does not necessarily make product A better than B. However, it provides information for comparison and judging performance which adds to the more traditional performance measures such as profit margin.

Advantages of Value Added Statements

Managers might be in a better position to control their organisations own inputs than the cost and usage efficiency of purchased material and services. Value added statements focus on what managers can do something about.
Value added statements can reflect the quality and success of management effort.
Value added statements also focus attention on how the benefits are shared out and in particular: Whether employees are paid too much for what they are doing.
Whether enough funds are being retained in the business in the form of internally funded growth.
Value added in relation to labour cost provide excellent measures of productivity and therefore can be used for comparing the relative productivity of two or more divisions.

BENCH MARKING

In the current business environment, organisations are under a lot of pressure to improve performance and that of their divisions or subsidiaries. Bench marking is therefore becoming increasingly popular and can be defined as a systematic analysis of one's own performance against that of another organisation with an overall objective of improving performance by learning form the experience of others.

Bench marking helps an organisation to understand its own business operations because of the detailed analysis that has to be carried out. Ideally performance should be compared with organisations known to be the best in the class of activities in question. From such an analysis, best practice can be identified and translated into use in the organisation.

Bench marking exercise should concentrate on areas of business which are of key strategic importance to the organisation and should be applied in activities where significant costs and efficiency improvements can be made.

Comparison can be made with direct competitors (competitive bench marking) or comparison can be made with the best external practitioner of the activity regardless of the industry within which they operate (functional bench marking). Internal bench marking involves comparing the performance of one part of the business with that of a different part of the same business with the major objective of establishing best practice through out the organisation. Bench marking establishes a desire to achieve continuous improvement and helps to develop a culture in which one admits mistakes and adopts or makes changes

BALANCED SCORE CARD

This is a popular approach in current management thinking which consists of a variety of indicators both financial and non-financial. The balanced scorecard focuses on 4 different perspectives:

Customer Perspective:

What do existing and new customers value from us? This perspective gives rise to target which matter to customers e.g. cost, quality, delivery, inspection of the goods, and handling of goods or customers.

Internal Perspective:

What processes must we excel at to achieve our financial and customer objective? This perspective therefore aims at improving internal processes and decision making.
Innovation and learning: Can we continue to improve and create future value? This perspective considers the business capacity to maintain its competitive position through the acquisition of new skills and development of new products. The organisation must set targets which will emphasise continuous change in customer needs such as the percentage of sales derived from new products. Compared with the established ones and the long-term investments undertaken.

Financial Perspective:

How do we create value for our shareholders? This perspective covers traditional measures e.g. growth, liability, shareholder value. But these are set once the key areas for improvement have been identified and the balanced score card is the main monthly report. The score card is balanced in the sense that managers are required to think in terms of all perspective to prevent improvement being made in one area at the expense of another. Important features of this approach are:

- It looks at both internal and external matters concerning the organisation
- It is related to the key elements of the company strategy
- Financial and non-financial measures are linked together.

TRANSFER PRICING AND PERFORMANCE EVALUATION

Introduction

Transfer pricing is simple in concept and yet complex in implementation. It provides a divisional output valuation where output from one division becomes the input of another division within the same organisation. This is often necessary to the operation of profit or investment centres. Complexity in implementation arises from the availability of a number of valuation bases, each with their own implications for the ways in which an organisation is to be managed. This section provides a description of the major valuation bases found in practice and in theory. Transfer prices are necessary to the operation of performance measurement based on profit and investment centres and the chapter includes a critical appraisal of performance measurement in general.

The chapter is divided into three. The first section describes the purpose of transfer pricing. In the second section, methods of transfer pricing are described and briefly evaluated. In the final section, a critical appraisal of performance measurement in general and transfer pricing in particular is provided.

1. TRANSFER PRICING: PURPOSES

Transfer pricing can contribute directly to the process of departmental performance measurement and indirectly to the measurement of product performance.
A transfer price is a value attached to the output of a department in order to measure the value of its trade with other departments inside the organisation. The transfer price of the supplying division is charged to the receiving division. Transfer prices do not affect overall organisational profit results but do affect the profits reported by divisions. The following example illustrates this point.

Example 1

Mwangi Inc. Plc. sells a single product at £5 per unit. The product is manufactured by passing raw materials through two departments, A and B, at costs of £1.50 and £2.50 respectively. A transfer price of £2 has been established to measure the profit achieved by department A.

Department A:		
Transfer price per unit		£2.00
Cost per unit		£1.50
Profit		£0.50
Department B:		
Selling price per unit		£5.00
Transfer price	£2.00	
Other costs	£2.50	£4.50
Profit		£0.50
Mwangi Inc.		
Selling price per unit		£5.00
Cost per unit:		
Department A	£1.50	
Department B	£2.50	£4.00
Profit		£1.00

The total cost of the product is £4 per unit providing a profit to the company of £1 per unit. Department A has costs, or inputs, of £1.50 per unit and a transfer price of £2 per unit as a measure of output value. It thus shows a profit of £0.50 per unit. Department B has input costs of £2.50 per unit, plus a transfer price of £2 per unit, and an output value of £5. Department B also shows a profit of £0.50 per unit, therefore, the profit of both departments together is £1 per unit (£ 0.50 plus £0.50). The organisation's profit of £1 per unit is unaffected by the transfer price because the output value attached to department A's production becomes an input value for department B.

If a transfer price of £1.50 per unit is used in this situation, department A appears to show £nil as a profit. The costs to department B are £ 1.50 plus £2.50, giving a £1 profit per unit for a selling price of £5 per unit. This transfer price ensures that department A's costs are transferred to department B but does not offer a profit motivation to department A's manager. Department A is unlikely to take action to improve performance if all credit for such effort is shown under department B's results. Different transfer prices allocate profit in different ways between divisions and it should be clear that:

1. Transfer pricing shares profits between divisions but does not, on its own, affect total profits;

2. Transfer pricing can motivate managers to take actions to improve profits for their divisions and for the organisation as a whole. The transfer price should allow the opportunity for effort to be translated into a positive measurement of performance.

- Transfer pricing is similar to cost apportionment and allocation in that values of one department are passed to another. For cost apportionment and allocation systems, costs of one department are passed to another with the objective of accumulating costs for product cost information purposes. In a sense, apportionment and allocation provides a mechanism of transfer pricing primarily based on input measures such as floor area or direct labour hours. Under transfer pricing arrangements, values of one department are also passed to another. Output measures of activities are used to charge departmental costs and allowable profits to other departments. The information produced can be used to accumulate product costs. Where transfer prices are cost based and make no allowance for profits, the results would differ from apportionment and allocation systems only with regard to the basis for transferring costs between departments.
- Where performance measurement is linked to rewards such as promotion or salary, the method of transfer price can have a direct impact on the motivation of the divisional manager. For example, a divisional manager appraised on a profit centre basis will be in a position of advantage where high transfer prices are established for the particular division. The effect of motivating managers to improve profits may lead to bargaining for transfer pricing methods which provide the highest transfer prices for their particular divisions. In a transfer pricing situation, as in a number of accounting situations, there will inevitably be winners and losers. The challenge to the accountant is to devise a transfer pricing methodology which ensures that the winners are those who will benefit the organisation most in the long term.

- Some multinational companies are in a position to use transfer pricing to reduce total taxation costs. This can be achieved by establishing transfer prices towards the higher end of the spectrum of allowable values in countries with low taxation. This would tend to lead to high profits in countries with low taxation and lower profits in countries with higher taxation. Governments in some countries take steps from time to time to regulate the operation of transfer pricing systems for this reason.

2. TRANSFER PRICING METHODS

Transfer pricing methods are concerned with the alternative means by which a transfer price can be set and its impact on organisations gauged. Emmanuel and Otley bring together a number of views of transfer pricing methods in practice. Essentially, they report that there are three categories of transfer price: cost based, market based and negotiated. Within the surveys reported, in terms of very rough approximations, about 20% of companies used negotiated prices, about 30% of companies used market values and about half used cost based prices. For each category, a good degree of discretion existed to develop alternative bases at a detailed level. For market based prices, for instance, competitors' prices, list prices, most recent bid and values adjusted by a discount provided alternative bases. The description which follows does not go to such a level of detail but concentrates on four main approaches: absorption cost bases, variable cost bases, market value bases and negotiated value bases. A final section describes the use of linear programming.

A. Absorption cost based transfer prices

Absorption, or full cost systems, transfer the full cost of the supplying department to the receiving department. Where a profit is to be allowed to the supplying division, it is necessary to determine a policy which can be consistently applied. Typical systems may allow a profit based on cost, sales or investment, as shown in the example below:

Example 2

A division has a product costing £5 which is transferred within a group of companies. Calculate a transfer price for the division for each of the following mutually exclusive divisional targets:

1. a net profit margin of 10%
2. a mark-up on cost of 10%
3. a net assets turnover rate of 5 and an ROCE of 30%.
4. an output of 1,000,000 units, a capital employed of £2,000,000 and an ROCE of 20%

Solutions:

1. A net profit margin of 10% is the same as a mark-up on cost of 10/90. The selling price is 100/90. Using a cost of £5, the transfer price should be:

   £5 X 100/90 or £5/0.9, which is £5.56, a profit of £0.56

2. The transfer price would be £5.50

3. Using the relationship:

   $$\text{Return on capital employed} = \frac{\text{Net profit}}{\text{Capital employed}}$$

   $$= \frac{\text{Net profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Capital employed}}$$

   The figures for the division would be:

   30% = Net profit margin X 5
   Net profit margin = 6% (30%/5)
   Transfer price = £5.32 (£5/0.94)

   This example illustrates a general procedure applicable in other situations.

4. Each unit of output utilises £2 of capital employed (£2,000,000/1,000,000). The required return is 20% profit per unit of £0.40. The required transfer price is therefore £5.40.

The two major drawbacks to the full cost approach concern its inability to motivate the supplying division's manager to improve performance and the danger of making incorrect decisions. Since all costs are passed on, irrespective of economy or efficiency in the supplying division, there is little incentive for managers of supplying divisions to cut costs or to operate more efficiently. Once costs are passed on, fixed costs of the supplying divisions are interpreted as variable costs of the receiving division and it is therefore possible for divisions to make short-term decisions which are suboptimal for the organisation as a whole. Consider a make or buy decision for which divisional variable costs are £2, including £0.30 fixed cost included in a transfer price, and the external supplier's costs are £1.90. the division would buy in the product, despite the fact that the company's variable costs, at £ 1.70, are lower than the buy-in price. The company would wish the division to continue to make, but can only do so by centralisation of the decision rule, with a loss of autonomy at the divisional level.

In order to ensure that inefficiencies are not passed on by suppliers, it has been suggested that standard absorption costing can produce reliable results. Underutilisation of plant capacity, inefficiencies and lack of price control remain in the division in which they occurred and are reported through the calculation of standard costing variances. However, the problem of suboptimisation would not be overcome by standard absorption costing transfer prices.

B. Variable cost based transfer prices.

Variable cost based systems overcome the decision-making problem of full cost system. Transfers from one division to another are made at variable cost. Standard variable cost overcomes the problem of passing on inefficiencies and diseconomies from division to division.

There are two ways by which profits can be created at a divisional level. The first approach is to apply the principles illustrated in *A* to marginal costing. Transfer pricing schemes would allow a suitable level of contribution, as measured in terms of contribution on sales ratio. An alternative approach is to create a two-part charging system. One part of the scheme would transfer a lump sum, representing an allowance for divisional fixed cost once a year to allow each division the chance of creating a final profit. The second part of the scheme would value transfers at variable cost.

C. Market value based transfer prices

There is universal agreement that in competitive markets a market value based transfer price should achieve optimal results. In this circumstance, it can be expected that:

1. The autonomy of the division is not undermined because markets determine the value of outputs, not centralised departments or divisional costs. Market prices would be seen to be objective and fair to all. The aim of creating an organisational structure where each division operates as an organisation in its own right can be achieved. Ideally, suppliers should be permitted to sell to external customers and receivers should be allowed to buy from external producers.

2. Managers' performance reflect their ability to compete with external companies in a free market. This may be a fair indication of the manager's ability and potential to perform at higher levels within the organisation and thus forms a fair basis for promotion and salary decisions;

3. From 2, it can be expected that the transfer pricing mechanism will be neutral in motivating managers to perform in accordance with organisational goals; that is, the transfer pricing mechanism will not be biased in any manner other than that created by market forces. Performance measurement schemes can thus be established to motivate managers to act in a goal congruent manner;

4. Reliable decisions would arise at divisional level. For instance, in a joint product situation, products, which should be sold at the split-of point would not be processed further since the post split-off processing division would show a loss based on the transfer price.

Unfortunately, two problems illustrate that market based prices may not be able to achieve these aims in all circumstances. The first problem arises because transfer pricing situations are not simply selling situations. The supplying division, for instance, does not have to incur the costs of selling normally associated with selling to external customers. The receiving division may be in a position to influence quality and delivery because it is in the interests of both divisions that the company as a whole prospers. It is sometimes desirable to adjust the market price to reflect such factors, with a commensurate loss in the objectivity which market prices can bring. The second problem is more fundamental; there may simply not be a perfect market in operation. A vertically integrated company, for instance, may not possess a market for its intermediary products. In this case, there is no market from which to establish market values. Other market imperfections would produce bias which would work to the benefit of either the supplying or the receiving division.

D. Negotiated prices

Where market based prices are not applicable, it has been argued that allowing managers to bargain with each other in order to establish transfer prices develops the kind of management skills which are necessary to the future of the enterprise. Managers would need to have detailed knowledge of their own resources and costs and would need to apply their inter-personal skills of communication, persuasiveness and bargaining in order to show a profit. Negotiated prices thus stress the human behavioural aspects of the organisation. Social and political skill can be translated into good divisional performance, as measured by the accounting system.

Unfortunately, negotiated prices can also lead to conflict, especially where two managers cannot agree on a transfer price. In such circumstances, a mechanism for resolving the dispute is required at a central level. The intervention of central authority to resolve conflicts clearly results in a loss of autonomy with dysfunctional consequences. A system which aims to reveal the behavioural skills of managers can as easily reward those who can manipulate the inherent tensions between centre and divisions to their best advantage.

E. Linear programming based transfers

Consider the following example:

Example 3

Two products, Exe and Wye are produced by Hifi, an electronics company which operates two departments. Both departments are limited to working a maximum of 10 hours per day per machine and both departments utilise 10 machines. A five-day week is in operation. Product cost information for Exe and Wye is as follows:

	Exe	Wye
	£	£
Selling price	9.50	8.50
Materials cost	1.00	1.00
Variable overheads cost:		
Department 1		
2 hours @ £1.50	3.00	
1 hour @ £1.50		1.50
Department 2		
1.25 hours @ £2	2.50	
2 hours @ £2		4.00
Contribution	3.00	2.00

Processing this example through a linear programming computer package reveals a total contribution of 181.8 units of Exe and 136.4 units of Wye, with shadow prices for department 1 and 2 of £1.2727 and £0.3636 respectively. Were the centralised management accounting department to run this package, they would instruct both departments to produce 182 units of Exe and 136 units of Wye. The transfer price would be calculated as the total of variable cost and opportunity cost. The opportunity cost of each product is given by the shadow price; in the case of Exe, calculated as follows:

Department 1	2 hours @ £1.2727	£2.545
Department 1	1.25 hours @ £0.3636	£0.455
		£3.000

There is no coincidence that this is the product contribution in this case. Transfer prices of £7 for Exe and £4.50 for Wye would be established. This implies that department 2 would not be able to show a profit.

A situation can be illustrated where the constraints of the linear programming model lead to the supplying division's manager being motivated to achieve optimum levels of production through the transfer price. In this case, the transfer price can itself motivate the achievement of optimum levels of output for both the supplying division and the organisation as a whole, where all output from one division is processed by the next. The supplying division is not allowed a profit and must be given production instructions from the centre, with loss of divisional autonomy.

International Transfer pricing

International transfer pricing refers to the determination of prices to be charged between related persons and in particular within a multinational enterprise for transactions between various group members (sales of goods, the provision of services, transfer and use of patents and know-how granting of loans etc.) As these prices are not negotiated in a free open market they may deviate from prices agreed upon by non-associated trading partners in comparable transactions under the same circumstances.

The above leads to a special interest on the part of tax authorities in intra-group transactions and especially in cross- border transactions. In many circumstances the tax authorities would seek to adjust the prices adopted in these transactions to arm's length prices. However, the intra-group trading partners themselves may find it difficult to settle on satisfactory transfer prices, even if they are in many cases no comparable transactions in the open market. In such circumstances the tax authorities may seek to arrive at the arm's length price by using cost -based methods or methods based on the price changed to the final customer – the 'resale minus' or resale price method or any other which can produce an acceptable result.

G. Transfer pricing with third party consequences

Transfer prices are used not only for internal record keeping and performance evaluation purposes. There are several settings where transfer prices have direct cash consequences for a company. The most widely cited case is in interstate and international transactions where transfer prices may affect tax liabilities, royalties or other payments due to different government jurisdiction. Since tax rates differ across states, or jurisdictions, companies have an incentive to establish a transfer price which will increase the income in the lower tax jurisdiction and decrease income in the higher tax jurisdiction.

Example:

Assume that the Kerbrook Shirt Company owns a manufacturing plant in Kenya where its marginal tax rate is 60 per cent of net income. These shirts are imported by Zambia where the marginal tax rate is 75 per cent of net income. For simplicity assume that there are no currency controls and that tax regulations concerning the definition of taxable income are the same between the two countries.

During the current year, the company incurred production costs equivalent to sh.2 million in Kenya. Costs incurred in Zambia aside from the costs of the shirts amounted to an equivalent of sh.6 million. Sales revenues in Zambia were sh. 24 million. Similar goods imported by independent companies in Zambia would have cost an equivalent of sh. 3 million.

However, Kerbrook Shirt Company points out that because of its special control over its operations in Kenya and the special approach it uses to manufacture its goods, the appropriate transfer price is sh. 10 million.

Required:

What would Kerbrook Shirt Company's total tax liability in both countries be if it used the sh. 3 million transfer price?
What would the liability be if it used the sh. 10 million transfer price?

Solution:

The solution is approached by determining the taxable income for each country under the alternative transfer price scenarios. The resulting taxable income is multiplied by the tax rate in each country to obtain the tax liabilities.

For the sh. 3 million transfer price, the tax liability is computed as follows;

	Kenya (sh)	Zambia (sh)
Sales revenues	3,000,000	24,000,000
Third party costs	2,000,000	6,000,000
Transferred goods costs	-	3,000,000
Total costs	2,000,000	9,000,000
Taxable income	1,000,000	15,000,000
Tax rate	60%	75%
Tax liability	600,000	11,250,000

Total tax liability = sh 11,850,000

For the sh.10 million transfer price;

	Kenya (Sh).	**Zambia (Sh).**
Revenues	10,000,000	24,000,000
Third party costs	2,000,000	6,000,000
Transferred goods costs	-	10,000,000
Total Costs	2,000,000	16,000,000
Taxable income	8,000,000	8,000,000
Tax rate	60%	75%
Tax liability	4,800,000	6,000,000

Total tax liability = sh. 10,800,000

The tax liability assuming a sh. 10 million transfer price is about 9% less than the liability that would be incurred if the transfer price was sh. 3 million.
International taxing authorities look closely at transfer prices when examining the tax returns of companies engaged in related party transactions which cross jurisdictional lines. Companies must therefore have adequate support for the use of the transfer price which they have chosen for such a situation.

Another situation where transfer pricing has direct economic consequences is where the owner of one entity holds a different ownership percentage than he or she holds in another entity. It is generally in the best interest of this person to transfer income to the entity in which he or she holds the higher ownership in percentage.
In situations where transfer prices have direct economic consequences it is important to develop transfer prices in a manner that will meet third party scrutiny since tax authorities may investigate transfer prices

which affect cross-border tax liabilities. In addition, in situations where an individual acts on both sides of a related party transaction, the possibility of litigation arises if transfer prices are not reasonable.

In general, transfer prices for goods and services between segments or companies located in different countries should reflect that countries have different tax rates and regulations. Due to these variances, companies have an incentive to transfer most of their income to the subsidiary that has a tax advantage over others within the corporate group. In addition some countries restrict payment of income or dividends to parties outside their national borders. In such cases, the company often increases the transfer prices so they pay more funds out of these countries while appearing to follow regulations.

Transfers from foreign countries where the wage level and/or tax rate is low may also be made at a domestic market price rather than on a cost basis because foreign economic conditions are so different from domestic conditions.

H. International transfer pricing - compliance and documentation

Transfer pricing is a perennial issue, within the international tax community (Richard Casna, Accounting and Business, February 1988, pp.30-31).

As multinationals become more sophisticated in employing transfer pricing techniques in their tax planning, the revenue authorities have increased their scrutiny of arrangements, putting transfer pricing at the forefront of international tax concerns.

It naturally follows that if profits can be shifted from a high tax jurisdiction to one of low tax through transfer pricing, the tax authorities will respond with rules designed to curtail tax avoidance and ensure tax payer compliance.

Revenue authorities around the globe have become more adept at countering the "profit-shifting" aspects of transfer pricing practices and are strengthening their statutory powers with ever more extensive and complex legislation and regulations.

To strengthen the tax authorities' position, regulations typically introduce specific rules to determine arms' length prices and require that tax payers maintain very extensive records documenting the methods used to determine their transfer prices (which often necessitates the employment of teams of both in-house and outside counsel, accountants and economists). Provision is made as well for the imposition of very stringent penalties in cases of non-compliance.

To achieve these ends, the statutes generally focus on guidelines set out by the OECD's Committee on Fiscal Affairs (the tax policy body of the OECD), first in its 1979 document "Transfer pricing and multinational Enterprises" and the 1995-1996 "Transfer pricing Guidelines for Multinational Enterprises and Tax Administrations." These guidelines generally stipulate the parameters of the arm's length pricing standard and the methodology to be followed in achieving arm's length prices.

The practitioner as adviser to multinationals which face the complexities of transfer pricing legislative and regulatory controls has therefore to simply consider the statutes in each country/state carefully, comply with the rules and maintain extensive documentation.

1.2. CRITIQUE OF PERFORMANCE MEASUREMENT

This section brings together material from preceding data in this lesson in order to provide a critical appraisal of performance measurement. In a sense, all of the material in the Management Accounting Notes can be related to the measurement of performance; good managers make good decisions, form good plans, establish good control practices and this should be reflected in measures of performance. An alternative view is that performance measurement drives the decision making , planning and control functions of management; managers manipulate performance results so that they can appear to be performing well. This provides an explanation for ROI approaches to the capital investment decision;

managers are more concerned to appear to be making the right decision than to be making the right decision in reality. Whatever the view, transfer pricing and performance measurement provides good material for assessing the problems facing the management accountant who is trying to devise systems which will benefit organisations.. The critique which follows provides a summary of the problems of ensuring that performance measurement systems achieve the purposes for which they are designed. The list which is provided can be considered as a coverage of some of the themes which influence management accounting as a whole.

A. Transfer pricing and performance measurement relies upon the judgement of the management accountant to make a suitable choice of approach and to calculate suitable values where appropriate.

Example 4

Alton division (A) and Birmingham division (B) are two manufacturing divisions of Conglom plc. Both of these divisions make a single standardised product; A makes product I and B make product J. Every unit of J requires one unit of I. The required input of I is normally purchased from division A but sometimes it is purchased from an outside source.

The following table gives details of selling price and costs for each product.

	Product I	Product J
	£	£
Established selling price	30	50
Variable costs		
Direct material	8	5
Transfers from A	—	30
Direct labour	5	3
Variable overhead	2	2
	15	40

Divisional fixed cost (per annum)	£500,000	£225,000
Annual outside demand with current selling prices (units)	100,000	25,000
Capacity of plant (units)	130,000	30,000
Investment in division	£6,625,000	£1,250,000

Division B is currently achieving a rate of return well below the target set by the central office. Its manager blames this situation on the high transfer price of product I. Division A charges division B for the transfers of I at the outside supply price of £30. The manager of division A claims that this is the price `determined by market forces'. The manager of B has consistently argued that intra group transfers should be charged at a lower price based on the costs of the producing division plus a `reasonable' mark-up.

The board of Conglom plc is concerned about B's low rate of return and the division manager has been asked to submit proposals for improving the situation. The board has now received a report from B's manager in which he asks the board to intervene to reduce the transfer price charged for product I. The manager of B also informs the board that he is considering the possibility of opening a branch office in rented premises in a nearby town, which should enlarge the market for product J by 5,000 units per year at the existing price. He estimates that the branch office establishment costs would be £50,000 per annum.

You have been asked to write a report advising the board on the response that it should make to the plans and proposals put forward by the manager of division B. Incorporate in your report a calculation in the rates of return currently being earned on the capital employed by each division and the changes to these that should follow from an implementation of any proposals that you would recommend.

An answer to this question would be provided in report style for examination purposes. The discussion which follows shows the influence of management accounting judgement rather than providing an ideal examination answer. It is anticipated that students will have the necessary skill to convert the points of discussion into an answer suitable for examination conditions.

If I and J are traded in a perfect market and both divisions are given complete autonomy, the present transfer price is optimal. Any increase in transfer price would lead to B purchasing from external sources, which would not be in the interests of the organisation. Any decrease in transfer price would lead to A selling to external customers, which would again not be in the best interests of the organisation.

It could be argued that A does not have to find the resources to market I and that some reduction from the external price is appropriate in setting the transfer price. The amount of the reduction could be a matter of negotiation between the managers of A and B or could be established through the judgement of the management accountant, bearing in mind any information available on competitor's selling costs.

If the market is imperfect then negotiated or cost based prices should be considered. It is a matter of judgement to determine whether negotiated prices would provide a suitable resolution to the problem, taking into account the personalities of the managers of A and B. Although the managers appear to be entrenched in their respective points of view, management training and/or an explanation of the purpose of transfer pricing may improve relations between the managers, lead to an acceptable transfer price and improve the future prospects for Conglom as a whole. Divisional autonomy would be maintained. Negotiated prices are thus to be recommended to the board of directors as a suitable alternative. The management accountant would have a role to play in educating managers in the purposes, benefits and limitations of management accounting systems. Cost based prices would require a degree of intervention from the centre, the part of the organisation where it could be expected that the necessary information is available.

Division A can meet B's demand for 25,000 units and the outside demand for 100,000 units, within its capacity of 130,000 units. Division B would meet the external demand. This would lead to the following financial statement under the present transfer price:

	Division	**Division**
	A	*B*
	£'000	**£'000**
Sales revenue (external customers)	3,000	1,250
Transfers	750	(750)
	3,570	500
Variable cost (excluding transfers)	1,875	250
Contribution	1,875	250
Fixed cost	500	225
Profit	1,375	25
Investment	6,625	1,250
Return on investment	20.8%	2 .0%

If autonomy is maintained, division A could make a decision on whether to sell to division B or not, and at what price on a short-term basis. The existence of surplus capacity should lead to any price in excess of variable cost being acceptable. Using variable cost as the transfer price would lead to the following results:

	Division A	**Division B**
	£'000	**£'000**
Sales revenue (external customers)	3,000	1,250
Transfers (at A's variable cost of £15 per unit)	375	(375)
	3,375	875
Variable cost (excluding transfers)	1,875	250
Contribution	1,500	625
Fixed cost	500	225
Profit	1,000	400
Investment	6,625	1,250
Return on investment	15.1%	32 .0%

Any cost based price between £15 and £30 would appear to be acceptable and the management accountant could apply judgement to decide on appropriate levels of profitability for each of the divisions.

If it is judged that an equal opportunity to achieve profit returns should be given, then the transfer price could be calculated as follows:

Total profit involved	£1,400,000
Total investment involved:	£7,875,000
Average return on investment:	17.8%
Total costs in department A are:	(1875 + 500) = £2,375,000

Applying the relationship:

$$\text{ROI} = \text{Net profit/investment}$$

$$= \frac{\text{Net profit}}{\text{Total costs}} \times \frac{\text{Total cost}}{\text{Investment}}$$

provides: 17.8 = Net profit mark-up X 2,375/6,625

Net profit mark-up = 49.7%

Cost per unit in A is (2375/125)	£19.00
Average selling price is (19 x 1.497)	£28.44
Total sales: 125,000 @ £28.44	£3,555,000.00
Less external sales:	£3,000,000.00
Transfer value:	£ 555,000.00
Transfer price (555/25)	£ 22.20

The reported financial statements would also be as follows:

	Division A £'000	Division B £'000
Sales revenue (external customers)	3,000	1,250
Transfers (at £22.20 per unit)	555	(555)
	3,555	695
Variable cost (excluding transfers)	1,875	250
Contribution	1,680	445
Fixed cost	500	225
Profit	1,180	220
Investment	6,625	1,250
Return on investment	17.8%	17 .6%

There are two aspects to the behavioural aspects of this situation which will be discussed. The first concerns the extent to which managers of A and B would find the transfer price `fair'. Any attempt by the management accountant to impose a transfer price would be perceived to be an infringement of autonomy and may lead to dysfunctional consequences. Wherever possible, if the autonomy of the division is to be guarded and an imperfect market operates, negotiated prices appear to offer most prospects of optimising the behavioural implications. The second behavioural implication concerns the motivation of managers to accept worthwhile projects. If it is accepted that managers are motivated to improve their reported performance, performance measures which lead to managers rejecting profitable projects are dysfunctional. This particular idea can be explored in relation to

Example 1.

At the existing transfer price of £30, the manager of B would produce the following calculations of the value of opening the branch office:

Additional sales 5,000 @£50		£250,000
Additional variable costs:		
Transfer price	£150,000	
Other variable costs	£ 50,000	
Fixed costs	£ 50,000	£250,000
Net profit		£ nil

On behavioural grounds, the project would be rejected by the manager because performance does not improve as a result of the effort necessary to open the branch. However, from Conglom plc's point of view the calculation would appear as follows:

Additional sales 5,000 @£50		£250,000
Additional variable costs:		
Transfer price	£ nil	
Other variable costs	£125,000	
Fixed costs	£ 50,000	£175,000
Net profit		£ 75,000

It is advantageous to the company as a whole, for the branch office to be opened. Since A has spare capacity sufficient to meet the additional requirement, a transfer price equal to the variable costs incurred in division A would lead to the manager of department B making the correct decision. A transfer price between £15 and £30 would lead to the branch being opened but a transfer price of £15 alone would ensure that all future decisions were evaluated correctly at divisional level. This leads to the second point in the critique performance measurement.

B. Values which are suitable for performance measurement purposes are not necessarily suitable for decision making, planning and control purposes. *Example 4* illustrated the problem of meeting both performance measurement and decision-making requirements. For planning purposes, reasonable future forecasts or targets which meet long-term planning requirements present two acceptable approaches and incremental budgeting offers a third means by which values can be established in practice. For control purposes, values should ideally be set just above aspiration levels. For performance measurement purposes, values should be set which avoid sub-optimisation and dysfunctional behaviour and which further the objectives of the performance measurement scheme and of the company in general. It is unlikely that a single value can meet all requirements.

In some circumstances, multiple values can be established. In overcoming the problem of setting up reliable and valid values for planning, control, decision making and performance measurement needs, however, further problems may arise. Imagine that a company establishes one target for performance measurement purposes and another, lower value, for planning purposes. The planning value must be kept secret from the divisional manager if it is to motivate since some types of manager may lower aspiration levels to the planned target. Secrecy can have detrimental effects to the coordination and communication objectives of budgeting. Again, the behavioural consequences of establishing values are of paramount importance and the management accountant finds that effective accounting is partly based on setting up sound systems at the technical level and partly based on setting up systems which work for the people within the organisation.

C. Emphasis on cost, profit and investment centre performance in the short term can have detrimental effects on the organisation in the long term. *Example 5* is taken from a situation which has occurred in practice.

Example 5

A company found it necessary for cash flow purposes to close one of its divisions. Two divisions were prime contenders for closure. Each would have brought in roughly equal amounts of cash and the amounts involved would have been sufficient to solve the cash flow crisis. Division A was set up ten years earlier and its assets were almost fully depreciated. Division B was set up two years earlier, incorporating the latest technology and had substantial balance sheet values because its assets were depreciated over a ten-year period. In the previous financial year, division A showed a 30% ROCE whilst division B showed a 20% ROCE. Which division should have been closed?

The company closed division B, because division A showed the best performance, as measured by ROCE. However, it found two years later that it needed to invest substantially in division A because of obsolete assets. A further cash flow crisis ensued.

This dysfunctional decision could have been avoided by applying a more appropriate valuation base for the assets than that provided by historic values derived from balance sheets designed primarily for financial accounting purposes.

Original cost, replacement value or an SSAP 16 philosophy have all been suggested as means by which ROCE can more reliably measure performance. The selection of asset valuation base is a matter of judgement.

Further examples of dysfunctional decisions arising from the need to meet short-term goals in terms of performance and/or budgetary control include postponing vital expenditure or investment. Postponement has the effect of ensuring that short-term goals are met but can disadvantage organisations in comparison with competitors who pursue long-term optimisation at some slight loss of optimisation in the short term.

D. Accounting figures can provide distorted information. Where a company imposes a cost based transfer price, results may be biased in favour of certain divisions at the expense of others, as *Example 4* illustrated. Where a company uses ROCE as a performance measure, performance appears to improve as assets age because the effect of depreciation is to reduce the asset base in the ROCE calculation. The accountant's figures on performance do not necessarily measure the true improvement or deterioration in divisional performance.

E. Financial measures of performance can give insufficient emphasis to non-financial and qualitative aspects of organisational management.

F. It is difficult to determine whether the manager's performance or department's performance is being measured in some circumstances. This is important where an organisation wishes to promote its most able managers to ensure the long-term successful management of the enterprise.

G. Independence and interdependence factors can lead to pseudo-profit and investment centres, where the accounting system treats divisions as autonomous despite the reality that autonomy cannot be achieved without detriment to the organisation as a whole. A transfer price which requires a decision from head office is likely to infringe divisional autonomy. Any system which requires a central accounting function to calculate a transfer price is therefore likely to lead to a loss of independence at divisional level.

H. The accounting models available to management accountants appear to create a potentially spurious sense of accuracy, reliability and validity. A budgeted target appears to have validity because it is visible and appears to be certain. In an uncertain world, deterministic targets may be invalid and probabilistic approaches may be more valid, but are unfortunately beset by problems, particularly the difficulty of establishing subjective probabilities. The section on forecasting suggested a number of reasons why managers tend to rely on relatively simple forecasting models in order to predict the workings of a complex world.

The problem facing the practical management accountant is to select an accounting model which most closely matches the reality of the situation faced by an organisation and for which data capture is feasible.

REINFORCING QUESTIONS

QUESTION ONE

Division A of Miujiza Ltd. is the only source of supply for an intermediate product that is converted by Division B into a saleable final product. Most of A's costs are fixed. For any output up to 1,000 units per day, its total costs are Shs 15,000 per day. Total costs increase by Shs 3,000 per day for every additional thousand units made. Division A judges that its own results will be optimised if it sets its price at Shs 12 per unit, and it acts accordingly.

Division B incurs additional costs in converting the intermediate product supplied by A into a finished product. These costs are Shs 37,500 for any output up to 1,000 units, and Shs 7,500 per thousand for outputs in excess of 1,000. On the revenue side, B can increase its revenue only by spending more on sales promotion and by reducing selling prices. Its sales forecast is

Sales in units	Net revenue per a thousand units Shs
1,000	52,500
2,000	39,750
3,000	33,000
4,000	27,750
5,000	24,000
6,000	20,000

Required:

(a) Prepare a schedule comparing B's costs including its purchases from A, revenue and net income at the following levels of output (1,000, 2,000, 3,000, 4,000, 5,000, 6,000). (5 marks)

(b) What is B's maximum net income? At that level, what is A's net income? At that level, what is Miujiza's aggregate net income? (5 marks)

(c) Suppose the company abandons its divisionalised structure. Thus the two profit centres A and B are combined into a single profit centre with responsibility for the complete production and marketing of the product. Prepare a schedule similar to that in (a). What volume level will provide the highest net income? (5 marks)

(d) Evaluate the results in (c). Why did the circumstances in requirement (a) lead to less income than in requirement (c)? How would you adjust the transfer-pricing policy to ensure that overall company net income will be maximised where separate profit centres A and B are maintained? (5 marks)

(Total: 20 marks)
(CPA JUNE 93)

QUESTION TWO

CD plc is a nationwide warehousing and distribution company, organised into eight geographical regions in each of which there is a depot and a fleet of vehicles.

These regions differ widely in respect of their area size, their mix of different types and sizes of shops and the major and minor roads that they comprise.

The remuneration of the general managers of each region comprises:

- Basic salary
  This is a starting figure of £12,000 per annum which increases by £1,000 per annum for every year of service as a manager up to a maximum of £22,000 per annum.

- Bonus of 0.75% of the excess of sales over target for the year.
  The target sales figure is calculated by a formula based on the value of vehicles operated by the region.

- Bonus based on the region's return on capital employed (ROCE)

  ROCE is calculated: $\dfrac{\text{Annual net profit before interest and tax}}{\text{End of year book value of net assets.}}$

  The bonus is the ROCE multiplied by:

  3% of the capital employed if the capital employed is £2 million or above.
  2% of the capital employed if the capital employed is below £2 million.

  This different percentage is to encourage expansion through the use of greater assets.

  For regions 3 and 7 the following figures show actual data for the year just ended (31 October 1990) and budgeted data for the year to 31 October 1991:

		Region 3		**Region 7**
	Actual	**Budgeted**	**Actual**	**Budgeted**
Year to:	**31 Oct 90**	**31 Oct 91**	**31 Oct 90**	**31 Oct 91**
	£000	**£000**	**£000**	**£000**
Sales	2,400	2,750	3,700	3,600
Cost of sales	1,872	2,172.5	3,034	2,844
Net profit*	123	147	166	241
*before interest, tax, basic salary and bonuses				
End of year capital employed:				
Working capital	180	210	230	230
Fixed assets:				
Building	640	680	820	820
Vehicles	1,030	1,370	1,750	1,850
Target sales	2,250	2,700	3,400	3,600
General manager's basic salary	£18,000		£22,000	

Your are required, as chief management accountant,

(a) to calculate what change there will be for each of the general managers of regions 3 and 7 between the remuneration based on the actual results for the year to 31 October 1990 and the remuneration based on the budgeted results for the year to 31 October 1991;
Show your workings. (6 marks)

(b) to explain whether you consider the changes calculated in (a) above show an appropriate reward for the performance of each of the two regional general managers;
Show relevant calculations. (9 marks)

(c) to recommend what changes in the basis of the remuneration scheme for regional general managers you would propose for discussion with the managing director. Briefly explain why you have included each recommendation.

Assume that

(i) your objective is to achieve rewards for the regional general managers that will more adequately recognise effective performance of benefit to the company as a whole;

(ii) the company does not wish to make changes in the operating methods of the regional managers or in the ways in which regions are financed. (10 marks)

(Total: 25 marks)

(CIMA)

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES

COMPREHENSIVE ASSIGNMENT NO.4

TO BE SUBMITTED AFTER LESSON 8

To be carried out under examination conditions and sent to the Distance Learning Administration for marking by the University

EXAMINATION PAPER: **TIME ALLOWED: 3 HRS.**

ANSWER ALL QUESTIONS.

QUESTION ONE

Ab plc is considering a new product with a three-year life. The product can be made with existing machinery which has spare capacity or by a labour saving specialised new machine which would have zero disposal value at the end of the three years.

The following estimates have been made at current prices:

Sales volume	1 million units per annum
Selling price	£15 per unit
Labour cost (without m/c)	£6 per unit
Material cost	£2 per unit
Variable overheads	£2 per unit

Additional fixed overheads for the new product are estimated to be £3 million per year. The new machine would cost £5 million now and would halve the labour cost per unit. Because of competition, selling price increases will be limited to 2% per annum although labour cost is expected to rise at 12% per annum and all her costs at 8% per annum. The company's money cost of capital is 15% and apart from the cost of the new machine all other cash flows can be assumed to arise at year ends.

Required:

To calculate the NPV of the new product assuming that manufacture uses existing machinery;

To calculate the NPV assuming that the new machine is purchased.

To recommend what action should be taken, and to comment on your recommendations;

To explain what changes if any there would be in your analysis if the existing machinery was already fully utilised on other production.

CIMA Management Accounting Techniques

QUESTION TWO

You are employed as a Management Accountant at the Head Office of a large conglomerate Group. For many years this organisation, which gives a considerable degree of autonomy to divisional managers, has rewarded them on ROI achieved.

This scheme has been broadly accepted by most managers but it is known that two managers are particularly unhappy with their ROI figures (and bonuses) and you are asked to investigate and report.

The following information is available for 1990:

	Division X		Division Y	
	Actual	Budget	Actual	Budget
	£m	£m	£m	£m
Sales	40	50	110	96
Assets employed	24	26.8	52	48.5
Profit before interest and tax	4.1	6.5	11.4	6.7
ROI	17.1%	24.3%	21.9%	13.8%

These figures include the apportioned costs for an automated warehouse shared by the two divisions. The data available for this facility for 1990 are:

	Warehouse	
	Actual	Budget
	£m	£m
Despatches	150	146
Assets employed at NBV	8	8
Operating costs:		
Depreciation	1.6	1.6
Other fixed costs	1.1	0.9
Variable storage costs	0.5	0.5
Variable handling costs	1.3	1.1
Total operating costs	4.5	4.1

The assets employed had been split between the divisions concerned in the proportions originally agreed when the warehouse investment had been authorised i.e. 50% each.
The operating costs had been split between the divisions in proportion to the actual usage, measured by actual sales.
Division X stocks occupy 40% of the space available.

Required

To recalculate the ROI of Division X using bases which maximise the return disclosed. Explain and briefly justify your calculations

To recalculate the ROI of Division Y using bases which maximise the return disclosed. Explain and briefly justify your calculation;

To recalculate the ROI figures for Divisions X and Y in the way you would recommend the group should use and justify this basis, explaining the motivational implications of your recommendations.

Later it becomes possible for the divisions to obtain comparable ware-housing facilities from outside suppliers at less cost:
Division X £1.1m
Division Y £2.3m
Explain how this additional information could affect your recommendations and discuss the impact of these recommendations on the managers of the two divisions and the warehouse.

QUESTION THREE
A group has two companies-
K Ltd which is operating at just above 50% capacity and
L Ltd which is operating at full capacity (7,000 production hours)
L Ltd produces two products, X and Y, using the same labour force for each product. For the next year its budget capacity involves a commitment to the sale of 3,000 kgs of Y, the remainder of its capacity being used on X.
Direct costs of these two products are:

	X	Y
Direct materials	18	14
Direct wages	15(1 production hour)	10(2/3 production hour)

The company's overhead is £126,000 per annum relating to X and Y in proportion to their direct wages, at full capacity, £70,000 of this overhead is variable. L Ltd. prices its products with a 60% mark-up on its total costs.
For the coming year, K Ltd wishes to buy from L Ltd 2,000 kgs of X which it proposes to adapt and sell, as product Z, for £100 per kg. The direct costs of adaptation are £15 per kg. K Ltd's total fixed costs will not change but variable overhead of £2 per kg will be incurred.

You are required to recommend, as group management accountant,

At what range of transfer prices, if at all, 2,000 kgs of product X should be sold to K Ltd.
what other points should be borne in mind when making any recommendations about transfer prices in the above circumstances.

CIMA Management Accounting – Decision making.

QUESTION FOUR

You are a Senior Audit Assistant of Abel Adongo & Co., Certified Public Accountant. The firm is one of the three accounting firms offering professional services in the city of Madolva. The other two firms are Beans Birundu & Co. Certified Public Accountants and Claudio Chege & Co. Certified Public Accountants. The three firms are referred to as A, B and C.

Your firm is worried about its market share and the Senior Partner in charge of Partnership development has collected the following data which she wants you to analyse. She further informs you that the observed client flow behaviour is expected to remain about the same for the foreseeable future.

	No. of Clients	Market Share	Flow of Clients						No. of Clients	Market Share
			Gains From			Losses To				
	31.12.90		A	B	C	A	B	C	31.12.91	
A	204	0.177	—	12	20	—	15	9	212	0.184
B	416	0.361	15	—	5	12	—	7	417	0.362
C	531	0.461	9	7	—	20	5	—	522	0.454

Required:

a. Convert the above data into a matrix of transition possibilities. (5 marks)
b. Estimate the firm's market shares for 2014 (5 marks)
c. Estimate the firm's steady state market shares. (7 marks)
d. Briefly comment on the advisability of using the Markov Technique in forecasting market shares in the market for accountancy services. (3 marks)

(TOTAL: 20 MARKS)
(CPA JUN `92)

QUESTION FIVE

a. Two opposing political parties are nominating candidates for presidency in the Peoples Republic of Yanke in separate conventions that are being held simultaneously. The following possibilities apply for the respective party winning the election for the indicated nominee pair. Naturally each party wishes to maximise its probability of winning the presidency.

Peoples Party	*Republican Party*	*Probability for the Peoples Party winning*	*Probability for the Republican Party winning*
Maina	Salat	0.75	0.25
Maina	Simiyu	0.25	0.75
Onyango	Salat	0.30	0.70
Onyango	Simiyu	0.60	0.40

Subtracting the Republican's win probability from the People's win probability gives the Peoples payoff measure so that the structure is converted into a zero-sum game. By carrying out this operation, construct the appropriate payoff table and then, using the minimax criterion, determine the value of this political game. (10 marks)

b. Two suspects are taken into police custody and separated. The prosecutor is certain that they are guilty of a specific crime, but he does not have adequate evidence to convict them at a trial. He points out to each suspect that each has two alternatives: to confess to the crime the police are sure they have committed, or not to confess. If they both do not confess then the prosecutor states that he will book them on some minor charge and they will both serve one year in prison. If they both confess, they will be jailed each for 8 years. If one confesses and the other does not the "co-operative" one will receive lenient treatment (i.e. three months jail term) while the other will receive the maximum jail term allowed by the law (i.e. fourteen years) for this crime.

Required:

i. Convert this situation into a non-zero sum game and determine how long each suspect will serve in jail. (5 marks)

ii. Discuss some of the managerial applications of this game. (5 marks)

(Total: 20 marks)
(CPA DEC `92)

END OF COMPREHENSIVE ASSIGNMENT No.4
NOW SEND YOUR ANSWERS TO THE DISTANCE LEARNING CENTRE FOR MARKING

LESSON NINE

EMERGING TRENDS IN MANAGEMENT ACCOUNTING

CONTENTS

- Strategic Management Accounting
- Target Costing
- Activity Based Predetermined Overhead Rates
- Activity Based Management (ABM).

INSTRUCTIONS

- Read chapters 10 and 23 of Management and Cost Accounting by Colin Drury 5th Edition
- Read the study text
- Attempt reinforcing questions at the end of the lesson
- Compare your answers to those provided in lesson 10.

STRATEGIC MANAGEMENT ACCOUNTING

Steps in Strategic Cost Analysis

- Identify the appropriate value chain and assign costs and assets to it.
- Diagnose the cost drivers of each value activity and how they interact.
- Identify competitor value chains, and determine the relative cost of competitors and the sources of cost differences.
- Develop a strategy to lower relative cost position through controlling cost drivers or reconfiguring the value chain and/or downstream value.
- Ensure that cost reduction efforts do not erode differentiation, or make a conscious choice to do so.
- Test the cost reduction strategy for sustainability.
- The three main elements of strategic cost management include:
- Value chain analysis
- Strategic positioning
- Cost driver analysis

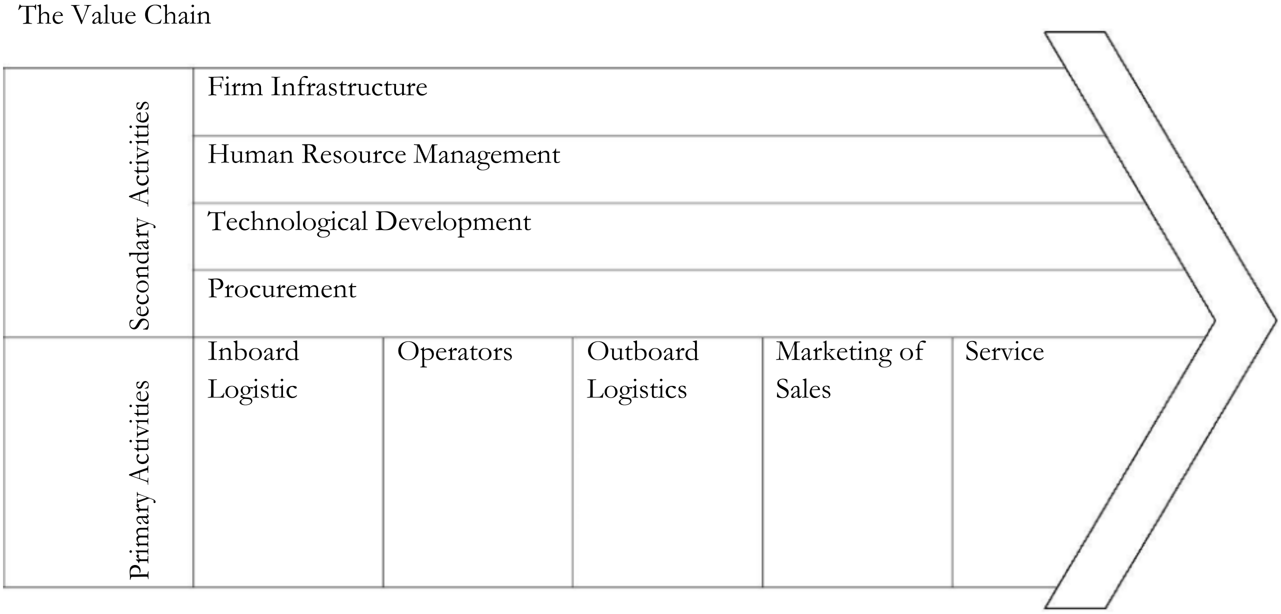
VALUE CHAIN ANALYSIS

Every firm is a collection of activities that are performed to design, produce, market, deliver and support its products/services. Value chain analysis is a systematic way of examining all activities that a firm performs and how they interact.

The value chain disaggregates the firm into strategically separable activities in order to understand the behaviour of costs so as to create competitive advantage. A firm creates competitive advantages by:

Finding new ways to conduct activities e.g. improving efficiency through automation.
Managing the linkages between activities better e.g. spending on better product design may reduce after sales service costs.
Managing the linkages between customers and suppliers better.

Value activities are physically and technologically distinct activities a firm performs. These are the building blocks by which a firm creates products and services valuable to its customers. The value chain has been shown by Michael Porter as follows.
The Value Chain



The Value Chain and Cost Advantage

Cost advantage is one of the two types of competitive advantage a firm may possess. Cost is also of vital importance to differentiation strategies because a differentiator must maintain cost proximity to competitors. Unless the resulting price premium exceeds the cost of differentiating, a differentiator will fail to achieve superior performance. The behaviour of cost also exerts a strong influence on overall industry structure.

Managers recognize the importance of cost, and many strategic plans establish "cost leadership or "cost reduction as goals. However, the behaviour of cost is rarely well understood. Wide disagreement often exists among managers about a firm's relative cost position and the reasons underlying it. Cost studies tend to concentrate on manufacturing costs and overlook the impact of other activities such as marketing, service, and infrastructure on relative cost position. Moreover, the cost of individual activities is analysed sequentially, without recognizing the linkages among activities that can affect cost. Finally, firms have great difficulty assessing the cost positions of competitors, an essential step in assessing their own relative positions. They often resort to simplistic comparisons of labour rates and raw material costs.

The absence of a systematic framework for cost analysis in most firms underlies these problems. Most cost studies address narrow issues and take a short -term viewpoint. Popular tools like the experience curve are often misused in cost analysis. The experience curve can serve as a starting point, but it ignores many of the important drivers of cost behavior and obscures important relationships among them. Cost analyses also tend to rely heavily on existing accounting systems. While accounting systems do contain useful data for cost analysis, they often get in the way of strategic cost analysis. Cost systems categorize costs in line items-such as direct labour, indirect labour, and burden-that may obscure the underlying activities a firm performs. This leads to aggregation of the costs of activities with very different economics, and to the artificial separation of labour, material, and overhead costs related to the same activity.

The value chain provides the basic tool for cost analysis. I begin by showing how to define a value chain for cost analysis purposes and how to associate costs and assets with value activities. I then describe how to analyze the behavior of cost, using the concept of *cost drivers. Cost* drivers are the structural determinants of the cost of an activity, and differ in the extent to which a firm controls them. Cost drivers determine the behavior of costs within an activity, reflecting any linkages or interrelationships that affect it. A firm's cost performance in each of its major discrete activities cumulates to establish its relative cost position.
The Value Chain and Cost Analysis

The behavior of a firm's costs and its relative cost position stem from the value activities the firm performs in competing in an industry. A meaningful cost analysis, therefore, examines costs within these activities and not the costs of the firm as a whole. Each value activity has its own cost structure and the behavior of its cost may be affected by linkages and interrelationships with other activities both within and outside the firm. Cost advantage results if the firm achieves a lower cumulative cost of performing value activities than its competitors.

The starting point for cost analysis is to define a firm's value chain and to assign operating costs and assets to value activities. Each activity in the value chain involves both operating costs and assets in the form of fixed and working capital. Purchased inputs make up part of the cost of every value activity, and can contribute to both operating costs (purchased operating inputs) and assets (purchased assets). The need to assign assets to value activities reflects the fact that the amount of assets in an activity and the efficiency of asset utilization are frequently important to the activity's cost.

For purposes of cost analysis, the disegregation of the generic value chain into individual value activities should reflect three principles that are not mutually exclusive:

The size and growth of the cost represented by the activity costs
The cost behavior of the activity
Competitor differences in performing the activity

Activities should be separated for cost analysis if they represent a significant or rapidly growing percentage of operating costs or assets. While most firms can easily identify the large components of their cost, they frequently overlook smaller but growing value activities that can eventually change their cost structure. Activities that represent a small and stagnant percentage of costs or assets can be grouped together into broader categories.

Activities must also be separated if they have different cost drivers. Activities with similar cost drivers can be safely grouped together. For example, advertising and promotion usually belong in separate value activities because advertising cost is sensitive to scale while promotional costs are largely variable. Any activity a business unit shares with others should also be treated as a separate value activity since conditions in other business units will affect its cost behavior. The same logic applies to any activity that has important linkages with other activities. In practice, one does not always know the drivers of cost behavior at the beginning of an analysis; hence the identification of value activities tends to require several iterations. The initial breakdown of the value chain into activities will inevitably represent a best guess of important differences in cost behavior. Value activities can then be aggregated or disaggregated as further analysis exposes differences or similarities in cost behavior. Usually an aggregated value chain is analyzed first, and then particular value activities that prove to be important are investigated in greater detail.

A final test for separating value activities is the behavior of competitors. Significant activities should be treated separately when a competitor performs them in a different way. Differences among competitors raise the possibility that an activity is the source of a relative cost advantage or disadvantage.

Assigning Costs and Assets

After identifying its value chain, a firm must assign operating activity and assets to value activities. Operating costs should be assigned to the activities in which they are incurred. Assets should be assigned to the activities that employ, control, or most influence their use. The assignment of operating costs is straightforward in principle, although it can be time-consuming. Accounting records must often be recast to match costs with value activities rather than with accounting classifications, particularly in areas such as overhead and purchased inputs. Since assets are expensive and their selection and use often involve tradeoffs with operating costs, assets must be assigned to value activities in some way that will permit an analysis of cost behavior. Assignment of assets to activities is more complex than assignment of operating costs.

First Cut Analysis of Costs

The allocation of costs and assets will produce a value chain that illustrates graphically the distribution of a firm's costs. It can prove revealing to separate the cost of each value activity into three categories: purchased operating inputs, human resource costs, and assets by major category. The proportions of the value chain can be drawn to reflect the distribution of costs and assets among activities.

Even the initial allocation of operating costs and assets to the value chain may suggest areas for cost improvement. Purchased operating inputs will often represent a larger proportion of costs than commonly perceived, for example, because all the purchased inputs in the value chain are rarely cumulated. Other insights can result from grouping value activities into direct, indirect and quality assurance activities, and cumulating costs in each category. Managers often fail to recognize burgeoning indirect costs and have a tendency to focus almost exclusively on direct costs. In many firms, indirect costs not only represent a large proportion of total cost but also have grown more rapidly than other cost elements. The introduction of sophisticated information systems and automated processes is reducing direct costs but boosting indirect costs by requiring such things as sophisticated maintenance and computer programmers to prepare machine tapes. In valve manufacturing, for example, indirect cost represents more than 10 percent of total cost. Firms can also find that the sum of all quality assurance activities in the value chain is strikingly large. In many industries, this has led to the growing conclusion that other approaches to quality assurance besides inspection, adjusting, and testing can yield large cost savings.

Cost Behavior

A firm's cost position results from the cost behavior of its value activities. Cost behavior depends on a number of structural factors that influence cost, which I term *cost drivers*. Several cost drivers can combine to determine the cost of a given activity. The important cost driver or drivers can differ among firms in the same industry if they employ different value chains. A firm's relative cost position in a value activity depends on its standing vis-à-vis important cost drivers.

Cost Drivers

Ten major cost drivers determine the cost behavior of value activities: economies of scale, learning, the pattern of capacity utilization, linkages, interrelationships, integration, timing, discretionary policies, location, and institutional factors. Cost drivers are the structural causes of the cost of an activity and can be more or less under a firm's control. Drivers often interact to determine the cost behavior of a particular activity, and the relative impact of cost drivers will differ widely among value activities. Thus no one cost driver, such as scale or the learning curve, is ever the sole determinant of a firm's cost position. Diagnosing the cost drivers of each value activity allows a firm to gain a sophisticated understanding of the sources of its relative cost position and how it might be changed.

Economies or Diseconomies of Scale

The costs of a value activity are often subject to economies or diseconomies of scale. Economies of scale arise from the ability to perform activities differently and more efficiently at larger volume, or from the ability to amortize the cost of intangibles such as advertising and R&D over a greater sales volume. Economies of scale can result from efficiencies in the actual operation of an activity at higher scale as well as from less than proportional increases in the infrastructure or overhead needed to support an activity as it grows. In a bauxite mine, for example, actual mining costs go down less with scale than do infrastructure costs.

Economies of scale must be clearly distinguished from capacity utilization. Increasing capacity utilization spreads the fixed costs of existing facilities and personnel over large volume, while economies of scale imply that an activity operating at full capacity is more efficient at larger scale. Mistaking capacity utilization for economies of scale can lead a firm to the false conclusion that its costs will continue to fall if it expands capacity once its existing capacity is full.

Increasing complexity and costs of coordination can lead to diseconomies of scale in a value activity as scale increases. When the number of lines in a metal can plant exceeds about *15,* for example, the complexity of the plant becomes unwieldy. Increasing scale also sometimes dampens employee motivation and may increase wage or purchased input costs. For example, a large plant may have a greater likelihood of unionization or lead to higher expectations and greater stridency of union negotiators. Diseconomies of scale in procurement can also occur if large requirements meet an inelastic supply, forcing up input prices. Diseconomies of scale appear to be present in many fashion-sensitive industries and professional services, which rely heavily on fast response times and creative individuals who do not function well in large organizations.

The scale sensitivity of activities varies widely. Value activities such as product development, national advertising, and firm infrastructure are typically more scale-sensitive than activities such as procurement and sales force operations because their costs are heavily fixed no matter what the firm's scale is. However, economies (and diseconomies) of scale can be found to some extent in virtually every value activity of a firm.

Learning and Spillovers

The cost of a value activity can decline over time due to learning that increases its efficiency. The mechanisms by which learning can lower cost over time are numerous, and include such factors as layout changes, improved scheduling, labor efficiency improvement, product design modifications that facilitate manufacturing, yield improvements, procedures that increase the utilization of assets, and better tailoring of raw materials to the process. Learning can also reduce the cost of constructing plants, retail outlets, or other facilities. Thus the possibilities for learning in an activity are much broader than learning by personnel to perform their functions more efficiently. The rate of learning varies widely among value activities because each offers differing possibilities for learning improvements. Learning is often the cumulation of many small improvements rather than major breakthroughs. The rate of learning may increase during slack periods when attention is focused on reducing costs rather than meeting demand. Moreover, learning tends to vary with the amount of management attention devoted to capturing it.

Learning can spill over from one firm in an industry to another, through mechanisms such as suppliers, consultants, ex-employees, and reverse engineering of products. Where spillover of learning among firms is high in a value activity, the rate of learning may stem more from total industry learning than from the learning of one firm.

Some of the policy choices that tend to have the greatest impact on cost include:

- Product configuration, performance, and features
- Mix and variety of products offered
- Level of service provided
- Spending rate on marketing and technology development activities
- Delivery time
- Buyers served (e.g., small versus large)
- Channels employed (e.g., fewer, more efficient dealers versus many small ones)
- Process technology chosen, independent or scale, timing, or other cost drivers
- The specifications of raw materials or other purchased inputs used (e.g., raw material quality affects processing yield in semiconductors)
- Wages paid and amenities provided to employees, relative to prevailing norms
- Other human resource policies including hiring, training, and employee motivation
- Procedures for scheduling production, maintenance, the sales force and other activities

Though policy choices always play an independent role in determining the cost of value activities, they also frequently affect or are affected by other cost drivers. Process technology is often dictated partly by scale and partly by what product characteristics are desired, for example. Moreover, other cost drivers inevitably affect the *cost* of policies. For example, an automated ticketing and seat selection system may well be subject to economies of scale that make such a system very costly for a small airline to adopt.

Policies typically play a particularly essential role in differentiation strategies. Differentiation often rests on policy choices that make a firm unique in performing one or more value activities, deliberately raising cost in the process (see Chapter 4). A differentiator must understand the costs associated with its differentiation and compare them to the price premium that results. This can be done only by isolating the effects of policies on cost.

STRATEGIC POSITIONING

The company must identify its strategic choices. This can be done from the firm's objectives, which emanates from the firms mission. Strategies have to be developed to achieve a competitive advantage over competitors, which may occur due to cost, price, quality, brand name, image of the product etc. Michael Porter highlighted two basic rules to competitive advantage
Cost Leadership strategy
Differentiation

Within each of these strategies a firm may decide to focus.

COST DRIVER ANALYSIS

Cost drivers are factors, which determine the costs of an activity i.e. a change in the cost driver will cause a change in the level of total cost related cost object. The cost drivers can either be volume based or transaction based. The company must therefore understand its cost drivers so as to control costs.

TARGET COSTING

This is another contribution to strategic management accounting. This is an approach to product pricing widely applied by Japanese companies and now being given a lot of attention in the USA and Europe. It is driven by external market factors.

A target market price is determined by marketing management prior to designing and introducing a new product. This target price is set at a level that will permit the company to achieve a desired market share and sales volume. A desired profit margin (target profit) is then deducted to determine the target maximum allowable product cost (target cost).

Product costs are computed based on design specification and compared with the target cost. If the projected product cost is above the target cost then product designers focus on it becomes cheaper to produce. Manufacturing engineers also focus on methods of improving production efficiency so that the target cost can be achieved even after a period of one to two years. A team of designers, engineers, marketing and production personnel, together with the management accountant, concentrate on producing a product that meets the target cost requirement. The role of the management accountant is to produce cost estimates for the various projected product designs, measure and monitor product costs once the production process begins.

ACTIVITY BASED PREDETERMINED OVERHEAD RATES

To allocate overhead costs more precisely (than is possible with traditional product costing methods), accountants use activity based overhead rates.

An activity based predetermined overhead rate is found by separating manufacturing overhead costs by activity and developing a predetermined overhead rate for each activity.

Advantages of activity based rate:

Only products using an activity are charged for its use.
Each activity uses the cost driver that best relates its costs to its production activity.

Costs traceable to an activity can be measured rather than allocated. Measurement is more precise than allocation.

Activity based allocation of overhead costs may be compared with plant wide allocation as shown below;

Plant wide overhead allocation

1. All factory overhead costs in a single pool. production activities.
2. Overhead costs allocated to products by using a single predetermined overhead rate activities.

.

Activity based overhead allocation

1. Overhead costs traced to particular service and
2. Service activity costs assigned to production
3. Overhead costs allowed to products by using a Separate predetermined rate for each activity.

Example:

San Juan car rental company has three car rental centres. It has two major cost drivers, number of rental centres and days of car rental. For the coming year, San Juan has made the following estimates:

Centres

	Town Centre	**Ngara**	**JKIA**
Estimated overhead	sh.892,400	736,000	343,000
Estimated rental days	4,400	6,800	200

Required:

calculate an overall (plantwide) overhead rate using rental days as the cost driver.

Calculate activity based overhead rates (one for each centre) using rental days as the cost driver.

Solution:

a.

	Estimated overhead (sh)	Estimated rental days
Town centre	892,400	4,400
Ngara	736,000	6,800
JKIA	343,000	2,000
Total	1,971,400	13,200

Plant wide rate = $\frac{\text{Total estimated overhead}}{\text{Total estimated rental days}}$
= Sh. 149.3

b. Activity based overhead rate

Overhead rate = $\frac{\text{Total estimated overhead}}{\text{total estimated rental days}}$

Town centre = $\frac{\text{sh.892,400}}{\text{4,400 days}}$ =sh.202.80

Ngara = $\frac{\text{sh. 736,000}}{\text{6,800 days}}$ = sh.108.20

JKIA = $\frac{\text{sh. 343,000}}{\text{2,000 days}}$ =sh.171.50

ACTIVITY BASED MANAGEMENT (ABM)

Also referred to as activity based cost management (ABCM). This is used to describe the cost management application of ABC.

To implement ABM, system, only the first three of the four stages of ABC system are required. These are;
Identifying the major activities that take place in an organisation
Assigning costs to cost pools/ cost centres for each activity.
Determining the cost driver for each major activity.

ABM views the business as a set of linked activities that ultimately add value to the customer. It focuses on managing the business on the basis of the activities that make up the organisation. It is based on the premise that activities consume costs. Hence by managing activities costs will be managed in the long run.

The goal of ABM is to enable the customer to be satisfied while making fewer demands on the organisations resources. The measurement of activities is a key role of the management accounting function. In particular activity cost information is useful for prioritising those activities that need to be studied closely so that they can be eliminated or improved. ABM is used in a variety of business applications such as cost reduction, benchmarking, activity based budgeting and performance measurement.

REINFORCING QUESTIONS

QUESTION ONE

Energy costs may include the following items in a company which manufactures and sells products:

1. Maintaining a statutory temperature range in the workplace
2. The operation of a specially humidified materials store
3. Power costs per unit of output
4. Power costs in the movement of raw materials and work in progress
5. Losses from steam pipelines and steam valves
6. Heat losses through windows.

Explain how management may be assisted in the implementation of an energy cost reduction strategy through the application of

a) Zero-base budgeting and
b) Total quality management. Your answers to a) and b) should each refer to any *three* of the energy cost examples given in the question.

ACCA, Information for Control and Design Making.

QUESTION TWO

CALTON LTD
Calton Ltd make and sell a single product. The existing product unit specifications are as follows:

Direct material X:	8 sq metres at £4 per sq metre
Machine time:	0.6 running hours
Machine cost per gross hour:	£40
Selling price:	£100

Calton Ltd require to fulfil orders for 5,000 product units per period. There are no stocks of product units at the beginning or end of the period under review. The stock level of material X remains unchanged throughout the period.

The following additional information affects the costs and revenues.

5% of incoming material from suppliers is scrapped due to poor receipt and storage organisation.

4% of material X input to the machine process is wasted due to processing problems.

Inspection and storage of material X costs 10 pence per sq metre purchased.

Inspection during the production cycle, calibration checks on inspection equipment, vendor rating and other checks cost £25,000 per period.

Production quantity is increased to allow for the downgrading of 12.5% of product units at the final inspection stage. Downgraded units are sold as 'second quality' units at a discount of 30% on the standard selling price.

Production quantity is increased to allow for returns from customers which are replaced free of charge. Returns are due to specification failure and account for 5% of units initially delivered to customers. Replacement units incur a delivery cost of £8 per unit. 80% of the returns from customers are rectified using 0.2 hours of machine running time per unit and are resold as 'third quality' products at a discount of 50% on the standard selling price. The remaining returned units are sold as scrap for £5 per unit.

Product liability and other claims by customers is estimated at 3% of sales revenue from standard product sales.

Machine idle time is 20% of gross machine hours used, i.e. running hours = 80% of gross hours.

Sundry costs of administration, selling and distribution total £60,000 per period.

Calton Ltd is aware of the problem of excess costs and currently spends £20,000 per period in efforts to prevent a number of such problems from occurring.

Calton Ltd is planning a quality management programme, which will increase its excess cost prevention expenditure from £20,000 to £60,000 per period. It is estimated that this will have the following impact.

A reduction in stores losses of material X to 3% of incoming material.

A reduction in the downgrading of product units at inspection to 7.5% of units inspected.

A reduction in material X losses process to 2.5% of input to the machine process.

A reduction in returns of products from customers to 2.5% of units delivered.

A reduction in machine idle time to 12.5% of gross hours used.

A reduction in product liability and other claims to 1% of sales revenue from standard product sales.

A reduction in inspection, calibration, vendor rating and other checks by 40% of the existing figure.

A reduction in sundry administration, selling and distribution costs by 10% of the existing figure.

A reduction in machine running time required per product unit to 0.5 hours.

Required

Prepare summaries showing the calculation of (i) total production units (pre-inspection), (ii) purchases of material X (sq metres), (iii) gross machine hours.

In each case the figures are required for the situation both before and after the implementation of the additional quality management programme, in order that the orders for 5,000 product units may be fulfilled. (13 marks)

Prepare profit an loss accounts for Calton Ltd for the period showing the profit earned both before and after the implementation of the additional quality management programme. (11 marks)

Comment on the relevance of a quality management programme and explain the meaning of the terms internal failure costs, external failure costs, appraisal costs and prevention costs giving examples for each, taken where possible from the information in the question. (11 marks)

(35 marks)

QUESTION TTHREE
TQM AND STANDARD COSTING

'It may be argued that in a total quality environment, variance analysis from a standard costing system is redundant.'
Discuss the validity of this statement. (8 marks)

Using the labour cost as the focus, discuss the differences in the measurement of labour efficiency/effectiveness where (i) total quality management techniques and (ii) standard cost variance analysis are implemented. (7 marks)

(15 marks)

QUESTION FOUR
TRITEX PLC (12/96
Tritex plc produces a number of products which pass through three consecutive processes – Making, Converting and Finishing – before sale to customers.

Until recently, Tritex plc has prepared standard product costs per unit for each product using (a) control standards and (b) current standards. Appendix 1 shows extracts from these standards for product A. These standards plus a detailed variance analysis have been the main focus of control information. The control standards based on industry average performance and the current standards based on the level of performance which it is anticipated should be attainable using the last year's actual performance as the starting point.

The control and current standard product costs per unit incorporate the following specifications.

Process losses (%) are expressed as a percentage of input to each process.

Material requirement is based on the input to the Making process.

Labour requirement is based on the total hours per unit of output from a process.

Variable overhead is absorbed on the basis of net processing hours (i.e. excluding idle time) per unit of output from a process.

Variable overhead is absorbed into all products using an average rate per net processing hour.

Work-in-progress is 100% complete at the end of each process.

Tritex plc has produced amended unit standard process costs which incorporate activity based costing (ABC) and the planned effects of a total quality management (TQM) programme. Appendix 1 shows an extract of such a standard cost for product A.

The ABC/TQM standard costs incorporate the following in their specification.

Zero idle time allowance for labour. Employees will carry out some rework, material handling and maintenance not previously included in the standard labour cost.

The cost driver for all variable overhead in the Making process is the number of steam operations per product unit and overhead is absorbed on this basis. For product A, this should result in an overhead cost reduction of 30% per product unit from the control standard cost. Any residual difference is due to improved work practices.

Excel Ltd. make and sell two products, VG4U and VG2. Both products are manufactured through two consecutive processes – making and packing. Raw materials is input at the commencement of the making process. The following estimated information is available for the period ending 31 March 1995.

(i)

	Making £000	**Packing £000**
Conversion costs:		
Variable	350	280
Fixed	210	140

40% of fixed costs are product specific, the remainder are company fixed costs. Fixed costs will remain unchanged throughout a wide activity range.

(ii) Product information:

	VG4U	VG2
Production time per unit:		
Making (minutes)	5.25	5.25
Packing (minutes)	6	4
Production / sales (units)	5,000	3,000
Selling price per unit (£)	150	180
Direct material cost per unit(3)	30	30

(iii) Conversion costs are absorbed by products using estimated time based rates.

Required:

Using the above information,
Calculate unit cost for each product, analysed as relevant.
Comment on a management suggestion that the production and sale of one of the products should not proceed in the period ending 31 March 1995.
Additional information is gathered for the period ending 31 March 1995 as follows:
The making process consists of two consecutive activities, moulding and trimming. The moulding variable conversion costs are incurred in proportion tot he temperature required in the moulds. The variable trimming conversion costs are incurred in proportion to the consistency of the material when it emerges from the moulds. The variable packing process conversion costs are incurred in proportion to the time required for each product. Packing materials (which are part of the variable packing cost) requirement depends on the complexity of packing specified for each product.

The proportions of product specific conversion costs (variable and fixed) are analysed as follows:
Making process: moulding (60%); trimming (40%)
Packing process: conversion (70%); packing material (30%)

An investigation into the effect of the cost drivers on costs has indicated that the proportions in which the total product specific conversion costs are attributable to VG4U and VG2 as follows:

	VG4U	**VG2**
Temperature (moulding)	2	1
Material consistency (trimming)	2	5
Time (packing)	3	2
Packing complexity	1	3

Company fixed costs are apportioned to products at an overall average rate per product unit bases on the estimated figures.
Required:
Calculate amended unit costs for each product where activity-based costing is used and company fixed costs are apportioned as detailed above.

Comment on the relevance of the amended unit costs in evaluating the management suggestion that one of the products be discontinued in the period ending 31 March 1995.

Management wish to achieve an overall net profit margin of 15% on sales, in the period ending 31 March 1995 in order to meet return on capital targets.
Required :
Explain how target costing may be used in achieving the required return and suggest specific areas of investigation .

CHECK YOUR ANSWERS WITH THOSE GIVEN IN LESSON 10 OF THE MANAGEMENT ACCOUNTING NOTES.

LESSON TEN

Revision Aid

MODEL ANSWERS TO REINFORCING QUESTIONS
CONTENTS

KASNEB SYLLABUS

MODEL ANSWERS TO REINFORCEMENT QUESTIONS IN

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SELECTED CPA PAST PAPERS

PILOT PAPER JULY 2011
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DECEMBER 2013

ANSWERS TO PAST PAPERS

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DECEMBER 2010
DECEMBER 2013

MOCK EXAMINATION PAPER

KASNEB SYLLABUS

OBJECTIVE

To examine candidates' ability to apply modern tools of analysis in the solution of management problems. The paper builds on material covered earlier in Economics; Cost Accounting; Business Finance; Systems Theory Analysis and Design; and Quantitative Techniques.

Content

The Nature of Managerial Decisions. The various decisions that a manager must make: planning, organising, directing and control. The key decisions to be made in the fields of production, marketing, financing and personnel. Evaluating the value of information that a manager needs to make decisions. Value of perfect and imperfect information.

Cost Estimation and Forecasting. Engineering methods, simulation methods and statistical methods. Simple and multiple regression, the statistical properties of regression. Time series models: smoothing and extrapolation, stochastic time series, linear time series models, forecasting with time series models.

Short-term Planning Decisions. Basic cost-volume profit analysis, limitations of CVP analysis, CVP under uncertainty, risk and measures of risk, risk analysis with multiple products, effect of product-mix decisions, learning curves, estimating the learning effect. Marginal costing and its application in analysis of special orders, make or buy decisions, selection of product mix and other similar short-run decisions.

Inventory Control. The cost of holding stock, stock replenishment models, quantity discounts, timing of replenishing orders, models involving shortage costs, stochastic inventory models, Pareto anaylsis, simulation of reorder decisions.

Capital Expenditure Decisions. Thenature of capital investments, costs and returns in financial appraisal, replacement and abandonment decisions. Treatment of uncertainty and risk. The use of computer spread sheets.

Sequential Decisions. Types of sequential process, representing activities by networks, analysis of activity times, types of floats, networks in planning and project control, uncertainty networks. Decision trees and their use in the analysis of sequential decisions.

Resource Allocation Decisions. The use of linear programming in resource allocation, slack and surplus variables, optimality analysis, treatment of integer variables amd multiple objectives, Non-linear programming models and their application in management.

Routing and Transportation Decision. The basic transportation model, extensions and applications. Transportation with fixed costs. Assignment models and their application in management. Dynamic programming including extensions to probabilistic situations.

Strategic and Transportation Decisions. Classification of games: Zero and non-zero sum games, prisoners dilemma games, battle of sexes games, n-person games. The concept of Nash equilibrium. Applications of game theory to management: collective bargaining, negotiations, tendering, diversification and retrenchment. The use of markov analysis in the formulation of strategic moves.

Performance Evaluation Decisions. Responsibility accounting and organisation design, interdependent responsibility centres, transfer pricing and risk-sharing in decentralizing firms. Budgets and participation, controllable and uncontrollable costs. The design of managerial incentive shemes. The agency problem

MODEL ANSWERS TO REINFORCING QUESTIONS

LESSON 1

QUESTION ONE

Petrol regular
premium
regular extra (at least 50% premium)

Value properly adjusted	50% premium	
	50% regular	minimum cost
Value out of adjustment	60% premium	
	40% regular	quantity required
	100,000 litres once value is adjusted	

Per litre

	Sh	
Cost premium	3.20	
Cost regular	3.00	
Cost checking value	800.00	
Cost adjusting the value	400.00	
		Probability
Event Value in adjustment		0.7
Value out of adjustment		0.3

(a) Expected cost of checking the value of adjusting if necessary

	Cost	Prob.	
Value OK	800	0.7	560
Value needs adjustment	1200	0.3	<u>360</u>
			Sh 920

OR

Cost of checking		=	800
+ Cost of adjustment	0.3 x 400	=	120
	800+ 120	=	Sh 920

(b) Value out of adjustment Prob. = 0.3

Cost/litre if value OK $= \frac{3.20 + 3.00}{2} =$ Sh 3.10/litre

Cost/litre if value not OK = 0.6 x 3.20 + 0.4 x 3.0

= 1.92 + 1.20

= Sh 3.12/litre

Cost of 10,000 litres if value OK	3.10 x	100,000	=	Sh 310,000
Cost of 10,000 litres if value not OK	3.12 x	100,000	=	Sh 312,000

Difference = Sh 2,000

The probability is 0.3

Expected cost = 2,000 x 0.3 = Sh 600

(c) The extra cost is Sh 2,000

Let the Probability be p

2010 x p = 800 + (400 x p)

p = 0.5

(d) Comment on the result (a) and (b) above

It is not worth checking the value

QUESTION TWO

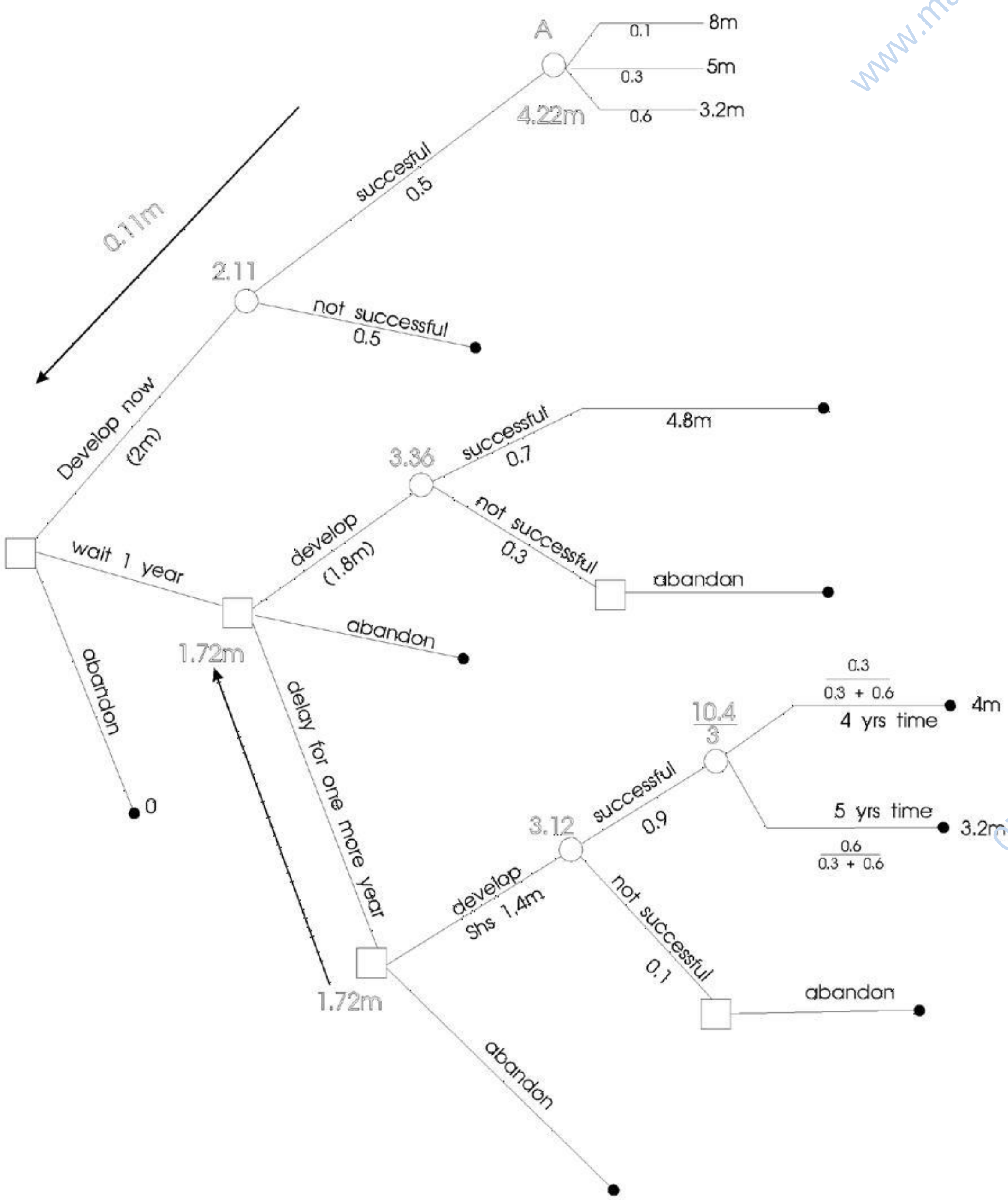
Chakula Engineering Company Limited (CECL)

	NPV pay off
10% new mixer reaches the Kenya market in a year's time	8m
30% new mixer reaches the Kenya market in 4 year's time	5m
60% new mixer reaches the Kenya market in 5 year's time	3.2m

P.V.

a. Making and selling the liquidizer Shs 8m if market develops
b. Making and selling the liquidizer Shs 4m if market is developed in 4 year's time
c. Making and selling the liquidizer Shs 3.2m if market is developed in 5 year's time

Immediate Development Cost Shs 2m



The probability of mixer coming into market in 4 years time = 30%
The probability of mixer coming into market in 5 years time = 60%
Total probability = 0.3 + 0.6 = 0.9

Pay-off Develop now = £ 0.11m
Wait for one year Pay-off = (3.36 - 1.8) = £ 1.56m
Develop after 2 years = £ 1.72

Best decision: Wait for one year, followed by a delay for one more year. Expected pay-off = Shs 1.72m

QUESTION THREE

(a)

Demand	Probability	Production Runs '000'			
		50	75	100	125
50	0.1	7,400	2,450	(3,600)	(8,550)
75	0.4	7,400	14,950	8,900	3,950
100	0.3	7,400	14,950	21,400	16,450
125	0.2	7,400	14,950	21,400	28,950
EMV		**7,400**	**13,700**	**14,900**	**11,450**

Profit (profit payoff) = (selling price x Quantity) – Total Costs

Working '000'
EMV at 50,000 Productions
= 7,400 x 1 = 7,400

EMV @ 75,000 production

= (-3,600 x 0.1) + (8,900 x 0.4) + (21,400 x 0.5) =

13,900 EMV @ 100,000 production

= (-3,600 x 0.1) + (8,900 x 0.4) + (21,400 x 0.5) = 13,900

EMV @ 125,000 Production

= (-8,550 x 0.1) + (3,950 x 0.4) + (16,450 x 0.3) + (28,950 x 0.2)
= 11,450

Decision
Produce at a production Run of 100,000 units because it yields the highest expected monetary value of Shs.13.9 million.

(b) Steps

Identify objectives
Search for alternative courses of Action
Gather data about alternatives
Select Alternative course of Action
Implement the decision
Compare actual and planned outcomes
Respond to divergencies from plan.

LESSON 2

QUESTION ONE

a. Expressing in Year 5 terms

Year 1	Sales	$200,000(1.10)^4$	292,820
	Costs	$100,000(1.07)^4$	131,080
		= Contribution	£ 161,740
Year 2	Sales	$260,000(1.10)^3$	346,060
	Costs	$132,000(1.07)^3$	161,706
		= Contribution	£ 184,354
Year 3	Sales	$300,000(1.10)^2$	363,000
	Costs	$156,000(1.07)^2$	161,706
		= Contribution	£ 184,354
Year 4	Sales	408,000(1.10)	448,800
	Costs	188,000(1.07)	201,160
		= Contribution	£ 184,354

Summary ('000s)

Output	**Contribution (to nearest whole number)**
150	162
180	184
200	184
230	248

x	y	x^2	xy
150	162	22,500	24,300
180	184	32,400	33,120
200	184	40,000	36,800
230	248	52,900	57,040
760	778	147,800	151,260

$$b = \frac{n\Sigma xy - \Sigma x\Sigma y \; n\Sigma}{x^2 - (\Sigma x)^2}$$

$$= \frac{4 \times 151,260 - 760 \times 778}{4 \times 147,800 - 760^2}$$

$$= \frac{13,760}{13,600} = 1.012$$

$$a = \frac{\Sigma y - b\Sigma x}{n}$$

$$= \frac{778 - 1.012 \times 760}{4} = 2.22$$

$$\therefore y = 2.22 + 1.012x$$

As the planned output is 260,000 the contribution = 2.22 + 1.012(260)
= 265.34 or £265.340

95% confidence interval for the point estimate for 260,000 units is:

$$265.34 \pm 4.303 \times 14.5$$
$$= 265.34 \pm 62.39$$

Upper limit £327,730
Lower limit £202,950

These are the limits within which we can be 95% certain that the actual value of contribution will be.

The limits are extremely broad because single point estimates (as opposed to the whole regression line) are relatively inaccurate especially in this case where there are only 4 readings from which to calculate the regression coefficients.

c. The regression line calculated by least squares is the line of best fit calculated mathematically. It utilises all the values and is statistically valid and can be used to show an average value of forecast provided that:

i. there is a genuine linear relationship between the dependent and independent variables;
ii. Conditions in the past continue into the future;

Extrapolation is not carried out too far into the future or too far beyond the base value.

QUESTION TWO

The experience curve states that the cost of production will decrease as greater experience is gained with a product or process. Although cost reduction will be a function of the learning curve the experience curve covers a greater number of areas such as product innovation and management skills. The experience curve can be used as a means of obtaining strategic advantage by forecasting cost reductions and consequently the selling price reductions of the competitors. Early experience with a new product can provide a means of conferring an unbeatable lead over competitors. Through the experience curve the leading competitor should be able to reduce its selling price for the product which should further increase its volume and market share and eventually force some lagging competitors out of the industry. Exploiting the principles of the experience curve can ensure that a firm has the lowest costs in the industry. It is therefore important that managers are aware of their organization’s position on the experience curve at the strategic planning stage.

By exploiting the cost reductions of the experience curve a firm can lower its selling prices and thus extend a products life cycle by stimulating demand from existing customers and/or enticing a new customers by price reductions. Furthermore, knowledge of an organization’s experience curve relative to that of its competitors will allow it to maximize market share and prolong the life cycle of its products or services.

A favorable position on the experience curve via product innovation and management skills will enable a firm to take appropriate steps to ensure that its products are competitive and prolong their profitable lives. In particular, it will enable managers to modify existing products and introduce new products that ensure that the organization is at the forefront of product development. This will help to delay the decline in demand for its products and prolong their life cycles. Also the experience of managers will enable them to react to environmental and technological changes so that the organization remains competitive. It will thus be able to respond effectively to changes in demand for its products and take steps to prolong their life cycles.

QUESTION THREE

The first stage is to convert all costs to Year 5 basis. The calculations are as follows:

	Year 1 (Sh.000)	Year 2 (Sh.000)	Year 3 (Sh.000)	Year 4 (Sh.000)
Raw materials, Skilled labor, Unskilled labor }	$242(1.2)^4$	$344(1.2)^3$	$461(1.2)^2$	477(1.2)
Factory overheads	$168(1.15)^3(1.2)$	$206(1.15)^2(1.2)$	246(1.15) (1.2)	265(1.2)
Power	$25(1.1)\ (1.25)^3$	$33(1.25)^3$	$47(1.25)^2$	44(1.25)
Raw materials, Skilled labor, Unskilled labor }	500.94	595.12	663.84	572.4
Factory overheads	306.432	326.304	339.48	318
Power	53.625	64.35	73.32	55
Total (2012 prices)	861000	986000	1077000	945000
Output (units)	160000	190000	220100	180000

The equation $Y = a + bx$ is calculated from the above schedule of total production costs (2012 prices) and output. The calculations are as follows:

Output In units (000) X	Total cost (Sh.000) Y	X^2	XY
160	861	25600	137760
190	986	36100	187340
220	1077	48400	236940
180	945	32400	170100
$\sum x = 750$	$\sum y = 3869$	$\sum x^2 = 142500$	$\sum xy = 732140$

We now solve the following simultaneous equations:

$$\sum y = Na + b\sum x$$
$$\sum xy = \sum xa + b\sum x^2$$

Therefore

$$= 4\,a + 750\,b \qquad (1)$$
$$732140 = 750\,a + 142500\,b \qquad (2)$$

Multiply equation (1) by 190(142500/750) and equation (2) by 1.Then equation (1) becomes

$$735110 = 760\,a + 142500\,b \qquad (3)$$

Subtract equation (2) from equation (3):

$$2970 = 10\,a$$
$$a = 297$$

Substitute for a in equation (1)

$$3869 = 4 \times 297 + 750\,b$$
$$2681 = 750\,b$$
$$b = 3.57$$

The relationship between total production costs and volume for 2012 is:

Y = $297000 + 3.57x

Where y = total production costs (at 2012 price) and x = output level.

(b) General company overheads will still continue whether or not product LT is produced. Therefore the output of LT will not affect general production overheads. Consequently, the regression equation should be calculated from cost data that includes general company overheads. General company overheads will not increase with increment in output of product LT. hence a short time decisions and cost control should focus on those costs that are relevant to production of LTs. Common and unavoidable general fixed costs are not relevant tot the production of LT, and should not be included in the regression equation.

LESSON 3

QUESTION ONE

a. Expected profit

	Football	Cricket
Volume (units)	40,000	300,000
	£	£
Contribution/unit	50	100
Total contribution	2,000,000	3,000,000
Less Fixed Costs	1,050,000	1,950,000
= Profit	£ 950,000	£ 1,050,000

b. Sensitivity analysis for volume, price, variable cost per unit and fixed costs. (Critical value £200,000 profit).

	Football Value	**Football** % Change	**Cricket** Value	**Cricket** % Change
Volume				
$\frac{1,050,000 + 200,000}{50}$	25,000			
$\frac{40,000 - 25,000}{40,000}$		37.5		
$\frac{1,950,000 + 200,000}{100}$			21,500	
$\frac{30,000 - 21,500}{30,000}$				28.3
Price				
$\frac{1,250,000 + 3,200,000}{40,000}$	£111.25			
$\frac{(130 - 111.25)}{130}$		14.4		
$\frac{2,150,000 + 3,000,000}{30,000}$			£171.67	
$\frac{(200 - 171.67)}{200}$				14.2
Variable cost/unit				
$\frac{5,200,000 - 1,250,000}{40,000}$	£98.75			
$\frac{(80 - 98.75)}{80}$		23.4		
$\frac{6,000,000 + 2,150,000}{30,000}$			£128.33	
$\frac{(100) - 128.33)}{100}$				28.3
Fixed Costs	£1.8M			
		71.4		
			£2.8M	
				43.6

c. Although the cricket game has the higher expected profit it has the higher risk in that smaller changes in price and volume cause its profit to drop to the critical value.

The most sensitive factors for the 2 products are:

	Football	Cricket
Most sensitive	Price	Price
	Variable cost	Volume & variable cost
	Volume	Volume
	Fixed cost	Fixed cost

Least sensitive

Other factors which need to be considered are:

1. the quality of the estimates;
2. reaction of competitors;
3. do these products fit in with the existing business?;
4. will demand increase/decrease?.

QUESTION TWO

b. i.Where demand in year to 31 July 2014 is 3,600 units;

Selling price/unit year to 31.7.91 £	year to 31.7.91 £000	Cash inflows year to 31.7.92 £000	Two year total £000	Year to 31.7.91 £000	Cash outflows year to 31.7.92 £000	Two year total £000	Net cash inflow two year total £000
20	40	80	120	32	32	64	56
30	48	96	144	32	32	64	80
40	48	112	160	32	32	64	96
50	55	116	171	32	32	64	107
60	60	120	180	32	32	64	116
70	49	132	181	32	32	64	117
80	32	144	176	32	32	64	112

In this situation the initial launch price should be set at £70 per unit in order to maximise the net benefit to Sniwe plc over the two year period.

ii. Where demand in year to 31 July 2014 is 1,000 units:

Selling price/unit year to 31.7.91 £	*year to 31.7.91 £000*	*Cash inflows year to 31.7.92 £000*	*Two year total £000*	*Year to 31.7.91 £000*	*Cash outflows year to 31.7.92 £000*	*Two year total £000*	*Net cash inflow two year total £000*
20	40	40	80	32	16	48	32
30	48	40	88	32	9.6	41.6	46.4
40	48	40	88	32	3.2	35.2	52.8
50	55	40	95	32	1.6	33.6	61.4
60	60	40	100	32	nil	32	68
70	49	40	89	32	nil	32	57

80	32	40	72	32	nil	32	40

In this situation the initial launch price should be set at £60 per unit in order to maximise the net benefit to Sniwe plc over the two year period.

Note:

The key to the choice of launch price is the fact that the opportunity cost/shadow price of the unsold stock per unit at the end of year 1 is £40 (sales price) where year 2 sales are greater than production capacity and £16 (variable cost) where year 2 sales are less than production.

b. We can see from the tabulation in (a.) (i.) that no spare capacity exists and all production in the year to 31 July 2014 will be sold.

The tabulation in (a) (ii) shows that nil production is required in the year to 31 July 2014 where the initial launch price was £60, £70 or £80. We may check for unsold stock at 31 July 2014 as follows:

Initial launch price	**£60**	**£70**	**£80**
Production in year to 31.7.91 (units)	2,000	2,000	2,000
Sales in year to 31.7.91 (units)	1,000	700	400
Stock on hand at 31.7.91 (units)	1,000	1,000	1,000
Unsold stock at 31.7.92 (units)	1,000	1,000	1,000
	nil	300	600

We require to calculate the selling price of the unsold stock so that the overall net benefit to Sniwe plc will be more than the £68,000 shown in the tabulation in (a.) (ii).

Let X = selling price per unit of unsold stock.

Where launch price = £70:

$$300 \times X = 68{,}000 - 57{,}000$$
$$X = £36.67$$

i.e. a disposal price of the unsold stock would have to be in excess of £46.67 before a change of launch price of £70 would maximise the net benefit to Sniwe plc over the two year period.

Where launch price = £80:

$$600 \times X = 68{,}000 - 40{,}000$$
$$X = £46.67$$

i.e. the disposal price of the unsold stock would have to be in excess of £46.67 before a change of launch price of £70 would maximise the net benefit to Sniwe plc over the two year period.

c. Where demand in year ending 31.7.92 is 1,000 units then there is spare production capacity throughout the year. If this spare capacity can be used for some other product the resulting contribution should be brought into the calculations.

An alternative possibility might be to go for a lower price in the year to 31.7.91 in the hope that this might lead to a larger market share to 31.7.92.

QUESTION THREE

(i) The first step is to calculate the average contributions margin at the assumed mix:

	Units	Revenue (sh)	Contribution margin (sh)
Product A	30,000	150,000	60,000
Product B	40,000	100,000	30,000
		250,000	90,000

Average contribution margin = $\frac{90,000}{250,000} \times 100 = 36\%$

The breakeven sales volume is obtained by dividing this average into the fixed costs i.e.

Sh 72,000 | 0.36 = Sh 200,000

Units of A ; $\left(\frac{150}{250} \times 200,000\right)$ | Sh 5 = 24,000 units

Units of B; $\left(\frac{100}{250} \times 200,000\right)$ | Sh 2.50 = 32,000 units

(ii) Margin of safety:

	Sh
Actual sales:	
Product A: 30,000 x Sh 5	150,000
Product B: 40,000 x Sh 2.50	100,000
Total sales	250,000
Breakeven volume	250,000
Margin of safety (difference)	50,000

Anticipated profit = Sh90,000 – 72,000 = Sh 18,000
= 50,000 x 0.36 = Sh 18,000

Net volume and contribution margin:

	Units	Revenue (Sh)	Contribution margin (Sh)
Product A	40,000	200,000	80,000
Product B	32,000	80,000	24,000
		280,000	104,000
Fixed costs			81,700
Profit			22,300

Decision: the proposal should be accepted as it results in a higher profit by an amount Sh 4,300 (i.e. 22,300 – 18,000)

Average contribution margin at the new mix is:

$\frac{\text{Sh } 104,000}{\text{Sh } 280,000} \times 100 = 37.14\%$

∴ Breakeven sales volume = Sh (72,000 + 9700) | 37.14%
= Sh 220,000

The two main assumptions are that as volume drops, sales of the two products will drop proportionally and fixed costs will remain at Sh 81700. Either of these assumptions can be challenged.

LESSON 4

QUESTION ONE

(a) **Advantages of Just-In-Time (JIT)**

Leads to substantial savings in stockholding
costs Elimination of waste
Savings in factory and warehouse space, which can be used for other profitable
activities Reduction in obsolete stocks
Considerable reduction in paper work arising from a reduction in purchasing stock
and accounting transaction or procedures.

Disadvantages

Additional investment costs in new machinery, changes in plant layout and goods services, thus affecting cash flow of the organization
Difficulty in predicting daily or weekly demand, which is a key feature of the JIT philosophy. Increased risk due to the greater probability of stock out costs arising from strikes, or other unforeseen circumstances, that restrict production or supplies.

(b)

Safety stock	Stock out	Stock out cost @ shs.100	Probability	Expected Cost (Shs)	Total (Shs)
500	0	0	0	0	0
400	100	10,000	0.04	400	400
300	200	20,000	0.04	800	
	100	10,000	0.07	700	1,500
200	300	30,000	0.04	1,200	
	200	20,000	0.07	1,400	
	100	10,000	0.10	1,000	3,600
100	400	40,000	0.04	1,600	
	300	30,000	0.07	2,100	
	200	20,000	0.10	2,000	
	100	10,000	0.13	1,300	7,000
0	500	50,000	0.04	2,000	
	400	40,000	0.07	2,800	
	300	30,000	0.10	3,000	
	200	20,000	0.13	2,600	
	100	10,000	0.16	1,600	12,000

SUMMARY

Safety Stock	Stock out cost	Holding Cost @ sh.10	Total Cost
0	12,000	0	12,000
100	7,000	1,000	8,000
200	3,600	2,000	5,600
300	1,500	3,000	4,500
400	400	4,000	4,400
500	0	5,000	5,000

The optional safety stock is <u>400 units</u>

P (being out of stock) i.e. at optimal safety stock of 400 units = 0.04

QUESTION TWO

(a) Annual Demand = 200,000 hooks

Cost per Order

	Hours	Shs.	
Cost per Ship Chartered		20,000	
Hours required to place an order	5		
Hours required to supervise on loading	4		
Total hours	9		
Labour cost 9 x 200		1,800	
Overhead cost 9 x 160		1,440	
Total		23,240	
Cost per unit of average Inventory			
Hours required per hook per day	½		
Labour costs (½ x 200)		100	
Overhead cost (½ x 160)		80	180.00
Cost of capital filed up in inventory variable			
Costs expected at the time of purchase			
Purchase price		400.00	
Shipping cost		40.00	
Equipment rental $\frac{1}{25}$ x Sh.100		4.00	
Hours required: on loading	$\frac{1}{25}$		
On storage	$\frac{1}{40}$		
Total	$\frac{13}{200}$		
Labour cost $\frac{13}{200}$ x 200		13.00	
Overhead cost $\frac{13}{200}$ x 160		10.40	
		467.40	
Cost of capital 20% x 467.40			93.48
Total cost per unit			273.48

$$EOQ = \sqrt{\frac{2x200,000 \text{ x } 23,240}{273.48}} = 5,830 \text{ hooks}$$

$$\text{Reorder Level} = \frac{DL}{360} = 200,000 \text{ x } \frac{1}{52} = 3,846 \text{ hooks}$$

Original decision order size is 5,830
Results of optimal decision, given alternative parameter
(a new rate = 16,000,000 + (Shs.240 x Total Labour hours)

The Shs.1,600,000 is irrelevant
Annual demand = 200,000 hooks

Actual cost per order = 23,240 + 9(240 – 160) =
23,960 Actual cost per unit of inventory
= 273.48 + ½(240 – 160) + 0.2(240 – 160) ($\frac{13}{200}$) = 314.52

$$EOQ = \sqrt{\frac{2Dco}{Ch}} = \sqrt{\frac{2 \times 200{,}000 \times 23{,}960}{314.52}} = 5{,}520 \text{ hooks}$$

$$TRC = \frac{D}{Q} Co + \frac{O}{2} Ch$$

Optimal

$$= \left(\frac{200{,}000}{5{,}520} \times 23{,}960\right) + \left(\frac{5{,}520}{2} \times 314.52 = 1{,}736191.142\right) = 1{,}736{,}191.142$$

Actual results, given original decision

$$\text{TRC Actual} \left(= \frac{200{,}000}{5{,}830} \times 23{,}960\right) \left(+ \frac{2{,}830}{2} \times 314.52\right) = 1{,}738{,}781.203$$

∴Cost prediction error = 1,738,781.203 – 1,736,191.142
= Shs.**2,590.061**

QUESTION THREE

a) No. of fish purchased

Purchases	No. of days	Probability	Cumulative	RN
100	30	0.1	0.1	0
200	60	0.2	0.3	1 – 2
300	90	0.3	0.6	3 – 5
400	90	0.3	0.9	6 – 8
500	30	0.1	1.0	9

Number of fish sold to consumers

Demand	**No. of days**	**Probability**	**Cumulative**	**RN**
100	45	0.15	0.15	00 –14
200	60	0.2	0.35	15– 34
300	90	0.3	0.65	35– 64
400	75	0.25	0.9	65– 89
500	30	0.1	1.	90– 99

Day	RN	SS	RN	DD	Sale	Balance c/f	Shortfall	Profit
1	5	300	73	400	300	-	100	4,000
2	4	300	23	200	200	100	-	2,000
3	7	400	09	100	100	300	-	(6,000)
4	7	400	51	300	300	100	-	(2,000)
5	4	300	83	400	400	-	-	6,000
6	6	400	81	400	400	-	-	8,000
7	3	300	20	200	200	100	-	2,000
8	9	500	31	200	200	300	-	(4,000)
Total profits for 8 days								**10,000**

Workings

Days	1	2	3	4	5	6	7	8
Sales	18,000	12,000	6,000	18,000	20,000	24,000	12,000	12,000
Less cost of sales	(12,000)	(10,000)	(12,000)	(20,000)	(14,000)	(16,00)	(10,000)	(16,000)
Less deficit cost	(2,000)	-	-	-	-	-	-	-
Net profit	4,000	2,000	(6,000)	(2,000)	6,000	8,000	2,000	(4,000)

b) Average profits = $\frac{10,000}{8}$ = Shs.1,250

LESSON 5

QUESTION ONE

(a) Produce (a) of A (b) units of B, (c) units of C (d) units of product D and (e) units of products E each week

Calculate the unit contribution of each product.

A: unit contribution is 40 – {(2.10 x 6) + (3.0 x 1.0) + (1.3 x 3) + 8.0 x 0.5)}
= Shs.16.50 per unit.

B: unit contribution is 42 – {(2.10 x 6.5) + (3.0 x 0.75) + (1.3 x 4.5) + (8 x 0.5)}
= Shs.16.25 per unit

C: unit contribution is 44 – {(2.10 x 6.10) + (3.0 x 1.25) + (1.3 x 6) + (8 x 0.5)}
= Shs.15.64 per unit

D: unit contribution is 48 – {(2.10 x 6.1) + (3.0 x 1) + (1.3 x 6) + (8 x 0.75)}
= Shs.18.39 units

E: unit contribution is 52 – {(2.10 x 6.4) + (3.0 x 1) + (1.3 x 4.5) + (8.0 x 1)}
= Shs.21.71 per unit

Maximize total weekly contribution, Shs. P where;

P = 16.5a + 16.25b + 15.64c + 18.39d + 21.71e Shs/Week

Subject to:
Materials: 6.0a + 6.5b + 6.1c + 6.1d + 6.4e ≤ 35,000 kg/week
Forming: 1.0a + 0.75b + 1.25c + 1.0d + 1.0e ≤ 6,000 hours/week
Firing: 3.0a + 4.5b + 6.0c + 6.0d + 4.5e ≤ 30,000 hours/week
Packing: 0.5a + 0.5b + 0.5c + 0.75d + 1.0e ≤ 4,000 kg/week
Non-negativity: a, b, c, d, e, ≥ 0

(b) (i) The optimum weekly production plan is to produce 3,357 units of product A,2,321 units of product E and none of B, C or D. The resulting maximum weekly contribution is Kshs.105,791.

(ii) There is spare capacity of 321 hours per week on the forming process and 9,482 hours per week on the firing process. All raw materials and all packing time are used up. Raw materials and packing time are the limiting constraints in the problem.

(iii) The shadow price is the amount, which would be added to the value of the total weekly contribution if one extra unit of a limiting resource were made available that:

No additional costs were incurred.
The resource remains limiting

Alternatively the shadow price is the amount by which the total weekly contribution would fall if the provision of a limiting resource was reduced by one unit.

From the table, we can see the shadow price for raw materials is Ksh.2.02 per kilogram and for packing time is Kshs.8.81 per hour. One additional kilogram of raw material will generate an extra Kshs.2.20 of contributions, subject to the conditions above. One extra hour of packing time will, similarly generate additional shs.8.81 of contribution.

The additional product would also have to be made at the expense of one or both of the other products, since all raw materials and packing time are currently used.

Unit contribution of the new product

$= 50 - \{(2.1 \times 1.6) + (3.0 \times 1) + (1.3 \times 5) + (8 \times 1)\}$
$= \underline{\text{Kshs.19.9}}$

If one unit of this new product was made, the provision of raw materials for the other two products would effectively be reduced by 6 kilogrammes, this would reduce the current total contribution by 6 x Kshs.2.02 = Kshs.12.12. Similarly, the available packing time would be reduced by 1 hour, this reduces the total contributions by Kshs.8.81. The total reduction is the weekly contribution which would be:

Kshs.12.12 + Kshs.8.81 = <u>Kshs.20.93</u>

The gain from one unit of the new product is shs.19.90 therefore, if one unit of the new product is made, there will be a net loss of
Shs.19.90 – Shs.20.93 = Shs.1.30. the proposition is not worthwhile.

QUESTION TWO

How can the transportation algorithm be modified to maximize rather than minimize?

Instead of minimizing the positive unit costs of all the cells, calculate the unit profits, make them negative and put these in each cell. Use the transportation algorithm as usual to minimize these negative profits.

Alternatively, load the cells with the largest profits (instead of smaller costs) to give an initial allocation. Test the empty cells as usual, but use any cell which has positive shadow price. If all the shadow prices are negative or zero, that allocation gives the maximum profit.

Factories P_1 P_2 P_3 supply outlets S_1 S_2 S_3 & S_4

The contribution per desk	=	selling price – at shop	variable cost – at the factory	factory outlet transport costs

e.g. the contribution per desk
Supplied from factory P = 2300 – 1500 – 220 = Sh.<u>580</u> to outlet S

The matrix for contribution is given below:

	S_1	S_2	S_3	S_4
P_1	580	610	530	600
P_2	510	600	520	570
P_3	540	650	490	660

The total demand from the four outlets is 850 + 640 + 380 + 230 = 2,100 desks.
The total supply from the three plants is: 625 + 825 + 450 = 1,900 desks.
There is therefore a need for a dummy factory to take up the 200 shortfall.
The transportation tableu is as follows:

	TO / FROM	K_1 = 58 S_1	K_2 = 67 S_2	K_3 = 59 S_3	K_4 =68 S_4	Total Capacity	
R_1 = 0	P_1	625 [58]	[61] -6	[53] -6	[60] -8	625	~~1~~ ~~1~~ 3 3
R_2 = -7	P_2	25 [51]	420 [60]	380 [52]	-4 [57]	825	38881
R_3 = -2	P_3	-2 [54]	220 [65]	-8 [49]	230 [66]	450	111
R_4= -58	Dummy	200 [0]	[0] -9	[0] -1	[0] -10	200	00000
Total Demand		850	640	380	230	2,100	
		~~4~~, ~~4~~, 7, 51,51	4, 4, 4, 60	1,1,1, 52, 52	6		

Note

The initial solution is determined by use of VAM.
The contributions are divided by 10 simplify the computations. The mode is used to solve for optimality.

Note

m + n – 1 = 7
No of filled cells = 7

The problem is not degenerate.

All the shadow prices are negative, therefore any change would reduce the contribution. This is thus the optimal solution. The optimal allocation is:

FROM	TO	Units	Contribution per unit	Total contribution
		Sh.	Sh.	Sh.
P_1	S_1	625	580	362,500
P_2	S_1	25	510	12,750
P_2	S_2	420	600	252,000
P_2	S_3	380	520	197,600
P_5	S_2	220	650	143,000

P_3	S_4	230	660	151,800
Dummy	S_1	200	0	0
Total contribution				**1,119,650**

QUESTION ONE

Standard costing variances should not be viewed in isolation because they may be inter-related, a variance in one costs might have caused a variance in another cost. Some examples of possible inter-relationship are:

Material price, material usage and efficiency variances.

Cheaper materials may produce a favourable material price variance but may be more difficult to process. The difficulties may lead to adverse material usage and efficiency variances.

Labour rate and efficiency

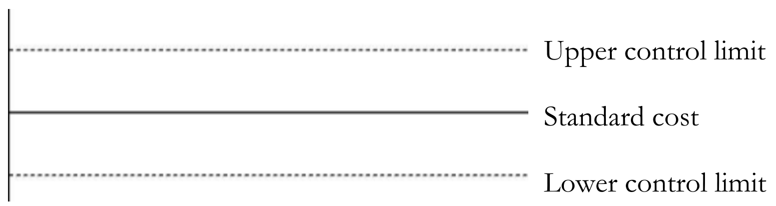
If a more highly skilled employee is used at a higher rate of pay this could result in an adverse labour rate variance. However, a favourable efficiency may also arise and therefore the two are interrelated. The case of a less skilled employee at a lower rate of pay is similarly true.

Sales price and sales volume

A reduction in sales price might stimulate sales volume so that the resulting adverse sales price variance and favourable sales volume variances are interrelated.

A number of factors should be considered in deciding whether or not to investigate a variance. Its significance

Management might set control limits for variances. If a recorded variance falls outside these control limits, then its deemed worthy of investigation. The control limits may be illustrated on a variance control chart.



The control limit may be set by a rule of the thumb or by use of statistical methods.

Cost and benefits of the investigation

Management must use their experience to judge the likely cost of an investigation and the benefit, which will arise if the investigation is successful in correcting the variance. An investigation can only be justified by its benefits exceeding its costs.

Controllability of the variance

The cause of some variances might be uncontrollable and therefore an investigation is not worthwhile. An example is where a price variance was due to fluctuations in market prices, which are out of the management's control.

The type of standard set

Some types of standards will often given rise to a variance, which need not necessarily be investigated. E.g. an ideal efficiency standard will almost always lead to adverse variances.

Your initial reaction might be that no variance should occur in a TQM environment because the organisation should be getting it right first time. However, do not forget the following:

The organisation may get right first time from its own point of view and yet find that variances arise due to factors beyond its control, for example, a world-wide pay rise, a change in government policy and so on.
TQM is also about continuous improvement. Favourable variances should therefore be the norm.

Traditional variance analysis can be unhelpful and potentially misleading in the modern organisation and can make managers focus their attention in the wrong issues. For example:
Adverse efficiency variances are regarded as a bad thing, which means that managers try to prevent idle time and to keep up production. Action to eliminate idle time could result in the manufacture of unwanted products that must be held in store and might eventually be scrapped. Efficiency variances could focus at management attention on the wrong problems or give rise to new problems.

In a JIT environment, the key issues with materials purchasing are supplier reliability, materials quality, and delivery in small order quantities. Purchasing managers should not be shopping around every month looking for the cheapest price. Many JIT systems depend on long term contractual links with suppliers, which means that material price variances are not relevant for managerial control purposes.

QUESTION TWO

Material		£
X	60 Kg x £2.00	120
Y	40 Kg x £1.00	40
Z	100 Kg x £1.40	140
	200	300

∴ Average standard cost per kg of material input = $\frac{£300}{200} = £1.50/\text{Kg}$

Direct material total variance

Material	Standard material cost for 1980 Kg output		Actual Material cost	(£)	Variance (£)
X	£ 120 x 11	1320	700 x £ 1.80	1260	60 (F)
Y	£ 40 x 11	440	440 x £ 1.10	484	44 (A)
Z	£ 140 x 11	1540	1120 x £ 1.30	1456	84 (F)
		3300		3200	100 (F)

Direct material price variance

	Standard price per Kg (£)	Actual price per Kg (£)	Difference (£)	Actual quantity	Variance (£)
X	2.0	1.8	0.2 (F)	700	140 (F)
Y	1.0	1.1	0.1 (A)	440	44 (A)
Z	1.4	1.3	0.1 (F)	1120	112 (F)
					208 (F)

Direct material usage variance

	Standard usage for Variance 1980 Kg output (Kg)		Actual Usage (Kg)	Difference (Kg)	Standard Price (£)	(£)
X	60 x 11	660	700	40 (A)	2.0	80 (A)
Y	40 x 11	440	440	-	1.0	-
Z	100 x 11	1100	1120	20 (A)	1.4	28 (A)

108 (A)

Direct material mix variance

		Actual quantity in standard mix (Kg)	Actual mix (Kg)	Difference (Kg)	At Standard price (£)	Variance (£)
X	(6)	678	700	22 (A)	2.0	44 (A)
Y	(4)	452	440	12 (F)	1.0	12 (F)
Z	(10)	1130	1120	10 (F)	1.4	14 (F)
		2260	2260			18 (A)

Direct material yield variance

	Kg
1980 Kg of output required	2260
but should have required (x 200/180)	2200
Yield variance in Kg	60 (A)
at average standard cost per kg	x £1.5
yield variance	£ 90 (A)

Direct labour efficiency variance

Output per hour for 10 employees	= 40 Kg of product
∴ Standard hours per kg	= 0.25 hours
∴ Standard hours for 1980 Kg	= 1980 x 0.25 hr
	= 495 hrs
Actual hours (45 x 10 employees)	= 450
Variance in hours	45 hrs (F)
At standard per hour	x £4
Variance	£ 180 (F)

QUESTION THREE

Sales quantity variance

Product	Actual sales in std mix (units)	Budgeted sales (units)	Difference (units)	Standard profit per unit (£)	Variance (£)
Dalek	750	500	250 (F)	2	500 (F)
Yeti	450	300	150 (F)	3	450 (F)
Cyberman	300	200	100 (F)	4	400 (F)
	1500	1000	500 (F)		1350 (F)

Sales mix variance

Product	Actual sales in actual mix (units)	Actual sales std mix (units)	difference (units)	Standard profit per unit (£)	Variance (£)
Dalek	700	750 (50%)	50 (A)	2	100 (A)
Yeti	300	450 (30%)	150 (A)	3	450 (A)
Cyberman	500	300 (20%)	200 (F)	4	800 (F)
	1500	1500	0		250 (F)

Total sales volume variance

Volume variance = quantity variance + mix variance

Dalek	£ 500 (F)	+ £100 (A)	= £400 (F)
Yeti	£ 450 (F)	+ £450 (A)	= 0
Cyberman	£ 400 (F)	+ £800 (F)	= £1200 (F)
			£1600 (F)

QUESTION FOUR

Price variance = (actual price – standard price) actual quantity

Note:

The quantity used in this case is that purchased as price variance is more meaningful at purchase than at usage.

Chemical	Actual price (sh)	Std price (sh)	Difference (sh)	Actual Qnty (l)	Price Variance (sh)
Echol	2.146	2.00	0.146(A)	25000	3650(A)
Protex	4.8	4.25	0.55(A)	13000	7150(A)
Benz	1.46	1.50	0.04(F)	40000	1600(F)
CT-40	2.96	3.00	0.04(F)	7500	300(F)
				Total material price variance	8,900(A)

(b) (i) Material mix = (Actual quantity at actual mix – Actual quantity at std mix) std price

Mix Chemical	Actual quantity at actual mix (l)	Actual quantity at std mix* (l)	Difference (l)	Standard price (sh)	variance (sh)
Echol	26600	28140	1540 (F)	2.0	3080 (F)
Protex	12880	14070	1190 (F)	4.25	5057.5 (F)
Benz	37800	35175	2625 (A)	1.50	3937.5 (A)
CT – 40	7140	7,035	105 (A)	3.00	315 (A)
Total	84,420	84,420		Total mix variance	3885 (F)

* The standard mix is

Echol; $\frac{200}{600} = \frac{1}{3}$; Protex; $\frac{100}{600} = \frac{1}{6}$ Benz; $\frac{250}{600} = \frac{5}{12}$ CT – 40; $\frac{50}{600} = \frac{1}{12}$

Material yield Variance = [Actual quantity at std mix – Standard quantity std mix] at std price

The standard quantity input for the production achieved has to be calculated first.

From the standard data, 600l produces 500l of gas gain. ∴ standard yield = 5/6 = 83.33%

∴ For the production of 140 batch x 500l/ batch = 70,000l
the standard input should be
standard input quantity = 70,000l x 6/5 = 84,000l

This is then stated at standard mix

Chemical	Actual quantity Yield at std mix (l)	Actual quantity at std mix (l)	Difference (l)	Standard price (sh)	variance (sh)
Echol	28140	28000	140 (A)	2.0	280 (A)
Protex	14070	14000	70 (A)	4.25	297.5 (A)
Benz	37800	35000	175 (A)	1.50	262.5 (A)
CT – 40	7035	7,000	35 (A)	3.00	105.0 (A)
Total	84,420	84,000		Total yield variance	945 (A)

Usage variance = yield variance + mix variance

Chemical	Yield variance (Sh)	Mix variance (Sh)	Usage variance (Sh)
Echol	280 (A)	3080 (F)	2800 (F)
Protex	297.5 (A)	5057.5 (F)	4760 (F)
Benz	262.5 (A)	3937.5 (A)	4200 (A)
CT – 40	105.0 (A)	315 (A)	420 (A)
	945.0 (A)	3885 (F)	2940 (F)

QUESTION FIVE

Garnet Ltd.

Let t be the month for which forecast is required so

that; t_0 = current month

t_1 = next month

t_{-1} = previous month

Let S be the sales for the current month

The equations for use in the cash budgeting model are as follows:

$$\text{Sales} = S(1.01)^t$$

Costs of sales = 0.75S (gross profit margin = 33 1/3% on cost of sales; therefore cost of sales = 75% of sales)

Cash collections t months from now;

$$0.2S\,(1.01)^t + 0.8[0.2S(1.01)^{t-1}] + 0.8\,[0.6S(1.01)^{t-2}] + 0.8[0.2S\,1.01)^{t-3}]$$

Purchases t months from now;

$$0.75S\,(1.01)^{t+2}$$

Payments for purchases t months from now;

$$0.75S\,(1.01)^{t+1}$$

Payments for expenses t months from now;

$$0.05\,S(1.01)^{t-1} + 3000 + 10{,}000$$

S for June = £100,000

t = 3 (month of September is t + 3 month from June)

Collections during September:

$$0.2S\,(1.01)^t + 0.8\,[0.2S(1.01)^{t-1}] + 0.8\,[0.6S\,(1.01)^{t-2}] + 0.8\,[0.2S\,(1.01)^{t-3}]$$

$= 0.2(100{,}000\ (1.01)^3 + 0.8(0.2)\ (100{,}000)\ (1.01)^2 + 0.8\ (0.6)\ (100{,}000)\ (1.01) + 0.8(0.2)\ (100{,}000)$

$= £\ 20606 + £\ 16{,}322 + £48{,}480 + £\ 16{,}000$
$= £101{,}408$

Payments for purchases during September;

$$0.75S\ (1.01)^{t+1} = 0.75\ (100{,}000)\ (1.01)^4 = £\ 78045$$

Payments for expenses during September;

$$= 0.05S(1.01)^2 + 3000 + 10{,}000$$
$$= £5100 + £3000 + £10{,}000 = £18{,}100$$

The cash flow statement for September is as follows:

	£	£
Receipts from sales		101,408
Payments; Purchases	78,045	
Payroll	5,100	
Utilities	3,000	
Other costs	<u>10,000</u>	<u>96,145</u>
Increase in cash		<u>5,263</u>

The following procedures can be applied to incorporate uncertainty;
Sensitivity analysis e.g. by "what if?" analysis,
Expected values
Simulation

LESSON 7

QUESTION ONE

(a) In general, given that the customer did not pay his bill in month n, the probability that he will not pay his bill in month n+1 is $p(NP_{n+1} | NP_n) = 20$, (where, NP denotes **Not Pay**).

Thus, the probability that the customer will not pay his bill in **any** of the next three months

is: $P(NP_1, NP_2, NP_3) = (0.20)(0.20)(0.20) = 0.008$

(b) Given that the customer did not pay his bill in month n, what are the probabilities of bill payment in each of the months n+1, n+2, n+3?

Month n+1:

$$[0.0 \quad 1.0] \begin{bmatrix} .90 & .10 \\ .80 & .20 \end{bmatrix} = [.80 \quad .20]$$

P (Payment in month n+1) = .80

Month n+2:

$$[0.80 \quad .20] \begin{bmatrix} .90 & .10 \\ .80 & .20 \end{bmatrix} = [.88 \quad .12]$$

P (Payment in month n+2) = .88

Month n+3:

$$[0.88 \quad .12] \begin{bmatrix} .90 & .10 \\ .80 & .20 \end{bmatrix} = [.89 \quad .11]$$

P (Payment in month n+3) = .89

(c) Steady State Conditions:

$$[\pi_1, \pi_2] = [\pi_1, \pi_2] \begin{bmatrix} .90 & .10 \\ .80 & .20 \end{bmatrix}$$

$\pi_1 \quad = \quad .90\pi_1 \; + \quad .80\pi_2$

$\pi_2 \quad = \quad .10\pi_1 \; + \quad .20\pi_2 \leftarrow$ eliminate

$\pi_1 \quad + \quad \pi_2 \quad = \quad 1.0$

$-.10\pi_1 + \quad .80\pi_2 \; = \quad 0$

$\pi_1 \quad + \quad \pi_2 \quad = \quad 1.0$

Since,

$-.10\pi_1 = \quad -.80\pi_2$

$\pi_1 \quad = \quad 8\pi_2$

Substituting:

$8\pi_2 \quad = \quad \pi_2 \quad = \quad 1.0$

$9\pi_2 \quad = \quad 1$

$\pi_2 \quad = \quad 1/9 \quad = \quad .111$

and,

$\pi_1 \quad = \quad 8\pi_2$

$\pi_1 \quad = \quad 8(.111)$

$\pi_1 \quad = \quad .888$

$[\pi_1, \pi_2] \quad = \quad [.89 \quad .11]$

QUESTION TWO

$$P.P = P^2 = \begin{bmatrix} .5 & .3 & .2 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix} \begin{bmatrix} .5 & .3 & .2 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} .30 & .38 & .32 \\ .15 & .54 & .31 \\ .14 & .18 & .68 \end{bmatrix}$$

$$P^3 = P.P^2 = \begin{bmatrix} .30 & .38 & .32 \\ .15 & .54 & .31 \\ .14 & .18 & .68 \end{bmatrix} \begin{bmatrix} .5 & .3 & .2 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} .220 & .388 & .392 \\ .160 & .454 & .386 \\ .156 & .236 & .608 \end{bmatrix}$$

$$[3000\ 5000\ \ 2010] \begin{bmatrix} .220 & .388 & .392 \\ .160 & .454 & .386 \\ .156 & .236 & .608 \end{bmatrix}$$

$$= \quad [1772\ \ 3906\ \ 4322]$$

(b)

$$[\pi_1, \pi_2, \pi_3,] = [\pi_1, \pi_2, \pi_3] \begin{bmatrix} .5 & .3 & .2 \\ .1 & .7 & .2 \\ .1 & .1 & .8 \end{bmatrix}$$

$$\pi_1 = .5\pi_1 + .1\pi_2 + .1\pi_3$$

$$\pi_2 = .3\pi_1 + .7\pi_2 + .1\pi_3$$

$$\pi_3 = .2\pi_1 + .2\pi_2 + .8\pi_3 \quad \leftarrow \text{Eliminate}$$

$$\pi_1 + \pi_2 + \pi_3 = 1.0$$

$$\begin{array}{lllll} -.5\pi_1 + & .1\pi_2 + & .1\pi_3 = & 0 & [1] \\ .3\pi_1 - & .3\pi_2 + & .1\pi_3 = & 0 & [2] \\ \pi_1 + & \pi_2 + & \pi_3 = & 1.0 & [3] \end{array}$$

Equation

$$\begin{array}{llllllll} [1] & .5\pi_1 + .1\pi_2 + .1\pi_3 = 0 & & -.5\pi_1 + .1\pi_2 + .1\pi_3 = 0 \\ [2] & .3\pi_1 - .3\pi_2 + .1\pi_3 = 0 & \rightarrow & -.3\pi_1 + .3\pi_2 - .1\pi_3 = 0 \\ \hline & & & -.8\pi_1 + .4\pi_2 = 0\ [A] \end{array}$$

$$\begin{array}{llll} [3] & \pi_1 + \pi_2 + \pi_3 = 1.0 & & -.1\pi_1 + .1\pi_2 + .1\pi_3 = .1 \\ [2] & .3\pi_1 - .3\pi_2 + .1\pi_3 = 0 & \rightarrow & -.3\pi_1 + .3\pi_2 - .1\pi_3 = 0 \\ \hline & & & -.2\pi_1 + .4\pi_2 = .1\ [B] \end{array}$$

Equation

[A] $-.8\pi_1 + .4\pi_2 = 0$		$-.8\pi_1 + .4\pi_2 = 0$
[B] $-.2\pi_1 + .4\pi_2 = .1$	→	$+.2\pi_1 - .4\pi_2 = -.1$
		$-.6\pi_1 = -.1$
		$\pi_1 = 1/6 = .167$

and,

[A] $-.8(.167) + .4\pi_2 = 0$

$.4\pi_2 = .134$

$\pi_2 = .335$

and,

[3] $\pi_1 + \pi_2 + \pi_3 = 1.0$

$.167 + .335 + \pi_3 = 1.0$

$\pi_3 = .498$

therefore, π = [.167 .335 .498], the steady state vector

To determine steady state enrolments in each college multiply each probability by 10,000:

$\pi_1 = 1670$

$\pi_2 = 3350$

$\pi_3 = 4980$

QUESTION THREE

$$\begin{array}{ccc} & \multicolumn{2}{c}{n+1} \\ & \text{Increase} & \text{Decrease} \end{array}$$

$$\begin{array}{ll} & \text{Increase} \\ n & \text{Decrease} \end{array} \begin{bmatrix} .3 & .7 \\ .8 & .2 \end{bmatrix}$$

$$[\pi_1, \pi_2] = [\pi_1, \pi_2] \begin{bmatrix} .3 & .7 \\ .8 & .2 \end{bmatrix}$$

$\pi_1 = .3\pi_1 + .8\pi_2$

$\pi_2 = .7\pi_1 + .2\pi_2$ ← eliminate

$\pi_1 + \pi_2 = 1.0$

$-.7\pi_1 + .8\pi_2 = 0$

$\pi_1 \quad - \quad \pi_2 \quad = \quad 1.0$

$-.7\pi_1 \quad + \quad .8\pi_2 \quad = \quad 0$

$-.8\pi_1 \quad - \quad .8\pi_2 \quad = \quad -.8$

$-.15\pi_1 \quad = \quad -.8$

$\pi_1 \quad = \quad 8/15 \quad = \quad .533$

$\pi_1 \quad + \quad \pi_2 \quad = \quad 1.0$

$.533 \quad + \quad \pi_2 \quad = \quad 1.0$

$\pi_2 \quad = \quad .47$

$\pi \quad = \quad [.533 \quad .47]$

LESSON 8

QUESTION ONE

REQUIREMENT A

Division B's output (units)	B's Own Processin g Costs	A's Charge to B for intermediate s	B's Total costs	B's Revenue (Net of selling Costs) per 1,000 units	B's Total revenue	B's Net income
(1)	(2)	(3)	(4) (2) + (3)	(5)	(6) (1) x (5)	(7) (6) - (4)
	Shs	Shs	Shs	Shs	Shs	Shs
1,000	37,500	12,000	49,500	52,500	52,500	3,000
2,000	45,000	24,000	69,000	39,750	79,500	10,500
3,000	52,500	36,000	88,500	33,000	99,000	10,500
4,000	60,000	48,000	108,000	27,750	111,000	3,000
5,000	67,500	60,000	127,500	24,000	120,000	(7,500)
6,000	75,000	72,000	147,500	19,980	119,880	(27,120)

B. The most profitable policy for Division B, in the circumstances, is to set its output at either 2,000 or 3,000 units a day and to accept a profit of Shs 10,500 a day. If its output is more than 3,000 or less than 2,000 it will make even less profit.

With Division B taking 3,000 units a day from it, Division A's revenue, at Shs 12 per unit = Shs 36,000 and its total costs = Shs 21,000. Therefore, A's separate profit is Shs 15,000.

C.

Output (units)	Cost of Producing intermediates	Cost of processing to completion	Total Costs	Total revenue	Net income
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Shs	Shs	Shs	Shs	Shs	
1,000	15,000	37,500	52,500	52,500	—
2,000	18,000	45,000	63,000	79,500	16,500
3,000	21,000	52,500	72,500	99,000	25,500
4,000	24,000	60,000	84,000	111,000	27,000
5,000	27,000	67,500	94,500	120,000	25,500
6,000	30,000	75,500	105,000	120,000	15,000

A single profit centre will operate more profitably than the two divisions formally did. By making and selling 4,000 units a day it can earn a profit of Shs 27,000 or Shs 1,500 a day in excess of the best result achieved by the combined activities of Divisions A and B.

D. "The company is seen to have been paying a price for the luxury of divisionalization. By suboptimizing (i.e. by seeking maximum profits for themselves as separate entities), the divisions have caused the corporation to less than optimize its profits as a whole. The reason was of course, that Division B reacted to the transfer price of Shs 12 a unit by restricting both its demand for the intermediate and its own output of the finished product. By making for itself the best of a bad job, it created an unsatisfactory situation for the company. But who can blame it? Assuming that the instructions to B were to maximize the division's separate profit, it did just that, given the conditions confronting it. Yet it is not fair to blame that division either, for it too was only carrying out instructions in seeking to maximize its own profit; and a transfer price of Shs 12, while it leads to a less than optimal result for the corporation, does maximize A's own profit.

"One further feature of this illustration is worth nothing. So far as its own profit was concerned, it was a matter of indifference to Divison B whether it sold 2,000 or 3,000 units. We assumed that it decided to sell 3,000. If it had chosen to sell only 2,000, its own profit would have been unaffected, while A's profit would have been cut from 15,000 to 6,000, so that the corporate profit would have been diminished by Shs 9,000. In a situation like this, negotiations about the price between A and B would probably have prevented this further damage to the corporation resulting from suboptimization. But it is unlikely that the divisions, left to themselves, would arrive at an optimal solution from the corporate point of view.

"The management of the single profit centre arrived at the conclusion that 4,000 units was its optimal output through comparison of incremental costs with incremental revenue for each prospective addition to output. Pushing output beyond 4,000 did not pay because an extra 1,000 units would have added Shs 10,500 to costs while adding only Shs 9,000 to revenues. The fact that incremental costs are made up of two parts (the cost of producing the intermediate product and the cost of processing it to completion) does not affect the result. Nor, from the point of view of the firm as a whole, should the result be affected if responsibility for the two operations happens to be split between two responsibility centres.

"The second responsibility centre (second, that is, in the chain of processes) can only do what is best for the company when deciding how much of the first division's production to take **if it has knowledge of the other division's incremental costs.** Leaving these decisions to divisions to work out for themselves implies that transferor divisions should offer their products to other responsibility centers at a figure not in excess of the incremental cost of producing them.

"This result appears to be a very far cry from the most common basis for fixing transfer prices, namely, the price of the transferred product on the outside market, provided the product, in fact, has an outside market. Actually, however, a close examination will show that **if the transferred product can be bought and sold in a competitive market**, the `incremental cost' rule and the `market price' rule for the transfer pricing are not in conflict.

"If there really is a competitive market for the transferred products, a transferee division can satisfy its needs for intermediate products by buying them outside at the going price. It will be in the company's interest that should do so to prevent another division from incurring **incremental** costs of a greater amount in supplying the intermediate. To do otherwise would cause the company to incur a greater cost in production of the intermediate than in buying it. If the transfer price of the intermediate is set at its market price, the transferor division can supply as much as it wishes (which will be as much as it can produce without incurring incremental costs in excess of the price it will get), leaving the transferee division to acquire any additional supplies it may need by outside purchase. Alternatively, the transferor division may be able and willing to supply more of the intermediate at the market price than the consuming division can use. In that case, the correct course is for the supplying division to go on producing so long as its incremental cost is below the market price. It can sell on the market any output not taken by the other division."

A. General Manager's Remuneration

Region 3	**1990 £000**		**1991 £000**
Basic salary	18,000		19,000
Sales Bonus:			
£2,400 - 2,250 x 0.75%	1,125	£2,750 - 2,700 x 0.75%	375
ROCE bonus:			
123 / 1,850 = 6.65%		147 / 2,260 = 6.50%	
2% of £1,850,000 x 6.65%	2,460	3% of £2,260,000 x 6.50%	4,407
	21,585		23,782

Region 7	**1990 £000**		**1991 £000**
Basic salary	22,000		22,000
Sales Bonus:			
£3,700 - 3,400 x 0.75%	2,250	£3,600 - 3,600 x 0.75%	—
ROCE bonus:			
166 / 2,800 = 5.93%		241 / 2,900 = 8.31%	
3% of £2,800,000 x 5.93%	4,981	3% of £2,900,000 x 8.31%	7,230
	29,231		29,230

Region 3 General Manager's remuneration increases by £2,197.
Region 7 General Manager remuneration reduces by £1.
Consideration of the appropriateness of the reward to the general managers. The significant figures are:

	Region 3 1991 v 1990	**Region 7 1991 v 1990**
Sales	+14.6%	-2.7%

Expenses	+6.3%	+3.0%
Profit	+19.5%	+45.2%
Gross profit/sales	-1.0%	+3.0%
Investment	+22.2% (now over £2 million)	+3.6%
ROCE	-2.3%	+40.1%
Sales/Target sales	1990 + 6.7% 1991 + 1.9%	1990 + 8.8% 1991 —
Remuneration	+10.2%	Slightly negative

b. Relative performance 1991 v 1990

Region 3

The general manager is expected to exceed his sales target, but by a smaller margin than in 1990. His return on capital employed is lower than in 1990. However, capital employed will grow in the year so that by year-end the book value will exceed £2 million. At this level bonus increases from 2% to 3%.

Region 7

The general manager is only expected to just reach his sales target in 1991 whereas he exceeded it in 1990. However his return on investment is expected to improve by over 40% mainly due to a lower cost of sales/sales ratio and a lower proportionate increase in his expenses.

Overall, I do not consider the changes in the remuneration are appropriate rewards for the results expected in 1991.

Region 3

general manager will receive a 10% increase. Half of this is due to the service increment of £1,000 and the remainder to the responsibility of handling a higher investment. Performance related to beating the sales target and ROCE are expected to be poorer than in 1990.

Region 7 general manager is expected to be slightly worse remunerated than in 1990. he has reached his maximum salary and probably based on his age is not expected to exceed his sales target. However, his experience appears to enable him to reduce expenses to give a greatly improved ROCE. The bonus for this does not completely offset his static sales performance. As ROCE should be a main criterion of performance this good work deserves a better remuneration.

Ignoring inflation an increase of 5% for Region 3 to reward experience and responsibility and 10% for Region 7 for probability would seem more appropriate.

c. Recommended changes in remuneration Basic

salary — company service and responsibility.

The basic salary with ten annual increments of £1,000 each based entirely on length of service is likely to lead to dissatisfaction between managers. For example, when a manager of ten year's service is replaced by a newcomer, the incoming manager's salary will only be 55% of his predecessors.

Smaller increment for service could be offset by a salary increment based on responsibility. This could be the size of the operation measured by investment in each depot. Thus, if the service element was limited to a 25% salary differential (£3,000), then £7,000 could be available for `responsibility'. A proposed allocation could be £1,000 for each £0.5 million investment, so that the £7,000 would be received at an investment level of £3.5 million.

Bonus — sales.

A bonus based on exceeding a pre-set sales target is a good method of rewarding performance. However, it does not seem satisfactory to have this based on the value of the vehicles operated by the region. First the `value' is the written-down book value which in itself leads to anomalies according to the age of the vehicles. a first major improvement would be to relate these to replacement cost. The replacement cost for the vehicle should be readily available.

A much more understandable sales target should be set for each region based on the potential business available in that area.

Bonus — return on capital employed

This is the ultimate test of effectiveness and should again be judged on performance against a pre-set target. This will encourage the managers to operate their vehicles as cost effectively as possible in the handling of the available traffic. Where a nationwide service is offered, many company policies are established which affect each region differently. For example:

`providing an overnight service'—this might be well-used in some regions, but sparsely used in others.

`accepting business from large manufacturers at national rates'—these rates tend to be averaged for the whole country and again may be more profitable in one region than another.

As a good basis, therefore, the budget preparation needs to be done carefully with a full analysis of the likely business, available vehicles and staff requirements. The resulting expected profit can the be set against the required investment. Returns on capital employed will vary between regions, but should average to an acceptable overall figure for the company. It is against these target returns for the regions that each manager's performance should be measured. The bonus should be a straight percentage based on the improved return. The present differential relating to investment above or below the £2 million level should be eliminated. the single step at £2 million is too blunt an incentive and may encourage unnecessary investment just to get above the figures.

Thus the general manager would be more fairly rewarded by:

- Basic salary to reflect company loyalty and also responsibility based on the total assets entrusted to the manager.
- Bonus based:
  - i. on achieving above target sales, and
  - ii. at undertaking these sales cost effectively to achieve above target return on capital employed.

LESSON 9

a. Whereas traditional incremental budgeting uses last year's budget as a starting point, zero-base budgeting (ZBB) theoretically starts from zero and makes it more likely that expenditure is justified. Thus, in the energy area the starting point is zero energy costs and increments justified stage by stage. For example, the first stage may be legally determined, i.e. to maintain the statutory workplace temperature. ZBB provides a systematic way of focusing attention on various facets of energy expenditure.

b. Total Quality Management (TQM) is an approach, which emphasises such things as: waste elimination
minimising cost
zero defects at lowest cost;
elimination of non-value added activities, and so on.

TQM is thus another systematic way of focusing attention on waste elimination, improving productivity. Energy related examples include: alternative ways of producing heat or insulating premises; reduction of movements to save energy; elimination of heat losses etc.

Calton Ltd.

a. (i) Production units required before inspection process

		Current situation units		With TQM procedures units
Perfect units required		5,000		5,000
Returns from customers	(5%)	250	(2.5%)	125
		5,250		5,125
Inspection process rejections	(12.5/87.5)	750	(7.5/92.5)	416
Units required before inspection		6,000		5,541

(ii) Purchase of material X

		Sq m		Sq m
Material in inspected units	(6000 x 8 sq m)	48,000	(5,541 x 8 sq m)	44,328
Losses due to processing faults	(4/96)	2,000	(2.5/97.5)	1,137
Process input required		50,000		45,465
Losses in stores and receiving	(5/95)	2,632	(3/97)	1,406
Purchases of material X required		52,632		48,871

Gross machine hours

		Hours		Hours
Hours for perfect units	(6,000 x 0.6)	3,600	(5,541 x 0.5)	2,771
Rectifying hours	(250 x 80% x 0.2)	40	(125 x 80% x 0.2)	20
Operating hours required		3,640		2,791
Idle time	(20/80)	910	(12.5/87.5)	399
Gross machine hours required		4,550		3,190

CALTON LTD

PROFIT AND LOSS ACCOUNT

		Current situation £		With TQM procedures £
Sales income:				
Perfect units	(5,000 x £100)	500,000		500,000
Second quality units	(750 x £70)	52,500	(416 x £70)	29,120
Third quality units	(200 x £50)	10,000	(100 x £50)	5,000
Scrap	(50 x £5)	250	(25 x £5)	125
Total sales income		562,750		534,245

Costs		Current situation £		With TQM procedures £
Material X	(52,632 x £4)	210,528	(46,871 x £4)	187,484
Goods receiving etc	(52,632 x £0.10)	5,263	(46,871 x £0.10)	4,687
Machine costs	(4,550 x £40)	182,000	(3,190 x £40)	127,600
Delivery costs for replacement	(250 x £8)	2,000	(125 x £8)	1,000
Inspection, vendor vetting etc		25,000	(x 60%)	15,000
Product liability insurance etc	(3% x 500,000)	15,000	(1% x 500,000)	5,000
Selling, distn and admin		60,000	(x 90%)	54,000
Prevention programme		20,000		60,000
		519,791		454,771
Monthly profit		42,959		79,474

The **TQM programme** will nearly double Calton's profits, the margin savings arising due to a **reduction in material costs and machine costs.** The TQM procedures concentrate on establishing management responsibility for each part of the internal process as well as for outputs. Every person in the organisation is expected to contribute to quality improvement and this can have a favourable impact on **employee morale.** The advantages are obvious in the substantial increase in Calton's profits, although the large fixed cost investment in the prevention programme can be risky if the projected quality improvements do not materialise.

Quality related costs can be classified as prevention cost, appraisal cost, internal and external failure cost.

Internal failure cost

The CIMA defines this as 'the cost arising from inadequate quality before the transfer of ownership from supplier to purchaser.' Examples from Calton's situation include the cost of material scrapped due to inefficiencies in goods receiving procedures and in stores control, the cost of material lost in process and the cost of units rejected during the inspection process.

(ii) **External failure cost**

The CIMA defines this as 'the cost arising from inadequate quality discovered after the transfer ofownership from supplier to purchaser…..'Examples from Calton's situation include the cost of product liability claims from customers and the cost of replacing and delivering returned units.

(iii) **Appraisal cost**

The CIMA defines this as 'the cost incurred, such as inspection and testing in initially ascertaining the conformance of the product to quality requirements.' Examples from Calton's situation include the inspection and calibration procedures and vendor vetting.

(iv) **Prevention cost**

The CIMA defines this as 'the cost incurred to reduce appraisal cost to a minimum.' These costs are incurred to try and improve the efficiency of checking procedures. A common example is the cost of training personnel in TQM procedures

Bushworks Ltd

a.

Elimination of synthetic stocks	**£**	**£**
Stores losses (68,711 x £1/100)		(687)
Goods inwards checks (given)		14,000
Savings on purchase quantity		
(2,748,450 – 2,090,651) x (£40/100)		263,120
Increased price(2,090,651 x ((£44 - £40)/100)		(83,626)
Curing/moulding costs		
Variable costs (2,679,739 – 2,090,651) x £20/100		117,818
Scrap sales forgone (267,974 – 20,907) x £5/100		(12,353)
Finishing process cost reduction		
Variable cost		
Existing cost (AX) (964,706 x £15/100)	144,706	
Existing cost (BX) (1,447,059 x £25/100)	361,765	
Less		
Amended cost (AX) (826,667 x £12/100)	(99,200)	
Amended cost (BX) (1,243,077 x £20/100)	(248,615)	
		158,656
Scrap sales forgone		
(144,706 + 217,059 - 20,677 - 31,077) x £10/100		(31,002)
Finished goods stock		
Holding costs		
((15,000 + 30,000) – (500 + 1,000)) x £15/1,000		653
		426,579
Cost of quality management programme		(250,000)
		176,579

b. **Internal failure costs**

Are costs arising within an organisation due to failure to achieve the quality specified. Examples in the question are the losses in the curing process and the finishing process, and the losses in stores.

External failure costs are costs arising outside the manufacturing organisation of failure to achieve specified quality after transfer of ownership to the customer. Thus the components that are returned by customers are examples.

Appraisal costs are the costs of assessing quality achieved. Examples are the goods inwards checks, the detection of flawed sub-components due to incorrect temperature and the detection of defective units following the finishing process.

Prevention costs represent the cost of any action taken to investigate, prevent or reduce defects and failures. Examples in the question are the JIT agreement, the improved temperature control procedures aimed at reducing the number of losses and the introduction of cellular manufacturing in finishing.

TQM AND STANDARD COSTING

There are a number of **reasons why** it may be argued that, in total quality environment, **variance analysis from a standard costing system is redundant.**

The ethos behind a system of standard costing is **that performance is satisfactory if it meets predetermined standards.** This is at odds with the philosophy of continual improvement inherent in a total quality environment.

For standard costing to be useful for control purposes, it requires a reasonably **stable environment**. Products or processes must be standardised and repetitive, so that standards can be established which will be useful for monitoring and control. In a total quality environment, however, continual improvements are likely to alter prices, quantities of inputs and so on.

Standard costs often incorporate a **planned level of scrap** in material standards. This is at odds with the TQM aim of zero defects and there is no motivation to 'get it right first time.'

Although ideal standards can be set (no wastage, no spoilage, no inefficiencies, no idle time, no breakdowns), **attainable standards**, which make some allowance for wastage and inefficiencies, are common. The use of such standards conflicts with the elimination of waste which is a vital ingredient of a TQM programme.

The control aspect of standard costing systems is achieved by making **individual managers responsible** for the variances relating to their part of the organisation's activities. A TQM programme, on the other hand, aims to make all personnel aware of, and responsible for, the importance of supplying the customer with a quality product.

It is these differences between the aims and ideas of standard costing and those of TQM which mean that variance analysis from a standard costing system is redundant in a total quality environment.

A system of standard costing analyses labour efficiency by comparing the standard labour input for the actual output achieved with the actual output. **Standard costing therefore concentrates on quantity** and ignores other factors contributing to effectiveness.

In a **total quality environment**, quantity is not an issue, however; **quality** is. Effectiveness in such an environment therefore centres on high quality output (produced as a result of high quality input and the elimination of non-value adding activities) and the cost of failing to achieve the required level of effectiveness is measured in terms of **internal and external failure costs**.

An internal failure cost might be the costs of re-inspecting items after they have been reworked due to poor quality workmanship.

An external failure cost might be the cost of repairing products returned from customers.

Neither of these costs would be identified by a traditional standard costing analysis of labour efficiency and effectiveness but are vital measures in a TQM environment.

Standard costing systems tend to measure labour efficiency in terms of individual tasks but, in a **total quality environment**, labour is more likely to be viewed as a number of multi-task teams who are responsible for the completion of a part of the production process. The **effectiveness of such a team** is more appropriately

measured, not in terms of output, but **in terms of re-working required, returns from customers, defects identified in subsequent stages of production and so on.**

TRITEX PLC

Calculation of missing values for 'product units' cells

To get one unit of a process that suffers losses it is necessary to put more than one unit in.

The Finishing process must have as its end result 1.0000 unit (100%),
so that input must be 100%/(100% - 70%) = 100/93 = 1.0753 units.

The Converting process must have as its end result 1.0753 units, so the input
must be 107.53%/ (100% - 13%) = 107.53/87 = 1.2360 units.

The Making process must have as its end result 1.2360 units, so the inputs
must be 123.6%/ (100% - 9%) = 123.6/91 = 1.3582 units.

Calculation of missing values for 'cost' cells

Raw material cost = units input to Making process x m^2 per unit x rate per m^2
= 1.3582 units x 6.72m^2 x £0.85
= £7.758

Labour cost = units of output from Making process x hours per unit x rate per hour
= 1.2360 units x 0.142hrs x £4.75
= £0.834

Overheads = units of output from Making process x net processing hours per unit x average rate per net processing hour
= 1.2360 x 0.125hrs x £7
= £1.082

Sub-total Making cost = £(7.758 + 0.834 + 1.082)
= £9.674

Calculation of missing value for 'WIP value per unit'

WIP value is the cost of the input units to the Making process divided by the output units
= £9.674 ÷ 1.236
= £7.827

At present, the company inputs around 12% more raw materials than specified by the control standard and pays £0.05 more per square metre for those raw materials. The company is possibly using higher quality raw material, but any advantage from this is wasted because of losses in the Making process of 9% as opposed to only 5% in the control standard. These losses may be due to lack of motivation in employees, poor training for employees, an emphasis on speed of throughput as opposed to quality and so on.

Labour hours are also higher: the company takes 0.142 hours compared with a control standard of 0.108 hours to produce one unit.

Idle time is 12% of total labour hours ((0.142 – 0.125)/0.142). The control standard requires that this should be 8% however. This implies that there is a certain amount of inefficiency at Tritex plc.
The actual **labour rate is 25p lower than the control standard.** Possibly the company employs less highly skilled labour: this would account for some of the losses and inefficiencies.

Overheads are also incurred at a higher rate than stipulated by the control standard (£7.00 as opposed to £6.32). This could be caused by inefficiency in the sourcing of overhead items. Moreover, overheads are absorbed on the basis of processing time, which is higher than set out in the control standard.

Losses in subsequent processes are much higher than required by the control standard, implying that insufficient care is taken to control the Converting and Finishing processes.

Overall, the result is that the actual WIP unit cost of Product A is nearly £1.60 more than that stipulated by the control standard.

The implementation of total quality management (TQM) involves establishing a programme of continuous improvement with the aim of producing zero defects, supplying high quality products and eliminating waste and non-value-added activities.
TQM has had the following impact on the standard cost of Product A.
The control standard cost is clearly not the ideal since the results reckoned to be achieved following the quality drive give a further reduction in unit cost of over 16p, and it looks as if the end product will be of a better standard.
Losses in the Making and Finishing processes are reduced to just 1% and in the Converting process to 2.5%, indicating a move towards zero defects and the elimination of waste.
Although raw materials input are the same under both the control standard and the ABC/TQM standard, the cost of the raw materials per m^2 to be used following the implementation of the TQM programme is 10p higher. This is likely to lead to less wastage and give a much higher quality end product, possibly one that can be sold at a premium price.
Labour time has been reduced by 0.13 hours per unit. The question implies that there is now no idle time: employees are employed on other tasks intended to improve the efficiency of overall operations. As a result the labour rate has increased from £5 to £6, so workers not only have more varied and interesting jobs but also earn more. The reduction in idle time means that the overall labour cost is still lower than that required by the control standard.
Overheads are now absorbed on a different basis. The figures indicate that this product was formerly overcharged, since the cost has reduced from over 70p to about 37p. The change in cost driver is thought to result in a 30% reduction in overhead cost and so the figures can be further analysed as follows.

	£
Control standard overheads (0.1 hrs x £6.32)	0.632
30% reduction (30% x £0.632)	(0.190)
ABC/TQM standard overheads (1 x £0.36)	(0.360)
Remainder, due to improved work practices of TQM programme	0.082

The adoption of a total quality philosophy will require the following additional information from the control system.
The ideal identified at present may not be as far as it is possible to go. Quality programmes generally aim at 'continuous improvement,' so more exacting targets may need to be established once the ABC/TQM standard has been met.
It may be desirable to categorise information in terms of the well-known, though perhaps old-fashioned, quality costs: prevention costs, appraisal costs, internal failure costs and external failure costs.
Other aspects of performance that the company may wish to measure include the following.
Customer satisfaction: Product A is likely to improve in quality and this ought to be measurable by means of opinion surveys, reports in the trade and consumer press and so on, as well as being reflected in sales figures. The question implies that attempts are being made to control non-value-added activities, and presumably this means that **non-financial indicators** relating to matters such as set-up times, materials handling and so on should be measured.
Competitive performance. It may be fruitful to benchmark performance against that of individual competitors (those considered 'best in class'), not only on cost and price but also on marketing performance, reputation and so on.

a. (i) Workings:

Total estimated minutes:

Making: 8,000 x 5. 25 = 42,000

Packing: 5,000 x 6 + 3,000 x 4 = 42,000

Absorption rate per product unit (both products):

Making : variable (£350,000/ 42,000) x 5.25 =£43.75

fixed (£210,000/42,000) x 5.25 =£26.25

Packing cost per minute:

variable £280,000/42,000 =£6.666

fixed £140,000/42,000 =£3.333

Unit costs are determined as cost per minute x minutes per unit. e.g. VG4U variable cost = £6.666 x 6 minutes =£40

VG2 fixed cost = £3.333 x 4 minutes = £13.33 (split 40% specific 60% company).

COST STATEMENT

		VG4U	VG2
		£	**£**
Direct material		30	30
variable conversion cost	-making	43.75	43.75
	-packing	40.00	26.67
		113.75	100.42
Product specific fixed costs:			
Making		10.50	10.50
Packing		8.00	5.33
Total product specific cost		132.25	116.25
Company fixed cost:			
Making		15.75	15.75
Packing		12.00	8.00
Total Cost		160.00	140.00
Selling price		150.00	180.00
Profit (loss)		(10.00)	40.00

VG4U makes a loss of £10 per unit, but makes a contribution over specific costs of £150 – 132.25 = £17.75 towards meeting general company fixed costs. This represents a total contribution of 5,000 x £17.75 = £88,750 for the period so that the product should be continued unless there is an alternative use for the capacity which produces more contribution.

Workings using ABC

		Total	VG4U	VG2
Product units			5,000	3,000
	%	£	£	£
Variable conversion cost:				
Moulding (temperature)	(60)	210,000	140,000	70,000
Trimming (consistency)	(40)	140,000	40,000	100,000
Packing (time)	(70)	196,000	117,600	78,400
Packing material (complexity)	(30)	84,000	21,000	63,000
			318,600	311,400
Variable cost per product unit			63.72	103.80
Product specific costs:				
Moulding (60% x £84,000)		50,400	33,600	16,800
Trimming (40% x £84,000)		33,600	9,600	24,000
Packing (70% x £56,000)		39,200	23,520	15,680
Packing material (30% x £56,000)		16,800	4,200	12,600
Specific fixed cost per product unit			14.18	23.03

Company fixed costs = £210,000 + 140,000 – 70,920 – 69,080

= £210,000

Overall average cost per unit = £210,000/8,000

= £26.25

Cost summary

	VG4U	VG2
	£	£
Direct material cost	30.00	30.00
Variable conversion costs	63.72	103.80
	93.72	133.80
Product specific fixed costs	14.18	23.03
	107.90	156.83
Company fixed costs	26.25	26.25
	134.15	183.08
Selling price	150.00	180.00
Profit (loss)	15.85	(3.08)

Comments

ABC is a different convection and therefore give different costs. It is claimed to be more realistic. All that can be said in this case is that both products have a margin over specific costs (£42.10 and 23.17) and thus contribute to meeting general fixed costs. They should be continued until higher earning products can be found.

Comments on target costing

PAST CPA EXAMINATION PAPERS

KENYA ACCOUNTS AND SECRETARIES NATIONAL EXAMINATIONS BOARD

CPA PART III

MANAGEMENT ACCOUNTING

JUNE 2013 **TIME ALLOWED: 3 HOURS**

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show all your workings.

QUESTION ONE

Briefly explain three methods that can be used to analyse uncertainty in cost-volume-profit (C-V-P) analysis. Aberdares Company Ltd. is a manufacturing company which produces and sells a single product known as T_1 at a price of Sh.10 per unit. The company incurs a variable cost of Sh.6 per unit and fixed costs of Sh.400,000. Sales are normally distributed with a mean of 110,000 units and a standard deviation of 10,000 units. The company is considering producing a second product, T_2 to sell at Sh.8 per unit and incur a variable cost of Sh.5 per unit with additional fixed costs of Sh.50,000. The demand for T_2 is also normally distributed with a mean of 50,000 units and standard deviation of 5,000 units. If T_2 is added to the production schedule, sales of T_1 will shift downwards to a mean of 85,000 units and standard deviation of 8,000 units. The correlation coefficient between sales of T_1 and T_2 is –0.9.

Required:

The company's break-even point for the current and proposed production schedules. (7 marks)
The coefficient of variation for the two proposals. (8 marks)
Based on your computation's in (i) and (ii) above advise the company on whether
To add T_2 to its production schedule. (2 marks)

(Total: 20 marks)

QUESTION TWO

"It is now fairly and widely accepted that conventional cost accounting, distorts management's view of business through unrepresentative overhead allocation and inappropriate product costing. This is because the traditional approach usually absorbs overhead costs across products solely on the basis of the direct labour involved in their manufacture. As direct labour cost expressed as a proportion of total manufacturing cost continues to fall, this leads to more an more distortion and misrepresentation of the impact of particular products on total overhead costs" (from Financial Times)

Required:

Briefly discuss the above statement and state what approaches are being adopted by management accountants to overcome such criticism. (8 marks) Traditional budgeting systems are incremental in nature and tend to focus on cost centers.
Activity based budgeting (ABB) links strategic planning to the overall
performance measurement aimed at continuous improvement.

Required:

Explain the weakness of traditional incremental budgeting systems. (4 marks)
Describe the main feature of activity based budgeting system and comment on its advantages. (8 marks)

(Total: 20 marks)

QUESTION THREE

Joan Odero, an independent movie producer, is negotiating with Roadshow Productions Limited on a contract for the production and marketing of her next film, titled "The rise and fall of a cock". The budget for the film is, Sh.100 million.

Roadshow Productions Limited is offering Joan Odero a choice of one of the three contracts.

Contract A

Roadshow Productions Limited will pay all the production and marketing costs. Joan Odero will receive a fixed fee of Sh.10 million.
Joan Odero will receive 10% of gross revenue from the film in excess of Sh.1 billion (no payment is made for gross revenue up to Sh.1 billion).

Contract B

Roadshow Productions Limited will pay 80% of all the production and marketing costs up to Sh.100 million and 30% of production and marketing costs in excess of Sh.100 million
Joan Odero will receive 10% of all gross revenue for the film.

Contract C

Roadshow Productions Limited will pay 50% of production and marketing costs up to Sh.100 million. Joan Odero will receive 30% of all gross revenue from the film.

Joan Odero estimates the following probabilities for the gross revenues:

P(high demand of Sh.2 billion)	0.1
P(medium demand of Sh.500 million)	0.3
P(low demand of Sh.100 million)	0.6

She estimates the following probabilities for the cost of production:

P(budgeted cost of Sh.100 million)	0.6
P(high cost of Sh.200 million)	0.4

Required:

The expected monetary value for Joan Odero under each contract for each of the six possible events. *(Hint: The possible events are high demand – budgeted costs, high demand – high costs, medium demand – budgeted costs, medium demand – high costs, low demand – budgeted costs, and low demand – high costs).* (15 marks)

Joan Odero will choose the contract that maximizes her expected monetary value from the film. Which contract should she choose? (Show calculations). (2 marks)

What information might Joan Odero use in assessing the probability distribution for the production and marketing costs of "The rise an fall of cock" film? (3 marks)

(Total: 20 marks)

QUESTION FOUR

High-tex Engineering Company Limited wishes to set flexible budgets for each of its operating departments. A separate maintenance department performs all routine and major repair works on the company's equipment and facilities. The company has determined that maintenance department performs all routine and major repair works on the company's equipment and facilities. The company has determined that maintenance cost is primarily a function of machine hours worked in the various production departments.

The maintenance cost incurred and the actual machine hours worked during the months of January, February, March and April 2013 were as follows:

Month	Machine hours in Production departments	Maintenance department's Costs Sh.
January	800	350
February	1,200	350
March	400	150
April	1,600	550

Required:

Determine the cost estimation function using:
High-low method. (5 marks)
Regression analysis (5 marks)

Using the regression function estimate:
The maintenance costs that would have been incurred if the machine hours were expected to be 900 in the month of May 2013. (1 mark)

The maximum machine hours that would have been worked If the maintenance cost incurred had been limited to Sh.400,000 for the month of May 2013. (6 marks)

Assuming that in the month of May 2013 machine hours were 900, establish a 95% confidence interval for this point estimate. (Assume t_c = 2.7764 and standard error of estimate, s_e = 63.25).

(3 marks)
(Total: 20 marks)

QUESTION FIVE

Construct a flowchart to show the logic solution of a zero-sum game. (6 marks)
Two manufacturers compete in a market for a specialized calculator. Company A controls 75% of the market while company B controls 25% of the market. Company A is considering a vigorous annual marketing campaign which will cost Sh.35,000,000. The total market for the specialize calculator is 100,000 units per year. The profit contribution per unit is Sh.3,000.
Company B is debating how much money to invest in research and development every year. It is considering three alternatives: Sh.25,000,000, Sh.50,000,000 and Sh.80,000,000. It is estimated that if company A runs a vigorous annual marketing campaign, its share of the market after one year will be either 79% or 73%, depending on company B's investment in research and development (Sh.25,000,000, 50,000,000 and Sh.80,000,000 respectively).

On the other hand, if company A does not run the marketing campaign, company B's share of the market will decrease by 1% of the total market if it invests Sh.25,000,000 in research and development, increase by 1% if it invests Sh.50,000,000 in research and development and increase by 3% if Sh.80,000,000 is invested.

Required:

Using the share of the market percentages only, convert the above into a zero sum game, and hence solve for the optimal strategies for both companies. (6 marks)

Obtain a pay off table consisting of contribution to profit in monetary terms, and hence solve the game. (8 marks)

(Total: 20 marks)

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATION BOARD
CPA PART III
MANAGEMENT ACCOUNTING

DECEMBER 2011 **TIME ALLOWED: 3 HOURS**

QUESTION ONE

Kiko Ltd. Is a large cash and carry warehouses which sells electronics. Kiko Ltd. Purchases the most popular model of calculators (FX 100) directly form the manufacturer at a cost of Sh.250 each. Average sales per a 300 day year are 475 calculators. Whenever an order with the manufacturers is placed, Kiko Ltd, Incurs a cost of Sh.50. The stock holding costs are estimated at Sh.12.50 plus 10% opportunity cost of capital. The lead-time is three days. During the last 50 stock cycles, the demand during the lead-time has generated the following frequency distribution:

Lead time demand	0	1	2	3	4	5	6	7	8
Number of stock cycles	1	2	6	8	10	8	8	5	2

Each time the warehouses runs out of stock, an emergency order is placed with an extra cost of Sh.20 per calculator.

Required:

The economic order quantity (EOQ) and the reorder level. (16 marks)
The total annual relevant costs for the order quantity in (a) above. (4 marks)
(Total: 20 marks)

QUESTION TWO

Boots Ltd. manufactures a range of five similar products, A, B, C, D and E. the table below shows the quantity of each of the required inputs necessary to produce one unit of each product, together with the weekly inputs available and selling prices of each product.

Inputs	A	**B**	**C**	**D**	**E**	**Weekly inputs available**
Raw materials (Kg)	6.0	6.5	6.1	6.1	6.4	35,000 Kgs
Forming (hours)	1.00	0.75	1.25	1.00	1.00	6,000 hours
Firing (hours)	3.00	4.50	6.00	6.00	4.50	30,000 hours
Packing (hours)	0.50	0.50	0.50	0.75	1.00	4,000 hours
Selling price (Sh.)	40	42	44	48	52	

The costs of each input is as follows:

Material	Sh.2.10 per Kg
Forming	Sh.3.00 per hour
Firing	Sh.1.30 per hour
Packing	Sh.8.00 per hour

Required:

Formulate this problem as a Linear Programming problem. (7 marks)
The problem has been solved using a computer package and the following final tableau of a simplex solution has been produced:

Basis	A	B	C	D	E	X	S	T	U	Value
A	1	1.18	1.04	0.46	0	0.36	0	0	-2.29	3,357
B	0	-0.34	0.23	0.02	0	-0.18	1	0	0.14	321
T	0	1.37	2.97	2.28	0	-0.27	0	1	-2.79	9,482
E	0	-0.09	-0.02	0.52	0	-0.18	0	0	2.14	2,321
Zj	0	1.26	1.06	0.51	0	2.02	0	0	8.81	105,791

Where A, B, C, D and E are the weekly production levels for the five products; X is the amount of raw material that falls short of the maximum available; S, T an U are the respective number of hours short of maximum weekly input of forming, firing and packing time.

Use this tableau to find the optimum weekly production plan. (4 marks)
Describe the implications of using this plan in terms of unused resources and overall contribution to profit. (3 marks)
In the context of this problem explain the meaning of "The dual or shadow price of a resource" (3 marks)
There is a proposition that the company manufactures an additional product which would sell at Sh.50 per unit. Each unit will need 6 kg of raw material, one hour of forming time, five hours of firing time and one hour of packing time. Is it s worthwhile proposition? (3 marks)

(Total: 20 marks)

QUESTION THREE

Briefly explain four ways in which competitive situations (or games) can be classified. (8 marks)
Kamau and Njoroge are two cousins specializing in hawking business along River road. Kamau specializes in second hand shirts while Njoroge specializes in cheap electronic goods. However, sales have been decreasing partly due to the harsh economic condition in Kenya and party due to restrictions by the City Council.

Each of the cousins is considering expanding to include in their lines of business, items on which their rivals now have a monopoly. Each knows that the other is considering this expansion and this influences each of their decisions.

Kamau figures out that if he does not expand his business and his cousin does, it will hurt his trade by Sh.500 of profit per day. If neither of them expands inventory to include the extra product, Kamau thinks it will boost his net profit by Sh.500 per day due to his superior location. If he expands and his cousin does also, he believes the combination of location and expanded inventory will increase his profits by Sh.1,000 per day. However, if he alone expands and his cousin does not, this will result in no net increase in business.

Required:

Prepare a game matrix and show that a pure strategy does not exist. (4 marks)
Solve the above game to determine the average winnings (or losses) each of the cousins would expect. (8 marks)

(Total: 20 marks)

QUESTION FOUR

Mega Techniques Ltd. makes special purpose equipment according to customer specifications. During the past year, one of its loyal customers, Pawa Ltd., ordered a specialized equipment to be fabricated for it. Mega

Techniques Ltd. Finished construction the equipment only to be notified that Pawa Ltd. Had recently gone into liquidation and will not therefore take the equipment.
The original price to Pawa Ltd. had been agreed at Sh.9,108,000 which included an estimated normal profit mark-up of 10 per cent on total costs. The costs incurred to manufacture the machine were

	Sh.
Direct materials	3,420,000
Direct wages	2,160,000
Overheads:	
Variable	540,000
Fixed; production	1,800,000
Fixed; selling and administration	360,000
	8,280,000

After a sustained search, the sales manager of Mega Techniques Ltd. Has managed to locate one potential buyer, Zimwi Systems Ltd, which has indicated that it could buy the machine if certain conversion work could be carried out.

Mega Techniques Ltd's production department has made a preliminary assessment which reveals that conversion would entail extra work costed as follows:

Direct materials	Sh.576,000	
Direct wages:		
Department X:		3 men for 4 weeks at Sh.27,000 per man/week
Department Y:		1 man for 4 weeks at Sh.21,600 per man/week
Variable overhead:		
20 per cent of direct wages		

Fixed production overhead:

Department X:	75 per cent of direct wages.
Department Y:	25 per cent of direct wages.

The following additional information is provided:
In the original machine, there were three types of basic materials:

Type P could now be sold to a scrap merchant for Sh.540,000.
Type Q could be sold to a scrap merchant for Sh.360,000 but it would take 120 hours of labour paid at Sh.270 per hour to put it into a suitable condition for sale.
Type R would need to be scrapped at a cost to Mega Techniques Ltd. of Sh.108,000

The materials for the conversion are at present in stock. If not needed for the conversion they could be used in the production of another machine in place of materials that would currently cost Sh.684,000.
The conversion would be carried out in two departments:

Department X is currently extremely busy and it is estimated that its contribution overheads and profits is Sh.2.50 for every Sh.1 of labour.

Department Y has idle staff, for organizational reasons its labour force cannot be reduced below its present level of four employees, all of whom are paid at the standard rate of Sh.21,600 per week.

The designs and specifications of the original machine could be sold in a neighbouring country for a sum of Sh.270,000 if the machine is scrapped.

An additional temporary supervisor would have to be engaged for the conversion work at a cost of Sh.162,000. It is the company's normal practice to charge supervision to fixed overhead.

Pawa Ltd. Had paid Mega Techniques Ltd. A non-returnable deposits of 12% of the selling price.

Required:

The minimum price that Mega Techniques Ltd. should accept from Zimwi Systems Ltd. for the converted machine. Explain clearly how you arrive at your figure. (16 marks) State clearly any assumptions that you have made in arriving at your conclusions in (a) above.

(4 marks)

(Total: 20 marks)

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATIONS BOARD

CPA PART III

MANAGEMENT ACCOUTING

DECEMBER 2010 **TIME ALLOWED: 3 HOURS**

QUESTION ONE

Differentiate between a feedback control system and a feed forward control system.

(4 marks)

In his study of: the impact of budges on people" C Argyris reported the following comment by a financial controller on the practice of participation in setting budgets in his company:
"We bring in the supervisors of budget areas, we tell them that we want their frank opinion, but most of them just sit there and no their heads. We know they are not coming out with exactly what they feel. I guess budget scares them".

Explain why managers may be reluctant to participate fully in setting budgets, indicating the negative side effects, which may arise from the imposition of budgets by senior management. (10 marks)

A critic has suggested that budgets should be abolished because they introduce rigidity and hamper creativity. Discuss. (6 marks)

(Total: 20 marks)

QUESTION TWO

Sola Ltd. Is a manufacturing company that requires component XLA20 in one of its production lines. The components are bought from outside suppliers. Form past experience, the company has determined that the demand for the component can be approximated by a normal distribution with a mean of 500 and a standard deviation of 10, over the range 470 to 530.

The unit is an initial stock of 2010 components and the company has decided to order in batches of 2500 whenever the stock level falls below 1500 components. Again, past experience indicates that the time between the order being placed and delivery varies as follows:

	Lead time distribution			
Lead time, weeks	1	2	3	4
Probability	0.02	0.50	0.25	005

The unit cost of holding stock is Sh.5 per week applied to the total stock held at the end of each week. The cost associated with placing an order is Sh.5.00 and the unit cost of being out of stock is Sh.200 per week. The company does all its accounting at the end of the week and all ordering and delivery occur at the beginning of a week.

Required:

Estimate the average cost per week of the above policy, using simulation analysis and the following random numbers:

For Demand:	034	743	738	636	964	736	614	698	637	
	162	332	616	804	560	111	410	959	774	246 762
For Leadtime:	95	73	10	76	51	74				

(Total: 20 marks)

Hint:

Use 15 trial runs
Round off the demand probabilities to 3 decimal places. (Estimate these probabilities in ranges of 5)

QUESTION THREE

Highlight how the transportation algorithm can be modified for profit maximization rather than minimization of costs. (3 marks)

The Executive Furnitures Ltd. (EFL) produces a unique type of computer desks. Four of EFL's main outlets are S_1, S_2, S_3, and S_4. These outlets already have requirements in excess of the combined capacity of its three production plants P_1, P_2, and P_3. The company needs to know how to allocate its production capacity to maximize profits.

Distribution costs (in Sh.) per unit from each production plant to each outlet are given in the following table:

		To			
		S_1 Sh.	**S_2 Sh.**	**S_3 Sh.**	**S_4 Sh.**
	P1	220	240	220	360
From	P2	240	200	180	280
	P3	260	200	260	240

Since the four outlets are in different parts of the country and as there are differing transportation costs between the production plants and the outlets along with slightly different production costs at different production plants there is a pricing structure which enables different prices to be charged at the four outlets. Currently, the price per unit charged is Sh.2,300 at S_1, Sh.2,350 at S_2, Sh.2,250 at S_3, and Sh.2,400 at S_4. The variable unit production costs are Sh.1,500 at plants P_1 and P_3 and Sh.1,550 at plant P_2. The demand at S_1, S_2, S_3 and S_4 are 850, 640, 380 and 230 desks respectively while the plant capacity at plant P_1, P_2 and P_3 are 625, 825 and 450 desks respectively.

Required:

Using the transportation algorithm, determine the contribution to profit for the optimal allocation.
(17 marks)
(Total: 20 marks)

QUESTION FOUR

Alvis Kiptoo has budgeted that output and sales of his single product will be 100,000 units in the coming year. At this level of activity, his unit variable costs are budgeted at Sh.50 and his unit fixed costs at Sh.25. His sales manager estimates that the demand for the product would increases by 1000 units for every decreased of Sh.1 in unit selling price (and vice versa) and that at a unit selling price of Sh.200 demand would be nil.
Information about two price increases has just been received from suppliers: one is for materials (which are included in Alvis Kiptoo's variable costs) and one is for fuel (which included in his fixed costs). Their effect will be to increase both the variable and fixed costs by 20% each over the budgeted figures.
Alvis Kiptoo aims at maximizing profits from his business.
Required:

Calculate before the cost increases the budgeted contribution and profit at the budgeted levels of 100,000 units. (3 marks)

Calculate the level of sales at which profits would be maximized and the amounts of these maximum profits before the cost increases. (4 marks)

Show whether and by how much Alvis Kiptoo should adjust his selling price in respect to increases in:

Fuel costs. (2 marks)

Material costs. (2 marks)

b) Some businesses which supply two or more separate markets from a single source may decide to charge a higher price for sales to home markets than for export sales. The businesses may justify their pricing policy by stating that they need to earn foreign exchange from foreign markets and recover their research and development costs, plus production overheads against home demand.

Required:

Critically explain briefly the rationale for such a differential pricing policy. (5 marks)

Should earning of foreign exchange be a factor in a firm's pricing policy. (4 marks)

(Total: 20 marks)

QUESTION FIVE

Explain the advantages of using Value Added Statements (VAS) for interdivision for comparisons in decentralized firm. (8 marks)

ABC Lt. Is a manufacturing company that makes only three products P, Q, and R. Data for the period ended last month are as follows:

	P	Q	R
Units produced and sold	12,000	16,000	8,000
	Sh.	**Sh.**	**Sh.**
Sales price per unit	50	70	60
Direct material cost per unit	16	24	20
Direct labour cost per unit	8	12	8

Production overheads costs	**Total**	**Cost drivers**
	Sh.	
Machining costs	102,000	Machine hours
Production scheduling	84,000	Machine hours
Set-up costs	54,000	Number of production runs
Quality control	49,200	Number of production runs
Receiving materials	64,800	Number of components receipts
Packing materials	36,000	Number of customer orders
	390,000	

Information on the cost drier is given as follows:

	P	Q	R
Direct labour hours per unit	1	1½	1
Machine hours per unit	½	1	1½
Number of components per unit	3	5	8
Number of component receipts	18	80	64
Number of customer orders	6	20	10
Number of production runs	6	16	8

Required:

Using activity based costing (ABC) show the cost and gross profit per unit for each product during the period. (12 marks)

(Total: 20 marks)

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATIONS BOARD

CPA PART III
MANAGEMENT ACCOUNTING

JULY 2010 TIME ALLOWED: 3 HOURS

QUESTION ONE

A processing company, Timao Co. Ltd., is extremely busy. It has increased its output and sales from 12,900 kg in 1st quarter of the year to 17,300 kg in the 2nd quarter. Although demand is still rising, it cannot increase its output more than an additional 5% from its existing labour force, which is now at its maximum.

Data for its four products in 2nd quarter were:

	Product P	Product Q	Product R	Product S
Output (Kg)	4560	6960	3480	2300
Selling price (Sh. Per kg)	162	116.40	99.20	136.80
Costs (Sh. Per kg)				
Direct labour @ Sh.60 per hour)	19.60	13.00	9.90	17.00
Direct materials	65.20	49.00	41.00	54.20
Direct packaging	8.40	7.40	5.60	7.00
Fixed overhead (Absorbed on basis of direct labour cost)	39.20	26.00	19.80	34.00
	132.40	95.40	76.30	112.20

The Kagocho Company has offered to supply 2010 kg of product Q at a delivered price of 90% of Timao's Co. Ltd. Selling price. Timao Co. Ltd., will then be able to produce extra of product P instead of product Q to the plant's total capacity.

Required:

State with supporting calculations, whether Timao Co. Ltd should accept the Kagocho Company's offer. (15 marks)

Which would be the most profitable combination of subcontracting 2010kg of one product at a price of 90% of its selling price and producing extra quantities of another product up to the plant total capacity? Assume that the market can absorb the extra output. (5 marks)

(Total: 20 marks)

QUESTION TWO

"Control theory offers valuable insights into the design and operation of management accounting information systems, but only under circumstances where an organization's environment is stable and predictable and outcomes are clearly measurable."

Required:

Comment on the relevance and validity of this statement within the analysis or established control theory systems within a business organization. **(Total: 20 marks)**

QUESTION THREE

The Z division of XYZ Ltd., produces a component which it sells externally, and can also be transferred to other divisions within the organization. The division has set a performance target for the coming financial year of residual income of Sh.5,000,000. The following budgeted information relating to Z division has been prepared for the coming financial year.

Maximum production/sales capacity 800,000 units.
Sales to external customers: 500,000 units at Sh.37.
Variable cost per component Sh.25.
Fixed costs directly attributable to the division Sh.1,400,000.
Capital employed: Sh.20,000,000 with cost of capital of 13%

The X division of XYZ Ltd has asked Z division to quote a transfer price for units of the component.

Required:

Calculate the transfer price per component which Z division should quote to X division so that its residual income target is achieved. (6 marks) Explain why the transfer price calculated in (i) above may lead to sub-optimal decision making from the point of view of XYZ Ltd taken as a whole. (4 marks)

A manufacturer produces and sells two products, A and B. The unit variable cost is sh.12 and sh.8 for A and B respectively. A review of selling prices is in progress and it has been estimated that, for each product an increase in the selling price would result in a fall in demand of Sh.500 units per every Sh.1 increase in price and similarly a decrease of Sh.1 in price would result in an increase in demand of 500 units.

The current sales prices and sales demand are:-

	Price (Sh.)	Demand (Units)
A	30	15,000
B	58	21,000

Required:

Calculate the profit-maximizing price for reach product. **(10 marks)**

QUESTION FOUR

Muthothi Ltd. Operates a conventional stock control system based on re-order levels and Economic Order Quantities (EOQ). The various control levels were set originally based on estimates which did not allow for any uncertainty and this has caused difficulties because, in practice, lead times, demands and other factors do vary.

As part of a review of the system, typical stock item, part no. X 206, has been studied in detail as follows:

Data for Part No.X 206

Lead times (Days)	**Probability**	**Demand (units)**	**Probability**
15	0.2	5000	0.4
20	0.5	7000	0.6
25	0.3		

The company works for 360 days per year and it costs Sh.1,000 to place an order. The holding cost is estimated at Sh.0.025 for storage plus 10% opportunity cost of capital. Each unit is purchased at Sh.2. The re-order level for this part is currently 150,000 units and it can be assumed that the demands would apply for the whole of the appropriate lead-time.

Required:

Calculate the level of buffer stock implicit in a re-order level of 150,000 units. 5 marks)
Calculate the probability of stock-outs. (2 marks)
Calculate the expected annual stock-outs in units. (4 marks)
Compute the stock-out costs per unit at which it would be worthwhile raising the re-order level to 175,000 units. (3 marks)
Discuss the possible alternatives to a re-order level EOQ inventory system and their advantages and disadvantages. (6 marks)

(Total: 20 marks)

QUESTION FIVE

Watt Lovell Ltd. (WLL) is trying to decide whether or not to drill for oil on a particular site in North Eastern Kenya. The Chief Engineer has assessed the probabilities that there will be oil as follow, based on past experience.

Oil	0.2
No oil	0.8

It is possible for WLL to hire a firm of international consultants to carry out a complete survey of the site. WLL has used the firm many times before and has made the following estimates:

If there really is oil, then there is a 95% chance that the report will be favourable.
If there is no oil then there is only a 10% chance that the report will indicate that there is oil.

The following additional information is also provided:

The cost of drilling is Sh.10 million.
The value of the benefits if oil is found is Sh.70 million
The cost of obtaining information is Sh.3 million.

Required:

Advise the company on whether to acquire additional information from the consultants. (16 marks) Compute the value of imperfect information. (4 marks)

(Total:20marks)

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATION BOARD
CPA PART III
MANAGEMENT ACCOUNTING

PILOT PAPER **OCTOBER 1991**
Time Allowed: 3 hours

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show all your workings.

QUESTION ONE

a. There are arguments that cost-volume-profit analysis is at best abstract and theoretical, with no relationship to reality. Its supporters argue that it is not only a useful management tool but also can be practically applied. Comment, using appropriate examples, on the usefulness and limitations of cost-volume-profit analysis when applied in a multi-product entity
(10 marks)

b. Mululu company produces only two products, Machungwa and Ndimu. These account for 60% and 40% of the total sales value of M respectively. Variable costs (as a percentage of sales) are 60% for Machungwa and 85% for Ndimu. Total fixed costs are Shs 150,000. There are no other costs.

Required:

i. Mululu's break-even point in Shillings.

ii. Assuming that Mululu's total fixed costs increase by 30%, what amount of sales in Shillings would be necessary to generate a net profit of Shs 9,000?
(10 marks)
(Total: 20 marks)

QUESTION TWO

A company requiring a certain machine has the options of EITHER buying a new machine OR buying a second-hand machine which may be 1, 2 or 3 years old. These respective purchase costs are as follows:

Age at purchase	**Price of Machine**
Years	*Shs 000*
New	40
One-old	31
Two-old	20

Three 11

You are also given the following information with regard to repair and maintenance costs for each machine and their respective scrap values.

Age of Machine	**Annual repair and Maintenance costs**	**Expected Scrap Value at Date of resale**
Years	*Shs 000*	*Shs 000*
1	3	28
2	4	18
3	6	40

4 9 4

Required:

a. Advise the management on the age at which the machine should be bought and when it should be replaced. (15 marks)

b. What is the role of abandonment value in decision making over projects? (5 marks)

(Total: 20 marks)

QUESTION THREE

Ben Ltd. has run a series of regressions as a first step towards designing a flexible budget for manufacturing overhead. With regard to machine hours and units of output the following results have been obtained:

Regression 1:Overhead costs as related to machine hours.

	Coefficient	Standard Error
Constant	-32,657	
Machine hours	16.57	4.32

$r^2 = 0.77$

$S_e = 3,456$

Regression 11: Overhead costs as related to units of output.

	Coefficient	Standard Error
Constant	17,865	
Machine hours	13.76.57	3.87

$r^2 = 0.61$

$S_e = 3,973$

Regression 111: Overhead costs as related to machine hours and units of output.

	Coefficient	Standard Error
Constant	-15,373	
Machine hours	7.37	3.31
	10.44	4.81

$r^2 = 0.79$

$S_e = 1,623$

The correlation matrix shows a 0.86 relationship between machine hours and units of output.

You are further informed that there were no data gaps or serious outlier problems.

Required:

a. Explain what is meant by the term "Computed t-value". (3 marks)

b. Determine the computed t-values for all the three regression runs. (3 marks)

c. Explain the meaning of r^2 in regression I and II and R^2 in regression III (4 marks)

d. Explain the meaning of the negative intercept in regressions I and II. (4 marks)

e. Critically evaluate each of the three regressions and on the basis of this evaluation advise management on which of them should form the basis for flexible budgeting. (6 marks)

(Total: 20 marks)

QUESTION FOUR

A newly incorporated company is developing compounds for use in the Agricultural Sector. The product codes for the three products are X1, X2 and X3 and the relevant information is summarised below:

i Chemical constituents: Percentage make up per tonne.

	Nitrate	Phosphate	Potash	Filler
X1	10	10	20	60
X2	10	20	10	60
X3	20	10	10	60

ii. Input prices per tonne:

	Shs
Nitrate	150
Phosphate	60
Potash	120
Filler	10

iii. Maximum available input in tonnes per month

Nitrate	1,200
Phosphate	2,000
Potash	22,00
Filler	No limit

iv. Selling Prices of fertilizer (per tonne)

	Shs
X1	83
X2	81
X3	81

The manufacturing costs excluding raw materials, are estimated at Shs 11 per tonne.

Required:

a. Formulate the above data into a linear programme with the objective as the maximisation of contribution. (3 marks)

b. Define X4, X5 and X6 as the slack variables for X1, X2 and X3 respectively and explain the meaning of these slack variables. (3 marks)

c. Construct the initial simplex tableau and indicate which will be the "entering variable" and the "leaving variables" in the first iteration. (5 marks)

d. The final matrix of the simplex solution is given below:

Basic Variables	X1	X2	X3	X4	X5	X6	Solution
X1	1	0	3	20	-10	0	4000
X2	0	1	-1	-10	10	0	8000
X6	0	0	-0.4	-3	1	1	600
Z	0	0	22	170	40	0	284000

Interpret the matrix and specify its significance with respect to the new product development. (9 marks)

(Total: 20 marks)

QUESTION FIVE

Consider a project which requires the following activities:

Activity	Preceding Activity	Activity Time (days)		Total Cost (Shs)	
		Norma	Crash	Normal	Crash
A	—	16	14	400	480
B	—	13	12	340	360
C	A	15	13	380	500
D	A	14	12	200	260
E	A	13	11	260	280
F	C	13	12	310	380
G	D	15	10	430	580
H	B,D,E	15	9	280	370
I	H	12	11	400	420
J	F,G,I	13	11	260	320

Required:

a. Using normal activity durations and costs:

i. Draw the critical path network for the project and determine the critical path and its duration.
ii. Calculate the cost of the project.
ii. Calculate the total floats of the non-critical activities. (15 marks)

b. It is required to reduce the project duration by one day at the minimum possible cost:

i. Which activity should be crashed?
ii. What will be the new cost of the project? (5 marks)

(Total: 20 marks)

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATION BOARD
CPA PART III
MANAGEMENT ACCOUNTING

Tuesday, 3 December 1991 **Time Allowed: 3 hours**

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show all your workings.

QUESTION ONE

The financial controller of Monica Industries Ltd. has determined the major activities involved in the preparation of the annual financial statements. These activities are shown below:

Activity		Duration (in weeks)	Immediately preceding activity
A:	Check outstanding purchase invoices	2	—
B:	Close cash book and post to general ledger	2	—
C:	Complete sales invoices	3	—
D:	Check accrued salaries and wages	1	—
E:	Count and verify stock levels	4	—
F:	Calculate stock valuation	2	E
G:	Close purchase ledger and post to general ledger	1	A
H:	Close sales ledger and post to general ledger	1	C
I:	Post salaries and wages to general ledger	1	D
J:	Close general ledger	1	B, G, H, I
K:	Prepare trial balance	3	J
L:	Apply closing adjustment to trial balance	2	F, K
M:	Re-open general ledger for new period	1	N
N:	Prepare final accounts	6	L
O:	Submit final accounts to board for approval	1	N

Required:

a. Draw a network to represent the inter-relationships between the above activities and insert the latest event times throughout. (6 marks)

b. Assuming that preparation of the account starts at the beginning of the first week in January, calculate the earliest date the final accounts can be presented to the board. (4 marks)

c. The end of week 13 is the end of the financial ear and the Board has indicated that the final accounts must have been approved by that time. The financial controller has suggested that to achieve this, activities A, C and D can all be completed by the end of December and also that the verification of stocks levels, E, could be done in only two weeks of additional labour available.

Comment on these suggestions (6 marks)

d. Explain how management may use network systems in the design of multi-stage production process. (4 marks)

(Total: 20 marks)

QUESTION TWO

The Oil Kenya Company currently sells three grades of petrol, regular, premium and `regular extra' which is a mixture of regular and premium. Regular premium is advertised as being "at least 50 percent premium". Although any mixture containing 50 per cent or more premium fuel could be sold as `regular extra' it is less costly to use exactly 50 per cent. The percentage of premium fuel in the mixture is determined by one small valve in the blending machine. If the valve is properly adjusted, the machine provides mixture which is 50 percent premium and 50 percent regular. Assume that if the valve is out of adjustment the machine provides a mixture which is 60 percent premium and 40 percent regular. Once the machine is started it must continue until 100,000 litres of `regular extra' have been mixed.

The following data is available:

	Shs
Cost per litre—premium	3.20
Cost per litre—regular	3.00
Cost of checking valve	800.00
Cost of adjusting valve	400.00

Subjective estimates of the probabilities of the valve's condition are estimated to be:

Event	Probability
Valve in adjustment	0.7
Valve out of adjustment	0.3

Required:

(a) The expected cost of checking the valve and adjusting it if necessary. (5 marks)

(b) The conditional cost of not checking the valve when it is out of adjustment (5 marks)

(c) Using the criterion of minimum expected cost, calculate the probability at which there will be need to check if the valve is out of adjustment. comment on the results. (5 marks)

(d) Comment on the results obtained in (a) and (b) above. (5 marks)

(Total: 20 marks)

3. (a) Many writers in process costing advocate ignoring normal spoilage in the computation of equivalent units.

Discuss the propriety of ignoring normal spoilage in the computation of equivalent units for developing per unit costs. consider in the answer the possibility that the point of inspection, and hence the identification of spoiled units, occurs:

i. at the end of a process;
ii at the beginning of a process; (10 marks)

(b) The Mathare Company had 8,000 units of work-in-process in its Department M as on 1 August 1991, which were 50% complete as to conversion costs. Materials are introduced at the beginning of the process. During August, 17,000 units were started, 18,000 units were completed and there were 2,000 units of normal spoilage. Mathare Company had 5,000 units of work-in-process at 31 August 1991, which were 60% complete as to conversion costs.

Under Mathare's cost accounting system, spoiled units reduce the number of units over which total cost can be spread.

Required:

Compute, using the weighted average method, the equivalent units for August. (10 marks)
(Total: 20 marks)

QUESTION FOUR

Westland Manufacturing Company Limited manufactures several different styles of children lunch boxes. Management estimates that during the months of July, August and September or 2014, the company will be operating at 80 percent of normal capacity. Because the company desires a higher utilisation of plant capacity, it will consider a special order.

Westland has received special-order inquiries from two companies. The first inquiry is from Sigh Plastic Limited (SPL) which would like to market a lunch box case similar to one of Westland's boxes. The SPL lunch box would be marketed under SPL's own label. SPL has offered Westland Shs 57.50 per box for 20,00 boxes to be supplied by 1 October 2014. Cost data are given below for the Westland lunch box that is similar to the lunch box desired by SPL.

	Shs
Regular selling price per unit	90.00
Cost per unit:	
Raw materials	25.00
Direct labour, 0.5 hours at Shs 60	30.00
Overhead, 0.25 machine hours at Shs 40	10.00
Total per unit	65.00

According to the specifications provided by SPL, the special order box requires less expensive raw materials. Consequently, the raw materials will only cost Shs 22.50 per box. Management has estimated that the remaining costs, labour time and machine time will be same as those for the Westland lunch box.

The second special order submitted by Odogo Foods Limited, was for 7,500 lunch boxes for Shs 75.00 per box. These boxes would be marketed under the Odogo label and would have to be supplied by October 2014. The Odogo lunch box is different from any lunch box in the Westland's line. The estimated cost per unit of this box are as follows:

	Shs
Raw materials	32.50
Direct labour, 0.5 hours at Shs 60	30.00
Overhead, 0.5 machine-hours at Shs 40	20.00
Total cost per unit	82.50

In addition, Westland will incur Shs 15,000 additional set-up costs and will have to purchase a Shs 25,000 special device to manufacture these boxes. The device will be discarded once the special order has been completed.

The Westland manufacturing capabilities are limited to the total machine-hours available. The plant capacity under normal operations is 90,000 machine-hours per year or 7,500 machine-hours per month. The budgeted fixed overhead for 2014 amounts to Shs 2,160,000. All manufacturing overhead costs (fixed and variable) are applied to production on the basis of machine-hours, at Shs 40 per hour.

During July, August and September, Westland will be able to use all of its excess capacity to work on special
orders. Management does not expect any repeat sales to be generated from either special order. Company practice precludes Westland from sub-contracting any portion of an order when special orders are not expected to generate repeat sales.

Required:

Should Westland Manufacturing Company Limited accept either special order?
Justify your answer and show calculations. **(20 marks)**

QUESTION FIVE

Discuss the meaning and applications of any FOUR of the following concepts in management accounting.

a. Program planning and budgeting systems.
b. Zero-base budgeting.
c. Just-in-Time Production.
d. Game Theory.
e. Information asymmetry. **(20 marks)**

MODEL ANSWERS TO PAST CPA EXAMINATION PAPERS

MODEL ANSWERS TO THE CPA PAPER JUNE 2013

QUESTION ONE

Methods used to analyses uncertainty in CV-P

analysis Sensitivity analysis

This is what if analysis that considers the effect of a marginal change on each of the relevant variables to the decision.

Point estimate of probability

This approach requires a number of different values for each of the uncertain variables to be selected. Usually three values are selected: these are the worst possible, most likely and best possible outcomes. For each of these values a probability of occurrence is estimated. The expected values and standard deviation can then be computed.

Continuous probability distribution

(e.g. normal distribution)
The uncertain variables can be estimated as a continuous probability distribution. Estimates are made of the mean and standard deviation, which can then be used to compute expected profit, standard deviation of profits and probability that the company will break even.

Simulation analysis

This is a method of analyzing a system by experimentally duplicating its behaviour. Simulation is used where analytical techniques are not available or would be very complex.

b) (i)**Current production: Ti only**

contribution = 10 – 6 = Sh.4
E (Profit) = 4 x 110,000 – 400,000 = Sh.40,000

δ profit = 4 x 10,000 = Sh.40,000

$$\text{BEP units} = \frac{\text{Total fixed costs}}{\text{Contribution margin}} = \frac{400{,}000}{4}$$

= 100,000 units

BEP sh. = 100,000 (10) = <u>Sh.1,000,000</u>

ii) **Coefficient of variation C.V**

$$C.V = \frac{\delta}{E(\text{profit})} = \frac{40{,}000}{40{,}000} = 1$$

(i) Proposed production: Ti and T2.

Expected profit = 4(85,000) + 3(50,000) – (400,000 + 50,000)

= <u>Sh.40,000</u>

$$\delta_{\Pi} = \sqrt{CM_1^2\ \delta_1^2 + CM_2^2\ \delta_2^2 + 2r_{12} + CM_1\ CM_2\ \delta_1\ \delta_2}$$

$$= \sqrt{4^2\ (8000)^2 + 3^2\ (5000)^2 + 2(-0.9)(4)(3)(8000)(5000)}$$

$$= 19621.4$$

$$B.E.P\text{ units} = \frac{\text{Total fixed costs average}}{\text{contribution margin}}$$

$$AV.CM = 4\left(\frac{85}{135}\right) + 3\left(\frac{50}{135}\right) = 3.62962963$$

$$\text{BEP units} = \frac{400{,}000 + 50{,}000}{3{,}62962963} = 123980 \text{ units}$$

$$T_1 = \frac{85}{135}(123980) = 78061 \text{ units}$$

$$T_2 = \frac{50}{135}(123980) = 45919 \text{ units}$$

in sh	T1		sh
BEP Sh =	78061 x Sh10	=	780,610
	T2		
BEP Sh =	45919 *x* 8	=	367,352
Total BEP Sh			1,147,962

(ii) Coefficient of variation (C.V)

$$C.V = \frac{\delta}{E(\mathit{profit})} = \frac{19621.4}{40{,}000} = \underline{\underline{0.49}}$$

(iii) Since the mean demand is greater than breakeven point then BEP is not a good criteria in making the decision. We should use the coefficient o f variation.

The decision therefore is to add T_2 to the production schedule since it r educes the coefficient of variation from 1 to 0.49.

QUESTION TWO

Overhead absorption is the technique of attributing departmental overhead costs to a cost unit.. Traditionally, the basis of overhead absorption was the number of labour hours expected within the budget period and this was then used to calculate an absorption rate per labour hour. This was then used to attribute costs to the cost units on the basis of the number of labour hours used to produce the cost unit.

Alternative bases of apportioning exist such as the number of machine hours or the percentage of particular elements of prime costs incurred in respect of cost units. If the method of manufacture is machine intensive for example, it is more realistic to absorb the overhead cost on the basis of the number of machine hours instead of the number of labour hours.

A further development is to divide the overheads into those costs, which are labour related, and those, which are machine hour, related and apply a separate absorption rate to each part of the overhead cost. This is the use of multiple rates similar to the principle of activity bases costing (ABC).

ABC is based on the principle that activities cause costs and therefore the use of activities should be the basis of attributing costs to cost units. Costs are identified with particular activities and the performance of those activities is linked with products.

b) (i) Incremental budgeting uses the previous year's budget as the starting point for the preparation of next year's budget. It assumes that the basic structure of the budget will remain unchanged and that adjustments will be made to allow for changes in volume, efficiency and price levels. The budget is therefore concerned with increments to operations that will occur during the period and the focus is on existing uses of resources rather than considering alternative strategies for the future budget period. Incremental budgeting suffers from the following weaknesses:

It perpetuates past inefficiencies

There is insufficient focus on improving efficiency and effectiveness.

The resource allocation tends to be based on existing strategies rather than considering future strategies. It tends to focus excessively on the short term and often leads to arbitrary cuts being made in order to achieve short-term financial targets

(ii) The answer should stress that:; The

focus is on managing activities

The focus is on the resources that are required for undertaking activities and identifying those activities resources that are un-utilized or which are insufficient to meet the requirements specified in the budget.

Attention is given to eliminating non-value-added activities.

The focus is on the control of the causes of costs (i.e. the cost drivers).

QUESTION THREE

For (a) and (b)

	Fixed Fee	Share of Revenue to Joan	Share of Costs Borne By Joan	Net profit to Joan	Joint Prob	Expected Monetary value
Contract A	**Sh. 'm'**	**Sh. 'm'**	**Sh. 'm'**	**Sh 'm'**		**Sh. 'm'**
HD – BC	10	100	-	110	0.06	6.6
HD – HC	10	100	-	110	0.04	4.4
MD – BC	10	0	-	10	0.18	1.8
MD – HC	10	0	-	10	0.12	1.2
LD – BC	10	0	-	10	0.36	3.6
LD – HC	10	0	-	10	0.24	2.4
					1.0	20.0
Contract B						
HD – BC	0	200	20	180	0.06	10.8
HD – HC	0	200	70	130	0.04	4.4
MD – BC	0	50	20	30	0.18	5.4
MD – HC	0	50	70	-20	0.12	-4.8
LD – BC	0	10	20	-10	0.36	-3.6
LD - HC	0	10	70	-60	0.24	-19.2
						-7.0
Contract C						
HD – BC	0	600	50	550	0.06	33.0
HD – HC	0	600	150	450	0.04	18.0
MD – BC	0	150	50	100	0.18	18.0
MD – HC	0	150	150	0	0.12	0
LD – BC	0	30	50	-20	0.36	-7.2
LD – HC	0	30	150	-12	0.24	-28.8
						33.0

Note
HD = High demand
MD = Moderate demand
LD = Low demand
BC = Budgeted Cost
HC = High Cost

b) From the solution above the
EMV for Contract A = Sh.20 million
EMV for Contract B = Sh.1 million
EMV for Contract C = Sh.3 million
Joan should choose contract C, given the objective of maximizing expected monetary value.

c) Three sources of information Joan might use are:
Her own track record of actual production and marketing relative to amounts
The tract record of the film station parts in "the rise and fall of cock" film running over budget
The geographical location of the technical nature of the stunts set the film. Examine the tract record costs against budgeted costs for films in the same location and of similar difficulty.

QUESTION FOUR

(a) (i)

Month	Machine Hours X	Maintenance Cost Y	XY	X^2	Y^2
1	800	350	280,000	640,000	122,500
2	1,200	350	420,000	1,440,000	122,500
3	400	150	60,000	160,000	22,500
4	1,600	550	880,000	2,560,000	302,500
Sum	4,000	1,400	1,640,000	4,800,000	570,000

High Low method

	X	Y
Highest point	1,600	550
Lowest point	400	150
Difference	1,200	400

$$b = \frac{400}{1200} = 0.33$$

$$\hat{Y} = a + bx$$

Substitute Highest point

$$550 = a + 0.33\,(1{,}600)$$

$$a = 17$$

The cost function is $\hat{Y} = 17 + 0.3X$.

Regression analysis

(ii)

$$b = \frac{n\varepsilon x\,y - \varepsilon x\,\varepsilon y}{n\varepsilon x^2 - (\varepsilon x)^2} = \frac{4(1{,}640{,}000) - 4{,}000(1{,}400)}{4(480{,}000) - (4{,}000)^2}$$

$$= 0.3$$

$$a = \frac{\Sigma Y}{n} - b\frac{\varepsilon x}{n} = \frac{1{,}400}{4} - 0.3\left(\frac{4{,}000}{4}\right) = 50$$

The function is $\hat{Y} = 50 + 0.3x$

b) (i)

$\hat{Y} = 50 + 0.3X$

If $x = 900$

$\hat{Y} = 50 + 0.3\,(900) = \underline{\underline{320}}$ (in Sh. '000')

$$\hat{X} = a^1 + b^1 Y$$

$$b^1 = \frac{n\Sigma XY - \Sigma X \Sigma Y}{N\Sigma Y^2 - (\Sigma Y)^2} = \frac{4(1,640,000) - 4000(1400)}{4(570,000) - (1400)^2}$$

$$= \frac{960,000}{320,000} = 3$$

(ii)

$$a^1 = \frac{\Sigma x}{n} - b^1 \frac{\Sigma Y}{n}$$

$$= \frac{4,000}{4} - 3\frac{1,400}{4} = \underline{-50}$$

$$\hat{X} = -50 + 3Y$$

If Y = 400

$$\hat{X} = -50 + 3(400) = 1,150 \text{ machine hours.}$$

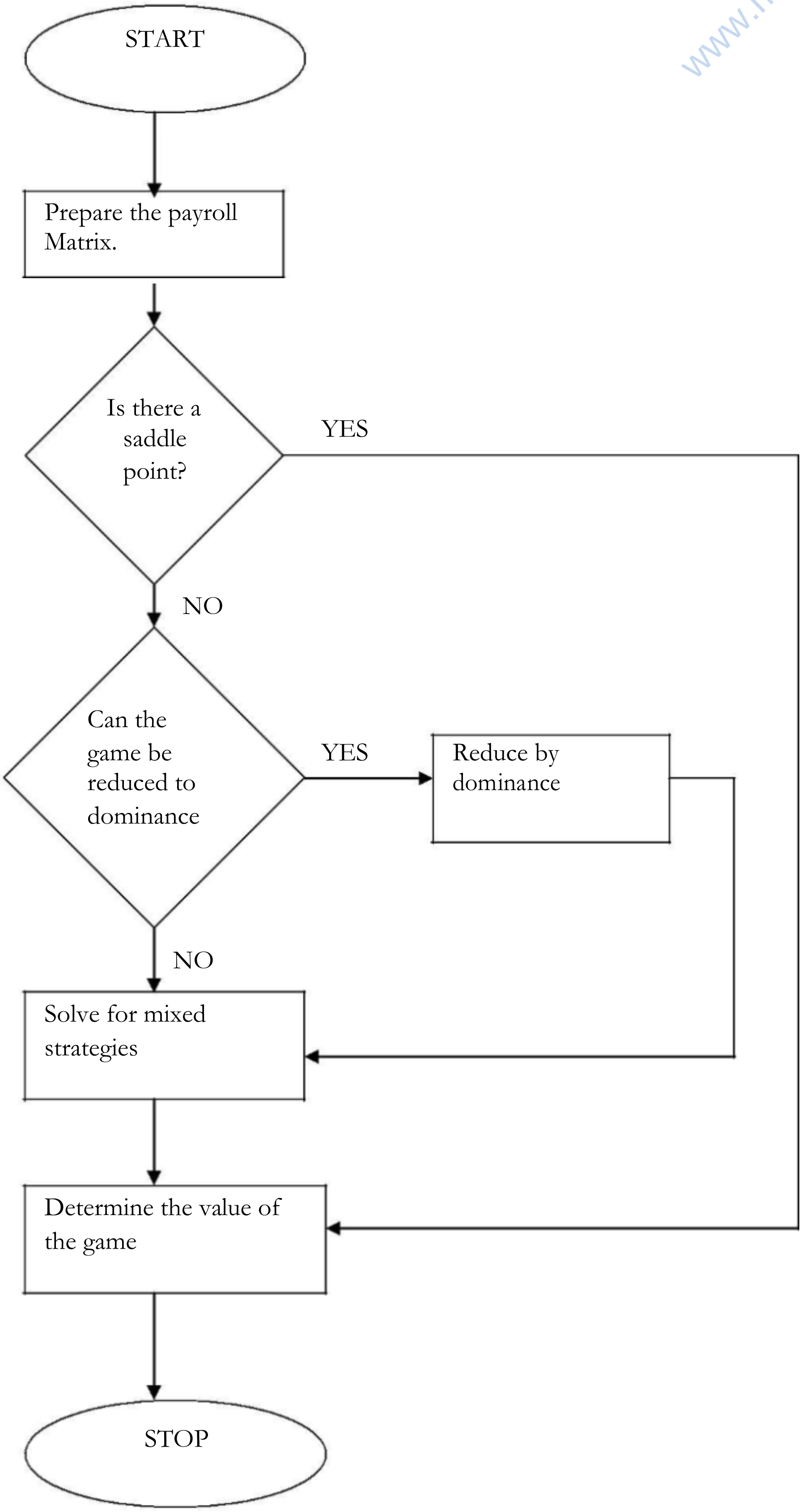
$$\hat{Y} - t_c s_e \le Y \le \hat{Y} + t_c s_e$$

$$320 - 2.7764(63.25) \le Y \le 320 + 2.27764(63.25)$$

$$144.39 \le Y \le 495.6$$

We are 95% confident that maintenance cost next period will lie between Sh.144,390 and Sh.495,600

QUESTION FIVE



b) (i)Let A_1 be company A undertakes a vigorous market campaign.

A_2 be Company A does not run the market campaign

B_1 = Company B invests Sh.25 m in Research and Development (R & D)

B_2 = Company B invests Sh.50 m in R & D

B_3 = Company B invests Sh.80 m in R & D

Game Matrix

Company B

		B_1	B_2	B_3b	
Company A_1	A_1	0.79	0.76	0.73	0.73
	A_2	0.76	0.74	0.72	0.72
	Max	0.78	0.76	0.73	

The game has a saddle point occurring at strategy A_3 . These are the optimal strategies with a game value of 73% of the market share of A implying that B will get a market share of 27%.

(ii) Payoff matrix

	B_1	B_2	B_3
A_1	(202, 38)	(193, 22)	(184, 1)
A_2	(228, 47)	(222, 28)	(216, 4)

<u>A's Reasonings</u>

If B plays strategy B_1 , then A should play strategy A_2 to maximize his winnings.

If B plays strategy B_2 , then A should play strategy A_2..

In all cases A plays strategy A_2 B's reasonings.

If A plays strategy A_1 then B should play strategy B_1 to maximize his profits.

If A plays strategy A_2 then B should play strategy B_1 .

The saddle point occurs when A plays strategy A_2 and B plays strategy B_1 .

The profit contribution will be:
Company A: Sh.<u>228</u> million
Company B: Sh<u>. 47</u> million

KENYA ACCOUNTANTS AND SECRETARIES NATIONAL EXAMINATIONS BOARD

CPA PART III

MANAGEMENT ACCOUNTING

DECEMBER 2011

QUESTIONONE

D = 475
C_o = Sh.50
C_h = Sh.12.50 + 10% (250) = Sh.37.50

Lead time demand	0	1	2	3	4	5	6	7	8
Probability	0.02	0.04	0.12	0.16	0.20	0.16	0.16	0.10	0.04

$$EOQ = \sqrt{\frac{2 X 475 X 50}{37.50}} = 35.59 \text{ units} \approx 36 \text{ units}$$

Safety stock	Stockholding Cost	Stock out cost		Total cost
0	0	1 x 0.04 x 20 x 13 =	10.4	
		2 x 0.12 x 20 x 13 =	48	
		3 x 0.16 x 20 x 13 =	124.8	
		4 x 0.20 x 20 x 13 =	208	
		5 x 0.16 x 20 x 13 =	208	
		6 x 0.16 x 20 x 13 =	249.6	
		7 x 0.10 x 20 x 13 =	182	
		8 x 0.04 x 20 x 13 =	83.2	
			1114	1114
1	37.50	1 x 0.12 x 20 x 13 =	31.2	
		2 x 0.16 x 20 x 13 =	83.2	
		3 x 0.20 x 20 x 13 =	156	
		4 x 0.16 x 20 x 13 =	166.4	
		5 x 0.16 x 20 x 13 =	208	
		6 x 0.10 x 20 x 13 =	156	
		7 x 0.04 x 20 x 13 =	72.8	
			873.6	911.1
2	75	1 x 0.16 x 20 x 13 =	41.6	
		2 x 0.20 x 20 x 13 =	104	
		3 x 0.16 x 20 x 13 =	124.8	
		4 x 0.16 x 20 x 13 =	166.4	
		5 x 0.10 x 20 x 13 =	130	
		6 x 0.04 x 20 x 13 =	629.2	704.2
3	3 x 37.50 = 112.5	1 x 0.20 x 20 x 13 =	52	
		2 x 0.16 x 20 x 13 =	83.2	
		3 x 0.16 x 20 x 13 =	124.8	
		4 x 0.10 x 20 x 13 =	104	
		5 x 0.04 x 20 x 13 =	52	
			416	528.5
4	4 x 37.5 = 150	1 x 0.16 x 20 x 13 =	41.6	

		2 x 0.16 x 20 x 13 = 83.2 3 x 0.10 x 20 x 13 = 78 4 x 0.04 x 20 x 13 = 41.6 244.4	394.4
5	5 x 37.5 = 187.5	1 x 0.16 x 20 x 13 = 41.6 2 x 0.10 x 20 x 13 = 52 3 x 0.04 x 20 x 13 = 31.2 124.8	312.3
6	6 x 37.5 = 225	1 x 0.1 x 20 x 13 = 26 2 x 0.04 x 20 x 13 = 20.8 46.8	271.8
7	7 x 37.5 = 262.5	1 x 0.04 x 20 x 13 = 10.4	272.9
8	8 x 37.5 = 300	0	300

The optimal safety stock is 6 units. The reorder level will be:

ROL = Cycle Stock + safety stock

$$= \frac{DL}{Q} + S$$

$$= \frac{475 \times 3}{36} + 6 = 45.58 \approx \underline{46 \text{ units}}$$

b) Total annual relevant Cost (TRC)

$$TRC = \frac{D}{Q} Co + \left[\tfrac{1}{2} Q + S \quad C_h \right]$$

$= 475 (50) + (½(36) + 6) 37.50 =$ **Ksh.1,559.72**

QUESTION TWO

(a) Produce (a) of A (b) units of B, (c) units of C (d) units of product D and (e) units of products E each week

Calculate the unit contribution of each product.

A: unit contribution is 40 – {(2.10 x 6) + (3.0 x 1.0) + (1.3 x 3) + 8.0 x 0.5)}
= Shs.16.50 per unit.

B: unit contribution is 42 – {(2.10 x 6.5) + (3.0 x 0.75) + (1.3 x 4.5) + (8 x 0.5)}
= Shs.16.25 per unit

C: unit contribution is 44 – {(2.10 x 6.10) + (3.0 x 1.25) + (1.3 x 6) + (8 x 0.5)}
= Shs.15.64 per unit

D: unit contribution is 48 – {(2.10 x 6.1) + (3.0 x 1) + (1.3 x 6) + (8 x 0.75)}
= Shs.18.39 units

E: unit contribution is 52 – {(2.10 x 6.4) + (3.0 x 1) + (1.3 x 4.5) + (8.0 x 1)}
= Shs.21.71 per unit

Maximize total weekly contribution, Shs. P where;

P = 16.5a + 16.25b + 15.64c + 18.39d + 21.71e Shs/Week

Subject to:
Materials: 6.0a + 6.5b + 6.1c + 6.1d + 6.4e ≤ 35,000 kg/week
Forming: 1.0a + 0.75b + 1.25c + 1.0d + 1.0e ≤ 6,000 hours/week
Firing: 3.0a + 4.5b + 6.0c + 6.0d + 4.5e ≤ 30,000 hours/week
Packing: 0.5a + 0.5b + 0.5c + 0.75d + 1.0e ≤ 4,000 kg/week
Non-negativity: a, b, c, d, e, ≥ 0

(b) (i)The optimum weekly production plan is to produce 3,357 units of product A,2,321 units of product E and none of B, C or D. The resulting maximum weekly contribution is Kshs.105,791.

(ii) There is spare capacity of 321 hours per week on the forming process and 9,482 hours per week on the firing process. All raw materials and all packing time are used up. Raw materials and packing time are the limiting constraints in the problem.

(iii) The shadow price is the amount, which would be added to the value of the total weekly contribution if one extra unit of a limiting resource were made available that:

No additional costs were incurred.
The resource remains limiting

Alternatively the shadow price is the amount by which the total weekly contribution would fall if the provision of a limiting resource was reduced by one unit.

From the table, we can see the shadow price for raw materials is Ksh.2.02 per kilogram and for packing time is Kshs.8.81 per hour. One additional kilogram of raw material will generate an extra Kshs.2.20 of contributions, subject to the conditions above. One extra hour of packing time will, similarly generate additional shs.8.81 of contribution.

The additional product would also have to be made at the expense of one or both of the other products, since all raw materials and packing time are currently used.

Unit contribution of the new product

= 50 – {(2.1 x 1.6) + (3.0 x 1) + (1.3 x 5) + (8 x 1)}
= <u>Kshs.19.9</u>

If one unit of this new product was made, the provision of raw materials for the other two products would effectively be reduced by 6 kilogrammes, this would reduce the current total contribution by 6 x Kshs.2.02 = Kshs.12.12. Similarly, the available packing time would be reduced by 1 hour, this reduces the total contributions by Kshs.8.81. The total reduction is the weekly contribution which would be:

Kshs.12.12 + Kshs.8.81 = <u>Kshs.20.93</u>

The gain from one unit of the new product is shs.19.90 therefore, if one unit of the new product is made, there will be a net loss of

Shs.19.90 – Shs.20.93 = Shs.1.30. the proposition is not worthwhile.

QUESTION THREE

a) Four ways in which competitive situation can be classified are:
Number of competitors e.g. two persons and N – persons game.
Nature of payoff e.g. zero sum and non zero sum games
Number of strategies available to each player e.g. 2 x 2 game, 2 x 3 game etc.
Amount of information the competitors have e.g. games with perfect information or Games with imperfect information.

b) (i)

Game matrix

Kamau		Njoroge N_1	N_2	Min
	K_1	500	-500	
	K_2	0	1,000	0
	Max	500	1,000	

Where:

K_1 is Kamau does not expand
K_2 is Kamau expands
N_1 is Njoroge does not expand
N_2 is Njoroge expand

Note:

Since there is no entry that simultaneously a maximum of the now minima and a minumum of the column maxima, then a saddle point does not exist. There is therefore no pure strategy.

(ii) Let K be proportion of time Kamau does not expand 1 – K is the proportion of time Kamau expands.

$500\ K_1 + 0\ (1 – K_1) = -500\ K_1 + 1{,}000\ (1 – K_1)$

$500K_1 + = -\ 1{,}500\ K_1 + 1{,}000$

$2{,}000\ K_1 = 1{,}000$

$K_1 = \underline{\underline{0.5}}$

$K_2 = \underline{0.5}$

Let N_1 be proportion of time Njoroge does not expand
$1 – N_1$ be proportion of time Njoroge expands.

$500N - 500\ (1 - N_1) = O_1N + 1{,}000\ (1 – N_1)$

$1{,}000\ N_1 - 500 = 1{,}000 – 1{,}000\ N$

$2{,}000\ N_1 = 1{,}500$

$$N_1 = \frac{1,500}{2,000} = \underline{0.75}$$

$$N_2 = \underline{0.25}$$

Strategies	Joint probability	Payoff	Weighted Pay offs
K_1 N_1	0.5 (0.75) = 0.375	500	187.5
K_1 N_2	0.5 (0.25) = 0.125	-500	-62.5
K_2 N_1	0.5 (0.75) = 0.375	0	0
K_2 N_2	0.5 (0.25) = 0.12	1,000	125
			250

Kamau would expect to increase his profits by Sh.250 per day on average while Njoroge expects to lose Sh.250 per day on average.

QUESTIONFOUR

(a) The minimum price of Mega Techniques Ltd is the price which reflects the relevant costs(opportunity costs) of the work. These are established as follows:

Cost of original machine. Past costs are not relevant, and the shs.8, 280,000 of costs incurred should be excluded form the minimum price calculation. It is necessary, however, to consider the alternative use of the direct materials (opportunity cost), which would be forgone if the conversion work is carried out.

	Shs.
Type P	
Revenue from sales as scrap (note 1)	540,000
Type Q	
Revenue from sales as scrap,	
Minus the additional cash costs necessary to	
Prepare it for sale (360,000 – {120 x 270}) note 1	327,600
Type R	
Cost of Disposal if the machine is not converted	
(a negative opportunity cost) note 2	108,000
Total opportunity costs of materials Types P, Q, R	885,600

By agreeing to the conversion of the machine Mega Techniques Ltd would therefore lose net revenue of Shs.885, 600 from alternative use of these materials.

Notes

1. Scrap sales would be lost if the work for Zimwi systems Limited goes ahead.
2. These costs would be incurred unless the work goes ahead.

The cost of additional materials for conversion is shs.576, 000 but this is an historical cost. The relevant cost of close materials is the Shs.684, 600 that would be spent on new purchases if the conversion is carried out. If the work in stock would be unavailable goes ahead, the materials in stock would be unavailable for production of the other machine mentioned item (2) of the question and so the extra purchases of khs.684, 000 would be needed.

Direct labour in Department X and Y is a fixed cost and the labour force will be paid regardless of the work they do or not do. The cost of labour for conversion in Department Y is not a relevant cost because the work could be done without any extra cost the company.
In Department X, however, acceptance of the conversion work would be oblige the company to divert production from other profitable jobs. The minimum contribution from using Department X labour must be sufficient to cover the cost of the labour and variable overheads and then make an additional Shs.2.50 in contribution per direct labour hour.

Department X – costs for direct labour hours spent in conversion;

3 men x 4 weeks x 27,000	Shs.324, 000
Variable overhead cost:	
Shs.324, 000 x 20%	Shs. 64,800
Contribution forgone by diverting	
Labour from other work	
Shs.2.5 per shs.1 of labour cost	
= 324,000 x 150%	Shs.486, 000

Variable overheads in Department Y are relevant costs because they will only be increased if production work is carried out (It's assumed that if the work done is idle, no variable overheads would be manned).

Department Y = 20% of (1 man x 4 weeks x shs.21,600) = 86,400

If the machine is converted, the company cannot sell the designs and specifications to the overseas companyhs.270,000 is relevant (opportunity) cost of accepting the conversion order.

Fixed overhead, being manly undercharged regardless of what the company decided to do should be ignored because they are not relevant (incremental) costs. The additional cost of supervision should, however, be included as a relevant cost of order because the shs.162,000 will not be spent unless the conversion work is done.

The money received from Pawa Limited should be ignored and should not be deducted in the calculation of the minimum price. Just as costs incurred in the past are irrelevant to a current decision about what to do in the future, revenue collected in the past are also irrelevant.

ESTIMATED MINIMUM PRICE FOR THE MACHINE

	Shs.	**Shs.**
Opportunity cost of using the direct		
Material types P, Q, R		885,600
Opportunity cost of additional materials for		684,000
Conversion		
Opportunity cost of work in Department X:		
Labour	324,000	
Variable overhead	64,800	
Contributions forgone	486,000	874,800
Opportunity cost: sale of design & specifications		270,000
Incremental costs:		
Variable production overheads in Department Y		86,400
Fixed production overheads		162,000
Minimum price		2,962,800

(b) (i) cost behavior patterns are known.

(ii) The amount of fixed costs, unit variable costs, sales price and sales demand are known with certainty.

The objective of decision-making in the short-run is to minimize "satisfaction" which is often regarded

MODEL ANSWERS TO THE CPA PAPER SET DECEMBER 2010

QUESTION ONE

Feed forward control

Describes a control system in which deviations in the system are anticipated in a forecast of future results, so that 'corrective action' can be taken in advance of any deviations actually happening while on the other hand, **Feedback control system** is information about actual achievements. In business organization, it is information about actual results, produced from within the organization (for example management accounting control reports) with the purpose of helping the control decisions.

In his statement Chris Argyris, he identified situations why mangers could be reluctant in setting budgets: as follows:
The budget is seen as a **pressure device**, based by management to force ' lazy' employees to work harder. The intention of such pressure is to improve performance, hut the unfavourable reactions of subordinates against is seems to be at the core of the budget problem.

The accounting department is usually responsible for recording actual achievement and comparing this against budget. Accountants therefore are 'budget man' is the failure of another manager and this failure causes loss of interest and declining performance. The accountant, on the other hand, fearful of having his budget derailed by factory management, obscures his budget and variance reporting, and deliberately makes it difficult to understand.

The budget usually sets targets for each department, achieving the departmental target becomes of paramount importance regardless of the effect this may have on the other departments and the overall company performance.

Budgets are used by mangers to express their character and patterns of leadership on subordinate; subordinates, resentful of their leadership style, blame the budget rather than the leader thus it looses meaning.

The decision calls for the analysis of benefits and problems of budgeting.
Benefits

It's the major formal way in which the organizational objectives are translated into specific plans, basics, and objectives related to individual managers and supervisors. It should provide clear guidelines for current operations.
It's an important medium of communication for organizational plans and objectives and the progress towards meeting those objectives.
The development of budgets (done properly) helps to achieve co-ordination between the various departments and functions of the organization.
The involvement of all levels of management with setting budgets, the acceptance of derived targets, the two way flow of information and other facets of a properly organized budgeting system all help to promote a coalition of interest and to increase motivation.
Management's time can be saved and alterations directed to areas of most concern by the 'exception principle' which is at the heart of budgetary control.
Performance of all levels is systematically reported and monitored thus aiding the control of current activities.

The investigation of operations and procedures, which is part of budgetary planning and the subsequent monitoring of expenditure, may lead to reduced costs and greater efficiency.

Problems

There may be too much reliance on the technique as a substitute for good management.
The budgetary system perhaps of undue pressure or poor human relations, may cause antagonism and decrease motivation.
Variances are just as frequently due to changing circumstances, poor forecasting or general uncertainties due to managerial performance.
Budgets are developed round existing organizational structures and departments, which may be inappropriate for current conditions and may not reflect the underlying economic realities.
The very existence of well documented plans and budgets may cause rigidity and lack of flexibility in adapting to change.
In conclusion, budget should not be abolished as a company or an organization might not adjust to its set objectives without a budget system.

QUESTION TWO

The variables in the problem are the demand and the lead time. Since the demand is approximated by the continuous normal distribution we will consider demand insteps of 5 x LA 20
Allocation of random numbers to lead time.

Lead Time Week	Probability	Cumulative probability	Random number
	0.20	0.20	00 –19
2	0.50	0.70	20 – 69
3	0.25	0.95	70 – 94
4	0.05	1.00	95 – 99

Allocation of random numbers ranges to weekly demand

Demand/ Week	Probability	Cummulative Probability	Random Number
470	0.003	0.003	000 – 002
475	0.009	0.012	003 – 011
480	0.028	0.040	012 – 039
485	0.066	0.106	040 – 105
490	0.121	0.227	106 – 226
495	0.175	0.402	227 – 401
500	0.197	0.599	402 – 598
505	0.175	0.774	599 – 773
510	0.121	0.895	774 – 894
515	0.066	0.961	895 – 960
520	0.028	0.989	961 – 986
525	0.009	0.998	989 – 997
530	0.003	1.000	998 – 999

Simulation of Stock Control

Number Week	Opening Stock	Demand RN	Amount	Closing Stock	Reorder? YES/NO	Lead-Time RN	Weeks	Shortage
1	2,000	034	480	1,520				
2	1,520	743	505	1,015				
3	1,015	738	505	510	YES	95	4	
4	510	636	505	5				
5	5	964	520	0				515
6	0	736	505	0				505
7	2,500	614	505	1,995				
8	1,995	698	505	1,490				

9	1,490	637	505	985	YES	73	3	
10	985	162	490	495				
11	495	332	495	0				
12	2,500	616	505	1,995				
13	1995	804	510	1,485				
14	1,485	560	500	985	YES	10	1	
15	3,485	111	490	2,995				
Total			7,525	15,475				1,020

Mean demand = $\frac{7525}{15}$ = 501.7 x LA20 / Week

Mean closing stock = $\frac{15,475}{15}$ = 1,031.6 x LA/Week

Mean shortage = $\frac{1,020}{15}$ = 68 x LA20/Week

Number of orders placed during the 15 weeks period = 3

Therefore, mean number of orders/week = 3/15 = 0.2

The expected Average cost per week

= (1,031.67 x Shs.5) +(68 x Shs.200) + (0.2 x 500)

= Shs.18,858.35

QUESTION THREE

How can the transportation algorithm be modified to maximize rather than minimize?

Instead of minimizing the positive unit costs of all the cells, calculate the unit profits, make them negative and put these in each cell. Use the transportation algorithm as usual to minimize these negative profits.

Alternatively, load the cells with the largest profits (instead of smaller costs) to give an initial allocation. Test the empty cells as usual, but use any cell which has positive shadow price. If all the shadow prices are negative or zero, that allocation gives the maximum profit.

Factories P_1 P_2 P_3 supply outlets S_1 S_2 S_3 & S_4

The contribution per desk = selling price at shop – variable cost at the factory – factory outlet transport costs

e.g. the contribution per desk
Supplied from factory P = 2300 – 1500 – 220 = Sh.580 to outlet S

The matrix for contribution is given below:

	S_1	S_2	S_3	S_4
P_1	580	610	530	600
P_2	510	600	520	570
P_3	540	650	490	660

The total demand from the four outlets is 850 + 640 + 380 + 230 = 2,100 desks.
The total supply from the three plants is: 625 + 825 + 450 = 1,900 desks.
There is therefore a need for a dummy factory to take up the 200 shortfall.
The transportation tableu is as follows:

	TO / FROM	K_1 = 58 S_1	K_2 = 67 S_2	K_3 = 59 S_3	K_4 =68 S_4	Total Capacity	
R_1 = 0	P_1	625 [58]	[61] -6	[53] -6	[60] -8	625	// ~~1~~3 3
R_2 = -7	P_2	25 [51]	420 [60]	380 [52]	-4 [57]	825	38881
R_3 = -2	P_3	-2 [54]	220 [65]	-8 [49]	230 [66]	450	111
R_4= -58	Dummy	200 [0]	[0] -9	[0] -1	[0] -10	200	00000
Total Demand		850	640	380	230	2,100	
		~~4~~, ~~4~~, 7, 51,51	4, 4, 4, 60	1,1,1, 52, 52	6		

Note

The initial solution is determined by use of VAM.
The contributions are divided by 10 simplify the computations.
The mode is used to solve for optimality.

Note

m + n – 1 = 7
No of filled cells = 7

The problem is not degenerate.

All the shadow prices are negative, therefore any change would reduce the contribution. This is thus the optimal solution. The optimal allocation is:

FROM	TO	Units	Contribution per unit	Total contribution
		Sh.	Sh.	Sh.
P_1	S_1	625	580	362,500
P_2	S_1	25	510	12,750
P_2	S_2	420	600	252,000
P_2	S_3	380	520	197,600
P_5	S_2	220	650	143,000
P_3	S_4	230	660	151,800
Dummy	S_1	200	0	0
Total contribution				1,119,650

QUESTION FOUR

a) (i) If the selling price is sh.200, demand will be zero. To increase demand by one unit, selling price must be reduced by Sh. $\frac{1}{1000}$ or Sh.0.001. Hence the demand function is P = 200 – 0.001Q

At the output level of 100,000 units.

P = 200 – 0.001 (100,000)

= Sh.100 per unit.

The total contribution at an output level of 100,000 units

	Sh
Contribution = 100,000 (100 – 50)	5,000,000
Less fixed cost	2,500,000
Profit	2,500,000

(ii) Profit is maximized when MC = MR

MC = Sh.50 per unit variable cost.

$MR = \frac{dTR}{dQ}$

$TR = 200Q - 0.001Q^2$

$\frac{dTR}{dQ} = 200 - 0.002Q$

The profit is maximized at

50 = 200 – 0.002Q

Q = 75,00 units

The profit maximizing selling price
= 200 – 0.001 (75,000) = Sh.125

The maximum profits	**Sh.**
Total contribution 75,000 x Sh.75 =	5,625,000
Less fixed costs	2,500,000
Maximum profit	3,125,000

(iii) Change in fuel costs
Revised fixed costs = Sh.3,000,000
The optimal output level will not be affected by a change in fixed costs. Therefore the selling price should not be changed. Profits will decline by Sh.500,000.

Change in Material Costs
Revised marginal costs = Sh.60
The new optimum is where 60 = 200 – 0.002
Q = 70,000

At this output level, P = 200 – 0.001
(70,000) = Sh.130

The price should be increased to Sh.130 to maximize profits.

b)
(i) The price in the home market is based on full absorption cost plus pricing, whereas the price in the overseas market is based on partial absorption or variable cost-plus pricing. Therefore both price methods are on cost-plus basis. The rationale for such an approach is as follows:

Home Market

Absorption cost-plus pricing is the norm in the home market, with all companies adopting this approach. Consequently the pricing method encourages price stability.
The home market provides high volume sales and can therefore bear the full costs.

Export Market

The export market is more competitive, and a price penetration policy might be adopted in order to obtain a significance share of the market. Consequently, the pricing objective might be to set a selling pricing in excess of incremental costs.
Firms might view export business as a means of utilizing any unused capacity. Consequently, overheads have already been recovered in the home market and contribution pricing methods are adopted in the overseas market.
The firm might consider sales in the export market to be uncertain, and short-term prices are set so as to cover short-run costs only.

(ii) The main objection to the above pricing methods is that they are cost-based and ignore price demand relationships. Prices should be set by equating the marginal cost schedule with the sum of the marginal revenue schedules of the two countries.

QUESTION FIVE

(a) Advantages of Value added Statement

Managers might be in a better position to control their organization’s own inputs than the cost or usage efficiency of purchased material and services. If this is so, value added statements focus attention on what managers can do something about. They would also reflect the quality of such management’s effort.
Value added statements also focus attention on how the benefits are shared out, and in particular: -

Whether the employees are getting paid too much for what they are doing. If the value added per unit of labour is declining, management will be made aware of the need to keep labour costs under control. On the other hand, an improving value added per shilling of labour cost would suggest that there is some scope for rewarding employees more highly.
Whether enough funds are being returned in the business (depreciation plus retained profits) to provide for asset replacement and internally –funded growth.

In organizations where the material cost content is a high proportion on total costs, the total profit will be influenced by changes in material prices (largely outside management control) and possibly also by occasional stock losses or profits when material prices alter value added statements, by taking out material costs as a separate item, allow alterations to be directed at activities within management's control.
Value added in relations to labour effort and labour costs provides excellent measures of productivity, and so far comprising the relative productivity of two or more divisions.

Overhead costs (Activities)	**P(Shs)**	**Q(Shs)**	**R(Shs)**	**TOTAL(SHS)**
Machinery cost	18,000	48,000	36,000	102,000
Production scheduling	16,800	44,800	22,400	84,000
Set up cost	10,800	28,800	14,400	54,000
Quality control	9,848	26,240	13,120	49,200
Receiving materials	7,200	32,000	25,600	64,800
Packing materials	6,000	16,000	8,000	36,000
Total overhead cost	**68,64**	**199,840**	**121,520**	**390,000**
Units produced	12,000	16,000	8,000	
Overhead cost/unit	**Shs.5.72**	**Shs.12.49**	**Shs.15.19**	

Workings for Recovery Rates

Machining cost = Budgeted machining cost = 102,000
Budgeted machine hours 34,000
= shs.3/machine hour

{Budgeted machine hours = ½ (12,0000 + (16,000) + 1½(8,000) = 34,000}

Production scheduling = budged production scheduling cost =
84,000 No. of production runs 30
= Shs.2,800/production run

iii. Set up costs = Budgeted Set Up Cost = 54,000 = shs.1,800/production run
No. of production runs 30,000

iv. Quality control = Budgeted Quality Control Cost = 49,200 = 1,640/production run
No. of production runs 30

v. Receiving materials = Budgeted Receiving Materials cost = 64,800 = 400 Receipt No. of components Receipts 162

vi. Packing Materials = Packing Material's cost = 36,000 = 1,000/Customer order
No. of customers orders 36

Total cost statement and profit (shillings per unit)

	P	**Q**	**R**
Direct materials	16.00	24.00	20.00
Direct labour	8.00	12.00	8.00
Overhead cost (as above)	5.72	12.49	15.19
Total production cost	29.72	12.49	15.19
Sales price	50.00	70.00	60.00

Gross profit per unit	**20.28**	**21.51**	**16.81**

MODEL ANSWERS TO THE CPA PILOT PAPER SET JULY 2010

QUESTION ONE

(a)Existing capacity

			Kshs
P	4560 x 19.6	=	89,376
Q	6960 x 13.0	=	90,480
R	3480 x 9.9	=	34,452
S	2300 x 17.0	=	39,100
Total Existing Capacity			253,408
Add 5% increase to full Capacity 5% x 253,408			12,670.4
Total Direct Labour of Full capacity			266,678.4

Switching of 2010 kg of Q releases Direct Labour cost by-: which is switch to P.

2010 x 13	26,000
Add 5% increase	12,670.4
Available cost to be switched	38,670.4

Labour cost of P = 19.6

Therefore units to be switched $= \frac{38,670.4}{19.6} = 1973$ Kg

Increased contribution therefore is: -

	Shs	Shs.
Sales 197 x 162		319,626
Less: Variable Cost		
Direct labour (1973 x 19.6)	38,670.8	
Direct materials (1973 x 65.20)	128,639.6	
Direct packaging 91973 x 8.4)	16,573.2	(183,883.6)
Contribution of P		135,742.4
Less: Lost contribution from		
Q = 2010{(0.9 x 116.40) – (13 + 49 + 7.4)}		(70,720)
Incremental Contribution		65,022.4

Decision

Timao Company Limited should subcontract 2010kg from Kagocho Company due to the incremental contribution of Kshs.65,022.4

(b)	P	Q	R	S
Timao's selling prices (A)	162	116.40	99.20	136.80
Subcontracts price = (90% x 4)	145.80	104.76	89.28	123.12
Less: Variable cost of marking				
Direct labour	19.60	13.00	9.90	17.00
Direct materials	65.20	49.00	41.00	54.20
Direct packing	8.40	7.40	5.60	7.00
Total Variable cost	93.20	69.40	56.50	78.20
Lost Contribution	52.60	35.40	32.90	44.90

Switching of 2010kg to different products. This can be done in a matrix form as follows.

Additional Production (Kg) from switching direct labour cost.

Source of units	P	Q	R	S
Shs.39,200 from P (a)	0	3015 (e)	3959 (f)	2305 (g)
Shs.26,000 from Q (b)	1326 (h)	0	2626 (i)	1529 (j)
Shs.19,800 from R (c)	1010 (k)	1523 (l)	0	1164 (m)
Shs.34,000 from S (d)	1734 (n)	2615 (o)	3434 (p)	0
Extra 5 % of capacity Shs.12,670.4	646 (q)	974 (r)	1280 (s)	745 (t)

Workings

2010 x 19.60	(e)	39,200 ÷ 13	(i)	26,000÷ 9.9
2010 x 13.00	(f)	39,200 ÷ 9.9	(j)	26,000÷ 17
2010 x 9.90	(g)	39,200 ÷ 17	(k)	19,800÷ 19.6
2010 x 17.00	(h)	26,000 ÷ 19.6	(l)	19,800÷13
19,800 ÷17	(q)	12,670.4 ÷ 19.6		
34,000 ÷ 19.60		(r) 12,760.4 ÷ 13		
34,000 ÷ 13	(s)	12,670.4 ÷ 9.9		
34,000 ÷ 9.9	(t)	12,670.4 ÷ 17.00		

Extra contribution gained in Shs.

	P	Q	R	S
Contribution per Kg/Sh.	68.80	47	42.70	58.6
2010Kg of P subcontract	0	82,283(i)	118,500(ii)	73,530
2010Kg of Q subcontract	64,954	0	96,070	62,536
2010Kg of R subcontract	48,373	51,788	0	46,307
2010 Kg of S subcontract	73,900	78,843	111,448	0

Workings

Incremental contribution - lost contribution
i.e. (i) { (3015 + 974) 47 } – { (2010 x 52.6) } = 82,280
(ii) {(3959 + 1280) 42.7)} – {(2010 x 52.6)} = 118,500 etc

Decision

The best profitable contribution is to subcontract 2010kg of P and replace it with 5239kg (3959 + 1280) kg of R leading to the highest contribution of shs.118,500.

QUESTION TWO

Two important concepts in control theory are firstly, that a system must have a purpose and must have controls if it's to remain cohesive and secondly, that is a system can be divided into a number of sub-systems and sub-sub-systems, each with it's own purpose and controls. Controls provide the binding force, which kept every various elements within the system all working towards a common objective. Control theory can be used to analyse or to establish control systems within a business organization. A model can be constructed and used as follow:

The system as a whole, and for each sub-system (and sub-sub-systems) one or more objectives are identified.
Actual achievements of the system and sub- system are monitored. Actual achievements are compared with the objective.
Reasons for any differences between the objectives and achievements are identified
Where suitable, corrective measures are taken to bring the system under control

When actual achievements are measured as actual outcomes and results, the comparison of results with objectives is called a **feedback** control loop. When actual achievements are measured as what the system is not expected to achieve and future expectations are compared with objectives (Such as projected completion dates for a project) we have **feed forward control.**
A control model can also be made to recognize environmental influences, and the ways in which environment can affect the system's achievements and objectives.
The concepts of control theory can provide valuable insights into the design and operation of a management accounting information system (MAIS) because:-

Business organizations need to be controlled by their management.
Management accounting provides an information system for control, based largely on system of a budgets. This information acts as a feedback loop.
The way in which a MAIS is structured and used can be determined by modeling techniques.

A control model for a MAIS would: -

Identify the sub-systems within the organization
Establish objectives for each sub-system, and for the system as a whole. These objectives must be measurable, and would usually take the form of budgets, with the control system being a budgetary control system.
Measure actual achievements for each sub-system, for the system as a whole. Compare actual results with the objectives (budgets)
Identify significant differences, and the reasons for them, indicating where control action should be taken.

A MAIS cannot initiate control action itself, but can only indicate where control measures might see appropriate, control measures must be taken by managers, perhaps using their judgment. In this respect, a MAIS falls short of the ideals' of a cybernetic control model.
The practical application of control principles to a MAIS, using a budgetary control system does depend on the stability of the environment and accurate measurement of results.
When an organization's environment is unstable ad unpredictable forecasts of achievements will be uncertain; and a comparison of actual results against plan might be meaningless. It might also be necessary to alter the system's objectives in response to environmental change.
If environmental changes are continual, or frequent, the problems of redefining systems objectives will be considerable and budgets would have to be revised at frequent intervals. In addition, the significance of differences between actual results and budget would be difficult to assess for control purposes.

Outcome needs to be fairly clearly measurable for control system to operate successfully. In practice, there may be problems in applying quantative measures to qualitative outputs, and control information might be imperfect and incomplete. This always the prospect that unless results can be measured objectively managers will manipulate and 'judge' the figures, so that the problems of human behaviour damage the operation of the control system.
In conclusion, control theory can provide a useful framework for a MAIS but control of a business is not "automatic". Business organizations are largely 'human systems,' ad there will inevitably be difficulties with applying the theoretical structure of a control model in practice. Further more, although some environmental change can be achieved for in a control mode, frequent changes caused by unstable environment could remove the practical value of feedback systems for control.

QUESTION THREE a)

			Sh. '000'
(i)	Desired Residual Income		5,000
	Current Income from external sales		
	Contribution = 500 (37 – 25)	6,000	
	Fixed costs	(1,400)	
	Capital cost 13% x 20 m	(2,600)	2,000
	Contribution to be generated by internal transfers		3,000

Contribution per unit $= \frac{3,000}{300} =$ Sh.10 per unit

Transfer price = Sh.25 + Sh.10 = Sh.35 per unit

(ii) The transfer price above may motivate the Z division manager to want to sell the components externally at Sh.37 rather than to transfer them to other divisions at Sh.35. This may result in the other divisions being forced to buy components externally and thus incur buying costs while Z will incur selling cots. The net effect is that the company as a whole losses.

b) The demand function can be determine as follows:

$P = A - bV$

Where P is the price per unit
V is the volume of sales at that price
A is the price at which V = O (Maximum price)
b is the rate at which the price falls for volume increases a proportion of sales volume.

Product A

Demand is currently 15,000 units at a price of Sh.30. The demand changes by 500 units for each Sh.1 change in price.

$A = 30 + \frac{15,000}{500} \times 1 = \text{Sh.}60$

The maximum price = Sh.60

$b = \frac{1}{500}$

The demand function will be

$P = 60 - \frac{1}{500}Q$

Total revenue $= PQ = 60Q - \frac{1}{500}Q^2$

Profit is maximized where MR = MC

$MR = \frac{dTR}{dQ} = 60 - \frac{2Q}{500} = 60 - \frac{Q}{250}$

MC is the unit variable cost = Sh.12

At Maximum profit MR = MC

$$60 - \frac{Q}{250} = 12$$

Q = 12,000 units

Substituting to find P

$$P = 60 - \frac{12{,}000}{500} = \underline{\textbf{Sh.36}}$$

The profit maximizing price is Sh.36 and profit maximizing Quantity is 12,000 units.

Product B

This is solved in the same way as A

$$A = 58 + \frac{21{,}000}{500} \times 1 = \text{Sh.}100$$

$$P = 100 - \frac{1\,Q}{500}$$

$$TR = 100Q - \frac{Q^2}{500}$$

$$MR = \frac{dTR}{dQ} = 100 - \frac{Q}{250}$$

MC = Sh.8

At maximum profit MR = MC

$$100 - \frac{Q}{250} = 8$$

Q = 23,000 units

Substituting

$$P = 100 - \frac{23{,}000}{500} = \underline{\textbf{Sh.54}}$$

The profit maximizing price is Ksh.54 while the profit maximizing quantity is Sh.23,000 units

QUESTION FOUR

Expected Value of Usage

Lead-Times (Days)	Probability	Demand (units)	Joint Probability	Expected Value (Usage)
15 working days 0.2	0.4	5,000	0.08	(15 x 5000) 0.08 = 6,000
	0.6	7,000	0.12	(15 x 7,000) 0.12 = 12,600
20 working days 0.5	0.4	5,000	0.20	(20 x 5,000) 0.20 = 20,000
	0.6	7,000	0.30	(20 x 7,000) 0.30 = 42,000
25 working days 0.3	0.4	5,000	0.12	(25 x 5,000) 0.12 = 15,000
	0.6	7,000	0.18	(25 x 7,000) 0.18 = 31,500
			1.00	127,100

Bufter stock at 150,000 units re- order level = (150,000 – 127,100)
= 22,900 units

(b) The P (stock out cost i.e. Demand in excess of 150,000 units)
= (25 x 7,000) = 175,000 units
∴ P (stock out cost) = 0.18

(c) EOQ = $\sqrt{\frac{2 \times (6{,}200 \times 360) \times 1{,}000}{0.0025 + (0.1 \times 2)}}$ = 140,855 units

Daily Demand = 5,000 (0.4) + 7,000 (0.6) = 6,200

No of average orders per annum = $\frac{6{,}200 \times 360}{140{,}855}$ = 15.85

∴ The expected annual stock outs in units per annum
= {(0.225) (175,000 – 150,000)} x 15.85 = 89,156 units

(d) The additional annual holding cost if the re-order level is increased to 175,00 units: 15 (175,000 – 150,000) (0.025 x 1.1 x 2) = 1,375
Therefore, are-order level of 150,000 units the expected value of stock outs per annum is 10,766 units.

Then the increase in stock is justified where stock out cost per unit is greater then Shs.0.3 (1,375/10,766)

JIT (Just in time) it involves a continuous commitment to re-pursuit of excellence in all phrases off manufacturing systems design an operation.

Advantages of JIT

- Leads to substantial savings in stockholding costs.
- Elimination of waste
- Savings in factory and warhorse space, which can be used for other profitable activities.
- Reduction in obsolete stocks
- Considerable reduction in paper work arising from a reduction in purchasing, stock and accounting transactions

Disadvantages of JIT

- Additional investment costs in new machinery, changes in plant layout and goods inwards facilities.
- Difficulty in predicting duty or weekly demand, which is a key feature of the JIT philosophy.
- Increased risk due to the greater probability of stock out costs arising from strikes, or other unforeseen circumstances, then restrict production or supplies.

QUESTION FIVE

Modification of the probability by use of Bayes Theorem

$$B\ (B/A) = \frac{P\ (B)\ x\ P}{(A/B)}\ P\ (A)$$

Steps to follow in modification of probabilities

Step 1

Interpretation of the formula into the question:

B is either oil (O) or not oil (N)

A is the result of the report either favourable (F) or unfavourable (U) under each of the above situations.

$$P\ (o/F) = \frac{P\ (O)\ x\ P\ (F/O)}{P\ (F)}$$

$$P\ (O/U) = \frac{P\ (O)\ x\ P\ (U/O)}{P\ (U)}$$

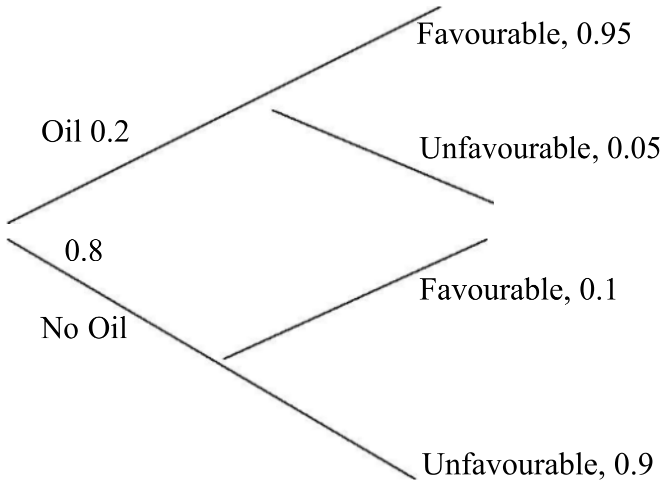
$$P\ (N/F) == \frac{P\ (N)\ x\ P\ (F/N)}{PF}$$

$$P\ (N/U) = \frac{P\ (N)\ x\ P\ (U/N)}{P\ (U)}$$

Step two

Construction of probability tree.

g



Step three

Derivation of probabilities from step two

P (O) = 0.2	P (F/O) = 0.1
P (N) = 0.3	P (U/N) = 0.9
P (F/O) = 0.95	P (F) = 0.95 (0.2) + 0.1 (0.8) = 0.27
P (U/O) = 0.05	P (U) = 1 – 0.27 = 0.73

Step four

Incorporation of the probabilities into the formulas in step 1

$$P(O/F) = \frac{P(O) \times P(F/O)}{P(F)} = \frac{0.2 \times 0.95}{0.27} = 0.704$$

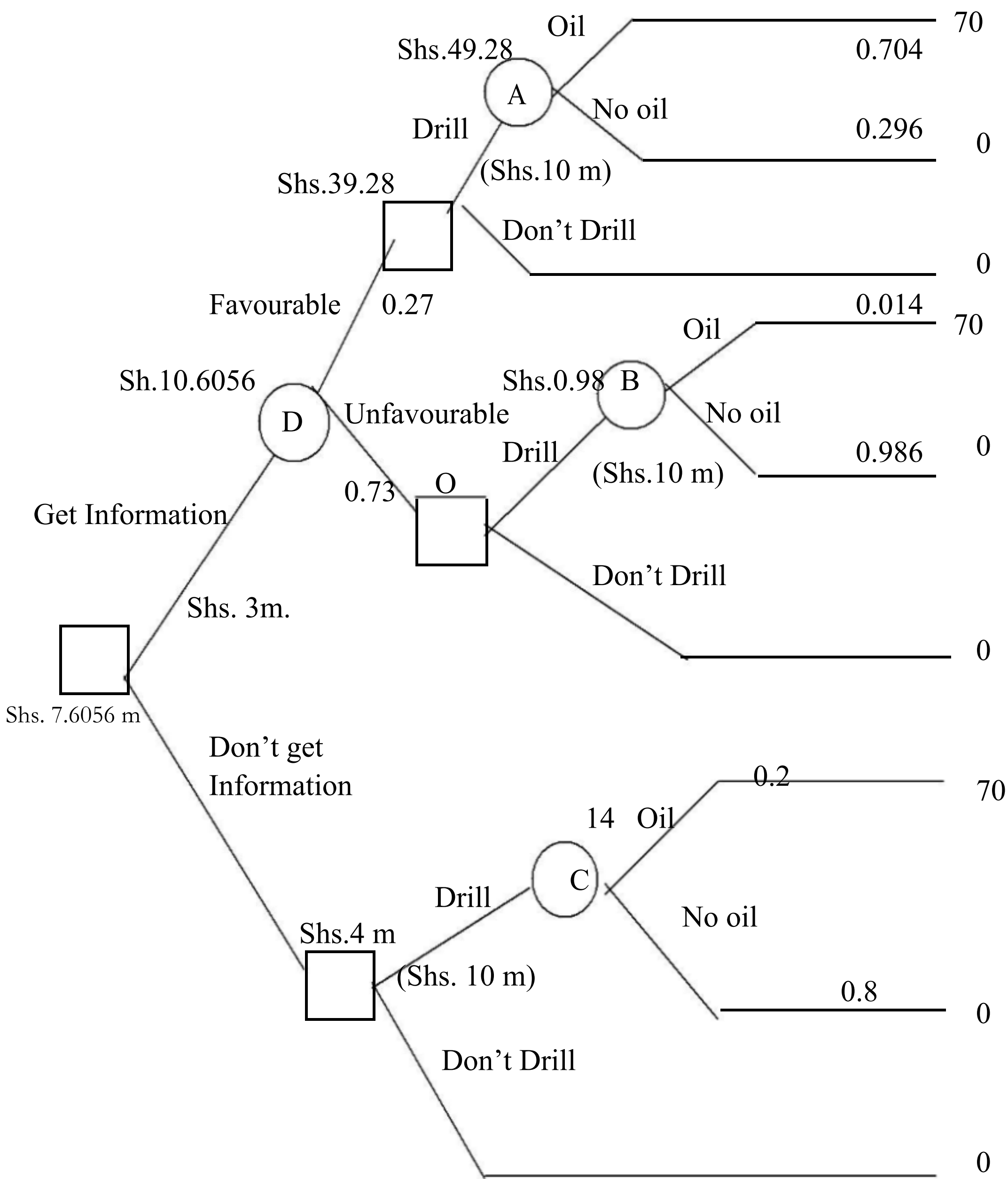
$$P(O/U) = \frac{P(O) \times 9U/O)}{P(U)} = \frac{0.2 \times 0.05}{0.73} = 0.014$$

$$P(N/F) = \frac{P(N) \times (F/N)}{P(F)} = \frac{0.8 \times 0.1}{0.27} = 0.296$$

$$P(N/U) = P\frac{(N) \times (U/N)}{P(U)} = \frac{0.8 \times 0.9}{0.73} = 0.0986$$

Step five

Construct a Decision tree and evaluate



Evaluation using EMV

Emv @ A = 70 (0.704) + 0 (0.296) = 49.28

Emv @ B = 70 (0.014) + 0 (0.986) = 0.98

Emv @ C = 70 (0.2) + 0 (0.8) = 14

Emv @ D = 39.28 (0.27) + 0 (0.73) = 10.6056

At D_2 ⇒ Drill = 49.28 – 10 m = 39.28
⇒ Don't Drill = 0

At D_3 ⇒ Drill = 0.98 – 10 m = -9.02
⇒ Don't Drill = 0

At D_4 ⇒ Drill = 14 – 10 m = 4
⇒ Don't Drill = 0

At D_1 ⇒ Hire 10.6056 – 3 = 7.6056
Don't Hire = 0

Note

1. At Decision Box choose the highest value
2. At outcome point use probabilities on values to get the expected monetary value.

Step six Make Decision or Advice –

Walt Lovell Limited (WLL) should use a consultants given the report is favourable drill as this will release a net benefit of Kshs. 7.6056 million.

The value of imperfect information is:

Value of imperfect (sample) information =
Expected monetary value with IPI – Expected monetary value without IPI

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MODEL ANSWERS TO THE CPA PILOT PAPER OCTOBER 1991

QUESTION ONE

A

The basic CVP **model is based on a number of assumptions:**

1. Selling price per unit remains constant during the period in which the model is being used irrespective of changes in activity.
2. The variable cost per unit remains constant during the period in which the model is being used irrespective of changes in activity.
3. Fixed cost remail?s unchanged throughout the possible activity range.
4. The analysis of costs into fixed and variable elements have been achieved with reasonable accuracy.
5. Changes in activity level is the only factor affecting costs and revenues. The CVP Model may be used to calculate

(a) Breakeven Point expressed in units or monetary value

(b) The units or sales value Shs at which a specific net profit will be earned.

The above assumptions are likely to apply only to a limited activity'range and are an over simplification of the conditions which are likely to apply even in the very short term.

The basic CVP Model should be seen as a crude decision making tool the longer the time scale the more likely it is that the pattern of costs and revenues will change thus reducing the validity of any decisions made.

The situation where multi-products are concerned adds to the possibility of departures from the basic model because of the increase in the factors/situations involved so emphasizing the models limitations.

B

1. Contribution per unit Machungwa 40% of Shs. Of Sales

Ndimu 15% of Shs. Of Sales

		Contribution per Sh of Sales
Sales mix weighting M	60% of 40%	24%
weighting N	40%of 15%	6%
Instaj Contribution per Shs of Sales		Shs0.3

SALES F Cost

Instaj click contribution in Shs = $\frac{150,000}{0.3}$ Shs500,000

PROOF SALES ANALYSIS

Sales	X	=	60% of 500,000	=	300,000
Sales	Y	=	40% of 500,000	=	200,000
Contribution	X	=	40% of 300,000	=	120,000
Contribution	Y	=	15% of 200,000	=	30,000
	Total Contribution on 500,000				
	Sales in required mix			=	150,000

2. To obtain contribution of KShs 150,000+30%+9,000 = 204,000

SALES = <u>204,000</u> Shs 680,000

Insta] 0.3

PROOF SALES ANALYSIS Shs 680,000

Sales	X	=	60% of 680,000	=	408,000
Sales	Y	=	40% of 680,000	=	272,000
Contribution	X	=	40% of 408,000	=	163,200
Contribution	Y	=	15% of 272,000	=	40,800
	Total contribution on 680,000				
	in required mix			=	208,000

QUESTION TWO

A.

Price New 1 Year Old 2 Year Old 3 Year Old

=

Sh 40,000 Sh 31,000 Sh 20,000 Sh 11,000

=

=

Age	Purchase Price	Sale Price	Repair & Maintenance	Total Cumulative Cost	Average Cost	Scrap Value (at date of resale	
0	40,000	-	-	-	-	-	0
1	31,000		3,000			2,800	1
2			4,000			18,000	2
3			6,000			10,000	3
4			9,000			4,000	4

When machine is new = Sh 40,000

	Depreciation cost	Repair & Maintenance Cost	Total Cost	A verage cost per yr
1	12,000	3,000	15,000	15,000
2	22,000	3,000 + 4,000	29,000/2	14,500
3	30,000	13,000	43,000/3	14,333.3
4	36,000	22,000	58,000/4	14,500.0

Buy 1 Year Old

	Total Depreciation cost	Repair & Maintenance	Total Cost	A vg. cost per year
2	31,000-18,000	4,000	13,000 + 4,000	17,000
3	31,000-10,000	6,000	21,000 + 10,000	31,000/2 = 15,500
4	31,000-4,000	9,000	27,000 + 19,000	46,000/3 = 15,333.3

Buy 2 Year Old

	Total Depreciation	Repair & Maintenance	Total Cost	Av. cost per year
3	20,000-10,000	6,000	16,000	16,000
4	20,000-4,000	9,000	16,000 + 15,000	15,500

Buy 3 Year Old

	Total Depreciation	Repair & Maintenance	Total Cost	A v. cost per year
I 4	11,000-4,000	9,000	7,000-9,000	16,000

Given the numbers revealed from the above table it would be advisable for the company to purchase new and sell at the end of Year 3 to reduce the aggregate average cost to 14.33 per annum for maintenance cost and depreciation.

The cost of capital interest has not been identified but assuming 10% the profits revealed in the foregoing is not significantly inpacted as the expected scrap value generally correspond to the price of second hand machines.

B.

Abandonment Value

The amount which could be recovered from an investment project if it were immediately to be abandoned. It is of importance in the field of **capital budgeting** (g.v.) as a concept relevant to the monitoring and review of ongoing capital projects. No decision to invest should be regarded as irrevocable, and if at any time the abandonment of a project is of greater value than its continuance then abandonment is indicated. The simple decision rule is that a project should be abandoned if its abandonment value exceeds the **net present value** (g.v.) of its projected cash flows. .

EXAMPLE. A certain project, nearing the end of its useful life, is expected to last for two further years and to yield at the end of these years positive cash flows of £8,000 and £6,000 respectively. If abandoned now, the plant and equipment used in the project could be sold for £12,500. After two further years of use it will have no value. The cost **of capital** (g. v) is 10% per annum.

Present value of future returns:

Year	*Returns*	*Discount factor*	*Present value*
	£		£
1	8,000	0.909	7,272
2	6,000	0.826	4,956
			12,228

Since it is worth only £12,228 to continue the project but £12,500 can be obtained by abandoning it, it should be abandoned. Failure to abandon is the equivalent of investing the £12,500 forgone into a project with a negative net present value.

QUESTION THREE

Multiple Regression

Regression I

Overhead costs -32657 + 16.57 x machine hours
Std error 4.32 (for machine hours)

r2 Se= 0.77 3.456

Regression II

Overhead costs =17.865 + 13.76 x units of output
Std error 3.87 (for units of outplit)
r2 Se=0.61 3.973

Regression III

Overhead costs =-15.373 + 7.37 x machine hours + 10.44 x units of output
Std error 3.31 (for machine hours) 3.81 (units of output)
r2 Se=0.79 1.623
=The correlation mix shows a 0.86 relationship between machine hours and units of output.

(a)

Meaning of Computed t-value
This is a sample t-statistic for the constant, the regression coefficient, correlation relation coefficient
For correlation coefficient This is compared against value from given tables at (n - 2) degrees of freedom
t-statistic for regression coefficient

$$t \quad \frac{b - 0}{\text{estimated std. error of b}}$$

In general

Sample statistic - parameter assumed in
Ho best estimate of standard error of
statistic

(b) t value for all the **three** regressions
.
J!... std. error not given Sea

. Machine hours

= JL 16.57 = 3.8356 (Std given 4.32)
Seb 4.32

H. Output

= 1.3.76 - 3.55 (Std given 3.87)
3.87

Machine hours and output

Machine hours 7.37 / 3.31 2.23

Outpu t 10.44 / 3.81 = 2.74

(c) Meaning r2 in i. 77% of the variation in overhead costs can be explained from changes in machinehours

61 % of variation in overhead costs can be explained from changes in units of output.

iii. Correlation mix

R or 0.79

R2 0.6241

Only 62% explained, due to both output and machine hours
Hence output does not significantly add to more accurate forecast

(d) Negative intercept has in general no value
Value of intercept in (i) when machine-hours = 0, what are overhead costs?, they can't be negative. We must have minimum machine hours (and maximum) in order that the data is valid.

(e) Overhead costs = constant + machine hours (because number of units produced does not significantly improve the forecast).

QUESTION FOUR

Three products
Xl X2

Nurate	Phosphate	Potash	Filler	II Fertilizer Selling price/tonne
0.1	0.1	0.2	0.6	R
0.1	0.2	0.1	0.6	81
0.2	0.1	0.1	0.6	81
Sh 150	Sh 60	Sh 120	Sh 10	
tonnes 1200	tonnes 2010	tonnes 2200	No limit	

X3
Price per tonne
Max. available
Selling Price per tonne
Manufacturing cost
Fixed Sh 11 per tonne (Excluding raw material)
Z 21 Xl + 25X2 + 16X3
0.1 XI + 0.1 X2 + 0.2 X3 :::; 1200 (1) 0.1 XI + 0.2 X2 + 0.1 X3 :::; 2010 (2) 0.2 Xl + 0.1 X2 + 0.1 X3 :::; 2200 (3)
Xj,X2,X3 ~ 0

Cost price product XI

0.1 x 150+0.1 x60+0.2x 120+0.6x 10=
15 + 6 + 24 + 6
= Sh 45 + Sh .6
Total price =51 + 11 Sh62

Product X2

0.1 x 150 + 0.2 x 60 + 0.1 x 120 + 0.6 x 10 +
11 15 + 12 + 12 + 6 + 11
Sh56

Product X3

0.2 x 150+0.1 x60+0.1 x 120+0.6x 10+ 11
= 30 + 6 + 12 + 6 + 11
= Sh 65

Contribution for Xl = 83 - 62 = Sh 21
X2 = 81 - 56 = Sh 25
X3 = 81 - 65 = Sh 16
Z = 21 Xl + 25 X2 + 16 X3

0.1 Xl + 0.1 X2 + 0.2 X3 +)4 = 1200 (Nitrate in tonnes) 0.1 X2 + 0.2 X2 + 0.1 X3 + Xs = 2010 (phosphate in tonnes) 0.2 XI + 0.1 X2 + 0.1 X3 + X<; = 2200 (potash in tonnes)
z 21 XI + 25 X2 + 16 X3 (Maxi
mise)

(b))4, when there is no production of XI> X2, X3,)4 give total nitrate available in tonnes.
When all)4 is consumed, the final tableau gives shadow price of Nitrate.
Similarly Xs and X6

(c)

Initial S'
Tabl

. , ---- nu -----n-

Basic Variable	Xl	X2	X3	X4	Xs	X6	Solution Quantity	Ratio
)4	0.1	0.1	0.2	1	0	0	1200	12,000
Xs	0.1	0.2	0.1	0	1	0	*2010*	10,000
X6	0.2	0.1	0.1	0	0	1	2200	22010
Z	-21	-25	-16	0	0	0	0	

(d) Final matrix as given in Q4 (d)

Interpretation

Production
Produce *4000* units of product Xl
Produce *8000* units of product X2
Do not produce product X3

The total contribution from this production is Sh 284,000

Calculated as follows:
z=21 Xl + 25 X2 + 16
X3 21 x 4000 + 25 x 8000 84,000 +
200,000 Sh 284,000

Dual Prices or Shadow prices

Chemical Nitrate has been fully, used and is a scarce quantity. Everyone tonne of this chemical available (above 1200 tonnes) will increase the profitability by Sh 170 (subject to maximum which can be calculated).

Similarly chemical phosphates has been fully used, every extra tonne over *2010* tonnes, will increase the profit by Sh 40 (subject'to a maximum which can be calculated).

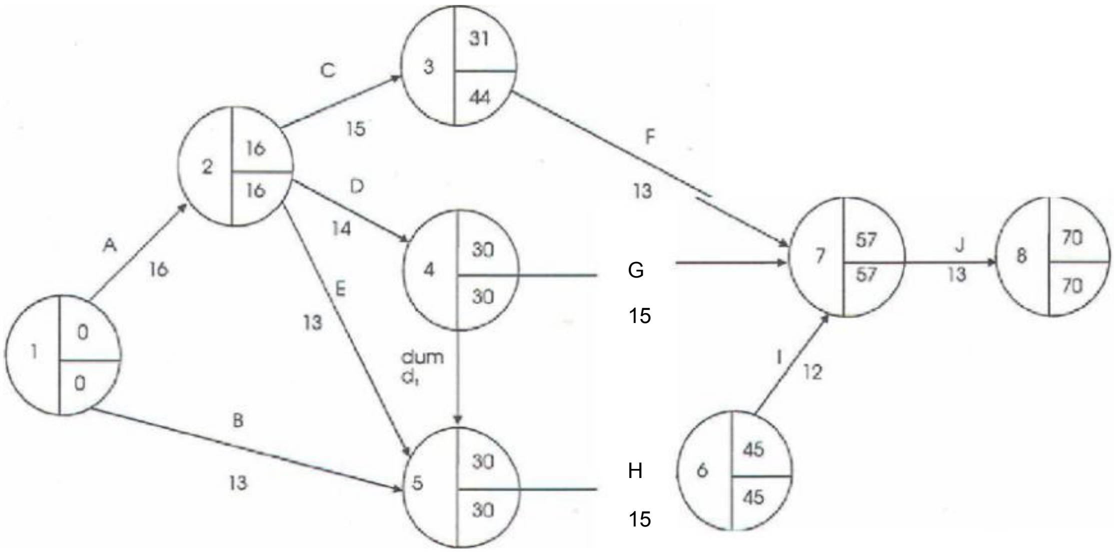
Potash not been fully used, there is still a surplus of *600* tonnes i.e. 2200 - *600* = 1600 tonnes has been used. Hence it has no scarcity value.

Production of X3 will reduce the overall profit by Sh 22 per unit. Hence on economic grounds it should not be produced.

Activity	Preceding Act	Activity Normal	Time (days) Crash	Total Cost (Sh) Normal	Crash	Crash cost per day Sh
A	-	16	14	400	480	40
B	-	13	12	340	360	20
C	A	15	13	380	500	60
D	A	14	12	200	260	30
E	A	13	11	260	280	10
F	C	13	12	310	380	70
G	D	15	10	430	580	30
H	B,D,E	15	9	280	370	15
I	H	12	11	400	420	20
J	F,G,I	13	11	260	320	30

1.

(a)



Critical Path A-D-d1-H-I-J = 70 days

ii.

III. Total Float for non-critical activities

Activity	Total Float LSTJ-ESTr-Dij
B	17
C	13
E	1
F	13
G	12

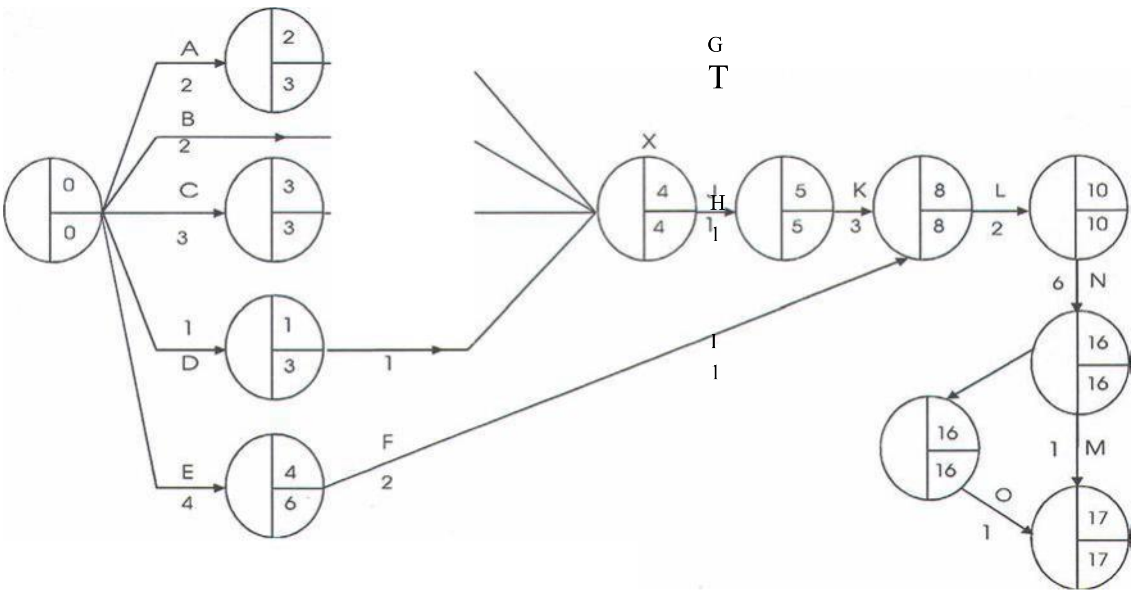
b)
1.
ii.
H (reduce H by 1 day)
New cost of project = 3,260 +
15 = Sh 3,275

MODEL ANSWERS TO PAST CPA PAPER DECEMBER 1991

QUESTION ONE

(a) Network Analysis

Time in weeks



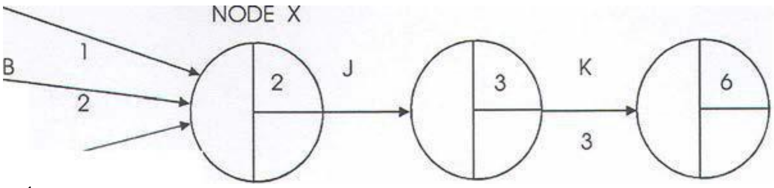
(b) Start Jan Week 1- May 1st Week

31 + 28 + 31 + 30 120 days 17 weeks

(End April/Beginning May)

(c) A, C, D can be finished end of December.

E may take only 2 weeks



c 1
I
51
G
B
~l/ 0
E is immaterial to the duration
The minimum, the project will take is 17 - 2 = 15
weeks Cd)

A network diagram may be defined as a diagram which illustrates the sequences in which activities must be done in a given project. The more complex the production process, the greater the need for precision in selecting the most cost effective sequence of operations. Discipline of the technique enables the production management to better appreciate and communicate to their peers the logic of the preferred routing

QUESTION TWO

Petrol regular
premium
regular extra (at least 50% premium)

Value properly adjusted	50% premium 50% regular	minimum cost
Value out of adjustment	60% premium 40% regular	quantity required
	100,000 litres once value is adjusted	

Per litre

	Sh	
Cost premium	3.20	
Cost regular	3.00	
Cost checking value	800.00	
Cost adjusting the value	400.00	
		Probability
Event Value in adjustment		0.7
Value out of adjustment		0.3

(a) Expected cost of checking the value of adjusting if necessary

	Cost	Prob.	
Value OK	800	0.7	560
Value needs adjustment	1200	0.3	360
		Sh	920

OR

Cost of checking			800
+ Cost of adjustment	0.3 x 400		120 Sh
	800 +	120	920

(b) Value out of adjustment Prob. = 0.3

Cost/litre if value OK= $\frac{3.20 + 3.00}{2}$ Sh 3.1O/litre

Cost/litre if value not OK 0.6 x 3.20 + 0.4 x 3.0

1.92 + 1.20

Sh 3. 12/litre

Cost of 10,000 litres if value OK	3.10 x 100,000	Sh 310,000
Cost of 10,000 litres if value not OK	3.12 x 100,000	Sh 312,000
	Difference	Sh 2,000

The probability is 0.3

Expected cost = 2,000 x 0.3 = Sh 600

(c) The extra cost is Sh 2,000

Let the Probability be p

2010 x p/p 800/0.5 + 400 p

(d) Comment on the result (a) and (b) above

It is not worth checking the value

The premise by which A normal . spoilage is ignored in the computation of equivalent units is that because of the nature of the processing operation and the anticipated efficiency of the machinery used in the process. Such factors are deemed to be outside management control. The level of loss which is selected as being the standard for the period under review may be based on past experience, quality control records from past periods or industry averages.

The cost of such losses is recovered by inflating the cost of output achieved to a~sorb the cost of the losses.

QUESTION THREE

The selection of the point of inspection should provide the quality control department and the management accountant with the most accurate valuation of work in progress totals consistent with considerations of cost effectiveness on the factory floor.

B. DEPARTMENT M

	Units		*Units*
Opening W.I.P	8,000	Normal Spoilage	2,000
Input material	17,000		
		Good Output	18,000
		Closing W.LP	5,000
	25.000		25.000

Equivalent units for August conversion cost

Good Output W.LP	18,000
5,000 x 60%	3,000
	21,000

Equivalent units for August material assuming all W. LP is 100% complete for material.

QUESTION FOUR

Q4.		SPL		ODOG O
	20,000 units @	57.50	7,500 units	75.00
	COSTS			
	MAT	22.50		32.50
	LAB	30.00		30:00
	ON 0.25	10.00		20.00
		62.50		82.50
	Plus special one-off	15,000		
	costs for start up	25,000		
	M/C has total	90,000		
	capacity p.a.			
	Monthly	7,500		

Available Capacity

July,August,September

20% of 3x 7,500 22500=4500 HRS

SPL Order Capacity Requirement Odogo Order

20,000 x 0.25 5000hrs 7500x 0.5 3750 3750hrs

Fixed OV/HD 2, 160000 OF 90000 =60%

60% of Shs 40 per hour = Shs24 contribution to fixed. Recovery per M/CHR

$\frac{2,160,000}{90,000}$ Shs24

Fixed OV/HD Contribution		Fixed OV/HD Contribution	
From Westland Order		From OdogoOrder	
5000hrs x Shs 24	Shs 120,000	3750hrs x 24	Shs 90,000
Less loss on sale		Less loss on sale	
57.50-62.50 Cost		7500-82.50	
Shs5x 20,000	=100,000	7.5x 7500	Shs56,250
		Less start up costs	Shs40,000
Net Contribution From order	Shs 20,000	=Shs 21875	Shs(6,250)

Given that the opportunities are the only ones available to Westland. The choice between the two opportunities can be identified reasonably easily. The SPL opportunity gives contribution of KShs 20,000 compared to Odongo Foods order of (6,250).
However in order to meet the SPL delivery date some rescheduling is necessary to meet the 1st order delivery date as only 4,500 hrs are available in July, August, September and the SPL orders requires 5,000. This relatively small imbalance may be managed either with negotiation with SPL or by adjustment to Westland Stocking Policy.

QUESTION FIVE

A. Programme planning and budgeting systems and priority based budgeting share some commonality with ZBB Below. PPBS establishes programs as opposed to decision units which must be justified before acceptance. The term program may relate to an educational program such as the integration of technology into schools and colleges or a local authority program say to improve safety on the roads by spending money on traffic control. PBB demands that activities be prioritised in order that

. the relative merits and costs of those activities can be assessed.

B

Zero -base budgeting (ZBB) is a cost-benefit whereby it is assumed that the cost al10wance for an item is zero, and will remain so until the manager responsible justifies the existence of the cost item and the benefits the expenditure brings. In this way a questioning attitude is developed wher~by each cost item and its level has to be justified in relation to the way it helps to meet objectives and how the expenditure benefits the organization. This is a forward looking approach as opposed *to* the all too common method of extrapolating past activities and costs, which is a feature of the incremental budgeting approach.
ZBB is formally defined by the CIMA thus, 'A method of budgeting whereby all activities are reevaluated each time a budget is formulated. Each functional budget starts with the assumption that the function does not exist and is at zero cost. Increments of cost are compared with increments of benefit, culminating in the planning maximum benefit for a given budgeted cost' *(Terminology)*
The use of ZBB was pioneered by P Phyrr in the United States in the early 1970s and has gained wide acceptance probably because it is a simple idea obviously based on common- sense. ZBB is concerned with the evaluation of the costs and benefits of alternatives and, implicit in the technique, is the concept of opportunity cost.

Where ZBB can be applied

ZBB can be applied in both profit seeking and non-profit seeking organization. The technique gained wide publicity when the then President Carter directed that all US government departments adopt ZBB. In a manufacturing firm, ZBB is best applied to service and support expenditure including; administration, marketing, personnel, information and computer services, research and development, finance and accounting, production planning and so on. These activities are less easily quantifiable by conventional methods and are more discretionary in nature. Manufacturing costs such as direct materials and labour and production overheads can be more easily controlled by well established methods which compare production outputs with resource inputs rather than using ZBB. Budgeting and controlling manufacturing expenditure uses techniques such as work study and standard costing which are described later in the manual.
ZBB can successfully be applied to service industries and to a wide range of non-profit seeking organizations. For example local and central government departments, educational establishments, hospitals and so on. ZBB could be applied in any organization where alternative levels of provision for each activity are possible and the costs and benefits can be separately identified. ZBB is concerned with alternatives and means that established activities have to be compared with alternative uses of the same resources. ZBB takes away the implied right of existing activities to continue to receive resources, unless it can be shown that this is the best use of those resources.

Just-In- Time Production

lIT Production works on a demand- pull basis and seeks to eliminate all waste and everything which does not add value to the product. As an example consider the lead times associated with making and selling a product. These include:
Inspection time - Transport time - Queuing time - Storage time - Processing time
Of these, only processing time adds value to the product whereas all the others add cost, but not value.
The ideal for **JIT** systems is to convert materials to finished products with a lead time equal to processing time so eliminating all activities which do no add value. A way of emphasizing the importance of reducing throughput time is to express the above lead time as follows:
Throughput time = Value-added time + Non-value added time
The JIT pull system means that components are not made until requested by the next process. The usual way this is done is by monitoring parts consumption at each state and using a system of markers (known as kanbans) which authorise production and movement to the process which requires the parts. A consequence of this is that there may be idle time at certain work stations but this is considered preferable to adding to work-in-progress inventory.

Poor and uncertain quality is a prime source of delays hence the drive in JIT systems for zero defects and Total Quality Control (TQc). When quality is poor, higher W.I.P is need to protect production from delays caused by defective parts. Higher inventory is also required when there are long set-up and changeover times. Accordingly there is continual pressure in JIT systems to reduce set-up times and eventually eliminate them so that the optimal batch size can become one. With a batch size of one, the work can flow smoothly to the next stage without the need to store it and schedule the next machine to accept the item.

D.

Game Theory

Game theory is used to determine the optimum strategy in a competitive situation.
When two or more competitors are engaged in making decisions, it may involve conflict of interests.
In such a case the outcome depends not only upon an individual's action but also upon the actions of the others. Both (competing) sides face a similar problem. Hence game theory is a science of conflict. Game theory does not concern itself with finding an optimum strategy but it helps to improve the decision process.
Game theory has been used in business and industry to develop bidding tactics, pricing policies, advertising strategies, timing of the introduction of new models into market, etc.

RULES OF GAME THEORY

i. The number of competitors is finite.
11. There is a conflict of interests between the participants.
iii. Each of these participants has available to him a finite set of available courses of action i.e. choices.
The rules governing these choices are specified and known to all the players.
iv. While playing each player chooses a singh~ course of action from the list of choices available to him.
v. The outcome of the game is affected by choices made by all of the players. The choices are to be made simultaneously so that no competitor knows his opponent's choices until he is already committed to his own.
vi. The outcome for all specific choices by all the players is known in advance and numerically defined.
vii. The players act rationally and intelligently.
When a competitive situation meets all these criteria above, we call it a game.
Note: Only in a few real competitive situations can game theory be applied because all the rules are difficult to apply at the same time to a given situation.

E.

Infonnation Asymmetry
Distortion and noise
A communication system is conceived as a transmitter connected to a receiver by a wire or, more correctly, by a sub-system comprising a communications channel. In the channel, the signal which originated at the transmitter is subject to change so that the signal arriving at the receiver may not coincide exactly with the signaI"leaving the transmitter. The two major causes for the change are distortion and noise.

Distortion can be illustrated by a classroom example commonly called 'Chinese Whispers'. One person in this class is given a message, without anyone else being able to know what the message is. That person has to whisper the message to a neighbour, without anyone else being able to hear. The neighbour cannot ask for the message to be repeated but must immediately pass the message to another neighbour, and so on, until all members of the class have heard the message. the final person in the chain then speaks the message out loud. A good example is given by the initial military message:
'Send three and four pence, we are going to dance.'
If the exercise is repeated in a classroom where everyone is shouting, the effects of noise can also be established.
Distortion changes the form of the message. Bias, discussed in the next chapter, is an example of a way in which information can be distorted. A manager may increase a planned cost budget by 10%, say so that apparently good results will be shown by budgetary control reports. Noise is random and can obliterate the message entirely, in extreme circumstances. The problems of accounting for the effec~ of random deviations from standard or budget will be discussed; in .other words, the problems facing the management accountant when faced with noisy data will be recognised. Noisy data includes incorrect entries on operational control documents such as goods received notes. Communications theory states that noise is unavoidable. Distortion can be countered if the effects that the communications channel creates are sufficiently well understood. For instance, good quality hi-fi does not distort input signals significantly. Less good quality hi-fi can distort input signals but the distortion can be corrected by means of graphic equalisers. In management accounting systems, if the effects of distortion can be corrected by equal and opposite corrections, the signal can be received in its proper form. If it is known that a manager regularly increases budgeted cost levels by 10%, the management accountant can reduce the final budgeted level by 10% to compensate

MOCK EXAMINATION

CPA PART III
MANAGEMENT ACCOUNTING

Time Allowed: 3 hours

Answer ALL questions. Show all your workings.

To be carried out under Examination conditions at home and is TO BE SENT to the Distance Learning Administrator for Marking by Strathmore University

QUESTION ONE

The Government of the Republic of Nyake has decided as a matter of top priority to build an oil pipeline joining the two main towns of Pwani and Victoria. Because of the need to complete the project as quickly as possible the work has been divided into five stages which are to be built simultaneously. Within Nyake there are six companies large enough to undertake the construction of any of the five stages and each company has been invited to submit a tender for each stage of the project. The tenders (in billions of Nyake shillings) are as follows:

Company	*1*	*2*	*3*	*4*	*5*
	Shs	Shs	Shs	Shs	Shs
Z	39	74	53	72	58
Y	43	82	52	no bid	57
X	44	76	57	68	58
W	36	76	52	66	no bid
V	47	84	56	73	60
U	40	72	55	70	62

Required:

a. Assuming that non of the six companies is large enough to undertake the work of more than one stage, advise the government on how the five contracts should be allocated. what is the minimum total cost for the project.

b. On speaking to representatives of the six companies, it is discovered that Z, Y W and U have the capacity to undertake any two stages simultaneously and that X can undertake any three stages simultaneously. Show how the problem may now be formulated and solved using the "transportation" algorithm. What is now the minimum cost allocation of contracts? *(JUN `94)*

QUESTION TWO

a. What is prisoners dilemma game?

b. Explain how the prisoners dilemma game can be applied in management problems.

c. Collective bargaining is relatively new among academic staff in institutions of higher learning. the collegial atmosphere in such institutions and universities, plus a strong commitment to professionalism would seem to make lecturers impossible to unionize. The pressure of "increasing enrolment, inflation, increased administrative controls, lack of facilities and other factors", however, have brought the idea of forming a union. One university, anticipating a vote on collective bargaining, established a senate sub-committee to investigate this unusual phenomenon. The committee was to refrain from any recommendations, pro or con, but was to examine the potential impact in the event a bargaining agent were elected to office. The committee's report ranged over a variety of topics. among some philosophical observations, two general features with respect of collective bargaining which are relevant to this report were apparent; the adversary nature of collective bargaining and the zero-sum game concept underlying collective bargaining.

It is the belief of this committee that although both sides in any collective bargaining situation should present their cases as ably as possible, the adversary nature of such proceedings should be "played down" and attempt should be made to arrive at conclusions at least partly satisfactory to both sides. Surely in academic, objectivity, nationality and the exercise of good manners and a decent respect for the view points of others are not outside the realm of the possible. This committee also takes the stand that collective bargaining may be a co-operative non zero-sum game wherein any losses to either or both parties can be offset by substantially larger gains.

Required:

i. Develop an argument supporting the notion that collective bargaining is a non zero-sum game between the union and management.

ii. Develop an argument supporting the notion that collective bargaining is a zero-sum game.

iii. Which do you believe collective bargaining really is: zero-sum game or non zero-sum game? *(DEC `94)*

QUESTION THREE

The Hatari Weapons Ltd. desires to submit a tender for 32 "string-to-surface" rockets required by Vita Ltd. It is estimated that each rocket will cost approximately Shs 40,000,000 for material and variable overhead costs. Total fixed costs will amount to approximately Shs 1,600,000,000 over the two years it will take to build the rockets, all of which would have to be recovered against this contract.

The company, as a result of past experience, anticipates it could expect a 75 per cent learning curve and that the steady state would not be achieved during this production run. Building the first rocket would require approximately 400,000 hours of direct labour at a direct labour cost of Shs 1.50 per hour. Variable overhead costs which vary with direct labour amount to Shs 50 per direct labour hour.

Eight rockets will be built during the first year of the contract and the remaining 24 will be completed during the second year. The Hatari Weapons Ltd. always adds 25 per cent profit margin to the estimated costs of the contract for which they tender.

Required:

a. Calculate the total labour hours that will be required to build the 32 rockets.

b. Draw up a quotation showing the total price to be quoted, with details of the constituent parts of the cost structure and the profit added.

c. Assuming the contract is awarded to the company, and no costs are deferred over the two year period, draft estimated income statements for the first and second years of the contract life. Revenue is to be recognised on the basis of completed rockets. Fixed costs are incurred equally each year.

(JUN `93)

QUESTION FOUR

Meeta Ltd. is faced with a scheduling problem for its three main product lines. Each of the three products requires the operations of casting, machining, assembly and packaging. Where specialised casting is required, the casting is done within the company otherwise casting could be sub-contracted.

The company has recently been re-organised and a Management Services Department has been established. This department is in the process of reviewing production policies with the objective of exploring opportunities for maximising profits. Towards this end the following data has been gathered:

Table I: Costs; Prices and Contribution Margins			
	Product 1 Shs	Product 2 Shs	Product 3 Shs
Cost of Casting:			
Produced at Meets Ltd.	0.3	0.5	0.4
Sub-contracted	0.5	0.6	—
Cost of Machining	0.2	0.1	0.27
Cost of assembly and Packaging	0.3	0.2	0.2
Price	1.5	1.8	1.97

Table II:Casting Machining, Assembly and Packaging times per Product (in minutes)			
	Casting	Machining	Assembly and Packaging
Product 1	6	6	3
Product 1 (S/C)	S/C	6	3
Product 2	10	3	2
Product 2 (S/C)	S/C	3	2
Product 3	8	8	2
Time available (minutes)	8,000	12,000	1,000

N.B. S/C = Sub-contracted.

Required:

a. Set up the problem as a linear programming model.
b. Using the Simplex Method, solve the linear program to obtain maximum profits.
c. Briefly explain the term "integer programming in the context of management accounting.

(JUN `92)

QUESTION FIVE

Your Finance Director has been invited by the University Management Accounting Students Association to address them on the marketing policies of your company with specific reference to:

a. Decentralisation and measurement of performance in both local and overseas markets.

b. Transfer pricing within the Eastern and Central African region where the company has manufacturing plants in six different countries.

Required:

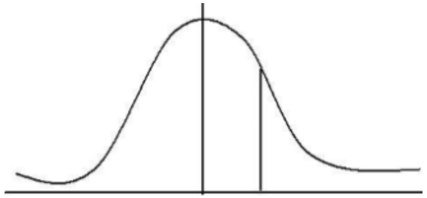
Write a detailed paper explaining in clear and concise terms the main issues the Financial Director should concentrate on. *(JUN `94)*

END OF THE MOCK EXAMINATION

NOW SEND YOUR ANSWERS TO THE DISTANCE LEARNING CENTRE FOR MARKING

Table I

Areas under the Standard Normal Curve from 0 to Z

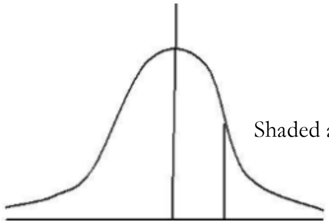


Z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1623	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3073	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441

1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4761	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4669	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4877	0.4978	0.4979	0.4979	0.4780	0.4781
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Table II

Normal distribution

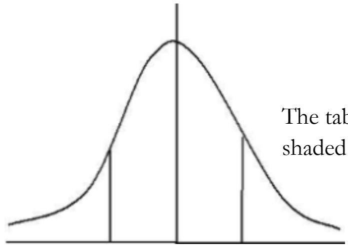


Z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
P	0.500	0.460	0.421	0.382	0.345	0.308	0.274	0.242	0.212	0.184
Z	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
P	0.159	0.136	0.115	0.097	0.081	0.067	0.055	0.045	0.036	0.029
Z	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
P	0.023	0.018	0.014	0.011	0.008	0.006	0.005	0.003	0.003	0.002
Z	3.0	3.1	3.2	3.3	3.4					
P	0.0013	0.0010	1.0007	0.0005	0.0003					

Table III

Percentage points of the t distribution.

The table gives the values for the area in both tails.



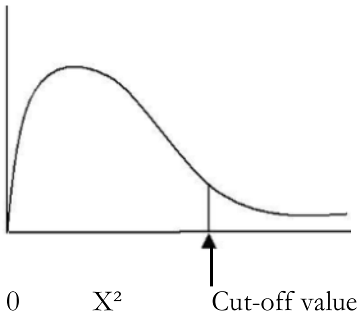
The table shows the total of the shaded area

Degree of freedom	Area in both tables combined .10	.05	.02	.01
v = 1	6.314	12.706	31.821	63.657
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.493	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
16	1.746	2.120	2.583	2.921
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.878
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845

Degrees of freedom	Area in both tables combined .10	.05	.02	.01
v = 21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
40	1.684	2.021	2.423	2.704
60	1.671	2.000	2.390	2.660
12	1.658	1.980	2.358	2.6170
∞	1.645	1.960	2.326	2.576

Table IV

The X^2 distribution



Degrees of freedom	Level of significance 5%	1%
v = 1	3.841	6.635
2	5.991	9.210
3	7.815	11.345
4	9.488	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209

Table V

Percentage points of the f distribution

V_1 = Degrees of freedom for numerator

V_2=Degrees of freedom for numerator

	1	2	3	4	5	6	7	8	9	10	11	12
1	161	200	216	225	230	234	237	239	241	242	243	244
	(4,052)	(4,999)	(5,403)	(5,625)	(5,764)	(5,859)	(5,928)	(5,981)	(6,022)	(6,056)	(6,082)	(6,106)
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41
	(98.49)	(99.01)	(99.17)	(99.25)	(99.30)	(99.33)	(99.34)	(99.36)	(99.38)	(99.40)	(99.41)	(99.42)
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74
	(34.12)	(30.81)	(29.46)	(28.71)	(28.24)	(27.91)	(27.67)	(27.49)	(27.34)	(27.23)	(27.13)	(27.05)
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91
	(21.20)	(18.00)	(16.69)	(15.98)	(15.52)	(15.21)	(14.98)	(14.80)	(15.68)	(14.64)	(14.45)	(14.37)
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.7	4.68
	(16.26)	(13.27)	(12.06)	(11.39)	(10.97)	(10.67)	(10.45)	(10.27)	(10.15)	(10.05)	(9.98)	(9.89)
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	(13.75)	(10.92)	(9.78)	(9.15)	(8.75)	(8.47)	(8.26)	(8.10)	(7.98)	(7.87)	(7.79)	(7.72)
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57
	(12.25)	(9.55)	(8.45)	(7.35)	(7.46)	(7.19)	(7.00)	(6.84)	(6.71)	(6.62)	(6.54)	(6.47)
8	5.32	4.46	4.07	3.34	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28
	(11.26)	(8.65)	(7.59)	(7.01)	(6.93)	(6.37)	(6.19)	(6.08)	(5.91)	(5.82)	(5.74)	(5.67)
9	5.12	4.26	3.83	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07
	(10.56)	(8.02)	(6.99)	(6.42)	(6.06)	(5.80)	(5.62)	(5.47)	(5.35)	(5.38)	(5.18)	(5.11)
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91
	(10.04)	(7.58)	(6.55)	(5.99)	(5.64)	(5.39)	(5.21)	(5.06)	(4.93)	(4.85)	(4.76)	(4.71)
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79
	(9.65)	(7.20)	(6.22)	(5.67)	(5.32)	(5.07)	(4.88)	(4.74)	(4.63)	(4.54)	(4.46)	(4.40)
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.70	2.73	2.69
	(9.33)	(6.93)	(5.95)	(5.41)	(5.06)	(4.82)	(4.65)	(4.50)	(4.39)	(4.30)	(4.23)	(4.18)

Values of f:

Right tail of the distribution for P = .05

Right tail of the distribution for P = .01 (in brackets).

Table VI

(a) Table of individual Poisson probabilities

	Number of occurrences (x)						
Mean (m)	0	1	2	3	4	5	6
0.1	0.9048	0.0905	0.0045	0.0002	0.000	0.000	0.000
0.2	0.8187	0.1637	0.0164	0.0011	0.0001	0.000	0.000
0.3	0.7408	0.2222	0.0333	0.0033	0.0003	0.000	0.000
0.4	0.6703	0.2681	0.0536	0.0072	0.0007	0.0001	0.000
0.5	0.6065	0.3033	0.0758	0.0126	0.0016	0.0002	0.000
0.6	0.5488	0.3293	0.0988	0.0198	0.0030	0.0004	0.000
0.7	0.4966	0.3476	0.1217	0.0284	0.0050	0.0007	0.0001
0.8	0.4493	0.3595	0.1438	0.0383	0.0077	0.0012	0.0002
0.9	0.4066	0.3659	0.1647	0.0494	0.0111	0.0020	0.0003
1.0	0.3679	0.3679	0.1839	0.0613	0.0153	0.0031	0.0005

Table shows probability of a given number of occurrences for a given mean (m).

(b) Table of cumulative Poisson probabilities

	Number of occurrences (x)						
Mean (m)	0	1	2	3	4	5	6
0.1	0.9048	0.9953	0.9998	1.00	1.00	1.00	1.00
0.2	0.8187	0.9824	0.9988	0.9999	1.00	1.00	1.00
0.3	0.7408	0.9630	0.9963	0.9996	0.9999	1.00	1.00
0.4	0.6703	0.9384	0.9920	0.9992	0.9999	1.00	1.00
0.5	0.6065	0.9098	0.9856	0.9982	0.9998	1.00	1.00
0.6	0.5488	0.8781	0.9769	0.9967	0.9997	1.00	1.00
0.7	0.4966	0.8442	0.9659	0.9943	0.993	1.00	1.00
0.8	0.4493	0.8088	0.9526	0.9909	0.9986	0.9998	1.00
0.9	0.4066	0.7725	0.9372	0.9866	0.9977	0.9997	1.00
1.0	0.3679	0.7358	0.9197	0.9810	0.9963	0.9994	0.9999

Table shows probability of finding x or fewer occurrences for a given mean (m).

Table VII
Compound interest

Table shows value of £1 at compound interest $(1 + r)^n$

	Interest rates (r) %										
Years (n)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080	1.090	1.100	1.110
2	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166	1.188	1.210	1.232
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	1.331	1.368
4	1.041	1.082	1.126	1.167	1.216	1.262	1.311	1.360	1.412	1.464	1.518
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469	1.539	1.610	1.685
6	1.061	1.126	1.194	1.265	1.340	1.419	1.501	1.587	1.677	1.772	1.870
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714	1.828	1.949	2.076
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851	1.993	2.144	2.304
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999	2.172	2.358	2.558
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159	2.367	2.594	2.839
11	1.116	1.243	1.384	1.539	1.710	1.898	2.105	2.332	2.580	2.853	3.152
12	1.127	1.268	1.426	1.601	1.796	2.012	2.252	2.519	2.813	3.138	3.498
13	1.138	1.294	1.468	1.665	1.886	2.133	2.410	2.720	3.066	3.452	3.883
14	1.149	1.319	1.513	1.732	1.980	2.261	2.578	2.937	3.342	3.797	4.310
15	1.161	1.346	1.558	1.801	2.079	2.397	2.759	3.172	3.642	4.177	4.785
20	1.220	1.486	1.806	2.191	2.653	3.207	3.870	4.661	5.604	6.727	8.062
25	1.282	1.641	2.094	2.666	3.386	4.292	5.427	6.848	8.623	10.835	13.585

	Interest rates (r) %										
Years (n)	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**25**	**30**
1	1.120	1.130	1.140	1.150	1.160	1.170	1.180	1.190	1.200	1.250	1.300
2	1.254	1.277	1.297	1.322	1.346	1.367	1.392	1.416	1.440	1.562	1.690
3	1.405	1.443	1.481	1.521	1.561	1.602	1.643	1.685	1.728	1.953	2.197
4	1.573	1.630	1.689	1.749	1.811	1.874	1.939	2.005	2.074	2.441	2.856
5	1.762	1.842	1.925	2.011	2.100	2.192	2.288	2.386	2.488	3.052	3.713
6	1.974	2.082	2.195	2.313	2.436	2.565	2.700	2.840	2.986	3.815	4.827
7	2.211	2.353	2.502	2.660	2.826	3.001	3.186	3.379	3.583	4.768	6.275
8	2.476	2.658	2.853	3.059	3.278	3.511	3.759	4.021	4.300	5.960	8.157
9	2.773	3.004	3.252	3.518	3.803	4.108	4.435	4.785	5.159	7.451	10.604
10	3.106	3.395	3.707	4.046	4.411	4.807	5.234	5.695	6.192	9.313	13.786
11	3.478	3.836	4.226	4.662	5.117	5.624	6.176	6.777	7.430	11.641	17.922
12	3.896	4.334	4.818	5.350	5.936	6.580	7.288	8.064	8.916	14.552	23.298
13	4.363	4.898	5.492	6.153	6.886	7.699	8.599	9.596	10.699	18.190	30.287
14	4.887	5.535	6.261	7.076	7.988	9.007	10.147	11.420	12.839	22.737	39.374
15	5.474	6.254	7.138	8.137	9.265	10.539	11.974	13.589	15.407	28.422	51.186
20	9.646	11.523	13.743	15.366	19.461	23.106	27.393	32.429	38.338	86.736	190.050
25	17.000	21.230	26.462	32.920	40.874	50.658	62.669	77.388	95.396	264.698	705.641

Table VIII

Present value factors. Present value of £1 $(1 + r)^{-n}$

Periods (n)	**1%**	**2%**	**4%**	**6%**	**8%**	**10%**	**12%**	**14%**	**15%**
				Interest rates (r) %					
1	0.990	0.980	0.962	0.943	0.926	0.909	0.893	0.877	0.870
2	0.980	0.961	0.925	0.890	0.857	0.826	0.797	0.769	0.756
3	0.971	0.942	0.889	0.840	0.794	0.751	0.712	0.675	0.658
4	0.961	0.924	0.855	0.792	0.735	0.683	0.636	0.592	0.572
5	0.951	0.906	0.822	0.747	0.681	0.621	0.567	0.519	0.497
6	0.942	0.888	0.790	0.705	0.630	0.564	0.507	0.456	0.432
7	0.933	0.871	0.760	0.665	0.583	0.513	0.452	0.400	0.376
8	0.923	0.853	0.731	0.627	0.540	0.467	0.404	0.351	0.327
9	0.914	0.837	0.703	0.592	0.500	0.424	0.361	0.308	0.284
10	0.905	0.820	0.676	0.558	0.463	0.386	0.322	0.270	0.247
11	0.0896	0.804	0.650	0.527	0.429	0.350	0.287	0.237	0.215
12	0.887	0.788	0.625	0.497	0.397	0.319	0.257	0.208	0.187
13	0.879	0.773	0.601	0.469	0.368	0.290	0.229	0.182	0.163
14	0.870	0.758	0.577	0.442	0.340	0.263	0.205	0.160	0.141
15	0.861	0.743	0.555	0.417	0.315	0.239	0.183	0.140	0.123
16	0.853	0.728	0.534	0.394	0.292	0.218	0.163	0.123	0.107
17	0.855	0.714	0.513	0.371	0.270	0.198	0.146	0.108	0.093
18	0.836	0.700	0.494	0.350	0.250	0.180	0.130	0.095	0.081
19	0.828	0.686	0.475	0.331	0.232	0.164	0.116	0.083	0.070
20	0.820	0.675	0.456	0.312	0.215	0.149	0.104	0.073	0.061
21	0.811	0.660	0.439	0.294	0.199	0.135	0.093	0.064	0.053
22	0.803	0.647	0.422	0.278	0.184	0.123	0.083	0.056	0.046
23	0.795	0.634	0.406	0.262	0.170	0.112	0.074	0.049	0.040
24	0.788	0.622	0.390	0.247	0.158	0.102	0.066	0.043	0.035
25	0.780	0.610	0.375	0.233	0.146	0.092	0.059	0.038	0.030

Periods (n)	16%	18%	20%	22%	24%	25%	26%	28%	30%
				Interest rates (r) %					
1	0.862	0.847	0.833	0.820	0.806	0.800	0.794	0.781	0.769
2	0.743	0.718	0.694	0.672	0.650	0.640	0.630	0.610	0.592
3	0.641	0.609	0.579	0.551	0.524	0.512	0.500	0.477	0.455
4	0.552	0.516	0.482	0.451	0.423	0.410	0.397	0.373	0.350
5	0.476	0.437	0.402	0.370	0.341	0.328	0.315	0.291	0.269
6	0.410	0.370	0.335	0.303	0.275	0.262	0.250	0.227	0.207
7	0.354	0.314	0.279	0.249	0.222	0.210	0.198	0.178	0.159
8	0.305	0.266	0.233	0.204	0.179	0.168	0.157	0.139	0.123
9	0.263	0.225	0.194	0.167	0.144	0.134	0.125	0.108	0.094
10	0.227	0.191	0.162	0.137	0.116	0.107	0.099	0.085	0.075
11	0.195	0.162	0.135	0.112	0.094	0.086	0.079	0.066	0.056
12	0.168	0.137	0.112	0.192	0.076	0.069	0.062	0.052	0.043
13	0.145	0.116	0.093	0.075	0.061	0.055	0.050	0.040	0.033
14	0.125	0.099	0.178	0.062	0.049	0.044	0.039	0.032	0.025
15	0.108	0.084	0.065	0.051	0.040	0.035	0.031	0.025	0.020
16	0.093	0.071	0.054	0.042	0.032	0.028	0.025	0.019	0.015
17	0.080	0.060	0.045	0.034	0.026	0.023	0.020	0.015	0.012
18	0.069	0.051	0.038	0.028	0.021	0.018	0.016	0.012	0.009
19	0.060	0.043	0.031	0.023	0.017	0.014	0.012	0.009	0.007
20	0.051	0.037	0.026	0.019	0.014	0.012	0.010	0.007	0.005
21	0.044	0.031	0.022	0.015	0.011	0.009	0.008	0.006	0.004
22	0.038	0.026	0.018	0.013	0.009	0.007	0.006	0.004	0.003
23	0.033	0.022	0.015	0.010	0.007	0.006	0.005	0.003	0.002
24	0.028	0.019	0.011	0.008	0.006	0.005	0.004	0.003	0.002
25	0.024	0.016	0.010	0.007	0.005	0.004	0.003	0.002	0.001